1.) a) $CDF(x;\lambda) = \int_x^\infty \lambda e^{-\lambda i} di = -e^{-\lambda i}\Big|_x^\infty = \lim_{i\to\infty}(-e^{-\lambda i}) - (-e^{-\lambda x})$

$= 0 + e^{-\lambda x} = e^{-\lambda x}$

b) mean $= \mathbb{E}[p(x|\lambda)] = \int_0^\infty \lambda e^{-\lambda x} x\, dx = -xe^{-\lambda x} - e^{-\lambda x}/\lambda \Big|_0^\infty$

$= \lim_{x\to\infty}(-xe^{-\lambda x} - e^{-\lambda x}/\lambda) + 0 + 1/\lambda = 0 + 1/\lambda = 1/\lambda$

c) variance $= \sigma^2 = \int_0^\infty (x - \text{mean}_{p(x|\lambda)})^2 \lambda e^{-\lambda x} dx$

$= \int_0^\infty (x - 1/\lambda)\lambda e^{-\lambda x} dx = \int_0^\infty x\lambda e^{-\lambda x} - e^{-\lambda x} dx$

$= -xe^{-\lambda x} - e^{-\lambda x}/\lambda - e^{-\lambda x}/\lambda \Big|_0^\infty = (-x - 2/\lambda)e^{-\lambda x}\Big|_0^\infty$

$= \lim_{x\to\infty}(((-x - 2/\lambda)e^{-\lambda x}) - (-(0) - 2/\lambda)e^{-\lambda \cdot 0} = 0 + (2/\lambda)\cdot 1 = 2/\lambda$

2.) a) $\mathbb{E}[p(x|\lambda)] = 1/\lambda = 6$ min

b) $CDF(20|1/6) = e^{-20/6} = 0.0357$. A one-sided test makes most sense here.

c) $CDF(1/60|1/6) = 1 - e^{-1/60 \cdot 6} = 0.00277$. According to the p-value test I am more surprised, but based on experience/logic I am less surprised. I still do not think it makes sense to do a two-sided test here.