2c Consider the function $f(x,y) = 2\sin(2x-3y)$, What is the maximum rate of change at $(0,\pi)$ and in what direction does it occur?

$$\begin{aligned}
\nabla f(x,y) &= \langle 4\cos(2x-3y), -6\cos(2x-3y)\rangle \\
\nabla f(0,\pi) &= \langle 4\cos(-3\pi), -6\cos(-3\pi)\rangle \\
\nabla f(0,\pi) &= \langle -4, 6\rangle \\
|\nabla f(0,\pi)| &= \sqrt{(-4)^2+6^2} \\
|\nabla f(0,\pi)| &= 2\sqrt{13}
\end{aligned}$$

Gradient of $f(x,y)$

Maximum rate of change occurs at the direction of $\nabla f(0,\pi)$

Maximum rate of change is $|\nabla f(0,\pi)|$

5 Evaluate $\int_C (xz - y^2)ds$ where C is the line segment from $(0, 1, 2)$ to $(-3, 7, -1)$

$x(t) = -3t$	Reparameterize line segment
$y(t) = 6t + 1$	
$z(t) = -3t + 2$	
$\frac{dx}{dt} = -3$	Derivatives with respect to t
$\frac{dy}{dt} = 6$	
$\frac{dz}{dt} = -3$	
$A = \int_C f(x, y, z)ds$	Line integral with respect to arc length
$ds = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2}dt$	Arc length reparameterized in terms of t
$A = \int_C f(x(t), y(t), z(t))\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2}dt$	Line integral with respect to t
$A = \int_0^1 (-3t)(-3t + 2) - (6t + 1)^2\sqrt{(-3)^2 + (6)^2 + (-3)^2}dt$	Substitute
$A = \int_0^1 \left((-3t)(-3t + 2) - (6t + 1)^2\right)(3\sqrt{6})dt$	Integrate
$A = \int_0^1 (-27t^2 - 18t - 1)(3\sqrt{6})dt$	
$A = 3\sqrt{6}\int_0^1 -27t^2 - 18t - 1dt$	
$A = 3\sqrt{6}(-9t^3 - 9t^2 - t)\vert_0^1$	
$A = -57\sqrt{6}$	

6 Find the absolute maximum and minimum values of $f(x,y) = x^4 + y^4 - 4xy + 2$ on the set $D = \{(x,y)|0 \le x \le 3, 0 \le y \le 3\}$.

$D = \{(x,y)\|0 \le x \le 3, 0 \le y \le 3\}$	Region is rectangular
$\frac{\partial f}{\partial x} = 4x^3 - 4y$	1st partial derivative with respect to x
$\frac{\partial f}{\partial y} = 4y^3 - 4x$	1st partial derivative with respect to y
$0 = 4x^3 - 4y$	Set partials equal to 0 and evaluate system
$0 = 4y^3 - 4x$	
$x = y^3$	Solve for x
$0 = 4(y^3)^3 - 4y$	Substitute
$0 = 4y^9 - 4y$	Simplify
$0 = 4y(y^8 - 1)$	Factor
$0 = 4y(y^2+1)(y^2-1)$	Difference of 2 squares
$0 = 4y(y^2+1)(y+1)(y-1)$	Difference of 2 squares
$y = 0, 1, -1$	Real roots
$0 = 4(0)^3 - 4x$	Let y = 0
$x = 0$	
$0 = 4(1)^3 - 4x$	Let y = 1
$x = 1$	
$0 = 4(-1)^3 - 4x$	Let y = -1
$x = -1$	
$(x,y) = (0,0), (1,1), (-1,-1)$	Critical Points
$f(0,0) = 2$	
$f(1,1) = 0$	
$f(-1,-1) = 0$	$(-1,-1)$ out of bounds
$x = 0$	Left edge of region
$f(0,y) = y^4 + 2$	Let x = 0
$f'(0,y) = 4y^3$	Differentiate with respect to y
$0 = 4y^3$	Set to 0 and solve
$y = 0$	
$f(0,0) = 2$	Bottom left corner, critical point (Local min)
$f(0,2) = 18$	Top left corner (Local max)
$x = 3$	Right edge of region
$f(3,y) = 81 + y^4 - 12y + 2$	Let x = 3
$f'(3,y) = 4y^3 - 12$	Differentiate with respect to y
$0 = 4y^3 - 12$	Set to 0 and solve
$y = \sqrt[3]{3}$	

6 (cont.)

