HW 1, Propagation

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1:

In Figure 1 we see the resulting membrane voltage as described by Eq 1, normalized to be within ± 1. This function only describes the upstroke of the action potential, and in this case we can use this to understand that the direction of propagation is from right to left in Figure 1.

$$V_m = 50 * tanh(x) \tag{1}$$

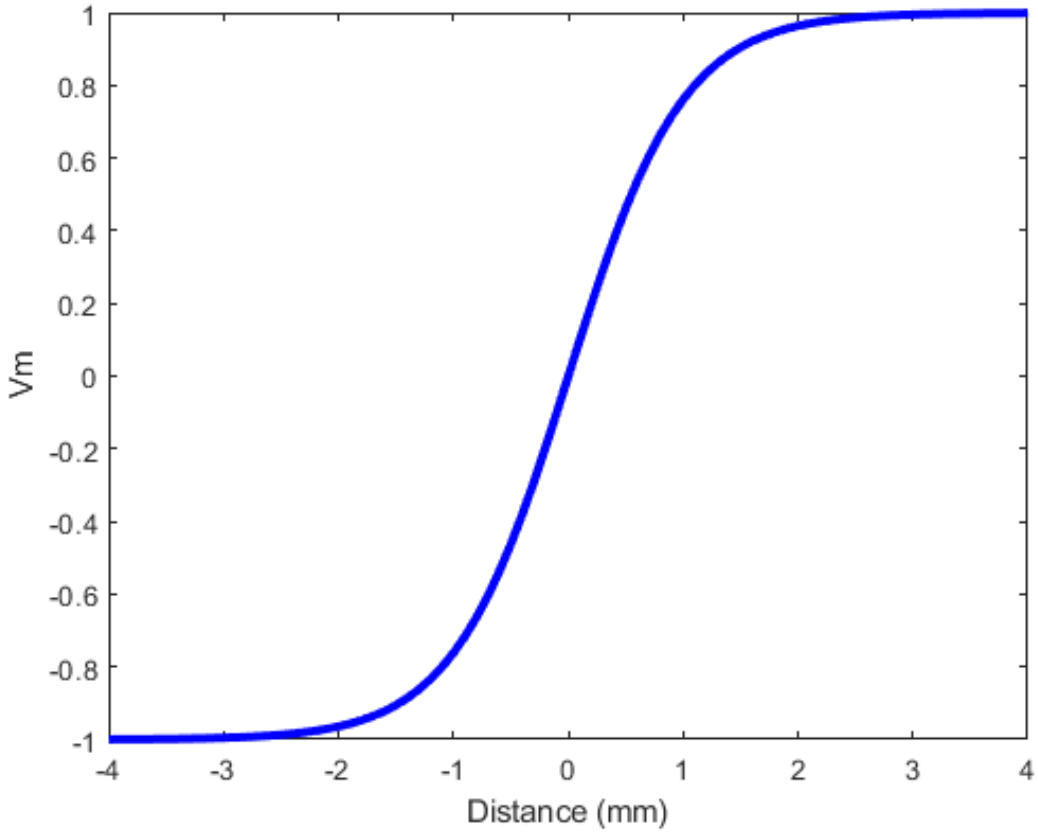

Figure 1: Membrane voltage as a function of space along the 1D fiber. Membrane voltage has been normalized such that the peaks are at ± 1.

2:

In Figure 2 we see the resulting intracellular current, I_i as described by Eq 2, normalized to be within ± 1. For this lab I chose $r_e = r_i = 1$ and $I = 0$ as there is no stimulus. At the peak current the flow is the same direction of the activation, as denoted by the negative sign of the extracellular current.

