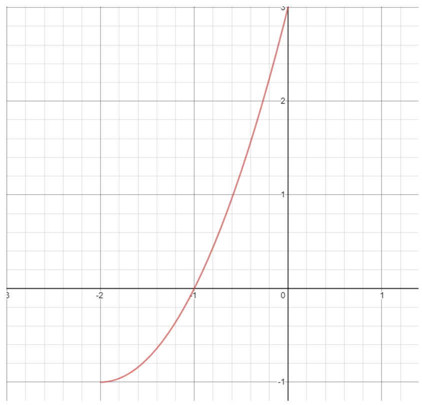
1 Reparametrize the curve $\vec{r}(t) = (4\cos t)\vec{i} + (4\sin t)\vec{j} + 3t\vec{k}$ with respect to the arc length measured from $(4, 0, 0)$ in the direction of increasing t.

$\vec{r}(0) = \langle 4\cos 0, 4\sin 0, 3(0)\rangle = \langle 4, 0, 0\rangle$	Let t = 0
$\vec{r}\,'(t) = \langle -4\sin t, 4\cos t, 3\rangle$	Derivative of $\vec{r}(t)$
$s(t) = \int_a^b \sqrt{[f'(t)]^2 + [g'(t)]^2 + [h'(t)]^2}\, dt$	Arc length formula
$s(t) = \int_0^t \sqrt{(-4\sin t)^2 + (4\cos t)^2 + 3^2} dt$	Substitute the components of $\vec{r}\,'(t)$
$s(t) = \int_0^t \sqrt{25} dt$	Evaluate the integral
$s(t) = \sqrt{25}\Big\vert_0^t$	
$s(t) = \sqrt{25}(t-0)$	
$t(s) = \dfrac{s}{5}$	t in terms of arc length
$\vec{r}(t(s)) = 4\cos\dfrac{s}{5}\vec{i} + 4\sin\dfrac{s}{5}\vec{j} + \dfrac{3s}{5}\vec{k}$	Curve reparametrized in terms of arc length

2c Given the plane curve $\vec{r}(t) = \langle t-2, t^2-1 \rangle$
Sketch the position vector $\vec{r}(t)$ and the tangent vector $\vec{r}\,'(t)$ at $t = 1$

t	x	y
0	-2	-1
1	-1	0
2	0	3
3	1	8
4	2	15
5	3	24

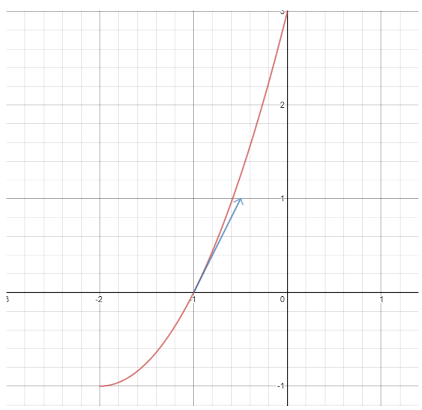
Table of values



Graph of $\vec{r}(t)$

$\vec{r}\,'(t) = \langle 1, 2t \rangle$ Take the derivative of $\vec{r}(t)$

$\vec{r}\,'(1) = \langle 1, 2 \rangle$ Substitute $t = 1$



Graph $\vec{r}\,'(1)$ at $t = 1$

3a Given $9y^2 + 4z^2 = x^2 + 36$

a. Rewrite the equation in standard form

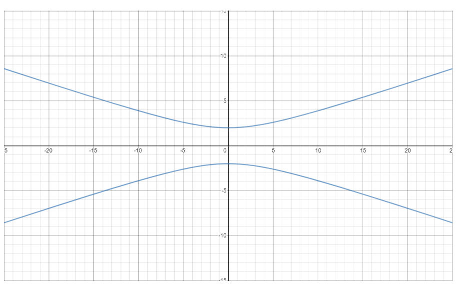
$$9y^2 + 4z^2 = x^2 + 36 \quad \text{Original equation}$$
$$9y^2 + 4z^2 - x^2 = 36 \quad \text{Subtract } x^2 \text{ on both sides}$$
$$\frac{y^2}{4} + \frac{z^2}{9} - \frac{x^2}{36} = 1 \quad \text{Divide by 36 on both sides}$$
$$-\frac{x^2}{36} + \frac{y^2}{4} + \frac{z^2}{9} = 1 \quad \text{Rearrange terms}$$
$$-\frac{x^2}{6^2} + \frac{y^2}{2^2} + \frac{z^2}{3^2} = 1 \quad \text{Rewrite denominators}$$

