3 Let Q(x, y) denote the statement ”x is the capital of y.” What are these truth values?

a Q(Denver, Colorado)

Solution Denver is the capital of Colorado: true

b Q(Detroit, Michigan)

Solution Detroit is the capital of Michigan: false

c Q(Massachusetts, Boston)

Solution Massachusetts is the capital of Boston: false

d Q(New York, New York)

Solution New York is the capital of New York: false

4 State the value of x fter the statement if P(x) then x := 1 is executed, where P(x) is the statement ”x > 1”, if the value of x when this statement is reached is

a $x = 0$

Solution $0 \not> 1, \therefore x = 0$

b $x = 1$

Solution $1 \not> 1, \therefore x = 1$

c $x = 2$

Solution $2 > 1, \therefore x = 1$

13 Determine the truth value of each of these statements if the domain consists of all integers.

a $\forall n(n+1>n)$

Solution True

b $\exists n(2n=3n)$

Solution True, when $n=0$, $2n=3n=0$

c $\exists n(n=-n)$

Solution True, when $n=0$, $n=-n=0$

d $\forall n(3n\leq 4n)$

Solution False for $n<0$

14 Determine the truth value of each of these statements if the domain of each variable consists of all real numbers.

a $\exists x(x^2=2)$

Solution True for $x=\sqrt{2}$

b $\exists x(x^2=-1)$

Solution False, only true for i

c $\forall x(x^2+2\geq 1)$

Solution False, only true for i

d $\forall x(x^2=x)$

Solution False, only true for 0 and 1

19 Suppose that the domain of the propositional function P(x) consists of the integers 1, 2, 3, 4, and 5. Express these statements without using quantifiers, instead using only negations, disjunctions, and conjunctions.

a $\exists x P(x)$

Solution $P(1) \vee P(2) \vee P(3) \vee P(4) \vee P(5)$

b $\forall x P(x)$

Solution $P(1) \wedge P(2) \wedge P(3) \wedge P(4) \wedge P(5)$

c $\neg \exists x P(x)$

Solution $\neg(P(1) \vee P(2) \vee P(3) \vee P(4) \vee P(5))$

d $\neg \forall x P(x)$

Solution $\neg(P(1) \wedge P(2) \wedge P(3) \wedge P(4) \wedge P(5))$

e $\forall x((x \neq 3) \rightarrow P(x)) \vee \exists x \neg P(x)$

Solution $P(1) \wedge P(2) \wedge P(4) \wedge P(5) \vee (\neg(P(1) \vee P(2) \vee P(4) \vee P(5)))$

23 Translate in two ways each of these statements into logical expressions using predicates, quantifiers, and logical connectives. First, let the domain consist of the students in your class and second, let it consist of all people, Where S(x) = x is in your class

a Someone in your class can speak Hindi.

Solution Domain: Students in your class
Let H(x) = x can speak Hindi
$\exists x H(x)$
Domain: All people
$\exists x(H(x) \wedge S(x))$

b Everyone in your class is friendly.

Solution Domain: Students in your class
Let F(x) = x is friendly
$\forall x F(x)$
Domain: All people
$\forall x(F(x) \wedge S(x))$

c There is a person in your class who was not born in California.

Solution Domain: Students in your class
Let C(x) = x is born in California
$\exists x \neg C(x)$
Domain: All people
$\exists x(S(x) \wedge \neg(C(x)))$

d A student in your class has been in a movie.

Solution Domain: Students in your class
Let M(x) = x has been in a movie
$\exists x M(x)$
Domain: All people
$\exists x(M(x) \wedge S(x))$

e No student in your class has taken a course in logic programming.

Solution Domain: Students in your class
Let L(x) = x has taken a course in logic programming
$\neg \forall x L(x)$
Domain: All people
$\neg \forall x(L(x) \wedge S(x))$

27 Translate each of these statements into logical expressions in three different ways by varying the domain and by using predicates with one and with two variables.

a A student in your school has lived in Vietnam.