$f(3, \sqrt[3]{3}) \approx 70.019$	Critical point (Local min)
$f(3, 0) = 83$	Bottom right corner (Local max)
$f(3, 2) = 75$	Top right corner
$y = 0$	Bottom edge of region
$f(x, 0) = x^4 + 2$	Let y = 0
$f'(x, 0) = 4x^3$	Differentiate with respect to x
$0 = 4x^3$	Set to 0 and solve
$x = 0$	
$f(0, 0) = 2$	Bottom left corner, critical point (Local min)
$f(0, 2) = 18$	Top left corner (Local max)
$y = 2$	Top edge of region
$f(x, 2) = x^4 + 16 - 8x + 2$	Let y = 2
$f'(x, 2) = 4x^3 - 8$	Differentiate with respect to x
$0 = 4x^3 - 8$	Set to 0 and solve
$x = \sqrt[3]{2}$	
$f(\sqrt[3]{2}, 2) \approx 10.44$	Critical point (Local min)
$f(3, 0) = 83$	Absolute maximum
$f(1, 1) = 0$	Absolute minimum

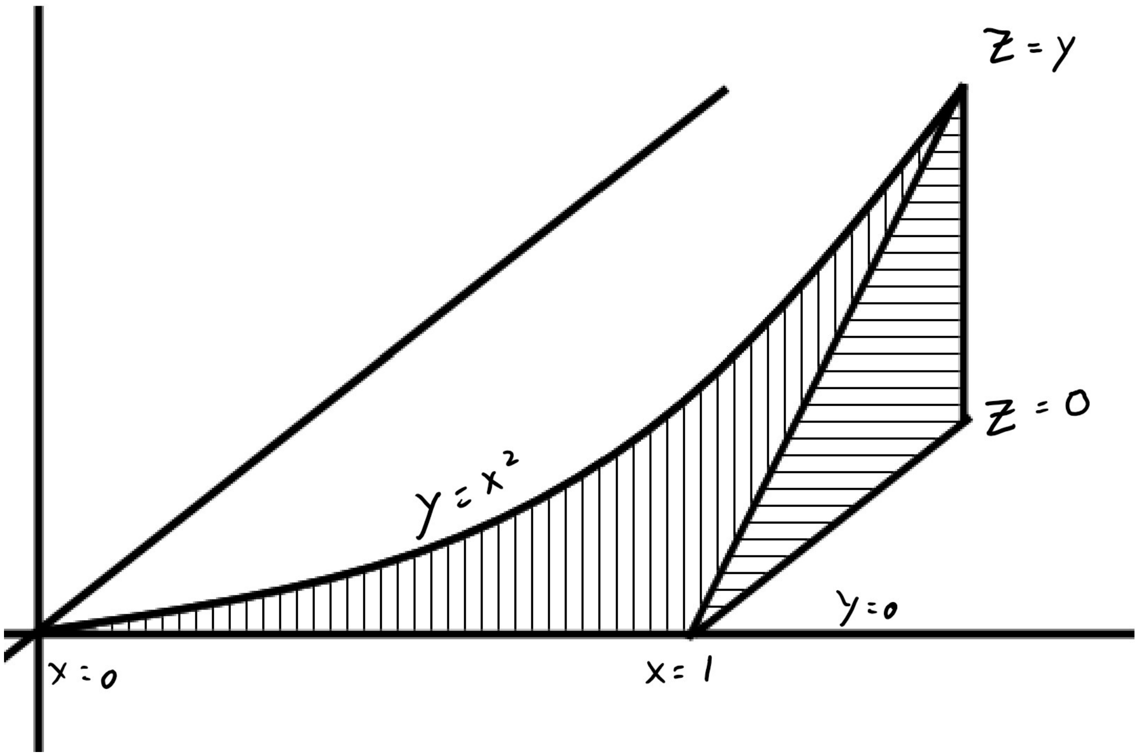
8 Use the method of Lagrange multipliers to find the maximum and minimum values and their locations of $f(x, y) = xy$ subject to the constraint $x^2 + y^2 - xy = 9$

