Exercise Sheet #3

due on Tuesday, 24 November 2020, 12 noon sharp

1. **EXAMple** Consider two random variables X, Y with joint probability measure $P(X, Y)$. Show that, if X and Y are *independent*, the entropy $\mathbb{H}$ (defined in the lecture) satisfies

$$\mathbb{H}(X, Y) = \mathbb{H}(X) + \mathbb{H}(Y).$$

2. Provide answers to the following variants of the "weighing question" discussed in the lecture:

   (a) You are given 16 balls, all of which are equal in weight except for one that is either heavier or lighter. You are also given a bizarre two-pan balance that can report only two outcomes: 'the two sides balance' and 'the two sides to not balance'. Design a strategy to determine which is the odd ball in as few uses of the balance as possible.

   (b) You have a two-pan balance; your job is to weigh out bags of flour with integer weights 1 to 40 kg inclusive. How many weights do you need? (You are to put weight on either pan. You are only allowed to put one flour bag on the balance at a time.)

   (c) You are given 12 balls and the three-outcome balance, as in the lecture. But this time, *two* of the balls are odd; each odd ball may be heavy or light, and we do not know which. We want to identify the odd balls, and in which direction they are odd. *Estimate* how many weighings are required by the optimal strategy. What if there are three odd balls?

3. **Practical Question:** You can find this week's sheet on Ilias as `Exercise_03.ipynb`