Domain: Students in your school

Solution Let V(x) = x has lived in Vietnam
$\exists x V(x)$

Domain: All people

Solution Let S(x) = x goes to your school
Let V(x) = x has lived in Vietnam
$\exists x(V(x) \wedge S(x))$

Domain: All people

Solution Let S(x, y) = x goes to y
Let V(x, z) = x has lived in z
$\exists x(V(x, Vietnam) \wedge S(x, your\ school))$

b There is a student in your school who cannot speak Hindi.

Domain: Students in your school

Solution Let H(x) = x can speak Hindi
$\exists x \neg H(x)$

Domain: All people

Solution Let S(x) = x goes to your school
Let H(x) = x can speak Hindi
$\exists x(\neg H(x) \wedge S(x))$

Domain: All people

Solution Let S(x, y) = x goes to y
Let H(x, z) = x can speak z
$\exists x(\neg H(x, Hindi) \wedge S(x, your\ school))$

c A student in your school knows Java, Prolog, and C++.

Domain: Students in your school

Solution
Let J(x) = x knows Java
Let P(x) = x knows Prolog
Let C(x) = x knows C++
$\exists x(J(x) \wedge P(x) \wedge C(x))$

Domain: Students in your school

Solution Let L(w, x, y, z) = w knows x, y, and z
$\exists w L(w, Java, Prolog, C++)$

Domain: All people

Solution
Let S(x) = x goes to your school
Let J(x) = x knows Java
Let P(x) = x knows Prolog
Let C(x) = x knows C++
$\exists x(S(x) \wedge J(x) \wedge P(x) \wedge C(x))$

d Everyone in your class enjoys Thai food

Domain: Students in your class

Solution Let T(x) = x enjoys Thai food
$\forall x T(x)$

Domain: All people

Solution Let S(x) = x goes to your class
Let T(x) = x enjoys Thai food
$\forall x(T(x) \wedge S(x))$

Domain: Students in your class

Solution Let F(x, y) = x enjoys y food
$\forall x F(x, Thai)$

e Someone in your class does not play hockey

Domain: Students in your class

Solution Let H(x) = x plays hockey
$\exists x \neg T(x)$

Domain: All people

Solution Let S(x) = x goes to your class
Let H(x) = x plays hockey
$\exists x(\neg H(x) \wedge S(x))$

Domain: Students in your class

Solution Let H(x, y) = x plays y
$\exists x \neg H(x, hockey)$

30 Suppose the domain of the propositional function P(x, y) consists of pairs x and y, where x is 1, 2, or 3 and y is 1, 2, or 3. Write out these propositions using disjunctions and conjunctions.

a $\exists x P(x, 3)$

Solution $P(1,3) \vee P(2,3) \vee P(3,3)$

b $\forall y P(1, y)$

Solution $P(1,1) \wedge P(1,2) \wedge P(1,3)$

c $\exists y \neg P(2, y)$

Solution $\neg P(2,1) \wedge \neg P(1,2) \wedge \neg P(1,3)$

d $\forall x \neg P(x, 2)$

Solution $\neg P(1,2) \vee \neg P(2,2) \vee \neg P(3,2)$

33 Express each of these statements using quantifiers. Then form the negation of the statement, so that no negation is to the left of a quantifier. Next, express the negation in simple English. (Do not simply use the phrase ”It is not the case that.”)

a Some old dogs can learn new tricks

Solution Domain: All old dogs
Let t(x) = x can learn new tricks
$\exists x t(x)$
Negation: $\forall x \neg t(x)$
”No old dogs can learn new tricks”

b No rabbit knows calculus

Solution Domain: All rabits
Let c(x) = x knows calculus
$\forall x \neg c(x)$
Negation: $\exists x c(x)$
”There exists a rabit that knows calculus”

c Every bird can fly.

Solution Domain: All birds
Let f(x) = x can fly
$\forall x f(x)$
Negation: $\exists x \neg f(x)$
”Some birds cannot fly”

d There is no dog that can talk.