$$I_i = \frac{-1}{r_i + r_e}\left(\frac{dV_m}{dx} - Ir_e\right) \tag{2}$$

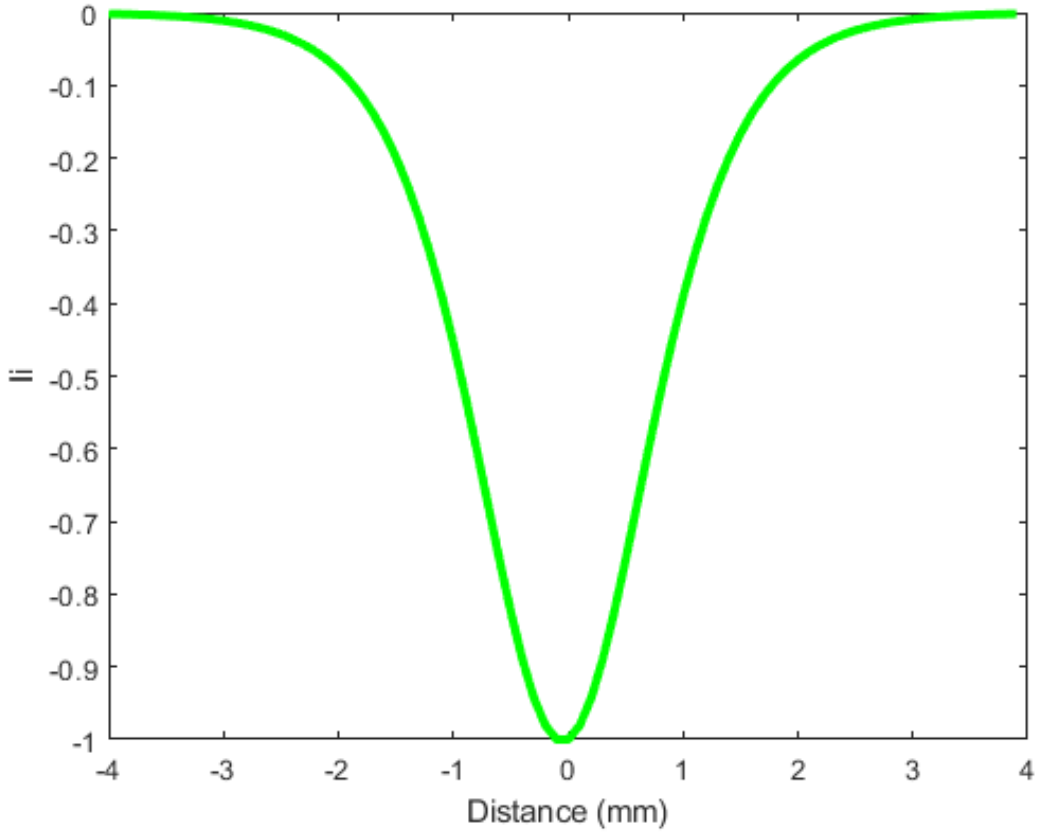

Figure 2: Intracellular current as a function of space along the 1D fiber. Current has been normalized such that the peak is -1.

3:

In Figure 3 we see the resulting extracellular current, I_e as described by Eq 3, normalized to be within ± 1. At the peak current the flow is the opposite direction of the activation, as denoted by the positive sign of the extracellular current.

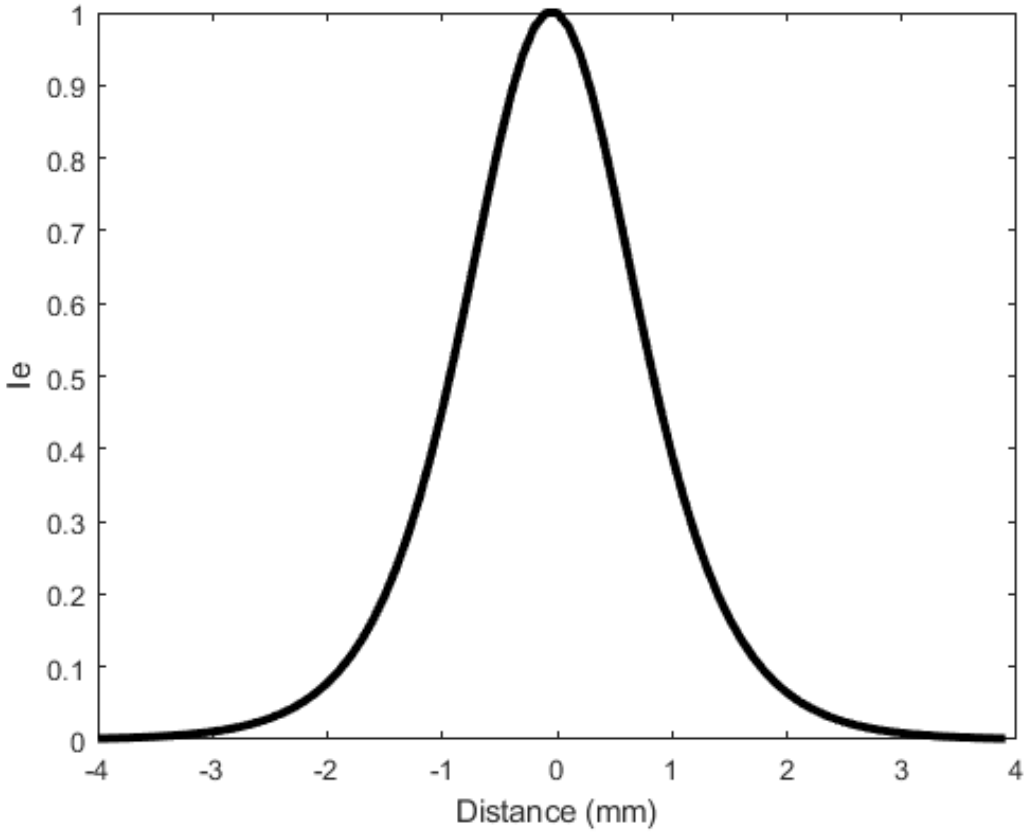

Figure 3: Intracellular current as a function of space along the 1D fiber. Current has been normalized such that the peak is 1.

