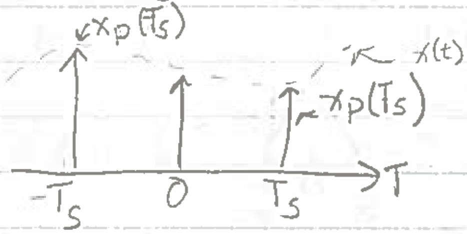
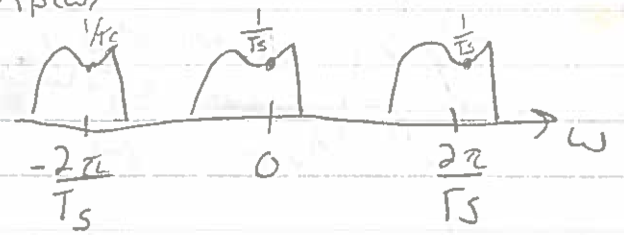
Jacob Riedel

PS08

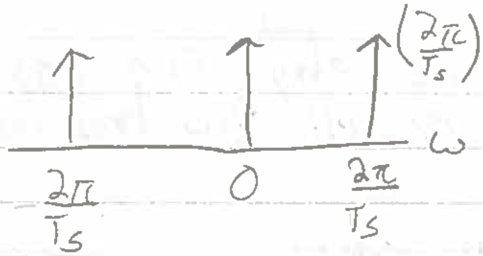
1a)



c) $X_p(\omega)$



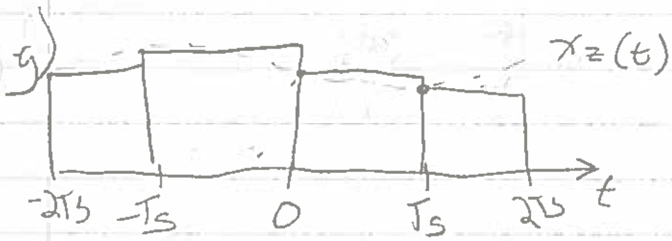
b)



d) $\omega_m = \frac{2\pi}{T_s}$

e) Use an ideal lowpass filter to just snip the frequency at $-\omega_m$ to ω_m. This is $X(\omega)$. Transform it to get $x(t)$

g)



$$\frac{\sin\left(2\pi \frac{T_s}{2} \frac{\omega}{2\pi}\right)}{\pi \cdot \frac{\omega}{2\pi}} \frac{T_s}{T_s} = T_s \operatorname{sinc}\left(T_s \frac{\omega}{2\pi}\right)$$

h) $Z(\omega) = \int_{-\infty}^{\infty} z(t) e^{-j\omega t} dt$

$= \int_{-\infty}^{0} \cancel{z(t) e^{-j\omega t}} + \int_0^{T_s} z(t) e^{-j\omega t} dt + \int_{T_s}^{\infty} \cancel{z(t) e^{-j\omega t}}$

$= \int_0^{T_s} e^{-j\omega t} dt$

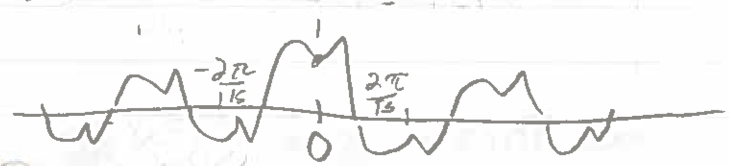
$= \frac{1}{-j\omega} \left[e^{-j\omega t} \right]_0^{T_s}$

$= \frac{1}{-j\omega} \left[e^{-j\omega T_s} - \cancel{e^{-j\omega 0}}^{1} \right]$

$= \frac{1}{j\omega} \left[1 - e^{-j\omega T_s} \right] \cdot \frac{2}{2}$

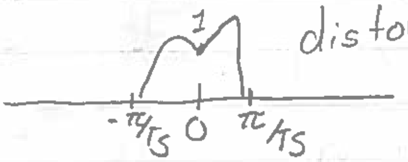
$= \frac{2}{2j\omega} \left[1 - e^{-j\omega T_s} \right] \left(e^{j\frac{\omega}{2}T_s} \right) \left(e^{-j\frac{\omega}{2}T_s} \right)$