b. Identify the surface

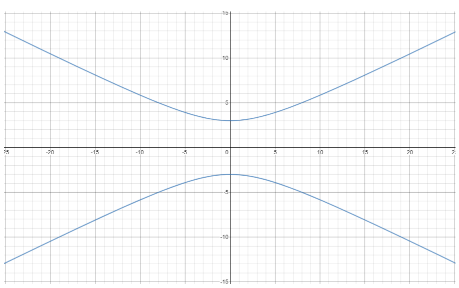
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1 \quad \text{Standard form of a hyperboloid of 1 sheet}$$
$$-\frac{x^2}{6^2} + \frac{y^2}{2^2} + \frac{z^2}{3^2} = 1 \quad \text{Equation matches the form}$$

Hyperboloid of 1 sheet

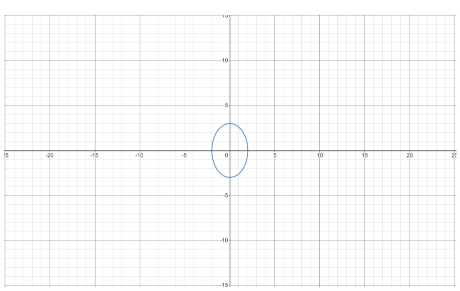
c. Draw each trace for the coordinate planes and discribe the conic section. Label and scale your axes.



xy axis, Let $z = 0$, Hyperbola

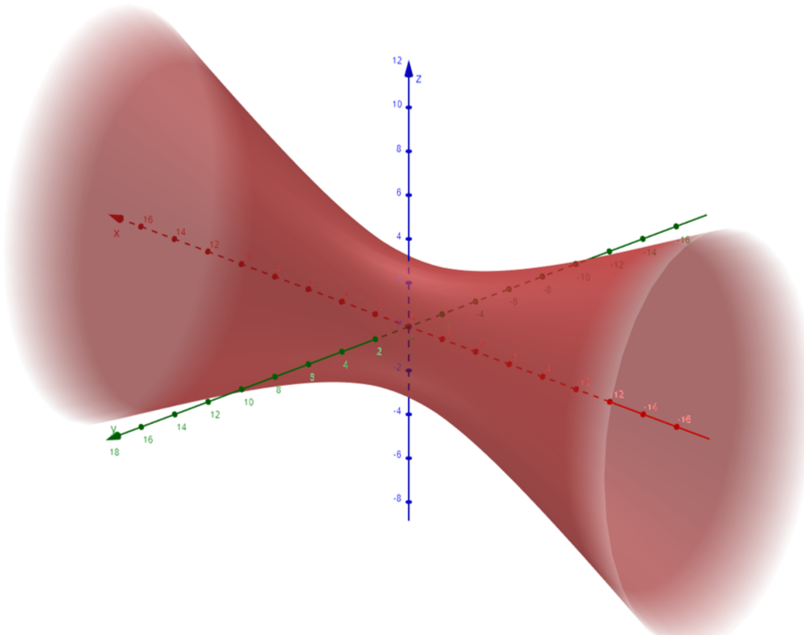


xz axis, Let $y = 0$, Hyperbola



yz axis, Let $x = 0$, Ellipse

d. Graph $9y^2 + 4z^2 = x^2 + 36$. Label and scale the axes.



4 Given $\vec{a}, \vec{b} \in \mathbb{R}^3$, show that $\vec{a} \times \vec{b}$ is orthogonal to $\vec{a}$ and $\vec{b}$.

