

MIE 262: Lab Project Report

Mirabel Luan | luanmira | (1010851632)
Javid Wu | wuhungja | (1011314267)

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University of Toronto

1.0 Abstract

Following the bankruptcy of a major competitor, WARP Shoe Company has consulted Industrial Engineering students at the University of Toronto to develop an optimized production plan to maximize their potential profit for February 2006. Using AMPL, the students developed a mixed-integer linear programming model and forecasted WARP's product demand for February 2006 using SQL queries. The model simulates shoe production based on actual shoe prices, forecasted demand, various production costs, and production constraints provided by WARP in addition to a series of assumptions. The students solved the model as an LP relaxation, yielding a maximum optimal profit of \$11,789,734.80. Additional explanation of the optimal solution and sensitivity analysis was performed to gain insights into how the model could be applied to the real world.

2.0 Introduction

The WARP Shoe Company, one of the oldest shoe companies in Canada, has consulted Industrial Engineering students from the University of Toronto to help optimize their profits in the past, reporting helpful and profitable results. One of WARP's major market competitors went bankrupt at the beginning of 2006, leading to WARP market analysts predicting that demand for all WARP shoes will double in the month of February 2006 [1].

The authors of this report were provided with a database [2] consisting of WARP's current and past company and production information, and have been consulted to determine the most profitable production plan for WARP in February 2006 following the bankruptcy.

The management of WARP has provided the following information, in addition to the database:

- The closing inventory of January 2006 was zero for all shoe types,
- All sales are assumed to happen at the end of the month,
- The budget for raw materials is \$10,000,000,
- Sales price on shoes remain consistent to the Product Master Table in the provided database,
- Failure to meet the demand on any type of shoe reflects a loss of potential customers, evaluated at a cost of \$10 per unsold pair,
- All machines at the WARP shoe production plant will work up to 12 hours a day and 28 days a month for the month of February 2006,
- Setup times and costs of machines are assumed to be negligible,
- The time per shoe from each machine is detailed in the Machine_Assign table,
- Workers are paid \$25 per hour and each machine has to be operated by one worker, and
- The transportation and manufacturing sequence can be ignored.

The objective of this project is to formulate and solve a mathematical optimization model that determines WARP's most profitable production plan for February 2006.

3.0 Methodology

To formulate the mathematical model that will maximize WARP's profits, each part of the problem needs to be analyzed.

3.1 Assumptions

3.1.1 Objective Function Assumptions

- Setup times and setup costs are negligible
- Transportation costs and distances between manufacturing plants, warehouses, and stores are negligible
- Repair costs and machine failure are ignored as failure data from the Failure_Data table shows that most (93.82%, 72238/77000) machines are operable beyond the 336 hour limit in February (12 hours * 28 days)
- Because the closing inventory of January 2006 was zero, we can safely assume that the opening inventory of February 2006 to also be zero.
- All sales take place at the end of the month, meaning that earned revenue during the month cannot be used to cover costs in the same timeframe. In addition, no revenue from previous months can be carried over.
- A unit of shoes is considered to be a pair of shoes, thus producing one unit of shoe is considered to be producing a pair of shoes
- Employees are paid based on their assigned machine's operating duration, and thus are paid by the second

3.1.2 Constraint Assumptions

- Manufacturing sequence can be ignored, and thus steps required to manufacture a pair of shoes can be performed simultaneously
- Warehouse capacities are cumulated and checked at the end of the month, and thus individual warehouse capacities are negligible

3.2 Definitions

In the following section we define all our sets, parameters and variables using the given database.

3.2.1 Sets

Table 1. Definition of relevant sets

Set	Name in Database	Description (each element corresponds to a...)
$p = \{1, 2, \dots, 557\}$	Product_Num	Set of all shoe types manufactured by WARP.

