Structured Products in France

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1 Introduction

1.1 Topic Presentation

1.1.1 Definition of Structured Products

1.1.1.1 What is a Structured Product ?

The first structured products were marketed in the 1980s in the United States and then in Europe in the 1990s. Since then, these products have experienced and continue to experience significant growth, particularly in France. Popularized thanks to a high-interest-rate environment, these products allow for investment strategies with returns different from traditional asset purchases.

They are designed to offer capital protection, generate income, or capitalize on growth opportunities while meeting specific financial objectives of investors. The value of structured products lies in their ability to be customized to match various risk profiles and market expectations.

These products are made up of two or more different components :

Maturity	**Initial Investment**
1	96,15%
2	92,46%
3	88,90%
4	85,48%
5	82,19%
6	79,03%
7	75,99%
8	73,07%
9	70,26%
10	67,56%

TABLE 1 – Price of a zero-coupon bond with a 4% interest rate

This table represents the price of a zero-coupon bond issued by the issuing banks and constitutes the bond component of the structured product. Since the final client purchases the product at 100%, the difference between this price and the bond portion represents the optional component. This residual portion is invested in one or more derivatives depending on the desired payoff. This component establishes the product's sensitivity to changes in the underlying asset and partly determines the product's overall valuation.

Therefore, these products are issued by banks and are intended for private or institutional investors. They can be traded on an organized exchange or sold directly by their issuing bank, which will provide bid and ask prices.

1.1.1.2 Classification of Structured Products

There is no single or universally agreed method for classifying structured products. Thus, the classification of these financial products can be made based on specific characteristics or peculiarities. Nevertheless, structured products are defined by three main parameters :

- **Underlying Asset** : This category includes the nature of the underlying asset, which may be a single asset, a basket of assets, or indices. Underlyings can also be classified by asset class, such as equities or FICC assets (Fixed Income, Currency, and Commodities). This classification helps differentiate structured products based on bonds, currencies, commodities, or equity indices, each having specific risk and return characteristics.
- **Wrapper** : This parameter involves the legal form of the product, such as its integration into life insurance or its distribution channel.
- **Payoff** : The payoff profile of a structured product can be convex or concave. A convex profile is characterized by a disproportionate increase in returns relative to gains in the underlying asset, ideal for maximizing potential gains. Conversely, a concave profile provides reduced sensitivity to variations in the underlying asset, limiting potential gains but offering greater protection against losses.

1.1.1.3 Types of Structured Products

It is now appropriate to present and describe the main categories of structured products, emphasizing the specific features distinguishing the various product types. This categorization was initially developed by the Swiss Structured Product Association (SSPA) in collaboration with several organizations, including the derivatives exchange SIX Structured Products. After its development, this classification was adopted by EUSIPA – the European Structured Investment Products Association – and has since been widely accepted throughout Europe.

Before providing a brief description of all categories, it is important to note that all instruments, regardless of their product class, can be constructed with various features. Consequently, choices should be evaluated based on their specific terms rather than by general product grouping. Likewise, the lack of comparability is apparent even within the same category, as products differ from each other in terms of payment structure and associated risks. Nevertheless, Martellini (2003) proposes dividing them into four categories :

- **Capital Protection Products :**
  As the name indicates, these products offer significant capital protection, typically between 90% and 100% of the total nominal value, even if the market moves contrary to investor expectations. However, the potential returns are lower compared to other products and may even be capped. They usually consist of a traditional bond that protects the initial investment at maturity, combined with a derivative, often options, responsible for generating returns above the initial investment. Examples include Capital Protection Certificates with Participation, Capital Protection Certificates with Barrier, Twin-Win Capital Protection Certificates, and Capital Protection Certificates with Coupon.

— **Yield Enhancement Products (Optimization) :**
These products offer no protection for the initial investment and provide a predefined return in exchange for downside risk. They are suitable when investors expect a stable or sideways-moving market since the risk occurs when underlying asset prices fall. Investors fully participate in the underlying asset's price increase up to a capped threshold or receive coupons. Typical products here include Reverse Convertible bonds (with or without barriers), which guarantee coupons while being exposed to downside risk.

— **Participation Products :**
Participation products primarily differ from the two previous categories by lacking a capped level. They suit investors interested in participating in the performance of an asset, such as equities, funds, bonds, indices, or combinations thereof. Generally, these products do not have upside or downside limits and usually pay no coupons or dividends, as issuers use these to finance the bonus. For instance, in outperformance certificates, investors waive dividends in favor of the outperformance mechanism once participation rates surpass a predetermined threshold. Some instruments include returns calculated at product maturity based on a participation rate in the underlying asset's performance, applicable to both upward and downward movements. Main products include Outperformance Certificates, Bonus Certificates, and Twin-Win Certificates.

— **Leverage Products :**
Leverage products allow short-term speculation or hedging, offering higher returns than yield enhancement products, often without upside participation limits. Common products include Call or Put Warrants with fixed lifetimes and Knockouts expiring once a barrier is reached. Another example is Leverage Certificates, offering enhanced participation in an underlying asset with intrinsic leverage. All leveraged products amplify the underlying asset's price movements through the leverage mechanism, allowing significant gains with limited capital. However, this leverage also applies negatively, potentially resulting in substantial losses if investor predictions fail. Consequently, these products often include a stop-loss to limit potential losses.

To conclude, considering all categories, capital protection products typically have low risk exposure, followed by yield enhancement products, participation products, and finally, leveraged products as the riskiest.

1.1.1.4 Risks Associated with Structured Products

From the Investors' Perspective

According to Vedeilhié (2008), three risk categories must be analyzed by investors :

— **Market Risk** : As with any investment, structured product investors are exposed to underlying asset fluctuations : interest rates, foreign exchange, equities, indices, etc.

— **Liquidity Risk** : Due to the customized nature of structured products, liquidity is a significant disadvantage. The issuer generally serves as the sole market-maker for the initiated product.

— **Issuer Risk** : Like any debt instrument, issuer solvency is crucial. Indeed, issuers are subject to credit ratings ranging from Aaa to C (Moody's Rating).

From the Issuers' Perspective

Three categories of risk must be analyzed by issuers :

— **Transaction Risk** : When quoting a structured product, interest rate and option components are considered. Once agreed, traders must quickly negotiate all components (interest rates, derivatives) before significant underlying movements occur.
— **Counterparty Risk** : At maturity, when investors purchase calls, they bear counterparty risk. If the seller is unable to meet obligations at the option's maturity, the promised return cannot be provided.
— **Settlement/Delivery Risk** : Each transaction involves managing financial flows on specific dates. Penalties, such as Libor-based charges, may apply if timelines are missed.

1.1.1.5 How to Value a Structured Product

Valuing structured products is a complex domain that relies on various quantitative methods to accurately estimate their value and manage associated risks. One of the most commonly used methods is Monte Carlo simulation, detailed by Glasserman (2003), which models the evolution of underlying asset prices by generating a large number of random market scenarios. This method is particularly useful for structured products with nonlinear features, such as barrier options and autocallable products. In parallel, stochastic volatility models, such as the Heston model (Heston, 1993), offer an analytical approach to valuing options by accounting for time-varying volatility. These models are often used in combination with numerical methods to obtain closed-form solutions for complex options.

Another notable approach is the use of Fast Fourier Transform (FFT) for option pricing, a method explored by Carr and Madan (1999). This technique allows efficient option pricing by utilizing the characteristics of the return distribution function of the underlying assets. Moreover, Gatheral (2006) emphasizes the importance of the volatility surface in valuing derivatives and structured products, providing a graphical representation of implied volatility variations across different maturities and strike prices.

Additionally, models based on the term structure of interest rates, such as those developed by Cox, Ingersoll, and Ross (1985) and Duffie and Kan (1996), are also essential for valuing interest-rate-dependent structured products, like floating-rate bonds and interest rate derivatives. These models capture the dynamics of interest rates across different horizons and are integrated into risk management platforms to evaluate the price of complex derivatives.

Finally, more recent models such as those described by Brigo, Pallavicini, and Torresetti (2010) examine correlations and dependencies between different market risks, particularly in the context of credit derivatives and Collateralized Debt Obligations (CDOs), to provide more consistent and robust valuation. These approaches illustrate the diversity of tools and methodologies available to finance professionals, each offering specific advantages depending on the product features and market conditions.

1.1.2 Importance and Relevance of Structured Products in the French Financial Context

In France, most retail structured products, known as "fonds à formule" (formula funds), typically have convex payoff profiles and do not pay any coupon during their life. A major feature of these funds is the absence of a formal secondary market, and even the issuing bank has no formal obligation to buy them back. If redemption is allowed, the investor may lose all the formula's benefits, unless this possibility is specified in the contract from the start. Investors can also access structured products on NYSE Euronext through warrants and certificates, which are not mutual funds but are listed on the exchange and traded like stocks. According to the Autorité des Marchés Financiers (AMF), the outstanding value of formula funds reached €11 billion in 2022, accounting for 0.6% of the total outstanding value of French funds.

1.2 Thesis Objectives

The objectives of this thesis are threefold. First, it aims to explore how structured products are perceived and used by investors and banks in France. It is essential to understand how investors view these products, what their objectives and expectations are, and what motivates their use—particularly in terms of capital protection, portfolio diversification, and yield seeking under various market conditions. Simultaneously, the development and marketing strategies of structured products by banks will be studied, as well as the innovations and new offerings introduced by financial institutions to meet investor needs.

Second, the thesis aims to analyze the different valuation methods for structured products, with particular emphasis on autocallable products. Valuing these financial instruments is essential to determine their fair price and manage associated risks. This part examines several valuation techniques, including classic financial models such as Black-Scholes and Monte Carlo simulations. While the Black-Scholes model is widely used for its simplicity and accuracy under stable market conditions, it has limitations in modeling the volatility and complex characteristics of structured products. To address these shortcomings, more advanced methods such as the Heston stochastic volatility model are also explored, allowing a better capture of real financial market dynamics. This section also explores how banks choose between different valuation methods based on the product's specific characteristics and market conditions, offering a comprehensive and practical view of valuation processes used in the financial sector.

Finally, the thesis seeks to examine the impact of regulation on structured products. The analysis will cover the main regulations affecting these products in France and Europe, such as MiFID II and PRIIPs, as well as the role of regulatory authorities such as the AMF and the European Securities and Markets Authority (ESMA). It is important to analyze how these regulations influence the development, marketing, and innovation of structured products, and to assess their consequences for investors and issuers in terms of transparency, investor protection, and risk management.

1.3 Research Question

The central question of this thesis is : "To what extent do the specificities of the French financial market influence the design, valuation, and acceptance of structured products, and how can these products be optimized to effectively meet investor expectations while complying with regulatory constraints ?" This central question guides the analysis of several aspects, including the unique features of the French market, regulatory requirements, valuation methods, and investor expectations.

To address this research question, three hypotheses are formulated. The first hypothesis concerns the influence of regulation on the design of structured products. It suggests that financial regulations in France, with their strict requirements for investor protection and market stability, limit innovation and flexibility in product design. Understanding this impact is crucial for identifying how issuers can navigate this regulatory framework while offering attractive and competitive products.

The second hypothesis focuses on the adaptation of structured product valuation methods. It is assumed that these methods must not only faithfully reflect market conditions but also comply with regulatory requirements. This adaptation can influence the accuracy of valuations and investors' understanding of products, thereby affecting their perception and adoption in the market.

Finally, the third hypothesis explores how, despite regulatory constraints, it is possible to optimize structured products to better meet the expectations of French investors. This could include incorporating capital protection mechanisms, creating return structures tailored to local preferences, and using diversification strategies. These optimizations aim to make structured products more attractive while ensuring compliance with current regulations.

By exploring these hypotheses, the thesis aims to provide clear insights into the challenges and opportunities associated with the use of structured products in France. The conclusions from this analysis will offer practical recommendations for structured product issuers, investors, and regulators to improve transparency, efficiency, and the attractiveness of these products on the French market. Ultimately, this study aims to enhance the understanding of how structured products can be adapted and optimized to meet the needs of French financial markets while complying with strict regulatory standards.

1.4 Methodology

The methodology of this thesis has been adapted to the constraints linked to the lack of specific studies and public data on structured products in France. To address this limitation, the methodological approach primarily relies on quantitative analyses, case studies, and the use of available relevant sources. A literature review was conducted based on international sources and industry reports, given the scarcity of academic work focused on the French market. This review made it possible to contextualize structured products in a global framework while identifying the particularities of the French market through limited but essential publications. Regulations such as the Markets in Financial Instruments Directive II (MiFID II) and the Packaged Retail and Insurance-based Investment Products (PRIIPs) were also studied to understand their influence on the design and valuation of these products.