$$I_e = \frac{\frac{dV_m}{dx} + I_i r_i}{r_e} \tag{3}$$

4:

In Figure 4 we see the resulting extracellular current, I_m as described by Eq 4, normalized to be within ± 1. At the positive peak of current we see that this aligns with the region at the wavefront of the upstroke, indicating positive current flow into the cell (sodium inflow at the wavefront as the activation spreads) while the negative peak of the membrane current corresponds to region of the wavefront that has just passed. The positive peak indicates the direction of propagation relative to the negative peak such that propagation flows in the direction dictated as negative peak towards positive peak.

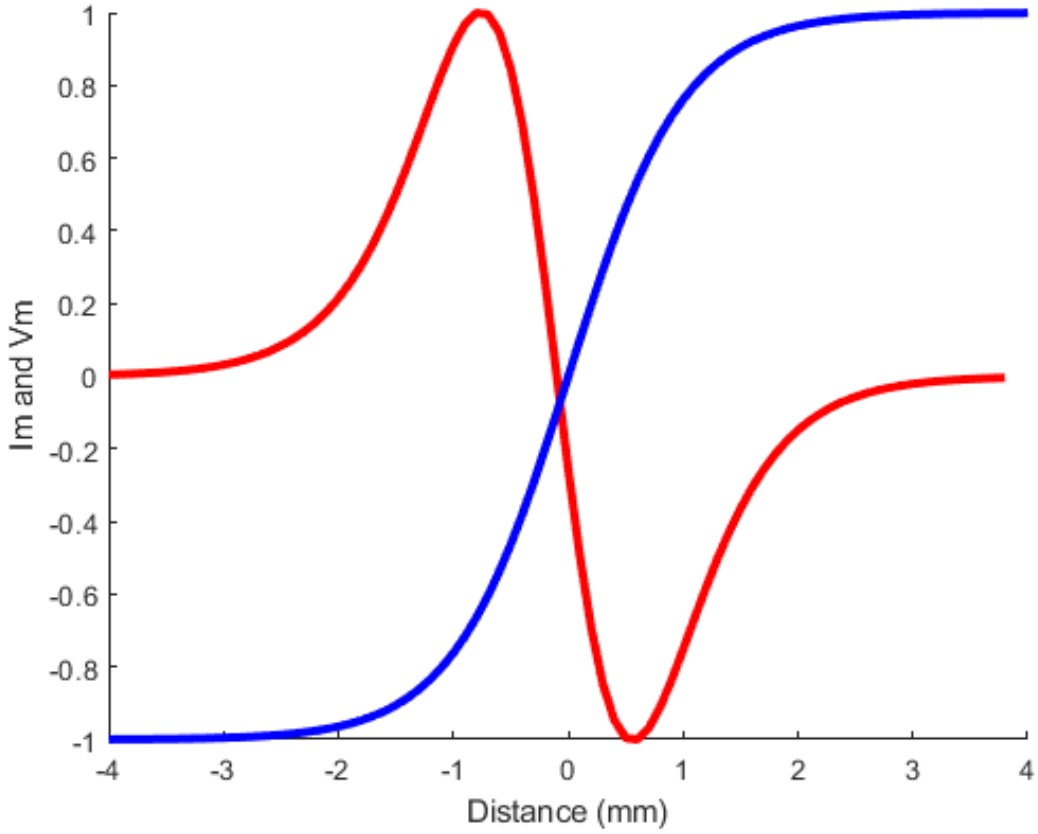

Figure 4: Membrane current (Red) as a function of space as well as membrane voltage (blue) along the 1D fiber. Current has been normalized such that the peaks are ± 1. Membrane voltage has been normalized such that the peaks are ± 1

$$I_m = \frac{1}{r_i + r_e}\left(\frac{d^2 V_m}{dx^2} - I r_e\right) \tag{4}$$

5:

We can solve this problem using the relationship shown in Eq 5, where S_R is the rheobase strength, τ is the membrane time constant, t is the stimulus time we desire to use, and $S(t)$ is the strength needed to elicit an action potential given the stimulus time t. We can validate this by plotting the associated strength duration curve (Figure 5) and see that the observed rheobase matches the $10\mu A/cm^2$ value perscribed, and that the experimentally observed strengths and durations fall on this curve. We can now compute the required strength when using a 0.2 ms stimulus, which comes out to be $125.1\mu A/cm^2$. This is done by evaluating Eq 5 using $t = 0.2ms$.