$m = \{1, 2, \dots, 72\}$	Machine_Num	Set of all machines used in the WARP production plant.
$r = \{1, 2, \dots, 165\}$	RM_Num	Set of all types of raw materials.
$w = \{1, 2, \dots, 8\}$	Warehouse_Num	Set of warehouse facilities.

3.2.2 Parameters

Table 2. Definition of Parameters

Parameter	Source	Description
P_p	Product_Master	Sale price of shoe p
D_p	Product_Demand	Forecasted demand for shoe type p in February 2006
t_{pm}	Machine_Assign	Time in seconds required to produce one unit of shoe p on machine m
c_m	Machine_Master	Operating cost per minute for machine m
c_r	RM_Master	Cost per raw material r
Q_r	RM_Master	Available quantity of raw material r
q_{pr}	BOM	Quantity of raw material r required for one unit of shoe p
W_{cap}	Warehouse_Master	Storage capacity of warehouse w
c_w	Warehouse_Master	Operational cost of Warehouse w
L_c	Project requirement	Labour cost per hour (constant)
B	Project Requirement	Maximum raw material budget (constant)
s_p	Project Requirement	Penalty per unit of unmet demand for p (constant)
T_m	Derived	Total minutes available on machine m (constant)

3.3.3 Decision Variables

Table 3. Definition of Decision Variables

Variable	Type	Definition
x_p	Integer	Number of units of shoe p produced
u_w	Binary	1 if warehouse location w is used, 0 otherwise

3.3 Objective Function Formulation

The objective function maximizes the profit, formulated as:

$$\max z = \text{profit} = \text{revenue} - \text{cost}.$$

WARP's sole source of revenue is shoe sales, which is determined by the number of pairs of shoes produced multiplied by the shoe's sale price for each respective shoe type,

$$\text{revenue} = \sum_{p=1}^{557} P_p x_p.$$

Warp has five sources of cost, which are detailed below:

1. **Total Machine operating cost**, determined by the sum of operating cost of all machine times used to make each shoe:

$$c_1 = \sum_{p=1}^{557} \sum_{m=1}^{72} \frac{1}{60} t_{pm} c_m x_p.$$

2. **Total Raw material cost**, determined by the sum of the cost of raw materials required for shoe production:

$$c_2 = \sum_{p=1}^{557} \sum_{r=1}^{165} q_{pr} c_r x_p.$$

3. **Total Labour cost**, the sum of hourly salary times the total machine operation duration:

$$c_3 = \sum_{p=1}^{557} \sum_{m=1}^{72} \frac{1}{3600} L_c t_{pm} x_p, \text{ where}$$

$\frac{1}{3600}$ converts t_{pm} from seconds into hour: $(sec)(\frac{1 min}{60 sec})(\frac{1 h}{60 min}) = \frac{1}{3600} h$.

4. **Total Warehouse operation cost**, the sum of the operation cost of all warehouses:

$$c_4 = \sum_{w=1}^8 u_w c_w.$$

5. **Total Penalty for unmet demands**, the sum of the cost of all unmet demands:

$$c_5 = \sum_{p=1}^{557} s_p \max\{0, D_p - x_p\}.$$

After substituting in the equations for revenue and costs, the complete objective function becomes:

$$\begin{aligned}
\max z = & \sum_{p=1}^{557} P_p x_p - \sum_{p=1}^{557} \sum_{m=1}^{72} \frac{1}{60} t_{pm} c_m x_p - \sum_{p=1}^{557} \sum_{r=1}^{165} q_{pr} c_r x_p - \sum_{p=1}^{557} \sum_{m=1}^{72} \frac{1}{3600} L_c t_{pm} x_p - \sum_{w=1}^8 u_w c_w \\
& - \sum_{p=1}^{557} s_p \max\{0, D_p - x_p\}
\end{aligned}$$

3.4 Constraint Formulation

The model needs to be confined within the constraints given from the project requirement [1] to ensure the model remains feasible, WARP's operations consists of six main types of constraints:

