

$$\begin{aligned}1.) H(X,Y) &= - \sum_i \sum_j P(x,y) \log_2 (P(x,y)) \\&= - \sum_i \sum_j P(x)P(y) \log_2 [P(x)P(y)] \\&= - \sum_i \sum_j P(x)P(y)(\log_2 P(x) + \log_2 P(y)) \\&= - [\sum_i \sum_j P(x)P(y) \log_2 P(x) + \sum_i \sum_j P(x)P(y) \log_2 P(y)] \\&= - \sum_i P(x) \log_2 P(x) - \sum_j P(y) \log_2 P(y) \\&= H(X) + H(Y)\end{aligned}$$