$\vec{a} \cdot \vec{b} = \|\vec{a}\|\|\vec{b}\| \cos(\theta)$	Angle between 2 vectors
$\vec{a} \cdot \vec{b} = \|\vec{a}\|\|\vec{b}\| \cos(\frac{\pi}{2})$	Let $\theta = \frac{\pi}{2}$
$\vec{a} \cdot \vec{b} = \|\vec{a}\|\|\vec{b}\| * 0$	
$\vec{a} \cdot \vec{b} = 0$	$\therefore$ When $\theta = \frac{\pi}{2}$, the dot product is equal to 0
$Let\ \vec{a} = \langle a_1, a_2, a_3 \rangle$	Given
$Let\ \vec{b} = \langle b_1, b_2, b_3 \rangle$	
$\vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$	Take the cross product
$\vec{a} \times \vec{b} = \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} \vec{i} - \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} \vec{j} + \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} \vec{k}$	
$\vec{a} \times \vec{b} = \langle a_2b_3 - a_3b_2, -a_1b_3 + a_3b_1, a_1b_2 - a_2b_1 \rangle$	
$\langle a_1, a_2, a_3 \rangle \cdot \langle a_2b_3 - a_3b_2, -a_1b_3 + a_3b_1, a_1b_2 - a_2b_1 \rangle$	Take the dot product with $\vec{a}$
$= a_1(a_2b_3 - a_3b_2) + a_2(-a_1b_3 + a_3b_1) + a_3(a_1b_2 - a_2b_1)$	
$= a_1a_2b_3 - a_1a_3b_2 - a_2a_1b_3 + a_2a_3b_1 + a_3a_1b_2 - a_3a_2b_1$	
$= (a_1a_2b_3 - a_1a_2b_3) + (a_1b_2a_3 - a_1b_2a_3) + (b_1a_2a_3 - b_1a_2a_3)$	Rearrange terms
$= 0$	$\therefore$ the angle between $\vec{a}$ and $\vec{a} \times \vec{b} = \frac{\pi}{2}$
$\langle a_1, a_2, a_3 \rangle \cdot \langle a_2b_3 - a_3b_2, -a_1b_3 + a_3b_1, a_1b_2 - a_2b_1 \rangle$	Take the dot product with $\vec{b}$
$= b_1(a_2b_3 - a_3b_2) + b_2(-a_1b_3 + a_3b_1) + b_3(a_1b_2 - a_2b_1)$	
$= b_1a_2b_3 - b_1a_3b_2 - b_2a_1b_3 + b_2a_3b_1 + b_3a_1b_2 - b_3a_2b_1$	
$= (b_1a_2b_3 - b_1a_2b_3) + (a_1b_2b_3 - a_1b_2b_3) + (b_1b_2a_3 - b_1b_2a_3)$	Rearrange terms
$= 0$	$\therefore$ the angle between $\vec{b}$ and $\vec{a} \times \vec{b} = \frac{\pi}{2}$

We know the angle between $\vec{a} \times \vec{b}$ and $\vec{a}$ is $\frac{\pi}{2}$ and the angle between $\vec{a} \times \vec{b}$ and $\vec{b}$ is $\frac{\pi}{2}$

$\therefore \vec{a} \times \vec{b}$ is orthogonal to $\vec{a}$ and $\vec{b}$ ■

6e Given $\vec{b} = \langle 0, 4, 5\rangle$ and $\vec{c} = \langle -1, 3, -2\rangle$, find $proj_{\vec{b}}\vec{c}$.

$proj_{\vec{u}}\vec{v} = \left(\dfrac{\vec{u}\cdot\vec{v}}{\lvert\vec{u}\rvert}\right)\left(\dfrac{\vec{u}}{\lvert\vec{u}\rvert}\right)$	Vector projection of $\vec{v}$ onto $\vec{u}$
$proj_{\vec{b}}\vec{c} = \left(\dfrac{\langle 0,4,5\rangle\cdot\langle -1,3,-2\rangle}{\lvert\langle 0,4,5\rangle\rvert}\right)\left(\dfrac{\langle 0,4,5\rangle}{\lvert\langle 0,4,5\rangle\rvert}\right)$	Substitute
$\langle 0,4,5\rangle\cdot\langle -1,3,-2\rangle = 2$	Dot Product of $\vec{b}$ and $\vec{c}$
$\sqrt{0^2+4^2+5^2} = \sqrt{41}$	Magnitude of $\vec{b}$
$proj_{\vec{b}}\vec{c} = \left(\dfrac{2}{\sqrt{41}}\right)\left(\dfrac{\langle 0,4,5\rangle}{\sqrt{41}}\right)$	Substitute
$proj_{\vec{b}}\vec{c} = \left\langle 0, \dfrac{8}{41}, \dfrac{10}{41}\right\rangle$	Vector projection of $\vec{c}$ onto $\vec{b}$