1. **Raw material budget;** the total raw material cost cannot exceed 10 million:

$$\sum_{p=1}^{557} \sum_{r=1}^{165} q_{pr} c_r x_p \leq 10,000,000.$$

2. **Available raw materials;** there is a limited quantity of raw materials that WARP can purchase:

$$\sum_{p=1}^{557} \sum_{r=1}^{165} q_{pr} x_p \leq Q_r \quad \forall r = 1, 2, \dots, 165.$$

3. **Warehouse capacity;** the sum of all shoes produced cannot exceed the total capacity of all warehouses' combined:

$$\sum_{p=1}^{557} x_p \leq \sum_{w=1}^8 W_{cap,w} u_w.$$

4. **Machine operating duration;** each machine can operate for a maximum of 12 hours and 28 days per month, which is calculated to be $T_m = (12)(28)(60) = 20160 \text{ min}$, and the total production time of each machine must not exceed T_m :

$$\sum_{p=1}^{557} \sum_{m=1}^{72} \frac{1}{60} t_{pm} x_p \leq T_m \quad \forall m = 1, 2, \dots, 72$$

5. **Integrality of production quantities;** Since x_p represents the number of shoes produced, the set of all x_p must be non-negative integers as it is impossible to produce half a shoe or negative shoes:

$$x_p \in \mathbb{Z}_+ \quad \forall p = 1, 2, \dots, 557.$$

6. **Binary warehouse usage;** Since u_w represents the usage of a warehouse w, it can only be either 1 or 0:

$$u_w \in \{0, 1\} \quad \forall w = 1, 2, \dots, 8.$$

3.5 Final Mathematical Formulation

Combining the objective function and constraints from Sections 3.3 and 3.4, the general form of the linear optimization model becomes:

$$\begin{aligned}
 \max z = & \sum_{p=1}^{557} P_p x_p - \sum_{p=1}^{557} \sum_{m=1}^{72} \frac{1}{60} t_{pm} c_m x_p - \sum_{p=1}^{557} \sum_{r=1}^{165} q_{pr} c_r x_p - \sum_{p=1}^{557} \sum_{m=1}^{72} \frac{1}{3600} L_c t_{pm} x_p - \sum_{w=1}^8 u_w c_w \\
 & - \sum_{p=1}^{557} s_p \max\{0, D_p - x_p\} \\
 \text{s. t. } & \sum_{p=1}^{557} \sum_{r=1}^{165} q_{pr} c_r x_p \leq 10,000,000 \\
 & \sum_{p=1}^{557} \sum_{r=1}^{165} q_{pr} x_p \leq Q_r \forall r = 1, 2, \dots, 165. \\
 & \sum_{p=1}^{557} x_p \leq \sum_{w=1}^8 W_{cap,w} u_w. \\
 & \sum_{p=1}^{557} \sum_{m=1}^{72} \frac{1}{60} t_{pm} x_p \leq T_m \forall m = 1, 2, \dots, 72 \\
 & x_p \in \mathbb{Z}_+ \forall p = 1, 2, \dots, 557. \\
 & u_w \in \{0, 1\} \forall w = 1, 2, \dots, 8.
 \end{aligned}$$

4.0 Results

4.1 AMPL Implementation

AMPL was used to implement the model to find the optimal profit for WARP. For each implementation, the code was spread across three files and one output file:

- A .dat file that assigns specific data from the provided database to our sets and parameters, and uses SQL to filter for necessary data,
- A .mod file defining all sets, parameters, variables, and constraints the model must have and adhere to,
- A .run file to solve the model given the provided data in the .mod and .dat files respectively, and
- A .out file containing the relevant outputs of the solution.

The naming scheme is:

- RelaxedLP.xxx for the relaxed LP model,
- RoundedIP.xxx for the rounded IP model, and

- Q7.xxx for the Question 7 formulation, which implements both the reduction in machine operation hours and reduction in total raw material budget.
- For consistency, the same .dat file was used for all relaxed LP, rounded IP, and Q7 implementations. .xxx are fillers for .mod, .run, and .out.

