

$$\begin{aligned}
1.) \quad H(x, y) &= - \sum_i \sum_j P(x, y) \log_2 (P(x, y)) \\
&= - \sum_i \sum_j P(x) P(y) \log_2 [P(x) P(y)] \\
&= - \sum_i \sum_j P(x) P(y) (\log_2 P(x) + \log_2 P(y)) \\
&= - \left[\sum_i \sum_j P(x) P(y) \log_2 P(x) + \sum_i \sum_j P(x) P(y) \log_2 P(y) \right] \\
&= - \sum_i P(x) \log_2 P(x) - \sum_j P(y) \log_2 P(y) \\
&= H(x) + H(y)
\end{aligned}$$