Solution Domain: All dogs
Let t(x) = x can fly
$\forall x \neg t(x)$
Negation: $\exists x t(x)$
”There exists a dog that can talk”

d There is no one in this class who knows French and Russian.

Solution Domain: All people in this class
Let f(x) = x knows French
Let r(x) = x knows Russian
$\forall x \neg (f(x) \wedge r(x))$
Negation: $\exists x (f(x) \vee r(x))$
”Someone in this class knows French or Russian”

36 Find a counterexample, if possible, to these universally quantified statements, where the domain for all variables consists of all real numbers.

a $\forall x(x^2 \neq x)$

Solution 0 or 1

b $\forall x(x^2 \neq 2)$

Solution Anything except for $\sqrt{2}$

c $\forall x(|x| > 0)$

Solution 0

39 Translate these specifications into English where F(p) is "Printer p is out of service," B(p) is "Printer p is busy," L(j) is "Print job j is lost," and Q(j) is "Print job j is queued."

a $\exists p(F(p) \wedge B(p)) \rightarrow \exists j L(j)$

Solution If there is a printer that is out of service and busy then there is a job that is lost

b $\forall p B(p) \rightarrow \exists j Q(j)$

Solution If all printers are busy then there is a job in the queue

c $\exists j(Q(j) \wedge L(j)) \rightarrow \exists p F(p)$

Solution If a job is queued and lost then a printer is out of service

d $(\forall p B(p) \wedge \forall j Q(j)) \rightarrow \exists j L(j)$

Solution If all printers are busy and all jobs are queued then there exists a job that was lost

44 Determine whether $\forall x(P(x) \leftrightarrow Q(x))$ and $\forall x P(x) \leftrightarrow \forall x Q(x)$ are logically equivalent. Justify your answer.

Solution These statements are logically equivalent because they use the same x for all x's for both P(x) and Q(x).

45 Show that $\exists x(P(x) \vee Q(x))$ and $\exists x P(x) \vee \exists x Q(x)$ are logically equivalent.

Solution Both statements are true for any x where P(x) is true or any x where Q(x) is true

51 Show that $\exists x P(x) \wedge \exists x Q(x)$ and $\exists x(P(x) \wedge Q(x))$ are not logically equivalent.

Solution The first statement says that there exists an x that satisfies P(x) and an x that satisfies Q(x). The second statement states that there exists an x that satisfies both P(x) and Q(x).

61 Let $P(x), Q(x), R(x)$, and $S(x)$ be the statements "x is a baby," "x is logical," "x is able to manage a crocodile," and "x is despised," respectively. Suppose that the domain consists of all people. Express each of these statements using quantifiers; logical connectives; and $P(x), Q(x), R(x)$, and $S(x)$.

a Babies are illogical.

Solution $\forall x(P(x) \rightarrow Q(x))$

b Nobody is despised who can manage a crocodile.

Solution $\forall x \neg(R(x) \wedge S(x))$

c Illogical persons are despised.

Solution $\forall Q(x) \rightarrow S(x)$

d Babies cannot manage crocodiles.

Solution $\forall P(x) \rightarrow \neg R(x)$

62 Let $P(x), Q(x), R(x)$, and $S(x)$ be the statements "x is a duck," "x is one of my poultry," "x is an officer," and "x is willing to waltz," respectively. Express each ofthese statements using quantifiers; logical connectives; and $P(x), Q(x), R(x)$, and $S(x)$.

a No ducks are willing to waltz.

Solution $\forall x(P(x) \rightarrow \neg S(x))$

b No officers ever decline to waltz.

Solution $\forall x(R(x) \wedge S(x))$

c All my poultry are ducks.

Solution $\forall Q(x) \wedge R(x)$

d My poultry are not officers.

Solution $\forall Q(x) \rightarrow \neg R(x)$

e Does (d) follow from (a), (b), and (c)? If not, is there a correct conclusion?

Solution No, the correct conclusion is "All my poultry are not willing to waltz"