Quantitative analysis was carried out using data from financial reports of major institutions, commercial databases, and other market sources. These data were used to evaluate the performance of structured products, their sensitivity to market condition variations, and the impact of current regulations. Particular attention was paid to autocallables, which represent a significant portion of the French market. To illustrate the challenges and characteristics of structured products in this market, a detailed case study was conducted, notably on products like the Phoenix autocallable. This case study provided an in-depth understanding of the design, valuation, and perception of products by investors, while taking into account regulatory constraints and local preferences. Stress scenarios were used to assess the resilience of these products in adverse market conditions.

Additionally, financial models such as the Black-Scholes model and Monte Carlo simulations were applied to evaluate the theoretical value of structured products and their sensitivity to market variables. These models were adjusted to integrate scenarios specific to the French market, taking into account regulatory requirements and the expectations of local investors. This approach made it possible to quantify the potential impact of regulatory constraints on the valuation and performance of these products.

However, the methodology was limited by the low availability of detailed public data on structured products in France. Most of the information used was obtained from market reports, financial institution publications, and internal analyses, which may limit the generalizability of the results. This constraint highlights the need for greater transparency and increased academic research in this area to better understand and develop the structured product market in France. Despite these limitations, this methodology helps bridge some gaps by providing a rigorous and organized analysis of structured products, their valuation, and regulation on the French market. The conclusions drawn from this study will offer recommendations to issuers, investors, and regulators to improve transparency, efficiency, and the attractiveness of these products.

2 The French Market for Structured Products

Structured products have emerged in recent decades as sophisticated financial instruments, offering investors a unique combination of potential yield and capital protection. In France, these products have experienced significant expansion, particularly in a low interest rate environment where investors are looking for alternatives to traditional investments. By combining derivatives and various underlying assets, structured products meet these expectations while offering personalized solutions tailored to different investor profiles.

The growing importance of structured products in France is inseparable from the evolution of the local financial market, where institutional players, notably insurers, play a key role. French insurers are among the main distributors of these products, often integrating them into their life insurance contracts in the form of unit-linked products. This allows them to offer their clients diversified investment solutions while optimizing their risk and performance management.

In this first section, we will focus on a detailed analysis of the structured products market in France. We will explore how this market has developed, the specific preferences of French investors, and the key role of insurers in the distribution and integration of these products into their portfolios. To illustrate these dynamics, we will also include an in-depth analysis of a structured product representative of the French market, examining its structure, valuation, and historical performance.

2.1 Overview of Structured Products in France

2.1.1 Market Evolution and Importance

The structured products market in France has experienced significant growth in recent years, driven by ongoing innovation in financial products and increasing investor interest in instruments that offer attractive returns while partially protecting capital. This trend has been particularly pronounced in a low interest rate environment, where institutional investors, such as insurers, seek alternatives to traditional investments.

The rise in interest rates, which began in July 2022 and reached 4.5% in September 2023, has renewed interest in structured products by increasing both the expected returns for savers and profit margins for financial institutions.

The increase in rates has also led to a shift in product structuring, with a growing share of products based on bond yields rather than equities. This allows the offering of higher coupons and often 100% capital protection.

This table, from an AMF study in 2022, presents a breakdown of UCITS (Undertakings for Collective Investment in Transferable Securities) held by French insurers, segmented by contract type (Euro Life, Unit-linked Life) and ISIN code (France, Luxembourg, Ireland, other countries). It details the total number of UCITS securities and their value in billions of euros (€bn) within insurer portfolios.

Country	Unit-linked Life (Number)	Unit-linked Life (Billion €)
FR	19 415	291
LU	20 309	128
IE	2 616	6
Others	1 180	12
Total	43 520	437

Table 2 – Evolution of Structured Products in Insurers' Assets (2015–2022)

In France, insurers hold a total of 27,138 UCITS securities for life contracts, divided into 5,684 in Euro and 19,415 in Unit-linked. These securities represent a total value of €599 billion, of which €291 billion are invested in Unit-linked products. Luxembourg and Ireland also appear as key jurisdictions, with 20,309 and 2,616 Unit-linked Life securities respectively, valued at €128 billion and €6 billion. This underlines the importance of these two jurisdictions in managing European insurance funds.

Although the table does not directly specify the share of structured products within Unit-linked investments, recent data help estimate their relevance. According to the latest Nortia barometer, in Q1 2024, structured products accounted for around 40% of Unit-linked inflows, compared to just 14.5% a year earlier. This significant rise highlights the growing appeal of structured products among investors via life insurance contracts.

Thus, the available data reflect their substantial weight in insurers' investments. This also emphasizes the importance for insurers to actively manage these products, taking associated risks into account, to maintain the balance and resilience of their portfolios in a constantly evolving economic environment.

2.2 Economic and Financial Context

The structured products market in France developed within an economic environment marked by a long period of low interest rates, particularly following the 2008 financial crisis. In such a context, both institutional and retail investors have sought alternatives to traditional investments like bonds and equities to enhance returns while managing risk. Structured products, which combine derivative instruments with diversified underlying assets, have become a favored solution, offering an attractive compromise between potential returns and capital protection.

Financial innovation has also played a crucial role in this dynamic, enabling the creation of increasingly sophisticated products tailored to specific market conditions. For instance, in a context of heightened volatility, some structured products were designed to capitalize on volatility by offering returns linked to indices or baskets of stocks, while integrating partial capital protection mechanisms. The flexibility offered by these instruments has attracted a wide range of investors, from individuals to institutional investors such as insurers.

The evolution of monetary policies, particularly with the implementation of quantitative easing by the European Central Bank (ECB), further reinforced this trend. These policies maintained historically low interest rates, making traditional products less attractive and thereby fostering the growth of structured products. This situation led to increased demand for instruments capable of offering competitive yields while effectively managing risks.

2.2.1 Presentation of the French Market Data

2.2.1.1 Number of Products Sold and Volumes

At the end of 2018, the outstanding amount of the French structured product market reached €52.8 billion, representing 1% of the total financial assets of French households. This market, primarily aimed at retail clients—who represented 77% of investors between 2001 and 2018—offers various access channels for investment. Investors can choose to work directly with a wealth management advisor (CGP) or invest via structured products offered in life insurance contracts.

The French structured products market has seen steady growth in the number of products offered, especially since the mid-2010s. This trend reflects ongoing innovation in the sector, with new products regularly introduced to meet specific market conditions and the diverse needs of investors. The year 2018 was particularly notable, setting a record for the number of structured products sold, although total volumes remained below the peak reached in 2007. This suggests that the average subscription amount per product has decreased, reflecting an increased strategy of portfolio diversification. Investors seem to favor risk minimization by spreading their capital across a broader range of structured products to optimize returns while controlling market exposure. This cautious approach reflects growing maturity in the French structured products market, marked by greater sophistication and understanding of complex financial instruments.

This dynamic is also explained by growing demand from retail investors for more sophisticated products, incorporating partial capital protection mechanisms while offering attractive return prospects. Most structured products sold in France are marketed through life insurance contracts, particularly in the form of Unit-Linked (UC) products. This distribution method allows insurers and wealth managers to offer products tailored to investors' various risk profiles.

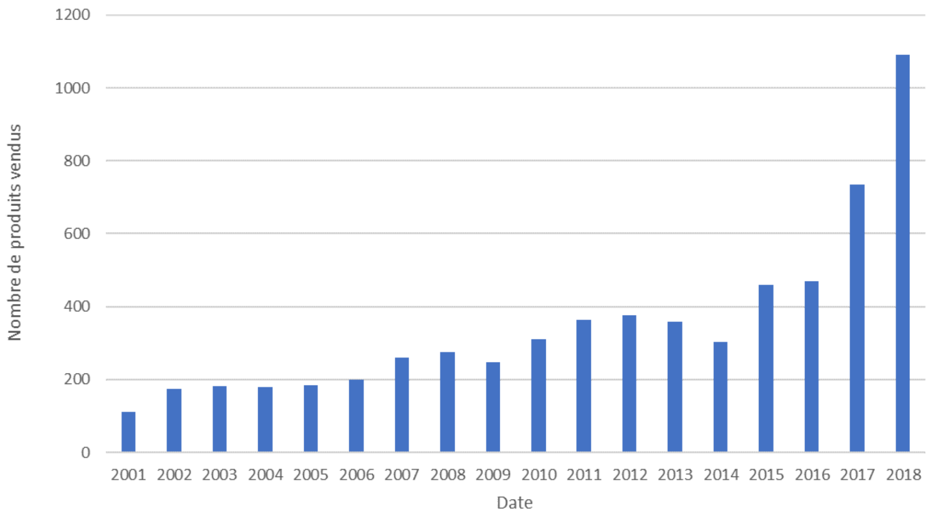

FIGURE 1 – Number of Structured Products Sold Per Year (2001–2018)

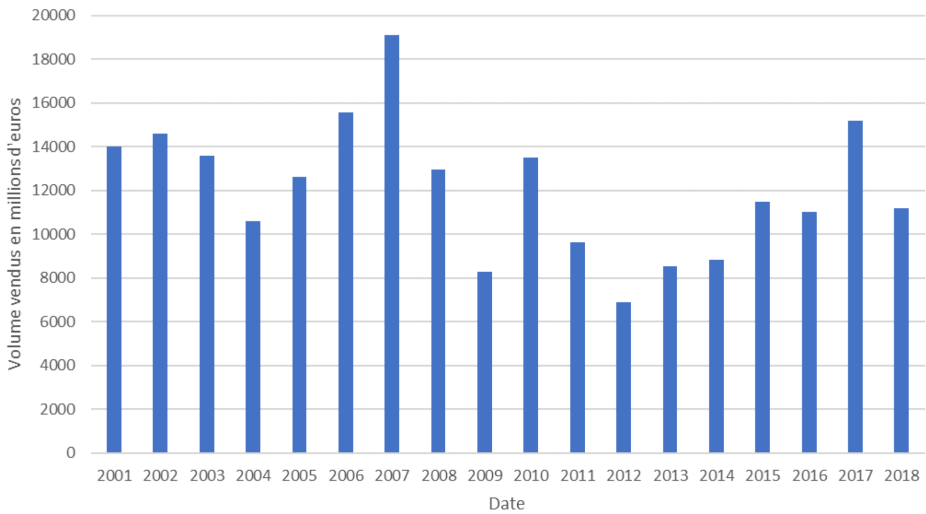

FIGURE 2 – Volumes of Structured Products Sold in France Per Year (2001–2018)

The previous graphs are based on data collected and calculated by the Autorité des Marchés Financiers (AMF). The AMF oversees and regulates financial products distributed in France, including structured products.

Data on structured products sold in France from 2001 to 2018 reveal interesting trends in both product volume and amounts invested. The first graph shows a notable increase in the number of structured products sold, with strong growth beginning in 2009 and peaking in 2012. This surge may be attributed to the rising popularity of these financial instruments due to their ability to provide competitive returns in a low interest rate environment. After 2013, while the market remained dynamic, a slight decline was followed by another increase.

The second graph shows the evolution of sales volumes in millions of euros, indicating a general upward trend. The steady growth includes peaks in 2007 and 2017, possibly pointing to periods of high demand or product innovation enhancing structured product attractiveness. Nonetheless, some stabilization or slight declines are also observed, perhaps reflecting market adjustments or shifting investor preferences.

These trends reflect the dynamics of the structured products market in France, highlighting increasing interest in these financial instruments while indicating market maturity and adaptation to economic cycles and regulatory changes.

2.2.1.2 Distinction Between Funds and Debt Securities

One of the most significant changes in the French structured products market is the shift from the dominance of collective investment funds to the prevalence of complex debt securities. This transition accelerated after the 2008 financial crisis, during which formula funds—once very popular—gradually gave way to debt securities. This evolution can be partly explained by the advantages that debt securities offer in terms of flexibility and customization of risk and return.

Structured debt securities allow issuers to design tailor-made products suited to the specific needs of investors, whether institutional or retail. They also provide a wider variety of underlyings—from stock indices to baskets of equities or bonds—thus enabling diversified sources of return while managing risk efficiently. However, the increased complexity of these securities requires greater expertise from investors to understand the underlying mechanisms and associated risks.

This transformation has also impacted market regulation, with authorities such as the AMF strengthening requirements regarding transparency and risk communication for these products. The aim is to ensure that investors—particularly retail ones—fully understand the characteristics and risks of the structured products they purchase.