$= \frac{2}{\omega} \frac{1}{2j} \left[e^{j\frac{\omega}{2}T_s} - e^{-j\frac{\omega}{2}T_s} \right] \left(e^{-j\frac{\omega}{2}T_s} \right)$



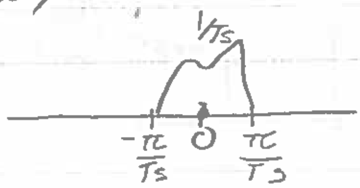
$\frac{T_s 2 \cdot 2}{2\omega T_s} \sin\left(\frac{\omega T_s}{2}\right) e^{-j\frac{\omega}{2}T_s} = T_s \operatorname{sinc}\left(\frac{\omega T_s}{2\pi}\right) e^{-j\frac{\omega}{2}T_s}$

$\bar{X}(\omega)$



distorted by $e^{-j\omega \frac{T_s}{2}}$

$\hat{X}(\omega)$



$\bar{X}(\omega)$ has a slight distortion and time delay. The amplitudes between $\bar{X}(\omega)$ and $\hat{X}(\omega)$ differ as well. $\hat{X}(\omega)$ has no distortion or time delay

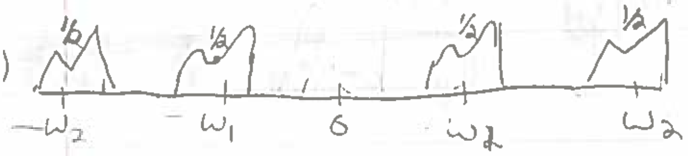
2) $y(t) = x_1(t)\cos(\omega_1 t) + x_2(t)\cos(\omega_2 t)$

$X_1(\omega) = 0 \quad X_2(\omega) = 0$ if $|\omega| > \omega_m \qquad \omega_1 >> \omega_M$

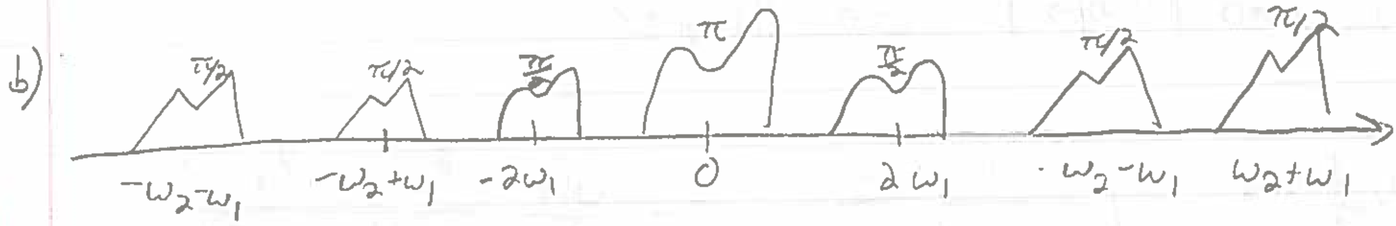
$\omega_2 >> \omega_M$

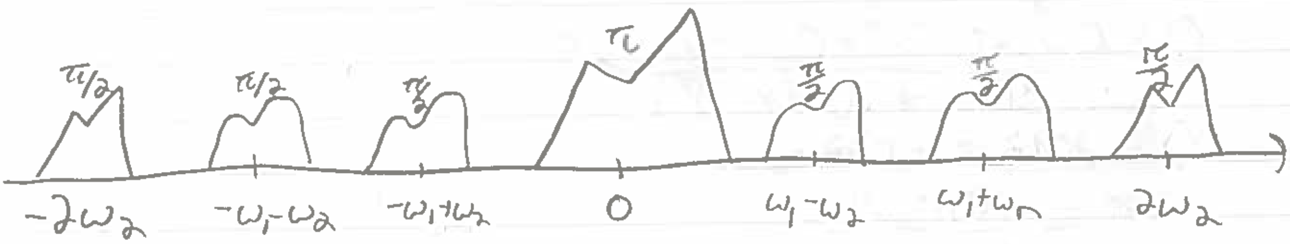
$\omega_1 + 2\omega_M < \omega_2$

a) $Y(\omega)$



b)





c) Multiply the incoming signal by $\cos(\omega t)$ where ω cooresponds to the appropiate signal. Use a lowpass signal from $-\omega_M$ to ω_M to capture just one signal. Final, divide by π to correct for amplitude

