

# Formularium

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## Basics

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### Symbols

- Absolute temperature:  $T$
- Boltzmann constant:  $k = 1.38064852 \times 10^{-23} \text{ m}^2 \text{ kg s}^{-2} \text{ K}^{-1}$
- Intrinsic free carrier concentration:  $n_i$
- Electron concentration:  $n$
- Hole concentration:  $p$
- Effective density of states in conduction band:  $N_c$
- Effective density of states in valence band:  $N_v$
- Fermi level:  $\phi_f$
- Fermi energy:  $E_f$
- Quasi Fermi energy for electrons (no equilibrium):  $E_{fn}$
- Quasi Fermi energy for holes (no equilibrium):  $E_{fp}$
- Intrinsic Fermi energy:  $E_i$
- Valence band energy:  $E_v$
- Conduction band energy:  $E_c$
- Donor ionization energy:  $E_D$
- Acceptor ionization energy:  $E_A$
- Band gap energy:  $E_g \triangleq E_c - E_v$
- Number of ionized donor ions:  $N_D$
- Number of ionized acceptor ions:  $N_A$
- Number of equivalent energy minima in conduction band:  $M_c$
- Density-of-state effective electron mass:  $m_{de}$
- Density-of-state effective hole mass:  $m_{dh}$
- Recombination:  $R$
- Thermal generation:  $G_{th}$
- External generation:  $G$
- Net recombination rate of electrons and holes:  $U \triangleq R - G_{th}$
- Equilibrium electron concentration:  $n_{po}$
- Equilibrium hole concentration:  $p_{no}$
- Absorption coefficient:  $\alpha$
- Planck's constant:  $6.62607015 \times 10^{-34} \text{ J s}$
- Photon frequency:  $\nu$
- ???:  $\gamma$
- Auger recombination rate:  $R_A$
- Auger coefficients:  $G_{An}, G_{Ap}$
- Impact ionization generation rate:  $G_i$
- Electron/hole ionization rate:  $\alpha_n, \alpha_p$
- Free electron/hole velocity:  $v_n, v_p$
- Impact ionization threshold energy:  $E_{i,n}, E_{i,p}$
- Minority carrier effective lifetimes:  $\tau_n, \tau_p$
- Electric field:  $\epsilon$
- Electrostatic potential:  $\Psi$
- Thermal voltage:  $V_t = kT/q$
- Electron/hole mobility:  $\mu_n, \mu_p$
- Drift current density:  $J_{\text{drift}}$
- Diffusion current density:  $J_{\text{Diffusion}}$
- Diffusion constants:  $D_n, D_p$

### Concentration

$$n = N_c \exp\left(\frac{E_f - E_c}{kT}\right)$$
$$p = N_v \exp\left(\frac{E_v - E_f}{kT}\right)$$

$$N_c = 2 \left( \frac{2\pi m_{de} kT}{h^2} \right)^{3/2} M_c$$

$$N_v = 2 \left( \frac{2\pi m_{dh} kT}{h^2} \right)^{3/2}$$

### Pure Semiconductor in Equilibrium

$$n = p = n_i$$

$$n_i^2 = np = N_c N_v \exp \left( \frac{-E_g}{kT} \right)$$

$$E_f = E_i = \frac{E_c + E_v}{2} + \frac{kT}{2} \ln \left( \frac{N_v}{N_c} \right)$$

### Doped Semiconductor in Equilibrium

Charge neutrality:

$$N_D + p = N_A + n$$

N-type

$$N_D > N_A$$

$$n = \frac{1}{2} \left( N_D - N_A + \sqrt{(N_D - N_A)^2 + 4n_i^2} \right)$$

$$\approx N_D - N_A$$

$$p = \frac{n_i^2}{n}$$

$$\approx \frac{n_i^2}{N_D - N_A}$$

$$E_f \approx E_i + kT \ln \left( \frac{N_D - N_A}{n_i} \right) > E_i$$

P-type

$$N_A > N_D$$

$$p = \frac{1}{2} \left( N_A - N_D + \sqrt{(N_A - N_D)^2 + 4n_i^2} \right)$$

$$\approx N_A - N_D$$

$$n = \frac{n_i^2}{p}$$

$$\approx \frac{n_i^2}{N_A - N_D}$$

$$E_f \approx E_i - kT \ln \left( \frac{N_A - N_D}{n_i} \right) < E_i$$

### Semiconductor out of Equilibrium

$$pn > n_i^2$$

$$n = N_c \exp \left( \frac{E_{fn} - E_c}{kT} \right)$$

$$= n_i \exp \left( \frac{E_{fn} - E_i}{kT} \right)$$

$$p = N_v \exp \left( \frac{E_v - E_{fp}}{kT} \right)$$

$$= n_i \exp \left( \frac{E_i - E_{fp}}{kT} \right)$$

$$np = N_c N_v \exp \left( \frac{(E_v - E_c) + (E_{fn} - E_{fp})}{kT} \right)$$

$$= n_i^2 \exp \left( \frac{E_{fn} - E_{fp}}{kT} \right)$$

### Carrier Recombination and Generation Mechanisms

$$U_n \approx \frac{n - n_{po}}{\tau_n}$$

$$U_p \approx \frac{p - p_{no}}{\tau_p}$$

$$\frac{dn}{dt} = \frac{dp}{dt} = -U + G = G_{th} + G - R$$

#### Shockley-Read-Hall generation and recombination

Impurities with energy levels within the forbidden gap.

