1 Use truth tables to verify these equivalences

a $p \wedge T \equiv p$

Solution

p	T	$p \wedge T$
T	T	T
F	T	F

b $p \vee F \equiv p$

Solution

p	F	$p \vee F$
T	F	T
F	F	F

c $p \wedge F \equiv F$

Solution

p	F	$p \wedge F$
T	F	F
F	F	F

d $p \vee T \equiv T$

Solution

p	F	$p \vee T$
T	T	T
F	T	T

e $p \vee p \equiv p$

Solution

p	$p \vee p$
T	T
F	F

f $p \wedge p \equiv p$

Solution

p	$p \wedge p$
T	T
F	F

6 Use a truth table to verify the first De Morgan law
$\neg(p \wedge q) \equiv \neg p \vee \neg q$

Solution

p	q	$p \wedge q$	$\neg(p \wedge q)$	$\neg p$	$\neg q$	$\neg p \vee \neg q$
T	T	T	F	F	F	F
T	F	F	T	F	T	T
F	T	F	T	T	F	T
F	F	F	T	T	T	T

7 Use De Morgan's laws to find the negation of each of the following statements.

a Jan is rich and happy.

Solution
Let p = Jan is rich
Let q = Jan is happy

$p \wedge q =$ Jan is rich and happy
$\neg(p \wedge q) = \neg p \vee \neg q =$ Jan is not rich or happy

b Carlos will bicycle or run tomorrow.

Solution
Let p = Carlos will bicycle tomorrow
Let q = Carlos will run tomorrow

$p \vee q =$ Carlos will bicycle or run tomorrow
$\neg(p \vee q) = \neg p \wedge \neg q =$ Carlos will not bicycle and run tomorrow

c Mei walks or takes the bus to class

Solution
Let p = Mei walks to class
Let q = Mei takes the bus to class

$p \vee q =$ Mei walks or takes the bus to class
$\neg(p \vee q) = \neg p \wedge \neg q =$ Mei does not walk and take the bus to class

d Ibrahim is smart and hard working.

Solution
Let p = Ibrahim is smart
Let q = Ibrahim is hard working

$p \wedge q =$ Ibrahim is smart and hard working
$\neg(p \wedge q) = \neg p \vee \neg q =$ Ibrahim is not smart or hard working

9 Show that each of these conditional statements is a tautology by using truth tables.

a $(p \wedge q) \to p$

Solution

p	q	$p \wedge q$	$(p \wedge q) \to p$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	T

b $p \to (p \vee q)$

Solution

p	q	$p \vee q$	$p \to (p \vee q)$
T	T	T	T
T	F	T	T
F	T	T	T
F	F	F	T

c $\neg p \to (p \to q)$

Solution

p	q	$\neg p$	$p \to q$	$\neg p \to (p \to q)$
T	T	F	T	T
T	F	F	F	T
F	T	T	T	T
F	F	T	T	T

d $(p \wedge q) \to (p \to q)$

Solution

p	q	$p \wedge q$	$p \to q$	$(p \wedge q) \to (p \to q)$
T	T	T	T	T
T	F	F	F	T
F	T	F	T	T
F	F	F	T	T

e $\neg(p \to q) \to p$

Solution

p	q	$p \to q$	$\neg(p \to q)$	$\neg(p \to q) \to p$
T	T	T	F	T
T	F	F	T	T
F	T	T	F	T
F	F	T	F	T

f $\neg(p \to q) \to \neg q$

Solution

p	q	$p \to q$	$\neg(p \to q)$	$\neg(p \to q) \to q$
T	T	T	F	T
T	F	F	T	T
F	T	T	F	T
F	F	T	F	T