$i(t) = C \frac{d}{dt} V_{out}(t)$ $\quad i(\omega) = C\, V_{out}(\omega)\, j\omega$

$V_L(t) = L\, d/dt\, i(t)$ $\quad V_L(\omega) = L\, i(\omega)\, j\omega = -\omega^2 LC\, V_{out}(\omega)$

i) $V_{in}(t) = V_R(t) + V_L(t) + V_{out}(t)$

$V_{in}(t) = R\,i(t) + V_L(t) + V_{out}(t)$

$V_{in}(\omega) = RC\,j\omega\, V_{out}(\omega) - \omega^2 LC\, V_{out}(\omega) + V_{out}(\omega)$

$$H(\omega) = \frac{V_{out}(\omega)}{V_{in}(\omega)} = \frac{1}{RCj\omega - \omega^2 LC + 1} \qquad |H(\omega)| = \frac{1}{|RCj\omega - \omega^2 LC + 1|} = \frac{1}{\sqrt{(RC\omega)^2 + (\omega^2 LC + 1)^2}}$$

$\omega \to 0 \quad |H(\omega)| \to 1 \qquad \omega \to \infty \quad |H(\omega)| \to 0$

$$\frac{d|H(\omega)|}{d\omega} = \frac{d}{d\omega}\left(\frac{1}{\sqrt{(RC\omega)^2 + (\omega^2 LC + 1)^2}}\right) = -\frac{C\omega(2CL^2\omega^2 + CR^2 + 2L)}{(C^2R^2\omega^2 + (CL\omega^2 + 1)^2)^{3/2}}$$

$$0 = -\frac{\cancel{C\omega}(2CL^2\omega^2 + CR^2 + 2L)}{\cancel{(C^2R^2\omega^2 + (CL\omega^2+1)^2)^{3/2}}}$$

