

$$1) \quad a) \quad \text{CDF}(x; \lambda) = \int_0^x \lambda e^{-\lambda i} di = -e^{-\lambda i} \Big|_0^x = \lim_{i \rightarrow \infty} (-e^{-\lambda i}) - (-e^{-\lambda x})$$

$$= 0 + e^{-\lambda x} = e^{-\lambda x}$$

$$b) \quad \text{mean} = [E[p(x|\lambda)]] = \int_0^\infty \lambda e^{-\lambda x} x dx = -x e^{-\lambda x} - \frac{e^{-\lambda x}}{\lambda} \Big|_0^\infty$$

$$= \lim_{x \rightarrow \infty} (-x e^{-\lambda x} - \frac{e^{-\lambda x}}{\lambda}) + 0 + \frac{1}{\lambda} = \frac{1}{\lambda}$$

$$c) \quad \text{variance} = \sigma^2 = \int_0^\infty (x - \text{mean}_{p(x|\lambda)})^2 \lambda e^{-\lambda x} dx$$

$$= \int_0^\infty (x - \frac{1}{\lambda})^2 \lambda e^{-\lambda x} dx = \int_0^\infty x^2 \lambda e^{-\lambda x} - e^{-\lambda x} dx$$

$$= -x e^{-\lambda x} - \frac{e^{-\lambda x}}{\lambda} - \frac{e^{-\lambda x}}{\lambda} \Big|_0^\infty = -x e^{-\lambda x} - 2 \cdot \frac{e^{-\lambda x}}{\lambda} \Big|_0^\infty = (-x - \frac{2}{\lambda}) e^{-\lambda x} \Big|_0^\infty$$

$$= \lim_{x \rightarrow \infty} ((-x - \frac{2}{\lambda}) e^{-\lambda x}) - ((-0) - \frac{2}{\lambda}) e^{-\lambda \cdot 0} = 0 + (\frac{2}{\lambda}) \cdot 1 = \frac{2}{\lambda}$$