11 Show that each of these conditional statements is a tautology by using truth tables.

a $(p \wedge q) \to p$

$(p \wedge q) \to p$	Given
$\neg(p \wedge q) \vee p$	Table 7
$\neg p \vee \neg q \vee p$	De Morgan's law
$\neg p \vee p \vee \neg q$	Commutative law
$T \vee \neg q$	Idempotent law
T	Domination law

b $p \to (p \vee q)$

$p \to (p \vee q)$	Given
$\neg p \vee (p \vee q)$	Table 7
$(\neg p \vee p) \vee q$	Associative Law
$T \vee q$	Negation Law
T	Domination Law

c $\neg p \to (p \to q)$

$\neg p \to (p \to q)$	Given
$p \vee (\neg p \vee q)$	Table 7
$(p \vee \neg p) \vee q$	Associative Law
$T \vee q$	Negation Law
T	Domination Law

d $(p \wedge q) \to (p \to q)$

$(p \wedge q) \to (p \to q)$	Given
$\neg(p \wedge q) \vee (p \to q)$	Table 7
$\neg p \vee \neg q \vee (p \to q)$	De Morgan's law
$\neg p \vee \neg q \vee (\neg p \vee q)$	Table 7
$\neg p \vee \neg p \vee \neg q \vee q$	Commutative Law
$\neg(p \wedge p) \vee \neg q \vee q$	De Morgan's Law
$\neg p \vee \neg q \vee q$	Idempotent law
$\neg p \vee T$	Negation law
T	Domination law

e $\neg(p \to q) \to p$

$\neg(p \to q) \to p$	Given
$\neg(\neg p \vee q) \to p$	Table 7
$p \wedge \neg q \to p$	De Morgan's law
$\neg(p \wedge \neg q) \vee p$	Table 7
$\neg p \vee q \vee p$	De Morgan's law
$\neg p \vee p \vee q$	Commutative law
$T \vee q$	Negation law
T	Domination law

f $\neg(p \to q) \to \neg q$

$\neg(p \to q) \to \neg q$	Given
$\neg(\neg p \vee q) \to \neg q$	Table 7
$p \wedge \neg q \to \neg q$	De Morgan's law
$\neg(p \wedge \neg q) \vee \neg q$	Table 7
$\neg p \vee q \vee \neg q$	De Morgan's law
$\neg p \vee T$	Negation law
T	Domination law

14 Determine whether $(\neg p \wedge (p \to q)) \to \neg q$ is a tautology

$(\neg p \wedge (p \to q)) \to \neg q$	Given
$(\neg p \wedge (\neg p \vee q)) \to \neg q$	Table 7
$\neg(\neg p \wedge (\neg p \vee q)) \vee \neg q$	Table 7
$\neg((\neg p \wedge \neg p) \vee q) \vee \neg q$	Associative law
$\neg(\neg p \vee q) \vee \neg q$	Idempotent law
$p \wedge \neg q \vee \neg q$	De Morgan's law
$p \wedge (\neg q \vee \neg q)$	Associative law
$p \wedge \neg q$	Idempotent law

$(\neg p \wedge (p \to q)) \to \neg q$ is the logical equivalent of $p \wedge \neg q$, which is not a tautology and is therefore is not a tautology

20 Show that $\neg(p \oplus q)$ and $p \leftrightarrow q$ are logically equivalent

Solution

p	q	$p \oplus q$	$\neg(p \oplus q)$	$p \leftrightarrow q$
T	T	F	T	T
T	F	T	F	F
F	T	T	F	F
F	F	F	T	T

27 Show that $p \leftrightarrow q$ and $(p \to q) \wedge (q \to p)$ are logically equivalent

Solution

p	q	$p \to q$	$q \to p$	$(p \to q) \wedge (q \to p)$	$p \leftrightarrow q$
T	T	T	T	T	T
T	F	F	T	F	F
F	T	T	F	F	F
F	F	T	T	T	T

32 Show that $(p \wedge q) \to r$ and $(p \to r) \wedge (q \to r)$ are not logically equivalent

$$(p \to r) \wedge (q \to r) \qquad \text{Given}$$
$$(p \to r) \wedge (q \to r) \equiv (p \vee q) \to r \qquad \text{Table 7}$$
$$(p \wedge q) \to r \not\equiv (p \vee q) \to r$$

41 Find a compound proposition involving the propositional variables p, q, and r that is true when exactly two of p, q, and r are true and is false otherwise.