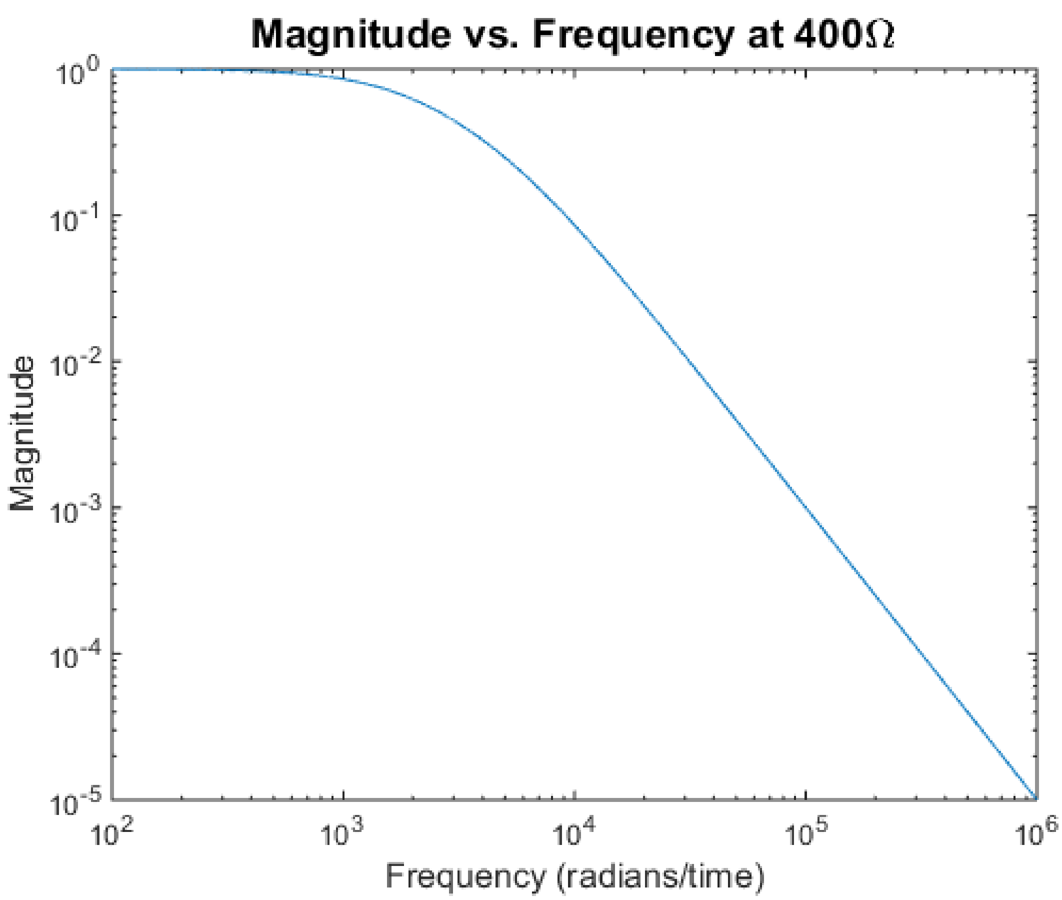
$$0 = 2CL^2\omega^2 + CR^2 + 2L$$

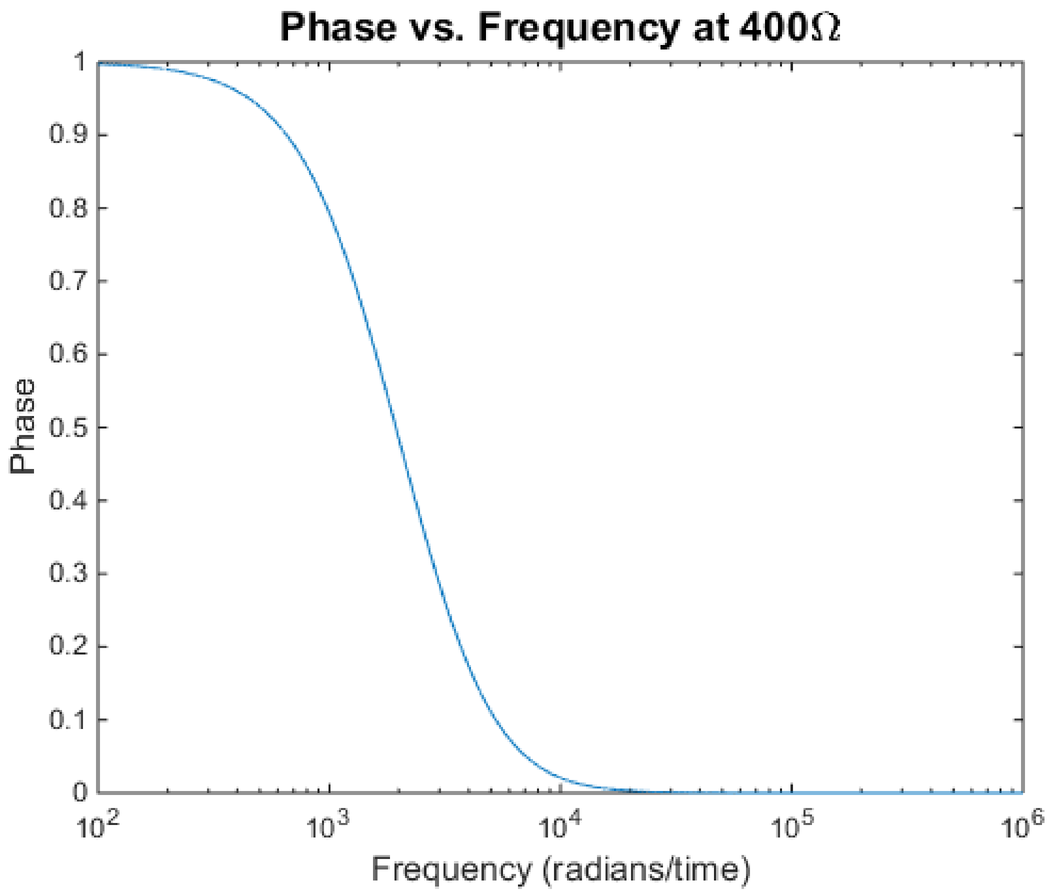
$$\omega^2 = \frac{-CR^2 - 2L}{2CL^2}$$

$$\omega = \pm\frac{\sqrt{-CR^2 - 2L}}{\sqrt{2C}\,L}$$

```
R = 400;
L = 10e-2;
C = 10e-7;

w = logspace (2,6,10000);

mag = 1./(sqrt((R.*C.*w).^2+(w.^2.*C.*L+1).^2));
phase = mag.*exp(-1i.*atan(w.*R.*C));

loglog (w,mag)
