Data Literacy Exercise Sheet 3

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2. Provide answers to the following variants of the "weighing question" discussed in the lecture:

(a) You are given 16 balls, all of which are equal in weight except for one that is either heavier or lighter. You are also given a bizarre two-pan balance that can report only two outcomes: 'the two sides balance' and 'the two sides to not balance'. Design a strategy to determine which is the odd ball in as few uses of the balance as possible.
Take 8 balls and put 4 on each side of the balance. If these balance, discard all 8 and move forward with the other 8, If these do not balance, discard the other 8 and move forward with these.
Take 4 balls and put 2 on each side of the balance. If these balance, discard all 4 and move forward with the other 4. If they do not balance, discard the other 4 and move forward with these.
Take two balls and put one on each side of the balance. If these balance, discard both and move forward with the other 2. If they do not balance, discard the other two and move forward with these.
Take one of the remaining balls and any single ball that was already discarded. Put one on each side of the balance. If they balance, the final ball was the unequal ball. If they do not, the ball on the balance which was never discarded is the unequal ball.

(b) You have a two-pan balance; your job is to weigh out bags of flour with integer weights 1 to 40 kg inclusive. How many weights do you need? (You are to put weight on either pan. You are only allowed to put one flour bag on the balance at a time.)
$log_2(40!) = 160$

(c) You are given 12 balls and the three-outcome balance, as in the lecture. But this time, two of the balls are odd; each odd ball may be heavy or light, and we do not know which. We want to identify the odd balls, and in which direction they are odd. Estimate how many weighings are required by the optimal strategy. What if there are three odd balls?
1 ball: $log_3(24) = 2.89 \implies 3$

2 balls: $log_3(24) + log_3(22) = 5.7 \implies 6$
n balls: $\sum_{i=1}^{n} log_3(2 * (13 - i))$