Solution $(p \wedge q \wedge \neg r) \vee (p \wedge r \wedge \neg q) \vee (q \wedge r \wedge \neg p)$

58 How many of the disjunctions $p \vee \neg q$, $\neg p \vee q$, $q \vee r$, $q \vee \neg r$, $\neg q \vee \neg r$ can be made simultaneously true by an assignment of truth values to p,q, and r?

Solution Let p, q, r = T
$p \vee \neg q = T$
$\neg p \vee q = T$
$q \vee r = T$
$q \vee \neg r = T$
$\neg q \vee \neg r = F$

58 How many of the disjunctions $p \vee \neg q \vee s$, $\neg p \vee \neg r \vee s$, $\neg p \vee \neg r \vee \neg s$, $\neg p \vee q \vee \neg s$, $q \vee r \vee \neg s$, $q \vee \neg r \vee \neg s$, $\neg p \vee \neg q \vee \neg s$, $p \vee r \vee s$, and $p \vee r \vee \neg s$ can be made simultaneously true by an assignment of truth values to p, q, r, and s?

Solution Let p, q, r, s = T
$p \vee \neg q \vee s = T$
$\neg p \vee \neg r \vee s = T$
$\neg p \vee \neg r \vee \neg s = F$
$\neg p \vee q \vee \neg s =$
$q \vee r \vee \neg s = T$
$q \vee \neg r \vee \neg s = T$
$\neg p \vee \neg q \vee \neg s = F$
$p \vee r \vee s = T$
$p \vee r \vee \neg s = T$

61 Determine whether each of these compound propositions is satisfiable.

a $(p \vee \neg q) \wedge (\neg p \vee q) \wedge (\neg p \vee \neg q)$

Solution
$p \vee \neg q$ can be satisfied when either p = T or q = F
$\neg p \vee q$ can be satisfied when either p = F or q = T
$\neg p \vee \neg q$ can be satisfied when either p = F or q = F
This proposition can be satisfied if both p and q are false.

b $(p \to q) \wedge (p \to \neg q) \wedge (\neg p \to q) \wedge (\neg p \to \neg q)$

Solution

p	q	$\neg p$	$\neg q$	$p \to q$	$p \to \neg q$	$\neg p \to q$	$\neg p \to \neg q$
T	T	F	F	T	F	T	T
T	F	F	T	F	T	T	F
F	T	T	F	T	T	T	T
F	F	T	T	T	T	F	T

This proposition can be satisfied if p = false and q = true

c $(p \leftrightarrow q) \wedge (\neg p \leftrightarrow q)$

Solution This proposition cannot be satisfied because p cannot be true if and only if q while being simultaneously false if and only if q

61 Determine whether each of these compound propositions is satisfiable.

a $(p \vee q \vee \neg r) \wedge (p \vee \neg q \vee \neg s) \wedge (p \vee \neg r \vee \neg s) \wedge (\neg p \vee \neg q \vee \neg s) \wedge (p \vee q \vee \neg s)$

Solution All propositions here are disjunctions and contain at least one negation so it can be satisfied if p, q, r, and s are false

b $(\neg p \vee \neg q \vee r) \wedge (\neg p \vee q \vee \neg s) \wedge (p \vee \neg q \vee \neg s) \wedge (\neg p \vee \neg r \vee \neg s) \wedge (p \vee q \vee \neg r) \wedge (p \vee \neg r \vee \neg s)$

Solution All propositions here are disjunctions and contain at least one negation so it can be satisfied if p, q, r, and s are false

c $(p \vee q \vee r) \wedge (p \vee \neg q \vee \neg s) \wedge (q \vee \neg r \vee s) \wedge (\neg p \vee r \vee s) \wedge (\neg p \vee q \vee \neg s) \wedge (p \vee \neg q \vee \neg r) \wedge (\neg p \vee \neg q \vee s) \wedge (\neg p \vee \neg r \vee \neg s)$

Solution All propositions here contain at least 1 non negated truth value except for $(\neg p \vee \neg r \vee \neg s)$, which requires either p r or s to be false. No other proposition depends on r being true so setting it to false satisfies this proposition.