title('Magnitude vs. Frequency at 400\Omega','FontSize',14)
xlabel('Frequency (radians/time)','FontSize',12)
ylabel('Magnitude','FontSize',12)

figure;
semilogx(w,phase)
title('Phase vs. Frequency at 400\Omega','FontSize',14)
xlabel('Frequency (radians/time)','FontSize',12)
ylabel('Phase','FontSize',12)
```

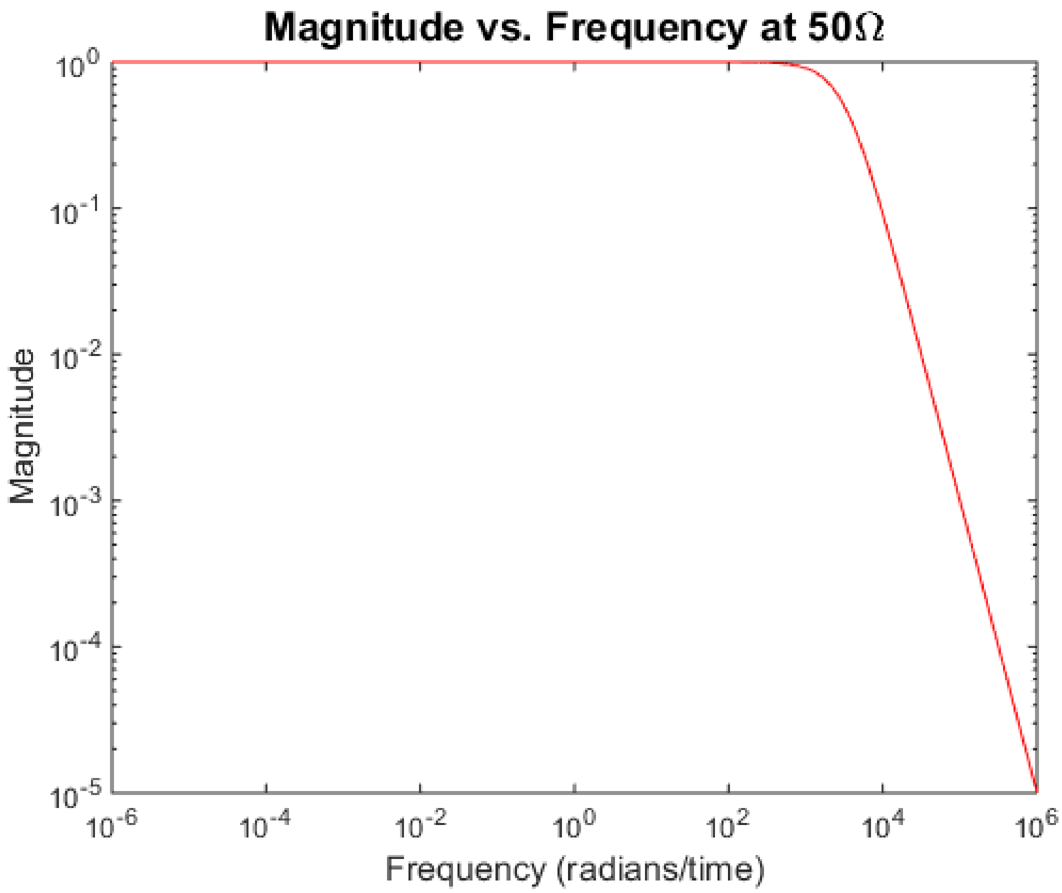
```
Warning: Imaginary parts of complex X and/or Y arguments ignored
```