Indeed, the composition of the total outstanding amount of structured products sold in France underwent a significant transformation during the period from 2001 to 2018. Initially, formula funds accounted for 100% of the total outstanding, with a breakdown of 90% funds and 10% debt securities. This dominance reflects investors' initial preference for products offering capital protection and return potential based on predefined formulas.

However, a clear trend toward diversification emerged over the years, particularly starting in 2013, when the share of formula funds in total outstanding fell to about 50%. This shift indicates a marked decline in the dominance of formula funds in favor of an increase in debt securities within structured product portfolios.

By 2018, the share of formula funds had further dropped to just 17% of the total outstanding. This sharp reduction demonstrates investors' clear preference for alternative and potentially more diversified investment strategies, reflecting changes in market conditions, economic outlooks, and risk preferences. This transformation marks a significant evolution in investment approaches in France's structured products sector, characterized by a growing demand for flexibility and a reduced dependence on traditional formula-based products.

2.2.1.3 Complexity and Maturity of Structured Products

Another notable aspect of the structured products market in France is the extension of the theoretical maturity of marketed products, a trend that has intensified since 2011. While the average maturity of structured products was relatively short in the 2000s, it gradually increased, often reaching 7 to 10 years for products issued after 2010. This extension of maturities is often associated with an increase in product complexity, making it more difficult for non-professional investors to assess future returns.

In 2018, for instance, 60% of structured products sold had a maturity greater than 8 years, compared to only 15% in 2010. This evolution can be attributed to several factors, notably the low interest rate environment, which encouraged investors to seek higher returns through longer-term investments. However, this trend also exposes investors to increased risks, particularly in terms of liquidity and volatility, as long-maturity products are more vulnerable to long-term economic and financial fluctuations.

In parallel, the composition of structured product underlyings has also evolved. In 2018, 90% of the notional amount sold was based on equity underlyings, up from 80% in 2009. This dominance of equities as underlyings reflects a trend where investors are increasingly willing to accept higher risk in exchange for potentially higher returns, despite the increasing complexity of the products.

The extension of maturities has also been accompanied by the generalization of early redemption clauses, a phenomenon that became nearly systematic by the end of the period studied. This mechanism allows issuers to call the products before their theoretical maturity if certain market conditions are met, which has contributed to a growing gap between the theoretical and effective maturity of matured products. For example, in 2017, 59% of products had a gap between theoretical and effective maturity ranging from 8 to 10 years. This figure was 16% in 2014.

This evolution, while offering some flexibility, further complicates the assessment of future returns by investors. The frequent use of early redemptions has not only changed the perception of risks associated with structured products, but also highlighted the need for investors to be particularly attentive to market conditions that may trigger these redemptions.

In conclusion, the extension of maturity, combined with increased complexity of underlyings and the widespread use of early redemption clauses, illustrates a significant evolution of the French market. While these trends partly respond to a low interest rate environment, they also pose major challenges for investors in terms of product understanding and risk management.

2.3 Market Dynamics and Distribution Strategies

2.3.1 Role of Insurers and Independent Financial Advisors (IFAs)

Insurers play a central role in the distribution of structured products in France, mainly through life insurance contracts in unit-linked format (UC). In collaboration with independent financial advisors (IFAs), insurers are the main channels through which these products are marketed to retail investors. This collaboration is essential as it combines the risk management expertise of insurers with the IFAs' understanding of their clients' specific needs.

The popularity of structured products within life insurance contracts has significantly increased, partly thanks to their ability to offer competitive returns while providing partial capital protection. IFAs play a crucial role in this process by helping investors select structured products that best match their risk profile and investment objectives. However, this role requires ongoing training and a deep understanding of the products available on the market, given the increasing complexity of these financial instruments.

2.3.2 Margins of Brokers and Independent Financial Advisors

Margins associated with structured products can vary significantly depending on the distribution channel and the actors involved. A key player in this process is the broker, who often uses IFAs as intermediaries to distribute these products. Brokers frequently design structured products specifically tailored for IFAs, enabling them to offer customized investment solutions to their clients.

Two main methods are used by brokers to reach investors. On the one hand, they offer structured products adapted to the specific needs of IFAs, allowing the latter to propose customized investment options. On the other hand, brokers use life insurance contracts as the main distribution vehicle. Thus, French brokers act as intermediaries between the issuing bank, insurers, and IFAs.

By working through IFAs, brokers benefit from a distribution network capable of presenting and explaining complex products to often non-expert clients. This collaboration not only increases the reach of structured products, but also maximizes margins by effectively targeting potential clients and adapting products to precise market requirements.

Therefore, the role of IFAs is central to brokers' distribution strategy, as brokers leverage this expertise to better penetrate the retail investor market, whether through life insurance contracts or other specialized distribution channels. Also, when a broker sells a structured product to an IFA, the latter often gains access to the broker's platform, which allows them to track the valuation of the product. However, it is important to note that the valuation initially displayed on the broker's platform may be slightly higher than the product's real value. This practice allows the broker to smooth out their initial margin on the transaction.

Indeed, IFAs and their end clients generally do not have access to tools like Bloomberg or FactSet, which would allow them to independently verify the precise valuation of the product using its ISIN code. Without direct access to external data sources, IFAs must rely on the information provided by the broker on its platform.

Thus, by initially displaying a slightly inflated price, the broker ensures some leeway to optimize their margins. Over time, the price normalizes to more accurately reflect the product's real value, but this initial gap can lead to a biased perception of product performance, especially for end clients who depend on the information provided by the IFA.

The fee structure for structured products typically includes structuring, hedging, and distribution fees, which are often embedded in the product's price. These fees can represent a significant portion of the total cost to the investor, thus reducing the net return of the products. Moreover, brokers' margins may vary depending on the product's complexity, the issuer, or even the IFA.

It is also important to note that not all issuers necessarily offer the best products. Issuers with lower credit ratings than their competitors may be tempted to offer more aggressive pricing to attract clients. These more competitive prices may seem appealing, but they often involve greater risk for the investor due to the issuer's lower creditworthiness.

As for IFAs, not all adopt the same approach. Some IFAs prefer to include an additional margin when marketing structured products, thereby increasing the final cost to the investor. Others, by contrast, choose not to add a margin, instead focusing on sales volume or other strategic advantages. This disparity between IFAs means that investors may end up with very different net returns depending on which advisor they work with.

This situation raises important questions in terms of transparency and investor protection, particularly for retail investors who may not be fully aware of the costs involved. Regulators such as the AMF closely monitor these practices to ensure that investors receive clear and accurate information on the costs and risks associated with structured products through the Term Sheet. This document provides a detailed description of the structured product, including its key features, investment strategy, return calculation mechanisms, associated risks, fees, and redemption terms.

2.3.3 Diversity of Underlyings and Product Categories

Structured products marketed in France are often indexed to a variety of underlyings, thus offering significant diversification for investors. The most commonly used underlyings are stock market indices such as the CAC 40 and the Euro Stoxx 50, as well as baskets of equities known as Best-of or Worst-of. The Best-of and Worst-of concepts refer to how a structured product's performance is determined based on a basket of assets.

- **Best-of :** In this type of structured product, the product's performance is based on the best-performing asset in the basket. For example, if a basket includes several stocks, the product's return will be calculated based on the one with the highest return over the specified period. This mechanism is advantageous for investors, as it maximizes potential returns by focusing on the top-performing option.
- **Worst-of :** Conversely, in a Worst-of structured product, the performance is based on the worst-performing asset in the basket. This means that even if several assets perform well, the product's return is determined by the one with the lowest performance. This type of product is generally riskier, as it exposes the investor to the weakest link in the basket, which can significantly reduce the overall return.

— **Settlement/Delivery Risk :** Each transaction involves the management of financial flows on specific dates ; penalties may apply based on LIBOR rates if deadlines are not met.

These mechanisms allow for the creation of products tailored to specific investment strategies, offering either relative safety (in the case of Best-of) or a higher return potential in exchange for increased risk (in the case of Worst-of).

2.3.4 Main Product Categories

In France, the regulation of structured products is particularly strict, especially regarding their complexity. The AMF (Autorité des Marchés Financiers) and the ACPR (Autorité de Contrôle Prudentiel et de Résolution) have defined specific criteria to assess this complexity, aiming to protect retail investors. Since 2017, a structured product's complexity has been evaluated not only by the number of mechanisms it incorporates but also by the complexity of its underlying asset. More specifically, if a product contains more than three different mechanisms to determine its overall return, it is considered complex and potentially difficult for a non-professional investor to understand. This classification directly affects how these products can be marketed, particularly their inclusion in life insurance contracts.

Structured products are classified into different categories, each designed to meet specific investment needs while complying with current regulations. These categories allow products to be tailored to the requirements of investors and insurers, who may limit the complexity of the products they accept.

— **Autocallables :** These products allow for early redemption under certain market conditions. Their complexity can vary, and if it exceeds the threshold set by the AMF and the ACPR, their distribution may be restricted, especially in life insurance contracts.

— **Formula funds :** Designed to offer a return based on a predefined formula, these products are often simplified to remain below the three-mechanism complexity threshold, making them easier to include in life insurance contracts.

— **Capital-guaranteed products :** These ensure the protection of the initial capital at maturity. Their simplicity makes them compatible with regulatory requirements, making them preferred options for life insurance contracts.

— **Leveraged products :** Aimed at amplifying gains, these products are often more complex due to the use of options or other derivatives, which may make them unsuitable for retail investors or life insurance contracts.

— **Hybrid products :** By combining multiple underlyings or mechanisms, these products may accumulate a high number of mechanisms, thus exceeding the complexity threshold and limiting their eligibility for inclusion in life insurance contracts.

— **Interest rate products :** Indexed to interest rates or bonds, these products are generally simple and regulation-compliant, making them suitable for life insurance contracts.

— **Volatility products :** These products, which exploit market volatility, are often deemed too complex to be included in standard life insurance contracts due to their dependence on volatile market conditions.

— **Custom structured products :** Although designed to meet specific needs, these products must also comply with complexity rules. If their structure exceeds the defined threshold, they are generally reserved for institutional investors or specific contracts.

The goal of this classification is to ensure that structured products offered to investors comply with the transparency and simplicity standards imposed by regulators. By limiting product complexity, the AMF and ACPR aim to minimize the risks of misinterpretation and mismanagement of financial products by investors, particularly within the context of life insurance contracts. This regulation therefore plays a crucial role in the design and marketing of structured products, ensuring that they remain accessible and understandable for a wide range of investors.

The regulation and classification of structured products directly influence their design and marketing, ensuring that these products remain transparent and comprehensible to investors. Once these regulatory requirements are met, the next step is to determine the product's value—a process known as pricing.

3 Valuation and Hedging Methodology for Structured Products

This section focuses on the pricing of a structured product—specifically, the autocallable—i.e., the calculation of its fair value. In a second part, we will examine how issuers hedge their positions.

3.1 Valuation of an Autocallable Structured Product

3.1.1 Definition of Autocallable Structured Products

Autocallable products are frequently used by investors seeking potentially high returns while benefiting from partial capital protection. These products are especially popular in life insurance policies and among wealth management advisors in France. Autocallables can be automatically redeemed before their final maturity if certain predefined conditions are met on specific dates, called observation or autocall dates. These conditions are usually based on the performance of an underlying asset, such as a stock or an equity index. If, on an observation date, the asset's price exceeds a certain threshold, the product is "called" (i.e., redeemed), and the investor receives a predefined payout.

Autocalls are among the most popular structured products in France, especially in life insurance policies. Their typical structure includes an underlying asset (stock, index, or basket), predefined observation dates, a call barrier (price threshold), and scheduled coupons. We distinguish two main types of autocallables :

— **Phoenix** : The coupon barrier is below the autocall barrier.
— **Athena** : The coupon barrier is equal to the autocall barrier.

The Phoenix structure allows the investor to receive coupons even if the product is not called. Additionally, a memory effect is often included. This feature ensures that missed coupons are not lost but deferred and paid later if conditions are met.

3.1.2 The Importance of Accurate Pricing Methods

Pricing a structured product—especially an autocallable—is critical for several reasons. First, it is key to market competitiveness. Issuers must provide clients with a realistic and accurate estimation of the embedded options. Overpricing makes the product less attractive than competitors' offers, while underpricing may result in losses, as issuers must honor contract terms even if market conditions are unfavorable. Second, accurate pricing helps issuers assess and manage product risks, including market, credit, and liquidity risks. Inaccurate pricing can lead to ineffective hedging strategies and financial losses. Lastly, transparent and precise pricing enhances investor confidence—they are more likely to invest in products whose risks and expected returns are well understood and properly evaluated.