$\nabla f(x,y) = \langle y, x\rangle$	Gradient of objective function
$\nabla g(x,y) = \langle 2x-y, 2y-x\rangle$	Gradient of constraint function
$\nabla f(x,y) = \lambda \nabla g(x,y)$	Lagrange multiplier formula
$\langle y, x\rangle = \lambda\langle 2x-y, 2y-x\rangle$	
$E1: y = \lambda(2x-y)$	Set up a system of equations
$E2: x = \lambda(2y-x)$	
$E3: 9 = x^2+y^2-xy$	
$E1: xy = \lambda(2x^2-xy)$	Multiply both sides of E1 by x
$E2: xy = \lambda(2y^2-xy)$	Multiply both sides of E2 by y
$\lambda(2x^2-xy) = \lambda(2y^2-xy)$	$\therefore$ E1 = E2
$x^2 = y^2$	
$x = y,\ x = -y$	
$9 = x^2+x^2-x^2$	Substitute x = y into E3
$\pm 3 = x$	
$9 = 9+y^2-3y$	Substitute x = 3 into E3
$0 = y(y-3)$	
$y = 0,\ y = 3$	
$(3,-3), (-3,3)$	Local Extrema
$f(3,-3) = f(-3,3) = -9$	
$Let\ x = 1$	Find a point within the constraint to determine if extrema are
$9 = 1+y^2-y$	minimum or maximum
$y \approx -2.37,\ y \approx 3.37$	
$f(2.37,-2.37) > -9$	$\therefore f(3,-3)$ and $f(-3,3)$ are local minimums

9 Express the iterated integral $\int_0^1 \int_0^{x^2} \int_0^y f(x,y,z)dzdydx$ as a triple integral in two different orders: $dxdzdy$ and $dydzdx$

	Express as $dxdzdy$
$x = \sqrt{y}$	Rewrite x bounds in terms of y
$0 \leq x \leq \sqrt{y}$	
$z = y$	z bounds already in terms of y
$0 \leq z \leq y$	
$y = 1^2$	Calculate the intersection between $y = x^2$ and $x = 1$
$0 \leq y \leq 1$	
$\int_0^1 \int_0^y \int_0^{\sqrt{y}} f(x,y,z)dxdzdy$	Rewrite integral
	Express as $dydzdx$
$y = x^2$	y bounds already in terms of x
$0 \leq y \leq x^2$	
$z = y = x^2$	Rewrite z bounds in terms of x
$z = x^2$	
$0 \leq z \leq x^2$	
$\int_0^1 \int_0^{x^2} \int_0^{x^2} f(x,y,z)dydzdx$	Rewrite integral

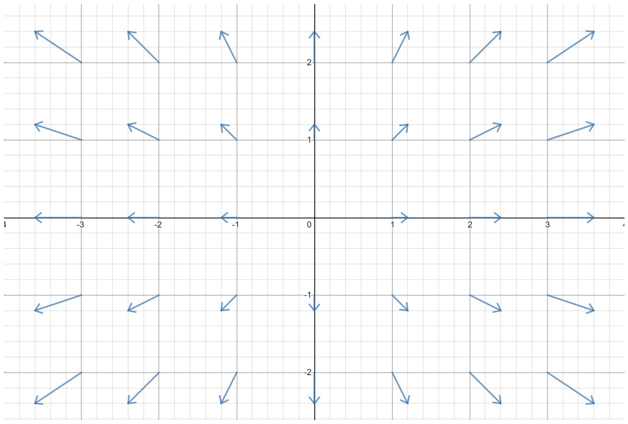
9 Drawing of region for reference



10b Given $f(x,y) = x^2 + y^2$, graph the gradient vector field

$\nabla f(x,y) = \langle 2x, 2y \rangle$

Calculate the gradient of the function



Graph vectors at points