7 Find the plane that goes through the points $(3, 0, -4)$, $(-2, -2, 5)$, and $(-1, 8, -3)$.

$$P = (3, 0, -4) \qquad \text{Define points}$$
$$Q = (-2, -2, 5)$$
$$R = (-1, 8, -3)$$

$$\overrightarrow{PQ} = \langle -5, -2, 9 \rangle \qquad \text{Vector through points}$$
$$\overrightarrow{PR} = \langle -4, 8, 1 \rangle$$

$$\overrightarrow{PQ} \times \overrightarrow{PR} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -5 & -2 & 9 \\ -4 & 8 & 1 \end{vmatrix} \qquad \text{Take the cross product}$$
$$\overrightarrow{PQ} \times \overrightarrow{PR} = \begin{vmatrix} -2 & 9 \\ 8 & 1 \end{vmatrix} \vec{i} - \begin{vmatrix} -5 & 9 \\ -4 & 1 \end{vmatrix} \vec{j} + \begin{vmatrix} -5 & -2 \\ -4 & 8 \end{vmatrix} \vec{k}$$

$$\overrightarrow{PQ} \times \overrightarrow{PR} = \langle -74, -31, -48 \rangle \qquad \text{Normal vector}$$
$$P = (3, 0, -4) \qquad \text{Point on plane}$$

$$0 = \vec{n} \cdot (r - r_0) \qquad \text{Equation of a plane}$$
$\vec{n}$: the normal vector
r_0: the point on the plane

$$0 = \langle -74, -31, -48 \rangle \cdot (\langle x, y, z \rangle - \langle 3, 0, 5 \rangle) \qquad \text{Equation of the plane}$$
$$0 = -74(x - 3) - 31y - 48(z - 4) \qquad \text{Simplified to scalar notation}$$

8 Find the curvature of $\vec{r}(t) = \langle 0, t^3, t^2 \rangle$

$$\vec{r}\,'(t) = \langle 0, 3t^2, 2t \rangle \qquad \text{Derivative of } \vec{r}$$

$$\vec{r}\,''(t) = \langle 0, 6t, 2 \rangle \qquad \text{2nd derivative of } \vec{r}$$

$$|\vec{r}\,'(t)| = \sqrt{(0)^2 + (3t^2)^2 + (2t)^2} \qquad \text{Magnitude of } \vec{r}\,'(t)$$

$$|\vec{r}\,'(t)| = \sqrt{0 + 9t^4 + 4t^2}$$

$$|\vec{r}\,'(t)| = \sqrt{t^2(9t^2 + 4)}$$

$$|\vec{r}\,'(t)| = t\sqrt{9t^2 + 4}$$

$$\kappa(t) = \frac{|\vec{r}\,'(t) \times \vec{r}\,''(t)|}{|\vec{r}\,'(t)|^3} \qquad \text{Curvature formula}$$

$$\kappa(t) = \frac{|\langle 0, 3t^2, 2t \rangle \times \langle 0, 6t, 2 \rangle|}{\left(t\sqrt{9t^2 + 4}\right)^3} \qquad \text{Evaluate curvature}$$

$$\langle 0, 3t^2, 2t \rangle \times \langle 0, 6t, 2 \rangle = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 3t^2 & 2t \\ 0 & 6t & 2 \end{vmatrix} \qquad \text{Cross Product}$$

$$\langle 0, 3t^2, 2t \rangle \times \langle 0, 6t, 2 \rangle = \begin{vmatrix} 3t^2 & 2t \\ 6t & 2 \end{vmatrix} \vec{i} - \begin{vmatrix} 0 & 2t \\ 0 & 2 \end{vmatrix} \vec{j} + \begin{vmatrix} 0 & 3t^2 \\ 0 & 6t \end{vmatrix} \vec{k}$$

$$\langle 0, 3t^2, 2t \rangle \times \langle 0, 6t, 2 \rangle = \langle 6t^2 - 12t^2, 0, 0 \rangle$$

$$\left|\langle 6t^2 - 12t^2, 0, 0 \rangle\right| = \sqrt{(6t^2 - 12t^2)^2} = \sqrt{\left(t^2(6 - 12)\right)^2} = \sqrt{\left(t^2(-6)\right)^2} \qquad \text{Magnitude}$$

$$\left|\langle 6t^2 - 12t^2, 0, 0 \rangle\right| = 6t^2$$

$$\kappa(t) = \frac{6t^2}{t^3(9t^2 + 4)^{\frac{3}{2}}} \qquad \text{Curvature of } \vec{r}(t)$$

$$\kappa(t) = \frac{6}{t(9t^2 + 4)^{\frac{3}{2}}} \qquad \text{Simplify}$$