4.2 Results and Explanation of Results

For the relaxed LP implementation, our optimal objective function value rounded to the nearest cent is \$11,789,734.80. This means that the maximum amount of profit WARP can make, inclusive of all revenues and costs, is \$11,789,734.80. The reported values of x_p represents the optimal number of pairs of each shoe type p WARP needs to produce in February 2006 in order to attain the stated amount of profit. However, not all shoes are economically viable to produce. x_p values of zero for certain shoe types indicate that WARP will not financially benefit from producing those shoes; this could be caused by low levels of demand for the shoe, making the cost of producing the shoe greater than the unmet production demand penalty. Conversely, shoe types with non-zero x_p values are financially desirable to produce and allow WARP to achieve its maximum feasible profit. In addition, observing the values of u_w , only u_2 , u_3 , u_4 , u_6 , and u_7 take on values of 1, whereas u_1 and u_5 take on values of 0. This means that WARP will only need to open warehouses 2, 3, 4, 6, and 7 to maximize profits and that warehouses 1 and 5 can stay closed. In addition, keeping warehouses 1 and 5 closed may suggest that their operation costs exceed the revenues they may help generate. These values can be found in RelaxedLP.out. Whereas some raw material quantity constraints are binding, warehouse capacity and raw material budget constraints have relatively large slack, possibly suggesting that an excess in allocation in these production areas and an opportunity to reallocate the raw material budget.

4.3 Answers to Questions

1. How should you estimate the demand for the month of February?

The demand was calculated using a SQL query, which was implemented in the warp.dat file. The Project document provides context, stating that “WARP market analysts predict that for the month of February the demand for all types of shoes will double” [1].

The following steps were performed:

1. The product demand in February for years 1997–2003 was filtered using (WHERE Month=2),
2. The average demand of each shoe across all the stores was taken using (AVG(Demand) AS [Demand_Feb] ... GROUP BY Product_Num ORDER BY Product_Num. The GROUP BY keyword takes the average of each shoe type from all the stores, and the ORDER BY keyword orders the type of shoes from 1 to 557.

3. The average demand of each shoe was then multiplied by 10, because the SQL query took the average of each store, and there are ten stores,
4. The average demand of each shoe was then multiplied by 2 to double the forecasted demand, as predicted by the market analyst.

Therefore the final SQL query for the forecasted demand for February 2006 is: SQL=SELECT Product_Num, 20*AVG(demand) AS [Demand_Feb] FROM Product_Demand WHERE Month=2 GROUP BY Product_Num ORDER BY Product_Num.

2. How many variables and constraints do you have?

The model consists of 565 decision variables: 557 from x_p , the types of shoes produced, and 8 from u_w , representing the operational state of warehouses. In addition, the model has 804 constraints: one raw material budget constraint, 165 available raw material constraints (one per raw material), one warehouse capacity constraint, 72 machine capacity constraints (one per machine), 557 positive integer constraints for each shoe production variable, and 8 binary constraints for each warehouse usage decision variable.

3. If you had to relax your integer program to an LP, how many constraints were violated after rounding the LP solution to the closest integer solution?

The integer program exceeded the 20 minute mark to execute, and therefore the model was relaxed into a linear program where the integer requirement for x_p was removed in RoundedLP.mod, and formulated as follows:

$$x_p \geq 0 \quad \forall p = 1, 2, \dots, 557.$$

After the optimal solution was found, the x_p values are then rounded into their nearest integers.

In addition, the slack of each constraint was calculated using the new rounded x_p values. We then determined if any constraints were violated by identifying those whose slacks were negative. This was performed in the RoundedLP.run file and the results can be found in RoundedLP.out. The results show that 55 raw material quantity constraints and 72 machine operating duration constraints were violated, for a total of 127 violated constraints. The violated raw material constraints represent the overconsumption of total available raw materials and use of all machines beyond their allowed operating duration. However, the budget constraint and warehouse capacity constraint were not violated and instead have excess slack, meaning that the rounding of the LP solutions does not violate these constraints.