Poor pricing can have serious implications for both issuers and investors. If a structured product is overpriced, investors may turn to more attractive alternatives offered by competitors, reducing the issuer's market share and revenues. Conversely, underpricing can lead to significant financial losses. For example, if the embedded options are undervalued, the issuer may have to pay higher-than-expected returns, affecting profitability.

In the following sections, we will detail the methodologies used to evaluate these complex products, as well as the hedging techniques employed to manage the associated risks. We first present a flexible framework based on partial differential equations (PDEs) to model autocallable structured products using the Black-Scholes model. This framework handles both discrete and continuous autocall dates. We value discrete autocall products using the finite difference method, and continuous ones via closed-form solutions. We then continue with Monte Carlo simulations and the Heston model.

Additionally, we estimate the probabilities of a product being called on each call date. Our model is presented later, where we assess a popular autocallable and quantify the additional cost to the investor of specific features (Athena and Phoenix structures).

3.1.3 Valuation Methodology for Autocallable Structured Products

The autocall feature is a key element in the valuation of autocallable structured products. It allows for the early redemption of the product if the price of the underlying asset reaches or exceeds a predefined level. The main characteristics of this feature include the autocall dates, which can be either discrete or continuous, influencing the trigger timing. If an autocall is triggered, investors receive a payment consisting of the nominal capital and a predetermined return. This feature reduces the effective duration of the product, thereby modifying expected returns. The probability of an autocall being triggered, based on market conditions and expected price movements, is essential for estimating the value of these products.

3.1.3.1 Introduction to the General Valuation Framework via Partial Differential Equations

In this section, we address the valuation of autocallable structured products by modeling them using partial differential equations (PDEs). While this PDE approach is flexible enough to handle both discrete and continuous autocalls, it is important to note that it is not the only valuation method available for these products.

This valuation model follows the Black-Scholes framework with risk-neutral assumptions. The price of the underlying asset is assumed to follow a generalized Brownian motion, defined by the following equation :

$$dS_t = (r - q)S_t\,dt + \sigma S_t\,dW_t$$

where r is the risk-free interest rate, q is the dividend yield, and σ is the volatility of the price process. Throughout this section, we assume that r, q, and σ are constant and continuously compounded over the product's duration $[0, T]$. For simplicity, we omit the time index from S_t.

Assuming that the structured product price $V(S,t)$ is a function of time $t \in [0,T]$ and of the underlying asset price $S \in [0,\infty)$, the Black-Scholes formula implies that the dynamic value of a structured product can be expressed by the following PDE :

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + (r-q)S\frac{\partial V}{\partial S} - (r + \overline{CDS})V = 0 \tag{1}$$

where CDS is the Credit Default Swap spread of the issuer. Structured products are unsecured debt instruments and, consequently, lose value if the issuer defaults. It is therefore essential to include the issuer's credit risk, represented by the CDS spread, in the PDE to calculate the present value of the structured product (Hull, 2008 ; Deng et al., 2010a).

Many different features of structured products can be modeled as variations of Equation (1). For instance, when the product is not called, the terminal payoff $f(S_T)$ is typically a function of the underlying asset's value at maturity :

$$V(S_T, T) = f(S_T)$$

For simplicity and without loss of generality, we assume that the initial principal of a structured product equals the initial value of the underlying asset S_0.

Embedded call and put options, as well as the autocall feature, can all be modeled as boundary conditions (Deng et al., 2010a). The boundary condition for the autocall feature is :

$$V(C,t) = P_t$$

for $t \in T_C$, where C is the call price (time-independent), P_t is the final payout if the product is called, and T_C is the set of discrete or continuous call dates. Note that once the autocall is triggered, the structured product immediately matures and the final payout is P_t. Autocallable structured products typically pay a fixed rate of return. Thus, the payout is defined as :

$$P_t = He^{Bt}$$

where B is the yield rate and H is a constant.

However, the Black-Scholes model is a powerful theoretical framework for evaluating derivative products, including autocallable structured products. In practice, though, several adjustments are often required to better reflect actual market conditions :

- **Stochastic volatility and jumps** : Real financial markets often exhibit sudden jumps and changing volatility, which the original Black-Scholes model does not capture. To enhance accuracy, stochastic volatility models (like the Heston model) and jump models (like the Merton model) can be used. These models adjust the base PDE to include additional terms for these factors.
- **Variable dividends** : The Black-Scholes model assumes a constant dividend yield q. In reality, dividends may vary. To incorporate this variability, one can model dividend yield as a time-dependent function or use models where dividends are paid at specific times, affecting the structured product's price.

— **Issuer credit** : The CDS spread reflects the issuer's credit risk. This spread must be accurately estimated for each issuer as it can significantly impact the valuation of the structured product. In volatile market conditions, CDS spreads can change substantially, requiring frequent adjustments in the valuation model.

Thus, these improvements enable issuers and investors to obtain more accurate estimates of prices and risks associated with structured products.

3.1.4 Valuation of autocallable products with discrete autocall dates

Starting from PDE (1), we define the following boundary conditions :

1. **Early redemption condition** : At each autocall date t_i, if the price of the underlying asset S reaches or exceeds the threshold C, the product is redeemed at the amount P_t :

$$V(C, t_i) = P_t \quad \text{for } t_i \in T_C, \text{ where } T_C \text{ is the set of autocall dates.}$$

2. **Zero price condition** : If the price of the underlying asset drops to zero, the value of the product becomes zero, as it can no longer be exercised :

$$V(0, t) = f(0)e^{-(r+\text{CDS})(T-t)}.$$

Next, to simplify the resolution of the PDE, a variable transformation is applied. This converts the complex PDE into a simpler diffusion equation, known as the heat equation. The new variables are defined as follows :

$$S = Ce^x \quad \text{and} \quad t = T - \frac{2\tau}{\sigma^2}$$

This allows us to express the value of the structured product in the form :

$$V(S, t) = Ce^{\alpha x + \beta\tau}u(x, \tau) + f(0)e^{-(r+\text{CDS})(T-t)},$$

where the parameters are defined as follows :

$$\alpha = -\frac{1}{2}(k_1 - 1),$$

$$\beta = -\alpha^2 - \frac{2(r + \text{CDS})}{\sigma^2},$$

$$k_1 = \frac{2(r - q)}{\sigma^2}.$$

These transformations simplify the PDE into a standard heat equation :

$$\frac{\partial u}{\partial \tau} = \frac{\partial^2 u}{\partial x^2}, \quad \text{for} \quad -\infty < x < \infty,\ \tau > 0.$$

Finally, we use the finite difference method to solve the transformed PDE. This involves discretizing the variables of the equation on a grid of points. The partial derivatives are approximated using finite differences :

$$\frac{\partial u}{\partial \tau} \approx \frac{u_n^{m+1} - u_n^m}{\delta\tau}, \quad \frac{\partial^2 u}{\partial x^2} \approx \frac{u_{n+1}^m - 2u_n^m + u_{n-1}^m}{(\delta x)^2}.$$

This approach makes it possible to compute the discrete values of $u(x, \tau)$, which represent the transformed value of the structured product. By using the discretized values, the equation is solved iteratively until the boundary conditions are satisfied across the entire grid. The accuracy of the solution depends on the discretization steps δx and $\delta\tau$.

3.1.5 Valuation of autocallable products with continuous autocall dates

For autocallable structured products with continuous autocall dates, the boundary conditions of the partial differential equation (PDE) are also continuous. These conditions define the early redemption of the product at any time the underlying asset's price reaches a certain threshold.

We recall here the boundary conditions and the heat equation :

1. **Autocall value condition** : $V(C, t) = P_t$, where P_t is the payment if the product is called at time t.
2. **Zero-value condition** : $V(0, t) = f(0)e^{-(r+\text{CDS})(T-t)}$, ensuring that the product has no value if the underlying asset price reaches zero.

To solve this PDE, we apply the same variable transformations as in the previous section, allowing us to reduce the equation to a standard heat equation :

$$\frac{\partial u}{\partial \tau} = \frac{\partial^2 u}{\partial x^2}, \quad \text{for} \quad -\infty < x < 0,\ \tau > 0.$$

The boundary conditions and the initial condition are defined as :

$$\begin{array}{lll} u(-\infty, \tau) & = & 0, \\ u(0, \tau) & = & h_1(\tau), \quad \text{for} \quad \tau > 0, \\ u(x, 0) & = & h_2(x), \quad \text{for} \quad -\infty < x < 0. \end{array}$$

To simplify these boundary conditions once again, we introduce a transformation to homogenize them. By defining $v(x, \tau) = u(x, \tau) - e^x h_1(\tau)$, the transformed Black-Scholes equation becomes :

$$\frac{\partial v}{\partial \tau} = \frac{\partial^2 v}{\partial x^2} + e^x \left(h_1(\tau) - h_1'(\tau)\right), \quad \text{for} \quad -\infty < x < 0,\ \tau > 0.$$

The new homogeneous boundary conditions are :

$$\begin{array}{lll} v(-\infty, \tau) & = & 0, \\ v(0, \tau) & = & 0. \end{array}$$

The initial condition is defined as :

$$v(x, 0) = h_2(x) - e^x h_1(0).$$

This transformation enables the PDE problem to be addressed with homogeneous conditions, thereby simplifying the resolution. The solution to the transformed PDE is obtained using numerical methods adapted to homogeneous conditions, as described by G. Evans (2010). The final solution $v(x, \tau)$ is then converted back into $u(x, \tau)$ to retrieve the original value of the structured product $V(S, t)$ using the relation :

$$u(x, \tau) = v(x, \tau) + e^x h_1(\tau).$$

Using this method, we can compute the value of the autocallable structured product at any time, accounting for the possibility of continuous early redemption, the volatility of the underlying asset, and other market parameters affecting the valuation.

3.1.6 Valuation of autocallable products using Monte Carlo simulation

The Monte Carlo method relies on generating a large number of possible scenarios for the evolution of the underlying asset price over the product's life. These scenarios are created by simulating price paths using a stochastic model, typically based on geometric Brownian motion, which models the dynamics of financial asset prices.

For an underlying asset S with initial price S_0, the price dynamics can be modeled by the following equation :

$$dS_t = S_t(\mu dt + \sigma dW_t)$$

where :

$$\begin{aligned} \mu \quad & \text{is the drift rate (expected return),} \\ \sigma \quad & \text{is the asset volatility,} \\ W_t \quad & \text{is a Wiener process (Brownian motion).} \end{aligned}$$

In Monte Carlo simulation, this equation is discretized to generate a sequence of simulated prices at different time points t. The price evolution over a time interval Δt is given by :

$$S_{t+\Delta t} = S_t \exp\left(\left(\mu - \frac{\sigma^2}{2}\right)\Delta t + \sigma\sqrt{\Delta t}Z\right)$$

where Z is a random variable drawn from a standard normal distribution $\mathcal{N}(0, 1)$.

Once the paths are simulated, the autocall conditions are applied. For each scenario, we check at each call date (for discrete autocallables) or continuously (for continuous autocallables) whether the asset price reaches or exceeds the call barrier. If it does, the product is called and the payoff is calculated as the nominal capital plus the accumulated coupon. If not called, the final payoff at maturity depends on the performance of the underlying asset relative to protection barriers and initial levels.

Finally, the value of the autocallable is estimated by averaging the payoffs across all simulated scenarios, discounted at the risk-free interest rate r to reflect their present value :

$$V_0 = e^{-rT}\frac{1}{N}\sum_{i=1}^{N} \text{Payoff}_i$$

where :

$$\begin{aligned} N \quad & \text{is the number of simulated scenarios,} \\ T \quad & \text{is the time to maturity,} \\ \text{Payoff}_i \quad & \text{is the payment received in scenario } i. \end{aligned}$$

However, the accuracy of the Monte Carlo simulation depends on the number of scenarios N. The higher the number of scenarios, the more accurate the valuation will be, as the results converge to the true expected value.

3.1.7 Valuation of autocallable products using the Heston model

Unlike the Black-Scholes model, which assumes constant volatility, the Heston model allows volatility to vary randomly over time. This is essential to capture market phenomena such as volatility smiles, where implied volatility varies with strike price and maturity. Volatility tends to be low at-the-money and increases for deep in- or out-of-the-money options.

