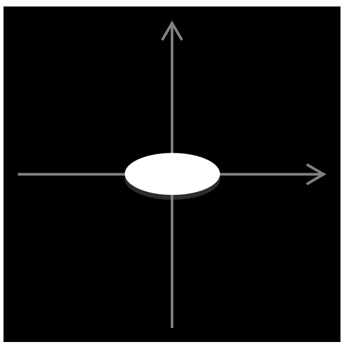
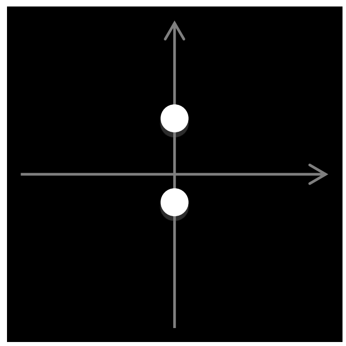
BIOEN 6401 Medical Imaging Systems
Homework #5
Due Monday, December 2, 2019

1. Be as precise as possible, and *without* checking your answers using a computer, sketch *by hand* the sinogram over 90° ≤ θ ≤ -90° for each of the following. Include explicit projection profiles for θ = 0°, ±45°, and ±90°. Unless otherwise stated, orientation convention of the projection angle as described in class is assumed.

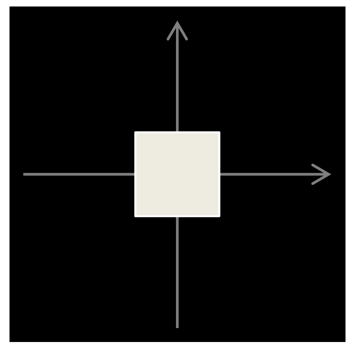
(a)



(b)

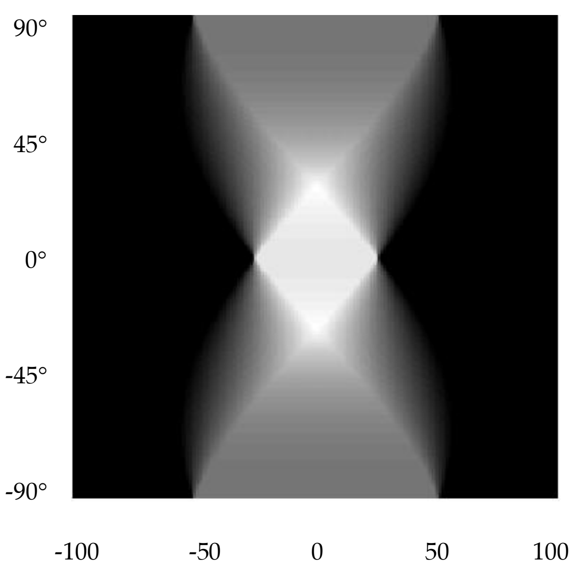


(c)

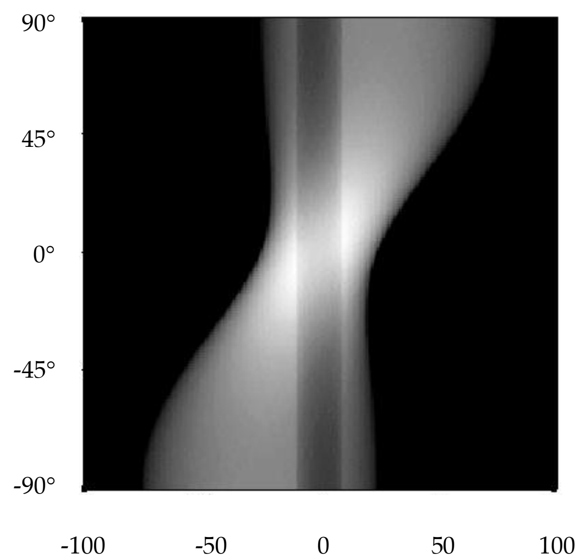


2. Again, be as precise as possible, using the same angular convention above, and *without* checking your answers on a computer, sketch *by hand* the object that would produce each of the following sonograms. For 2a, the projection profiles for all angles are either a rectangle, triangle, or trapezoid. For 2b, the main lobe of the profiles are half ellipses.

(a)



(b)



For Problems 3a, which will be used later for Problems 3—4, you'll receive 10 extra credits if you write your own function, and another 5 if the function does not use 'for' or similar loops).

3. Using the same convention used in Problem 1 for the projection angle (i.e., counterclockwise from the positive horizontal axis):
   (a) Write a user-defined function or use Matlab's built-in iradon function to perform direct backprojection of a *single* projection for a given angle (i.e., find $b_\theta(x,y)$), with the angle θ and projection profile $g_\theta(R)$ as input, and $b_\theta(x,y)$ as output argument. Use linear interpolation or better to estimate value of the arguments at non-integer indices. Demonstrate that your function works by showing the direct backprojections of a rectangular profile at 3 different angles (e.g., $\theta = 0°, 30°$ and $45°$).
   (b) Load into Matlab the file *Prob3.mat*, which contains two variables: *sinogram* and *thetas*. The former is a sinogram, and the latter is a list of angles (in degrees) at which the projections are taken. Show the sinogram as an image with the radial distance plotted horizontally and the projection angle vertically. Mark on the vertical axis where θ is 0°, 90°, and 180°.
   (c) Based on the function you used for Part (a), perform and show the *direct* backprojection reconstruction of the entire sinogram.
   (d) Repeat part (c) using *filtered* backprojection. You must perform your own ρ-filter, including any window function used, even if you use Matlab's iradon function. Plot your ρ-filter and show your corrected image.

4. From the sinogram given in Problem #3 and the FBP technique developed in Part (d), reconstruct images where the sinogram is downsampled 25%, 50% and 75% in the projection azimuthal angles (suffice to select the nearest angles that are given). Repeat the same for the sinogram down-sampled in the radial direction (no azimuthal down-sampling), using linear interpolation to simulate downsampling when necessary. Describe the artifacts associated with downsampling in each azimuthal and radial dimensions.

5. Suppose during a CT calibration, pre-calibration Hounsfield numbers -20 and -1200 were measured for water and air, respectively. Determine the coefficients *a* and *b* such that the true Hounsfield number can be determined via the equation $H_{true} = a\, H_{measured} + b$. What is the true Hounsfield number for a tissue that has a measured Hounsfield number of 1850?