4. Which constraints are binding, and what is the real-world interpretation of those binding constraints?

When consulting the LP relaxation solution, we found 48 violated and 70 binding raw material constraints out of our total 804 constraints. Because violated constraints indicate overuse of a specific resource (i.e. raw materials), they may or may not be binding under the IP model. If we assume that the violated constraints are binding, we have a total of 118 binding constraints. We can interpret binding raw material constraints as the production process using the total available stock of the respective raw material to work at its optimal. This suggests that the process depends on and is bottlenecked by the availability of the respective raw materials whose constraints are binding. In the real world, if demand for shoes that require the corresponding raw materials whose constraints are binding increased, WARP would either not be able to meet that demand or could increase their profits if they increased the limiting amounts.

5. Assume that some additional warehouse space is available at the price of \$10/box of shoes. Is it economical to buy it? What is the optimal amount of space to buy in this situation?

Computing the slack for the warehouse space constraint in the relaxed LP model shows that it has a slack of 4406. This indicates that not all of the warehouse is being used to allow the production to operate at optimal capacity, so increasing the amount of warehouse space would not be economical; shoe production would not be impacted, and doing so would incur unnecessary costs.

6. Imagine that machines were available for only 8 hours per day. How would your solution change? Which constraints are binding now? Does the new solution seem realistic to you?

If the total amount of time machines could operate was reduced to 8 hours a day, then machines can operate for 13440 total minutes in February, corresponding to a decrease of $20160 - 13440 = 6720$ less minutes than previously. However, the slack value for each machine operation time constraint is greater than 6720, indicating that even if the total time machines could run for was limited to 8 hours a day, there would be no effect on WARP's shoe production. This means that our solution and binding constraints would not change. This seems realistic provided that there is enough slack for each machine to handle the new decrease in maximum operating time.

7. If in addition there was a \$7,000,000 budget available to buy raw materials, what would you do? Change your formulation and solve it again.

Assuming that this includes both the decrease in machine operation time from Q6 and that the question implies that the budget decreases from \$10,000,000 to \$7,000,000, we would choose to allocate the budget differently than the original relaxed LP. For instance, raw materials that do not have high demand (i.e. have a larger slack) will receive less of the budget, while high demand raw materials whose constraints are binding will be prioritized. The formulation and

output for this question is included in the files Q7-time-and-budget.run, Q7-time-and-budget.mod, and Q7.out.

5.0 Conclusion

This project formulated a mathematical model that determined WARP Shoe Company's maximum optimal profit production plan for February 2006. Using AMPL, we extracted data from the WARP database [2] and formulated a linear optimization model subject to constraints related to available raw materials, warehouse and machine operation costs, and machine work capacities. The model produced an optimal profit of \$11,789,734.80 under the given production plan, constraints, and additional assumptions. Sensitivity analysis of the relaxed LP revealed that all binding constraints are raw material quantity constraints, which guide WARP toward the critical constraints that directly contribute towards and bottleneck WARP's profit gains. In addition, analysis of the rounded and relaxed LP solutions reveal several violated constraints in both raw material quantity and machine operation times. However, warehouse capacity and raw material budget constraints are not binding, allowing for a reduction in allocation towards these areas. Although this simplified model relies on several assumptions, its results provide valuable insights on how WARP should allocate its resources most profitably. Overall, the results demonstrate how the linear optimization framework can support WARP in making important production planning decisions in a market with changing demands.

6.0 References

- [1] D. M. Aleman, "Lab project: WARP Shoe Company," Quercus. [Online]. Available: https://q.utoronto.ca/courses/411715/files/38891325?module_item_id=7049468. [Accessed Dec 2, 2025].
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