Under the Heston model, the price of the underlying asset S_t and its variance v_t follow the stochastic processes :

$$dS_t = \mu S_t dt + \sqrt{v_t} S_t dW_t^S$$

$$dv_t = \kappa(\theta - v_t)dt + \sigma_v \sqrt{v_t} dW_t^v$$

where :

$$\begin{aligned} \mu \quad & \text{is the drift rate (expected return),} \\ \kappa \quad & \text{is the speed of mean reversion of the variance,} \\ \theta \quad & \text{is the long-term variance level to which } v_t \text{ reverts,} \\ \sigma_v \quad & \text{is the volatility of the variance (vol of vol),} \\ dW_t^S \text{ and } dW_t^v \quad & \text{are correlated Wiener processes with correlation coefficient } \rho. \end{aligned}$$

The Heston model, with its ability to capture more realistic market dynamics, is particularly useful for valuing autocallable products, which are sensitive to volatility fluctuations and non-linear behaviors of underlying assets. It enables more accurate valuation under real market conditions and enhances risk management for investors.

3.1.8 Which valuation methods do banks use ?

When valuing autocallable structured products, banks use various methods tailored to specific needs. The finite difference method is preferred for its accuracy in modeling well-defined features such as discrete call dates and barriers. Although computationally intensive, it offers precise solutions, especially for simpler products or when analytical benchmarks are available.

Monte Carlo simulation is widely favored for its flexibility. It is particularly effective in managing complex market dynamics and exotic products involving multiple barriers and multi-asset options. This method can model nonlinear scenarios and unique product features that are hard to capture otherwise. Its main drawback is the high computational cost, as it requires a large number of simulations for high accuracy. Still, it is broadly used for the most complex products.

The Heston model, which incorporates stochastic volatility, is especially relevant when fluctuating volatility significantly impacts product valuation. It can replicate volatility smiles—curves showing implied volatility variations with strike prices. This model, though complex and computationally demanding, is often combined with Monte Carlo methods to value exotic equity options and products requiring precise volatility modeling.

In summary, the choice of valuation method depends on the product's specific features and operational constraints. The finite difference method is ideal for precise detail, Monte Carlo is preferred for complex scenario handling, and Heston is used when stochastic volatility plays a key role. Financial institutions often adopt a hybrid approach, selecting the most suitable method based on the product's requirements and available resources.

However, it is worth noting that it is virtually impossible to know precisely which valuation methods banks use for structured products, as they generally do not disclose this information.

3.2 Practical Application

In this section, we illustrate the application of pricing methods on a real-world example of an autocallable product, very popular in the French market, by comparing the two types of products — Phoenix and Athena — based on the same underlying asset. We focus here on valuation using the Heston model.

3.2.1 Parameter Definition :

The key parameters of the Phoenix and Athena structured financial products are defined as follows :

— **Underlying asset :** Saint-Gobain
— **Current level :** 79.98
— **Issue date :** 9 mai 2025
— **Duration :** 5 years
— **Observation dates :** Semi-annual
— **Coupon barrier (Phoenix) :** 70% of initial level
— **Protection barrier :** 60% of capital
— **Autocall barrier :** 100% of initial level
— **Risk-free rate :** 4.5%
— **Annual volatility :** 35.65%
— **Dividend yield :** 2.62%
— **Memory effect :** Yes
— **Coupon :** 5% (10% annualized)

3.2.2 Payoff Mechanism

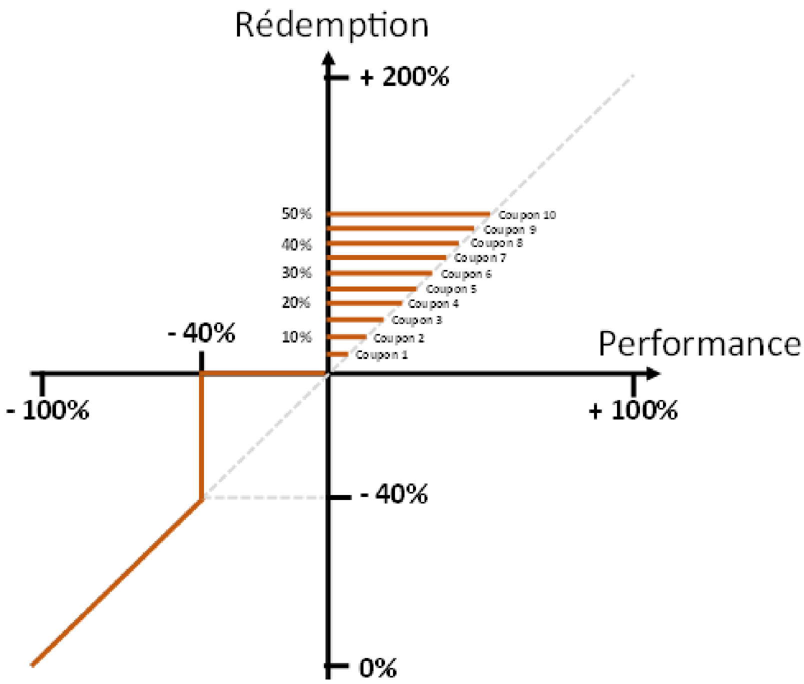

FIGURE 3 – Structured product payoff

Phoenix Product

At each semi-annual observation date i (where $i = 1$ to 9) :

- If the underlying closes at or above its initial level : early redemption at 100% + 5% + previously missed coupon(s).
- If the underlying closes at 70% or more of its initial level : next coupon is paid at 5% + missed coupons.
- Otherwise, no coupon is paid and the product continues.

At the final observation date $i = 10$:

- If the underlying closes at 70% or more of its initial level : redemption at 100% + 5% + previously missed coupons.
- If the underlying closes at or above 60% of its initial level : redemption at 100%.
- Otherwise, redemption equals : Final level / Initial level of the underlying.

Athena Product

At each semi-annual observation date i (where $i = 1$ to 9) :

- If the underlying closes at or above its initial level : early redemption at 100% + 5% + previously missed coupon(s).
- Otherwise, no coupon is paid and the product continues.

At the final observation date $i = 10$:

— If the underlying closes at 100% or more of its initial level : redemption at 100% + 5% + previously missed coupons.
— If the underlying closes at or above 60% of its initial level : redemption at 100%.
— Otherwise, redemption equals : Final level / Initial level of the underlying.

3.2.3 Structured Product Composition

The structured product studied is composed of :

— **Zero-Coupon Bond (ZC) :** A significant part of the invested capital is used to purchase a zero-coupon bond that guarantees principal repayment at maturity.
— **Down-and-In Put Option (PDI) :** This option is integrated to manage downside risk. The PDI activates only if the underlying price falls below a predetermined barrier (here 60%) at maturity, since in our example it's a European option.
— **Digital Options (Digit) :** Digital options are used to structure coupon payments. These are binary options. If the underlying is above the barrier, the coupon is paid ; otherwise, no coupon is paid.

3.2.4 Modeling with the Heston Model

The Heston model is a stochastic volatility model used to simulate the behavior of financial assets, especially for pricing options. The initial parameters used for calibration of the model are as follows :

Initial Parameters
— v_0 (initial variance) : 0.01
— θ (long-term variance mean) : 0.01
— κ (speed of mean reversion) : 0.5
— σ (volatility of volatility) : 0.2
— ρ (correlation between underlying and variance) : -0.5

Calibrated Parameters After calibrating the Heston model, the updated parameters are :
— v_0 (initial variance) : 0.127092
— κ (speed of mean reversion) : 9.99435
— θ (long-term variance mean) : 0.0100123
— σ (volatility of volatility) : 0.000330621
— ρ (correlation between underlying and variance) : 0.127094

3.3 Presentation of Results

To evaluate the Athena and Phoenix products, we used the calibrated Heston model to simulate 10,000 paths of the underlying asset's price. This allows for a more realistic consideration of market volatility fluctuations.

Structured Product Valuations

— Value of Athena product : €816,500
— Value of Phoenix product : €902,800

These values represent the amounts that investors could expect to receive if the product were purchased on the valuation date, accounting for fluctuations in the underlying's volatility and the specific characteristics of each product.

Purchase Percentages

With a notional amount of €1,000,000, the purchase percentages are :
— Athena purchase percentage : 81.65%
— Phoenix purchase percentage : 90.28%

These percentages indicate the valuation of the products under simulated market conditions. The Phoenix product shows a slightly higher value, reflecting more favorable redemption conditions or a lower perceived risk. This difference is explained by the coupon barrier level — the probability of receiving coupons with Phoenix is higher than with Athena.

In practice, when pricing structured products such as Athena and Phoenix, issuers often aim to align their values to maintain comparable appeal, even if the payout structures differ (e.g., coupon or protection barriers).

To achieve this value equivalence, issuers mainly adjust the coupon rates. For example, a Phoenix product with a lower coupon barrier might offer a different coupon rate than an Athena product to compensate for perceived risk differences.

As a result, the nominal value of these products, in terms of purchase percentage or present value, remains relatively stable and similar, as issuers adjust the coupon to balance value propositions. This is essential since issuers must preserve margins, especially when products are distributed through intermediaries such as insurers or wealth advisors. These intermediaries usually apply fixed fees or spreads, regardless of the product structure.

In conclusion, during the design phase, issuers strive to maintain value parity between Athena and Phoenix products while adjusting coupon rates to reflect each product's characteristics. This enables investors to choose based on risk appetite and personal preferences, while ensuring consistency in market offerings.

Lastly, it is crucial to understand how issuers manage the risks associated with structured products like Athena and Phoenix. Once priced and offered to investors, issuers must hedge inherent risks, such as market fluctuations and default risk. The hedging process is essential to securing commitments made to investors and protecting issuers' margins.

In the next section, we explore the different hedging strategies used to manage these risks, the financial instruments involved, and the implications for issuers and investors. Hedging is a critical component of structured product management, helping ensure financial stability and predictable outcomes in uncertain markets.

4 Hedging Structured Products

Hedging is a fundamental component in managing structured products. This process helps manage various types of risk, such as market fluctuations, volatility changes, and correlation risks. Banks bear the risk that hedging costs may exceed expectations or that unexpected losses may arise from hedging operations. To offset this, they apply a margin over expected hedging costs to ensure long-term profitability.

During the design and pricing of structured products, banks go through several iterations and evaluations to test different configurations. This is essential for identifying the most attractive product features while managing risks.

For example, if a trader is already heavily exposed to Saint-Gobain stock, they will be less inclined to offer competitive pricing to the sales team to avoid increasing this exposure. Pricing decisions are thus influenced by the need to maintain balance in the bank's portfolio and manage risks proactively.

That said, the hedging process begins once the product has been negotiated between the two parties — that is, between the seller and the buyer.

4.1 Design and Pricing of Structured Products

The design and pricing phase of structured products is a complex process that includes detailed modeling of various market scenarios using models such as Black-Scholes and Monte Carlo simulations. In addition to transaction costs, investment banks must also consider regulatory requirements such as Basel III, which influence the level of capital required to cover associated market risks.

Hedging plays a central role in this phase. Indeed, hedging considerations — such as transaction costs and associated market risks — directly affect the bank's willingness to offer a product and at what price. Banks must ensure that the products they offer are competitive while sufficiently covering risks to avoid significant losses. Consequently, product design and pricing are often adjusted based on available hedging strategies and forecasted hedging costs.

These initial hedging steps are essential to ensure that structured products provide value to investors while minimizing risk to issuers. They lay the groundwork for specific hedging strategies such as delta hedging and vega hedging, which will be discussed in the following sections.

4.2 Hedging Strategies

The trader, responsible for hedging the structured products issued by the bank, focuses on managing the risks related to the optional components of these products. We will now explore the major strategies developed by traders.

4.2.1 Delta Hedging

Delta hedging is a strategy used to neutralize price variation risks of the underlying assets in structured products. The Delta (Δ) of a derivative measures the sensitivity of its price to a change in the underlying asset price. Mathematically, Delta is defined as the partial derivative of the product price (V) with respect to the underlying asset price (S) :

$$\Delta = \frac{\partial V}{\partial S}$$

A Delta of 0.5 indicates that if the underlying asset price increases by 1 unit, the option price will increase by 0.5 units.

The goal of delta hedging is to maintain a delta-neutral position, where the sum of Deltas of all components in a portfolio is zero or close to zero. This involves continuously adjusting positions in the underlying asset to offset changes in Delta caused by asset price movements.

In its hedging portfolio, the bank holds shares of the underlying asset to manage the risk associated with structured products like Phoenix. The bank uses these shares to adjust its delta position and neutralize exposure to changes in the underlying asset. A delta-neutral position is achieved when positions are adjusted such that variations in the underlying asset do not significantly affect the total portfolio value.