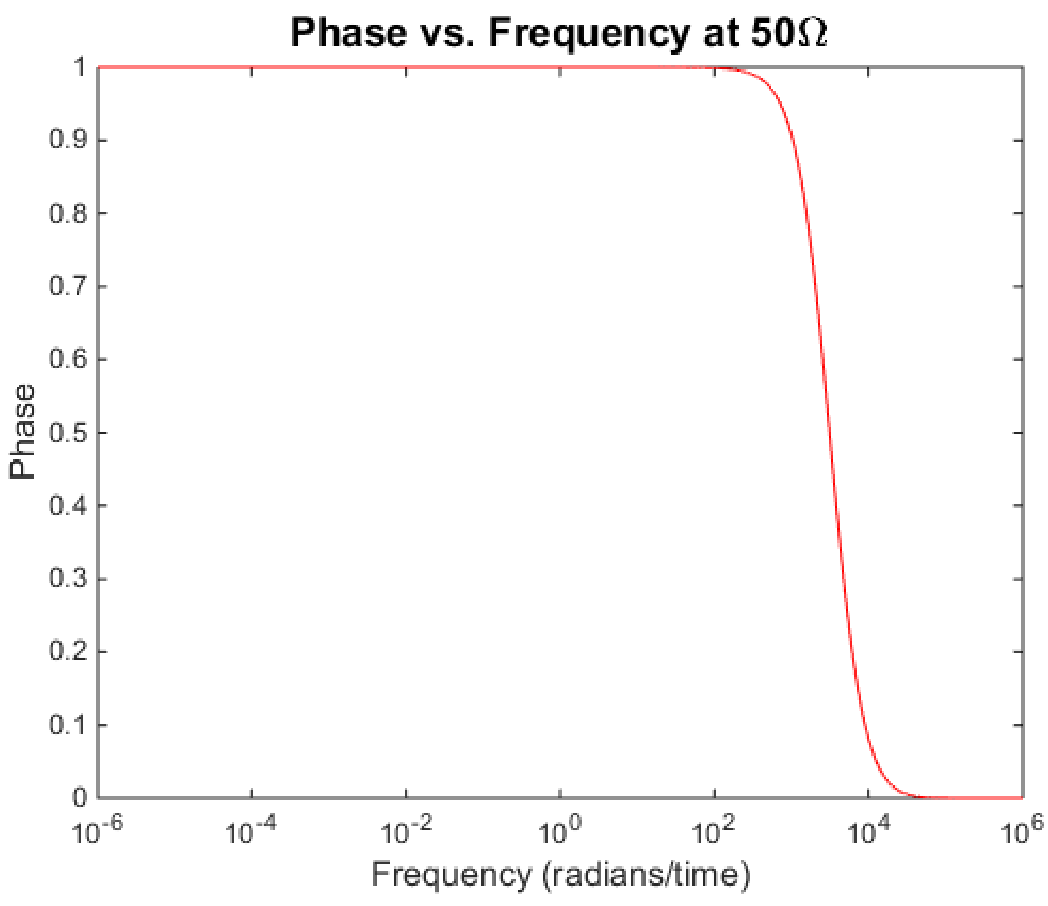





```
R = 50;
L = 10e-2;
C = 10e-7;

w = logspace (-6,6,10000);
mag = 1./(sqrt((R.*C.*w).^2+(w.^2.*C.*L+1).^2));
phase = mag.*exp(-1i.*atan(w.*R.*C));


loglog(w,mag,'r')
title('Magnitude vs. Frequency at 50\Omega','FontSize',14)
xlabel('Frequency (radians/time)','FontSize',12)
ylabel('Magnitude','FontSize',12)
figure;
semilogx(w,phase,'r')
title('Phase vs. Frequency at 50\Omega','FontSize',14)
xlabel('Frequency (radians/time)','FontSize',12)
ylabel('Phase','FontSize',12)
```

```
Warning: Imaginary parts of complex X and/or Y arguments ignored
```


Magnitude vs. Frequency at 50Ω
Magnitude
Frequency (radians/time)


Phase vs. Frequency at 50Ω
Phase
Frequency (radians/time)