2) a) $\log\left[\dfrac{p(\ell_1 | x_n)}{p(\ell_2 | x_n)}\right] = \log\left[\dfrac{\dfrac{1}{S_1\sqrt{2\pi}}\exp\left(-\dfrac{x_n - m_1}{2S_1^2}\right)\cdot\dfrac{N_1}{N_1+N_2}}{\dfrac{1}{S_2\sqrt{2\pi}}\exp\left(-\dfrac{x_n - m_2}{2S_2^2}\right)\cdot\dfrac{N_2}{N_1+N_2}}\right]$

$0 = \log\left[\dfrac{S_2\cdot N_1\cdot \exp\left(\dfrac{x_n - m_2}{2S_2^2} - \dfrac{x_n - m_1}{2S_1^2}\right)}{S_1\cdot N_2}\right]$

$1 = \dfrac{S_2\cdot N_1}{S_1\cdot N_2}\exp\left(\dfrac{S_1^2(x_n - m_2) - S_2^2(x^n - m_1)}{2S_2^2 S_1^2}\right)$

$\ln\left(\dfrac{S_1 N_2}{S_2 N_1}\right) = \dfrac{x_n(S_1^2 - S_2^2) - S_1^2 m_2 + S_2^2 m_1}{2S_2^2 S_1^2}$

$2S_2^2 S_1^2 \ln\left(\dfrac{S_1 N_2}{S_2 N_1}\right) + S_1^2 m_2 - S_2^2 m_1 = x_n(S_1^2 - S_2^2)$

$x_n = \dfrac{2S_2^2 S_1^2 \ln\left(\dfrac{S_1 N_2}{S_2 N_1}\right) + S_1^2 m_2 - S_2^2 m_1}{S_1^2 - S_2^2}$

$* \quad x_n = \dfrac{2S_2^2 S_1^2 \ln\left(\dfrac{S_1 N_2}{S_2 N_1}\right) + S_1^2 m_2 - S_2^2 m_1}{S_1^2 - S_2^2}$