For example, when the underlying price drops, Delta may decrease. In that case, the bank must reduce its short position in the shares since the hedging need has lessened. It can then buy back the shares at a lower price, benefiting from the buy-low strategy. Conversely, if the underlying price rises and Delta increases, the bank must increase its short position. That means selling more shares at a higher price, following the sell-high strategy. This process is advantageous as it maximizes profits on the shares.

Thus, by adjusting its positions based on market movements, the bank can not only maintain a delta-neutral stance but also profit from market fluctuations. This demonstrates the importance of active and rigorous hedging portfolio management, not only to stabilize premium income but also to minimize potential losses.

Ultimately, delta hedging requires active management, especially in volatile markets. For example, adjustments may need to be made several times per day to maintain a delta-neutral position, particularly for structured products with barriers. This minimizes risks related to unexpected barrier triggers, which can lead to substantial losses.

4.2.2 Gamma Hedging

Gamma measures the sensitivity of Delta to changes in the underlying asset price. It is defined as the second derivative of the product price with respect to the underlying asset price :

$$\Gamma = \frac{\partial^2 V}{\partial S^2}$$

A high Gamma means that Delta changes rapidly with market movements, requiring frequent hedging adjustments. Ultimately, the trader seeks to control both Delta and Gamma when hedging the sale of a structured product to neutralize risks associated with fluctuations in the underlying asset price (Delta) and the sensitivity of that risk to market movements (Gamma). Accurate management of these parameters minimizes potential losses from unforeseen market movements, optimizes capital resource usage, and maintains revenue stability. It also allows for quick responses to changing market conditions, ensuring efficient and proactive risk management.

For example, in the context of a Phoenix product, the bank acts as the seller of options, while the product buyer is effectively the buyer of a put option. As the seller of this implicit option, the bank receives a premium from the buyer, providing immediate revenue. However, the bank also takes on the risk that the underlying exceeds the coupon barrier, which would trigger a payment to the investor or a loss to the bank. To manage this risk, the responsible trader must carry out hedging operations, such as buying or selling the underlying asset according to market movements to remain delta-neutral.

Practical Example :

1. **Selling the implicit option :**
   — The bank sells a Phoenix product with a Delta of 0.6 and a Gamma of 0.02.
2. **Receiving the premium :**
   — In exchange for this sale, the bank receives a premium, which represents the cost of the option for the investor. This premium compensates the bank for the risk taken.
3. **Delta management :**
   — To hedge, the trader must sell shares of the underlying asset to offset the effect of the put sale. With a Delta of 0.6 on 1,000 units sold, the trader sells 600 shares.
4. **Hedging adjustments :**
   — If the underlying asset price rises, Delta might increase to 0.62, requiring the sale of 20 additional shares to adjust the hedge.
   — If the price falls and Delta drops to 0.58, the trader buys 20 shares to adjust the hedge.

This process allows the bank to manage the risk related to selling the Phoenix product while stabilizing the income generated by the received premium. The trader must closely monitor market movements and adjust the hedge accordingly to minimize potential losses.

4.2.3 Vega Hedging

Vega hedging is a critical strategy for managing volatility risk associated with structured products. This technique focuses on hedging against changes in the implied volatility of underlying assets, which can significantly affect the price of embedded options within these products.

Vega represents the sensitivity of a derivative's price to changes in the implied volatility of the underlying. More specifically, it measures the impact of a 1% change in volatility on the option price. The Vega of an option is calculated from the parameters of the Black-Scholes model, and is expressed as :

$$\text{Vega} = S\sqrt{T}\,\phi(d_1)$$

where :

S	is the current price of the underlying asset,
T	is the time to expiration of the option (in years),
ϕ	is the standard normal probability density function,
d_1	is defined as $\frac{\ln(S/K)+(r+\sigma^2/2)T}{\sigma\sqrt{T}}$,
σ	is the implied volatility,
K	is the option strike price.

Thus, to achieve Vega neutrality in a portfolio, the sum of the Vegas of all positions must be zero or close to zero. This is generally done by adjusting option positions or using volatility derivatives. The Vega of an option can range from 0 to a theoretical maximum, but is generally between 0 and 1 for standard options. Here are some key points regarding the magnitude of Vega :

— **Proximity to at-the-money options :** Vega is generally highest when the option is at-the-money, meaning the strike price is close to the current price of the underlying asset. In such cases, Vega can reach high values, often close to 0.5 or more, depending on volatility and time to expiration.
— **Deep in-the-money or out-of-the-money options :** Vega decreases as the option becomes deeply in-the-money or out-of-the-money. In these cases, Vega may fall well below 1 and approaches 0 as the option loses sensitivity to implied volatility.
— **Time to maturity :** The longer the time to expiration, the higher the Vega, as changes in volatility have a greater impact on long-term options. For short-term options, Vega tends to decrease quickly.

Buying options—whether calls or puts—increases the Vega of the portfolio, while selling options decreases it. Thus, to reach a Vega-neutral position, a trader can balance the Vegas by adjusting long and short option positions.

In summary, Vega hedging is essential to protect against changes in implied volatility, especially in portfolios containing complex structured products. Traders can use volatility options or variance swaps to adjust their exposure. Dynamic Vega management is often complex and requires frequent adjustments, particularly for portfolios composed of options with different expiration dates and underlying assets.

However, adjusting positions to maintain Vega neutrality can become challenging, especially when options have different maturities or when the underlying assets differ. In such cases, a trader may use dynamic hedging, constantly adjusting the portfolio in response to changes in implied volatility and underlying prices.

To manage these complexities, traders use dynamic hedging strategies that involve several key practices. First, frequent re-evaluation is essential : traders must regularly assess the Vega of their portfolio and adjust positions accordingly, which may involve buying or selling options to compensate for changes in total Vega. Additionally, the use of complementary derivative instruments such as variance swaps or volatility futures is another commonly used tactic. These instruments allow volatility exposure to be adjusted without directly modifying the option positions, offering greater flexibility in risk management.

Finally, managing correlated risks is crucial when the portfolio contains options on multiple underlyings. It is important to understand how volatility movements of different assets may be correlated, as this affects the effectiveness of the dynamic hedging strategy. By integrating these elements, traders can better navigate the challenges posed by fluctuations in implied volatility and other market variables.

4.3 Specific Hedging Issues

In managing structured products, traders and risk managers face unique challenges related to hedging. These challenges are often amplified by unpredictable external factors and complex market variables. The two main concerns are gap risk, which can lead to substantial losses due to sudden price movements, and carry effects, which influence product valuation based on interest rates and dividends. A thorough understanding of these issues and the application of appropriate hedging strategies are essential to mitigate risk and protect traders' portfolios.

4.3.1 Gap Risk

Gap risk, also known as "jump risk," refers to sudden and unexpected movements in the prices of underlying assets. This risk is particularly relevant during events such as corporate earnings announcements or monetary policy decisions. Traders can use barrier options and exotic products to hedge against these risks. For example, purchasing a knock-in option (which only becomes active if the underlying price hits a predetermined level) can provide protection in case key price levels are breached.

4.3.2 Carry Effects

Carry effects relate to the impact of interest rates and dividend yields on the valuation of structured products. Structured products are often sensitive to interest rate changes, as these rates influence the level of the zero-coupon bond (ZC). Dividends, in turn, affect the value of equity-linked derivatives since distributed dividends reduce the stock price, which must be accounted for in hedging strategies.

Proper hedging of carry effects often involves using index futures or interest rate swaps to manage changes in financing costs. For equity-linked products, hedging strategies must also consider expected dividend yields to avoid underestimating risks. Adjustments should be dynamic, reflecting changes in interest rates or dividend policies of the underlying companies.

In conclusion, effective management of gap risk and carry effects is crucial for the stability of structured products. Banks and traders must employ a combination of hedging strategies and quantitative tools to anticipate and manage these risks, while remaining flexible to adapt to changing market conditions.

4.4 Hedging using Futures

4.4.1 Principles

Hedging using futures is an essential strategy for structured product traders, particularly useful for managing risks related to the maturity dates of underlying assets. Futures are standardized contracts obligating parties to buy or sell an asset at a predetermined price on a specific future date. For structured products with defined maturities, futures allow stabilization of returns by locking in future asset prices, thereby reducing uncertainty linked to market fluctuations.

For example, suppose the structured product is linked to a specific stock index. The bank can decide to purchase futures contracts on this index with maturities matching regular intervals throughout the four-year period. If the nearest futures contract matures in three months, the bank can buy this contract and, as it approaches expiry, sell it and purchase a new three-month contract. This process, known as rollover, ensures continuous hedging throughout the life of the structured product.

Additionally, if the bank anticipates a potential decline in the index in the coming months, it can take a short position on futures to hedge against this drop. For instance, if the current index is at 1,000 points and the three-month future is trading at 990 points, the bank can sell the futures contract to lock in a selling price of 990 points. If the index indeed drops to 950, the value of the short futures position will rise, offsetting potential losses on the structured product.

4.4.2 Margins and Risks

However, it is crucial to note that futures positions require margin deposits. The initial margin is a collateral amount the trader must deposit to open a position, calculated as a percentage of the total contract value. In addition to the initial margin, maintenance margins exist — minimum balances that must always be maintained. If prices move unfavorably for the trader's position, margin calls may be triggered, requiring additional funds to cover potential losses. This margin mechanism is designed to limit counterparty risk in futures markets.

There is also basis risk, which arises when the price of the futures contract and the underlying asset are not perfectly correlated. This risk implies that futures price movements may not fully offset underlying asset movements. For example, if the index drops unexpectedly but the futures price does not reflect this movement proportionally, the hedge will be less effective, potentially resulting in losses. Therefore, for futures hedging to be effective, traders must closely monitor market movements, proactively manage margins, and adjust their futures positions in response to price changes and market conditions.

In conclusion, hedging is a cornerstone of structured product risk management, helping to minimize financial exposure to market fluctuations, volatility, and other risks. By employing strategies such as delta hedging, vega hedging, and futures usage, issuers can protect their positions and stabilize investor returns. However, these strategies require rigorous management and constant adjustment to align with evolving market conditions.

Moreover, specific risks such as gap risk, carry effects, and basis risk when using futures highlight the inherent complexity of risk management in structured products. Traders must not only be proficient in applying these strategies but also remain alert to market risks and margin requirements that can impact their effectiveness. While hedging plays a crucial role in risk management, it is also strongly influenced by the prevailing regulatory environment. Financial regulations such as Basel III and other international directives impose strict requirements on capital and risk management. These regulations aim to strengthen overall financial stability and protect investors, but they also increase compliance costs and reporting obligations for structured product issuers.

In the next section, we will analyze the overall regulatory impact on the structured product market. We will explore how capital requirements, transparency standards, and other regulatory constraints influence not only pricing and hedging, but also product design, distribution, and management. This analysis will provide insight into how issuers navigate this complex regulatory environment to optimize their operations and ensure compliance while maximizing profitability.

5 Regulation of Structured Products

5.1 Introduction to Structured Product Regulation

5.1.1 Historical Context and Regulatory Objectives

In France, the structured product market experienced significant growth in the early 2000s, driven by increasing investor demand for potentially higher-yielding products in a low-interest-rate environment. According to AMF data, in 2022, structured products accounted for approximately 20% of investment offerings for retail investors, underscoring the segment's importance in the French market. However, this rapid growth also raised concerns about the complexity of these products and investors' ability to understand the associated risks.

The 2008 financial crisis served as a catalyst for sweeping reforms in the regulation of complex financial products, including structured products. Prior to the crisis, the market was largely under-regulated, with limited requirements for transparency and investor protection. The significant losses incurred by many retail investors highlighted the shortcomings of the existing regulatory framework. As a result, regulation of structured products in France was significantly strengthened to protect investors from disproportionate and often misunderstood risks.

The AMF took proactive steps to tighten regulation around structured products, with the primary aim of enhancing transparency and ensuring stronger investor protection. Key reform objectives included making products more comprehensible to investors, ensuring clear and complete communication of risks, and strengthening oversight of issuer and distributor business practices. For example, the AMF imposed stricter documentation and disclosure obligations to ensure investors are fully aware of the risks before subscribing to these products.

5.1.2 Regulatory Framework in France and Europe

The regulation of structured products in France is strongly influenced by European directives, yet it also has its own national specificities that reflect the priorities of the French regulator. While the Basel III agreements help to strengthen the stability of the banking sector, it is MiFID II and the PRIIPs regulation that form the cornerstones of this regulatory framework. MiFID II, implemented in France in January 2018, imposes strict requirements in terms of transparency, investor protection, and product governance. Approximately 75% of structured products marketed in France had to be reassessed to ensure their compliance with the new MiFID II requirements, particularly regarding cost and risk transparency. This directive obliges financial institutions to provide clear and detailed information on product features, associated risks, and costs, leading to a better understanding by investors of the products they purchase.