$$U_{SRH} = \text{staat hopelijk in het formularium}$$

#### Radiative generation and recombination

Usually negligible for indirect semiconductors

$$R_r = Bnp$$

N-type

$$R_r = Bpn_0 = \frac{p}{\tau_p}$$

$$\tau_p = \frac{1}{Bn_0} \approx \frac{1}{BN_D}$$

$$U_r = \frac{p - p_0}{\tau_p}$$

P-type

$$R_r = Bnp_0 = \frac{n}{\tau_n}$$

$$\tau_n = \frac{1}{Bp_0} \approx \frac{1}{BN_A}$$

$$U_r = \frac{n - n_0}{\tau_n}$$

Radiative generation

$$\alpha = (h\nu - E_g)^\gamma \quad (h\nu > E_g)$$

#### Auger recombination and Impact Ionization

Auger recombination: Energy released by band-to-band recombination is given to another free hole or electron

$$R_A = G_{An}n^2p + G_{Ap}p^2n$$

$$R_A \approx G_{An}N_D^2p = \frac{p}{\tau_p} \quad \text{n-type}$$

$$R_A \approx G_{Ap}N_A^2n = \frac{n}{\tau_n} \quad \text{p-type}$$

Impact ionization: Kinetic energy of an electron or hole is released in a collision to a neutral atom and generates an electron-hole pair

$$G_i = \alpha_n n v_n + \alpha_p p v_p$$

$$\alpha_n(\varepsilon) = \frac{q\varepsilon}{E_{i,n}} \exp\left(-\frac{b}{\varepsilon}\right)$$

$$\alpha_p(\varepsilon) = \frac{q\varepsilon}{E_{i,p}} \exp\left(-\frac{b}{\varepsilon}\right)$$

Effective minority carrier lifetime

$$\frac{1}{\tau} = \frac{1}{\tau_{SRH}} + \frac{1}{\tau_{rad}} + \frac{1}{\tau_{Auger}}$$

#### Carrier Transport

$$\varepsilon = \frac{1}{q} \nabla E_i$$

$$\varepsilon = -\nabla \Psi$$

$$\Psi = -\frac{E_i}{q}$$

$$\phi_f = -\frac{E_f}{q}$$

Equilibrium:

$$n = n_i \exp\left(\frac{\Psi - \phi_f}{V_t}\right)$$

$$p = n_i \exp\left(\frac{\phi_f - \Psi}{V_t}\right)$$

Out of equilibrium:

$$n = n_i \exp\left(\frac{\Psi - \phi_{fn}}{V_t}\right)$$

$$p = n_i \exp\left(\frac{\phi_{fp} - \Psi}{V_t}\right)$$

$$pn = n_i^2 \exp\left(\frac{\phi_{fp} - \phi_{fn}}{V_t}\right)$$

Drift

$$v_n = -\mu_n \varepsilon$$

$$v_p = -\mu_p \varepsilon$$

TODO: p. II-24 - II-26

$$J_{\text{drift},n} = q\mu_n n \varepsilon = \sigma_n \varepsilon$$

$$J_{\text{drift},p} = q\mu_p p \varepsilon = \sigma_p \varepsilon$$

Diffusion

$$J_{\text{diffusion},n} = -qD_n(-\nabla n)$$

$$J_{\text{diffusion},p} = qD_p(-\nabla p)$$

$$D_n = \mu_n \frac{kT}{q}$$

$$D_p = \mu_p \frac{kT}{q}$$

Current equations

$$J_n = q\mu_n \left( n \varepsilon + \frac{kT}{q} \nabla n \right) = -q\mu_n n \nabla \phi_{fn}$$

$$J_p = q\mu_p \left( p \varepsilon - \frac{kT}{q} \nabla p \right) = -q\mu_p p \nabla \phi_{fp}$$

PN-junction

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Symbols

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MOS-cap

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MOSFET

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N-FET

Fermi level

$$\phi_f = V_t \ln\left(\frac{N_A}{n_i}\right)$$

Threshold voltage

$$V_T = V_{FB} + 2\phi_f + \frac{\sqrt{2q\epsilon_s N_A}}{C_{ox}} \sqrt{2\phi_f - V_{BS}}$$

Drain-Source current:

$$I_{DS,\text{lin}} = \frac{\mu C_{ox} W}{L} \left( V_{GS} - V_T + \frac{V_{DS}}{2} \right) V_{DS}$$
$$I_{DS,\text{sat}} = \frac{\mu C_{ox} W}{2L} (V_{GS} - V_T)^2$$

BJT

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PV cell

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$$V_{oc} = V_t \ln \left( \frac{J_{sc}}{J_0} \right)$$

$$J_0 = q \sqrt{\frac{V_t \mu_p}{\tau_p}} \frac{n_i^2}{N_D}$$

$$I_{np} = I_{sc} - I_S \left[ \exp \left( \frac{V_{pn}}{V_t} \right) - 1 \right]$$