$$S(t) = \frac{S_R}{1 - e^{\frac{t}{\tau}}} \tag{5}$$

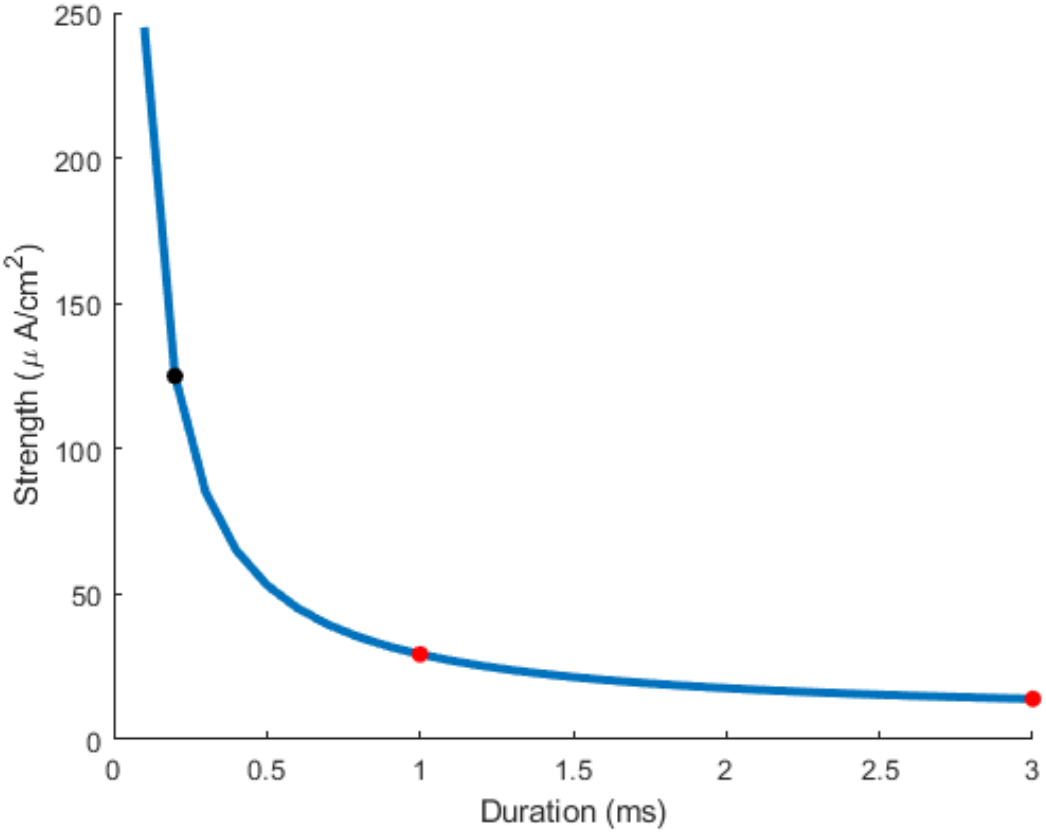

Figure 5: Strength Duration curve using Eq 5 with $\tau = 2.4ms$, $S_R = 10\mu A/cm^2$. Measured points shown in red, desired measurement location shown in black.

Appendix

All MATLAB code used to generate the figures and calculations presented in this assignment is shown below.

```
%1)
clear;close all
re = 1;
ri = 1;
x = [-4:.1:4];
vMx = 50*tanh(x);
dvMx = diff(vMx);
ddvMx = diff(dvMx);
I = 0;%No stimulation
Ii = (-1/(ri+re))*(dvMx-I*re);
Im = (1/(ri+re))*(ddvMx-I*re);
Ie = (dvMx+Ii.*ri)./re;
figure(1);
```

```
plot(x,vMx./50,'b','linewidth',3);
xlabel('Distance (mm)');
ylabel('Vm');

figure(2);
plot(x(1:end-1),Ii./(max(abs(Ii))),'g','linewidth',3)
xlabel('Distance (mm)');
ylabel('Ii');

figure(3);clf();
hold on;
plot(x(1:end-2),Im./(max(abs(Im))),'r','linewidth',3)
plot(x,vMx./50,'b','linewidth',3)
xlabel('Distance (mm)');
ylabel('Im and Vm');

figure(4);plot(x(1:end-1),Ie./(max(abs(Ie))),'k','linewidth',3)
xlabel('Distance (mm)');
ylabel('Ie');

%
%2)
rheobase = 10;%uA/cm2
durations = [1,3];
strengths = [29.346,14.015];
tc = 2.4;

t = [.1:.1:3];
Str = rheobase./(1-exp(-t/tc));
StrReq = rheobase./(1-exp(-0.2/tc));
figure(5);
hold on;
plot(t,Str,'linewidth',3)
scatter(durations,strengths,'ro','filled')
scatter(0.2,StrReq,'ko','filled');
xlabel('Duration (ms)');
ylabel('Strength (\mu A/cm^2)')
```