In France, the AMF has ensured strict application of MiFID II, placing emphasis on the training of financial advisors and client segmentation. For example, structured products can only be sold to clients whose risk profiles match the characteristics of the product, which has reduced the sale of these products to inexperienced or non-sophisticated investors.

The PRIIPs regulation, which came into effect at the same time as MiFID II, introduced an increased transparency requirement by mandating the creation of a Key Information Document (KID) for each structured product. In 2022, the AMF noted that 95% of structured products marketed in France complied with PRIIPs requirements, especially concerning the clarity of information provided in the KIDs. These documents allow investors to better understand the potential returns, risks, and costs of the products before making an investment decision.

The KID must be written in a way that is understandable even for non-professional investors, which has led to the simplification of many products to meet this requirement. The AMF has also intensified its oversight to ensure that the information provided in the KIDs is accurate and not misleading, sanctioning institutions that fail to comply with these standards.

In addition to European regulations, the AMF has developed specific guidelines for the French market, taking into account local specificities and the behavior of French investors. For example, the AMF has issued recommendations aimed at limiting the excessive complexity of structured products offered to retail investors. In 2022, an AMF study revealed that 60% of structured products marketed in France had been simplified to better meet investor expectations regarding transparency and understanding.

Furthermore, the AMF closely monitors the distribution of structured products to ensure that commercial practices meet suitability and appropriateness standards, meaning that products are aligned with the investor's risk profile. In 2017, Natixis Asset Management (NAM), one of the leading asset management firms in France, was subject to a historic fine of 35 million euros imposed by the AMF. This sanction, accompanied by a reprimand, concerned practices deemed non-compliant with asset management and transparency rules. NAM was accused of collecting and recording excessive management fees, particularly through redemption fees applied to unitholders wishing to retrieve their funds before maturity. These fees, considered *unduly charged* by the AMF, allegedly allowed NAM to build up an excessive financial cushion, deducted from the fund's net asset value and credited to the company's accounts. Moreover, NAM was accused of providing *partial* and *misleading* information to investors, by applying management fees above the 2% cap stated in the prospectus, which violates MiFID II regulations.

5.2 Impact of Regulation on the Design and Distribution of Structured Products

5.2.1 Transformation of Structured Product Design Practices

The implementation of MiFID II and PRIIPs has had a significant impact on how structured products are designed and marketed in the French market. Prior to these reforms, structured products were often designed with considerable flexibility, allowing issuers to create highly customized yet complex financial instruments. However, the complexity of these products often led to poor risk comprehension among investors, creating a climate conducive to unexpected losses, especially during the 2008 financial crisis.

With the introduction of MiFID II, structured product issuers had to adapt their design processes to comply with new product governance requirements. This includes the obligation to define a specific target market for each product, considering its characteristics, risk level, and compatibility with the financial objectives of potential clients. This requirement led to a reduction in the complexity of structured products, as issuers must now ensure that the products are suitable for the defined target market and not excessively risky for the intended investor profiles.

One direct consequence of this regulation has been increased efforts to standardize certain structured products to make them more accessible and understandable to a broader client base. For instance, products previously designed with complex payout structures based on exotic derivatives have been simplified to enhance transparency in terms of costs and risks. This standardization has helped reduce compliance and distribution costs while increasing transparency for investors. However, it has also limited the ability of issuers to offer highly tailored products, which may be seen as a constraint on financial innovation.

5.2.2 Impacts on Structured Product Distribution

The distribution of structured products has also been deeply affected by the new regulations, particularly through MiFID II's requirements concerning transparency and alignment of interests between distributors and investors. The directive requires financial product distributors to ensure that the products offered truly match the needs and risk profiles of their clients. This has led to a paradigm shift in the way products are sold.

Under MiFID II, financial advisors must now provide a detailed risk profile assessment of each client before recommending a structured product. This process includes an in-depth evaluation of the client's knowledge, experience with complex financial instruments, and risk tolerance. This requirement has led to greater customer segmentation, where only clients with sufficient knowledge and risk tolerance can access more complex products.

Additionally, MiFID II introduced heightened transparency obligations, requiring distributors to provide clear and detailed information on the total costs associated with structured products, including sales commissions, management fees, and implicit costs related to hedging strategies used by the issuer. This enhanced transparency aims to eliminate abusive sales practices where hidden costs could significantly erode investor returns. Recent data from the AMF indicates that since MiFID II came into effect, the number of complaints regarding structured products has decreased, suggesting better investor understanding.

The PRIIPs regulation, with its requirement to provide a Key Information Document (KID), has also strengthened investor transparency and protection. The KID, which must be provided before subscription, presents the product's key features in a standardized and accessible format, including potential performance scenarios, associated risks, and costs. In France, the AMF observed that PRIIPs compliance led to the simplification of structured products, with issuers modifying or withdrawing certain products considered too complex to meet the KID clarity and comprehensibility standards.

5.2.3 Consequences for Banks and Structured Product Issuers

MiFID II and PRIIPs regulations have also imposed significant compliance costs on banks and structured product issuers. The costs associated with adapting design processes, training financial advisors, and producing transparency documents have been considerable. Many banks have had to invest heavily in updating their IT systems to meet MiFID II reporting and transparency requirements. Additionally, issuers have had to thoroughly review their product ranges to meet the new transparency and governance requirements, which has sometimes led to a reduction in the supply of structured products on the market.

Another notable impact is the increased requirements for risk management. MiFID II mandates continuous monitoring of product compatibility with the target market, as well as regular assessment of the effectiveness of hedging strategies used for structured products. This has led to increased operational costs for issuers, who must now allocate more resources to risk management and regulatory compliance.

Finally, the regulations have also affected the profitability of structured products for banks. The need to reduce hidden costs and improve transparency has often led to a compression of profit margins on these products. In response, some banks have adopted diversification strategies, offering alternative products or focusing on less regulated market segments. However, the strict application of regulations has also helped strengthen investor confidence in structured products, which may ultimately reinforce their market position.

5.3 Impact of Regulation on Banks' Hedging Strategies

The post-2008 financial crisis regulation of financial products—through frameworks such as MiFID II, PRIIPs, EMIR, and Basel III—has profoundly influenced banks' hedging strategies, especially those related to structured products. These new regulations have imposed increased requirements in terms of transparency, risk management, and product governance, which has forced banks to adapt their hedging strategies to comply with these new standards.

5.3.1 Effects of MiFID II on Hedging Strategies

Under MiFID II, banks must not only ensure that the structured products they issue are suitable for the target market, but also that the hedging strategies used for these products align with investor expectations and product characteristics. This has driven banks to reassess their traditional approaches, especially by aligning their hedging strategies more closely with the specific risk profiles of end investors.

MiFID II enforces strict transparency obligations regarding costs, including those associated with hedging strategies. Banks are now required to provide detailed disclosures of the hedging costs embedded in the pricing of structured products. This includes transparency on transaction costs and operational fees related to risk management. Increased transparency has led banks to reevaluate their hedging approaches to minimize costs while complying with regulatory obligations. As a result, many banks have been forced to simplify their hedging strategies, favoring more standardized models that facilitate reporting and reduce costs.

The requirement to adapt hedging strategies under MiFID II has also impacted the design of the structured products themselves. Banks have moved toward simplifying product structures to reduce the complexity of required hedges. For instance, the use of exotic derivatives—which require complex and expensive hedging—has declined. Instead, banks now favor more linear structures, such as structured coupon bonds or barrier notes, where risks can be more easily hedged using standardized derivatives like vanilla options and swaps.

5.3.2 Effects of EMIR and Basel III on Hedging Strategies

EMIR mandates central clearing for over-the-counter (OTC) derivatives, which has significantly impacted banks' hedging strategies. Derivative transactions used to hedge structured products must now pass through central clearing houses, increasing costs due to initial and variation margins. This has prompted banks to reassess their derivative portfolios used for hedging, favoring centrally cleared instruments—even at the expense of higher costs and liquidity demands. EMIR also imposes detailed reporting obligations for every derivative transaction, requiring substantial investment in banks' risk management and reporting systems. The increased administrative burden associated with these obligations has led to a streamlining of derivative portfolios used for hedging, with a preference for the most liquid and easily reportable instruments.

Similarly, Basel III imposes stricter capital requirements for banks, especially regarding risk-weighted assets (RWAs). Derivatives used to hedge structured products, particularly those not centrally cleared, are subject to higher capital charges, making these strategies more expensive. In response, banks have adjusted their hedging strategies by reducing the use of capital-intensive derivatives and focusing on less demanding instruments, such as centrally cleared swaps or standardized options.

This capital constraint has also led banks to revisit the pricing of structured products, as the additional costs related to hedging must be integrated into the final product price. This has sometimes reduced the competitiveness of structured products, particularly those requiring complex and costly hedges.

In the same vein, regulations such as the Fundamental Review of the Trading Book (FRTB) impose stricter requirements on risk and hedging practices, notably through Profit and Loss (P&L) attribution tests. These tests are essential to validate the risk models used by banks and to ensure alignment with front-office practices. Financial institutions must demonstrate that their hedging strategies are not only effective but also compliant with new regulations, potentially requiring frequent adjustments and innovation in risk management.

5.4 Strategic Adaptations by Banks to New Regulations

In response to the heightened capital and liquidity requirements under Basel III and the operational costs imposed by EMIR, banks have had to reassess their hedging models. This includes optimizing derivative portfolios to minimize capital costs while maintaining effective risk coverage. Some banks have developed more sophisticated internal models to better assess capital requirements related to hedging strategies and to identify the most cost-efficient instruments. Banks have also adjusted their capital allocation based on the risk levels of structured products. For example, products with more complex or riskier exposures may now be paired with more conservative hedging strategies or restricted to institutional clients capable of bearing increased costs.

In response to regulatory constraints, banks have also sought innovation in the design of structured products and hedging strategies. This includes developing new product structures that are easier to hedge or that utilize centrally cleared derivatives to reduce capital and liquidity burdens. Hybrid products—combining features from different asset classes—have also gained popularity, as they offer more diverse and potentially cost-effective hedging opportunities. Banks have further explored the use of new technologies, such as automation in hedging processes and blockchain, to enhance transparency and reduce transaction costs. These technologies enable more efficient management of hedging portfolios and facilitate compliance with MiFID II and EMIR reporting obligations.

Regulations have also affected how banks interact with clients regarding hedging. Transparency and product governance obligations have strengthened the dialogue between banks and clients, particularly concerning risk management and hedging-related costs. Banks are now required to provide detailed explanations of how hedging costs are incorporated into structured product pricing and how these strategies align with clients' risk objectives. Furthermore, banks must be more proactive in educating clients about regulatory implications, especially regarding the limitations imposed by Basel III and EMIR on the types of hedging products available. This has led to a more consultative relationship between banks and clients, with a focus on tailoring hedging strategies to specific client needs while complying with regulatory constraints.

5.5 Long-Term Consequences and Future Outlook

In the long run, regulations such as MiFID II, PRIIPs, EMIR, and Basel III have reshaped banks' hedging strategies by imposing significant constraints but also creating opportunities for innovation. Banks must continue to adapt to these regulations while maintaining effective and economically viable hedging strategies. The emphasis on transparency and governance has improved the safety and resilience of the financial system, but has also reduced banks' flexibility in managing complex risks. In the future, banks may need to develop more integrated hedging solutions using advanced technologies to optimize costs and resources while remaining compliant with regulatory requirements. Furthermore, the evolution of regulation—including the potential introduction of new requirements under Basel IV or revisions to EMIR—will continue to shape banks' hedging strategies.

In conclusion, post-crisis regulations have had a profound impact on banks' hedging strategies, forcing them to rethink traditional approaches, innovate in product design, and improve transparency toward clients.

6 Conclusion

This thesis has explored the domain of structured products, highlighting their growing role in investment strategy in France as well as the challenges and opportunities associated with their use. By combining derivatives and underlying assets, structured products have proven to be complex yet effective financial instruments, enabling investors to achieve return objectives while benefiting from certain protections against market risks. The analysis of various valuation methods, including Black-Scholes, Monte Carlo, and Heston models, demonstrated the importance of rigorous risk management and advanced technical expertise to ensure accurate pricing and reliable projected returns.

