

# The Z-transform

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## Signals as a Sum of Delta Functions

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Any discrete signal can be written as an infinite sum of scaled Kronecker delta functions.

$$x[n] = \sum_{k=-\infty}^{+\infty} x[k] \delta[n - k]$$

You can easily see that all terms where  $n \neq k$  are zero, because the Kronecker delta is zero in that case. Only the term for  $n = k$  is non-zero, in which case the Kronecker delta is one, so the result is just  $x[k]$ . This is a consequence of the sifting property of the delta function, covered in the [previous page](#).

## DTLTI Transformations as Convolutions

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You can express the output of any discrete-time linear time-invariant system  $T$  as the convolution of the input with the impulse response of the system,  $h[n]$ :

$$T(x[n]) = x[n] * h[n]$$

### Proof

The proof itself is very simple: We just decompose the input as a sum of delta functions, as described in the previous paragraph, and then we use the linearity and time-invariance to bring the  $T$  operator inside of the summation.

$$\begin{aligned} y[n] &= T(x[n]) \\ &= T\left(\sum_{k=-\infty}^{+\infty} x[k] \delta[n - k]\right) \\ &= \sum_{k=-\infty}^{+\infty} T(x[k] \delta[n - k]) \\ &= \sum_{k=-\infty}^{+\infty} x[k] T(\delta[n - k]) \\ &= \sum_{k=-\infty}^{+\infty} x[k] h[n - k] \\ &\triangleq x[n] * h[n] \end{aligned}$$

The  $*$  symbol in the last step is called the convolution operator, and it is defined as the sum in the step before it.  $\square$

## Eigenfunctions - ⚠️ Under Construction ⚠️

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