Regulatory impact was another crucial point addressed in this thesis. Regulatory frameworks such as MiFID II and PRIIPs, by imposing strict standards of transparency and investor protection, have not only strengthened confidence in structured products but also introduced additional constraints for issuers. These regulations have led to improvements in risk management practices and increased operational costs, at times limiting innovation within the sector. Stress testing, integrated into these regulations, has played an essential role by providing an assessment of the resilience of structured products under adverse market scenarios, thereby contributing to financial stability. However, despite the use of high-quality data from the AMF, some critiques can be made. Although extensive, the data is not always fully transparent or accessible to retail investors, raising concerns about non-professionals' understanding of the associated risks. It is therefore imperative to improve transparency and communication regarding the costs and valuation of structured products in order to strengthen investor confidence.

To address the identified challenges and emerging trends, several avenues for future research have been proposed. Integrating ESG (Environmental, Social, and Governance) criteria into structured products could respond to growing demand for sustainable and responsible investments. Moreover, the adoption of emerging technologies such as artificial intelligence and blockchain could transform how structured products are priced and managed, improving the accuracy of valuations and transaction transparency. In addition, comparative analysis of international practices and deeper exploration of investor behavior could provide valuable insights for adapting structured products to the specific needs of global markets.

In conclusion, structured products will continue to play a key role in the financial landscape in France and beyond by offering investment solutions tailored to different investor profiles. However, their future development will depend on issuers' ability to innovate while complying with increasingly stringent regulations, integrating sustainability components, and enhancing transparency for investors. A proactive approach to understanding and adapting to market dynamics, technological advancements, and investor expectations will be essential to ensuring the sustainability and growth of this sector in an ever-evolving financial environment.

7 Bibliography

7.1 Newspaper Articles

Références

[1] Laurence Boccara, *Placements : les nouveaux atouts des produits structurés*, Les Echos, May 10, 2022.

[2] Laurence Boisseau, *L'AMF requiert une sanction record contre Natixis AM*, Les Echos, June 26, 2017.

[3] Laurence Delain, *Placements : fonds structurés et obligataires datés, le meilleur du rapport rendement-risque*, Les Echos, June 16, 2023.

[4] Lucas Downey, *Vega Neutral : What it is, How it Works, Example*, Investopedia, September 11, 2022.

[5] Dusan Isakov, *Le coût des produits structurés n'est pas aussi élevé qu'on le croit !*, Le Temps, January 15, 2016.

[6] Amélie Laurin, *Assurance-vie : les produits structurés dans le viseur des superviseurs français*, Les Echos, July 2, 2024.

[7] Tim Mortimer, *Hedging structured products*, FVC, May 4, 2022.

[8] Tim Mortimer, *Risk management : over the hedge*, Structured Retail Products, July 19, 2023.

[9] Sandra Sebag, *Les produits structurés s'invitent dans la gestion de trésorerie*, Option Finance, November 9, 2023.

7.2 Newspaper Articles

Références

[1] Laurence Boccara, *Placements : les nouveaux atouts des produits structurés*, Les Echos, May 10, 2022.

[2] Laurence Boisseau, *L'AMF requiert une sanction record contre Natixis AM*, Les Echos, June 26, 2017.

[3] Laurence Delain, *Placements : fonds structurés et obligataires datés, le meilleur du rapport rendement-risque*, Les Echos, June 16, 2023.

[4] Lucas Downey, *Vega Neutral : What it is, How it Works, Example*, Investopedia, September 11, 2022.

[5] Dusan Isakov, *Le coût des produits structurés n'est pas aussi élevé qu'on le croit !*, Le Temps, January 15, 2016.

[6] Amélie Laurin, *Assurance-vie : les produits structurés dans le viseur des superviseurs français*, Les Echos, July 2, 2024.

[7] Tim Mortimer, *Hedging structured products*,
FVC, May 4, 2022.

[8] Tim Mortimer, *Risk management : over the hedge*,
Structured Retail Products, July 19, 2023.

[9] Sandra Sebag, *Les produits structurés s'invitent dans la gestion de trésorerie*,
Option Finance, November 9, 2023.

7.3 Academic Papers and Reports

Références

[1] Benhami, K., et al., *Cartographie des marchés et des risques 2022*,
Published : June 2022.

[2] Brigo, D., Pallavicini, A., Torresetti, R., *Credit Models and the Crisis : A Journey into CDOs, Copulas, Correlations and Dynamic Models*,
Published : March 2010.

[3] Carr, P., Madan, D.B., *Option valuation using the fast Fourier transform*,
Published : December 1999.

[4] Cox, J.C., Ingersoll, J.E., Ross, S.A., *A theory of the term structure of interest rates*,
Published : May 1985.

[5] De Goria, C., *Étude sur l'intégration de produits structurés dans l'allocation d'actifs d'un institut de prévoyance*,
Published : September 2017.

[6] Deng, G., Husson, T., Mccann, C., *Valuation of Structured Products*,
Published : February 2014.

[7] Demartini, A., Mosson, N., *La complexité des produits structurés commercialisés en France : Quel bilan de l'action de l'AMF ?*,
Published : January 2020.

[8] Duffie, D., Kan, R., *A yield-factor model of interest rates*,
Published : October 1996.

[9] Gatheral, J., *The Volatility Surface – A Practitioner's Guide*,
Published : April 2006.

[10] Glasserman, P., *Monte Carlo Methods in Financial Engineering*,
Published : February 2003.

[11] Hansson, F., *A pricing and performance study on auto-callable structured products*,
Published : May 2012.

[12] Henderson, B.J., Pearson, N.D., Wang, L., *Pre-Trade Hedging : Evidence from the Issuance of Retail Structured Product*,
Published : February 2019.

[13] Kifle, H., *The Impact of Regulation on Corporate Hedging Activities and the Response of Corporates – A Preliminary Conceptual Framework*,
Published : March 2017.

[14] Heston, S.L., *A closed-form solution for options with stochastic volatility with applications to bond and currency options*,
Published : June 1993.

[15] Labbé, A., et al., *Structured Products Performance Review*,
Published : May 2019.

[16] Martellini, L., Priaulet, P., Priaulet, S., *Fixed-Income Securities : Valuation, Risk Management, and Portfolio Strategies*,
Published : October 2003.

7.4 Books

Références

[1] John Hull, *Pack Options, futures et autres actifs dérivés*,
Pearson, 2021.

[2] Raymond H., et al., *Financial Mathematics, Derivatives and Structured Products*,
Springer, 2022.

[3] Roland Portait, Patrice Poncet, *Finance de marché : Instruments de base, produits dérivés, portefeuilles et risques*,
Dalloz, 2023.

[4] Robert Vedeilhié, *Tout savoir sur les Produits Structurés*,
Gualino, 2008.

7.5 Websites

Références

[1] Autorité des Marchés Financiers, *Généralisation de l'établissement d'un document d'informations clés à tous les placements collectifs : l'AMF met à jour sa doctrine*,
`https://www.amf-france.org/fr/actualites-publications/actualites/generalisation-de-`

[2] Deloitte, *Structured Products and FRTB*,
`https://www2.deloitte.com/ch/en/pages/risk/solutions/structured-products-and-FRTB.h`

[3] Faster Capital, *Vega Hedge - Atténuation du risque dans le trading d'options*,
`https://fastercapital.com/fr/contenu/Vega-Hedge-The-Vega-Hedge-attenuation-du-risqu`

[4] Moody's Rating, *Moody's Ratings*,
`https://ratings.moodys.io/ratings`.

[5] SRP, *Structured Retail Products API*,
`https://www.structuredretailproducts.com/api`.

7.6 Websites

Références

[1] Autorité des Marchés Financiers, *Généralisation de l'établissement d'un document d'informations clés à tous les placements collectifs : l'AMF met à jour sa doctrine*, https://www.amf-france.org/fr/actualites-publications/actualites/generalisation-de-

[2] Deloitte, *Structured Products and FRTB*, https://www2.deloitte.com/ch/en/pages/risk/solutions/structured-products-and-FRTB.h

[3] Faster Capital, *Vega Hedge - Atténuation du risque dans le trading d'options*, https://fastercapital.com/fr/contenu/Vega-Hedge-The-Vega-Hedge-attenuation-du-risqu

[4] Moody's Rating, *Moody's Ratings*, https://ratings.moodys.io/ratings.

[5] SRP, *Structured Retail Products API*, https://www.structuredretailproducts.com/api.

A Python Source Code : Heston Pricing

The following Python code was used for the simulation of paths and calibration of the Heston model, as well as the evaluation of the autocallable product.

```python
import QuantLib as ql
import numpy as np
import scipy.optimize as opt

# Heston path generator
def HestonPathGenerator(dates, dayCounter, process, nPaths):
    t = np.array([dayCounter.yearFraction(dates[0], d) for d in dates])
    nGridSteps = (t.shape[0] - 1) * 2
    sequenceGenerator = ql.UniformRandomSequenceGenerator(nGridSteps, ql.UniformRandomGenerator())
    gaussianSequenceGenerator = ql.GaussianRandomSequenceGenerator(sequenceGenerator)
    pathGenerator = ql.GaussianMultiPathGenerator(process, t, gaussianSequenceGenerator, False)
    paths = np.zeros(shape = (nPaths, t.shape[0]))

    for i in range(nPaths):
        multiPath = pathGenerator.next().value()
        paths[i,:] = np.array(list(multiPath[0]))

    return paths

# Heston model calibration
def HestonModelCalibrator(valuationDate, calendar, spot, curveHandle, dividendHandle,
```

```
    v0, kappa, theta, sigma, rho, expiration_dates, strikes, data, optimizer, bounds):

    helpers = []
    process = ql.HestonProcess(curveHandle, dividendHandle,
        ql.QuoteHandle(ql.SimpleQuote(spot)), v0, kappa, theta, sigma, rho)
    model = ql.HestonModel(process)
    engine = ql.AnalyticHestonEngine(model)

    def CostFunction(x):
        parameters = ql.Array(list(x))
        model.setParams(parameters)
        error = [helper.calibrationError() for helper in helpers]
        return np.sqrt(np.sum(np.abs(error)))

    for i in range(len(expiration_dates)):
        for j in range(len(strikes)):
            expiration = expiration_dates[i]
            days = expiration - valuationDate
            period = ql.Period(days, ql.Days)
            vol = data[i][j]
            strike = strikes[j]
            helper = ql.HestonModelHelper(period, calendar, spot, strike,
                ql.QuoteHandle(ql.SimpleQuote(vol)), curveHandle, dividendHandle)
            helper.setPricingEngine(engine)
            helpers.append(helper)

    optimizer(CostFunction, bounds)
    return process, model

# Autocallable product valuation
def AutoCallableNote(valuationDate, couponDates, strike, pastFixings,
    autoCallBarrier, couponBarrier, protectionBarrier, hasMemory, finalRedemptionFormula,
    coupon, notional, dayCounter, process, generator, nPaths, curve):

    if valuationDate >= couponDates[-1]:
        return 0.0

    if valuationDate >= couponDates[0]:
        if max(pastFixings.values()) >= (autoCallBarrier * strike):
            return 0.0

    dates = np.hstack((np.array([valuationDate]), couponDates[couponDates > valuationDate]))
    paths = generator(dates, dayCounter, process, nPaths)[:,1:]

    pastDates = couponDates[couponDates <= valuationDate]
    if pastDates.shape[0] > 0:
        pastFixingsArray = np.array([pastFixings[pastDate] for pastDate in pastDates])
        pastFixingsArray = np.tile(pastFixingsArray, (paths.shape[0], 1))
        paths = np.hstack((pastFixingsArray, paths))

```

```
    global_pv = []
    expirationDate = couponDates[-1]
    hasMemory = int(hasMemory)

    for path in paths:
        payoffPV = 0.0
        unpaidCoupons = 0
        hasAutoCalled = False

        for date, index in zip(couponDates, (path / strike)):
            if hasAutoCalled:
                break
            payoff = 0.0

            if date == expirationDate:
                if index >= couponBarrier:
                    payoff = notional * (1 + (coupon * (1 + unpaidCoupons * hasMemory)))
                elif index >= protectionBarrier:
                    payoff = notional
                else:
                    index = index * strike
                    payoff = notional * finalRedemptionFormula(index)

            else:
                if index >= autoCallBarrier:
                    payoff = notional * (1 + (coupon * (1 + unpaidCoupons * hasMemory)))
                    hasAutoCalled = True
                elif index >= couponBarrier:
                    payoff = notional * (coupon * (1 + unpaidCoupons * hasMemory))
                    unpaidCoupons = 0
                else:
                    unpaidCoupons += 1

            if date > valuationDate:
                df = curve.discount(date)
                payoffPV += payoff * df

        global_pv.append(payoffPV)

    return np.mean(np.array(global_pv))
```

Listing 1 – Pricing Heston