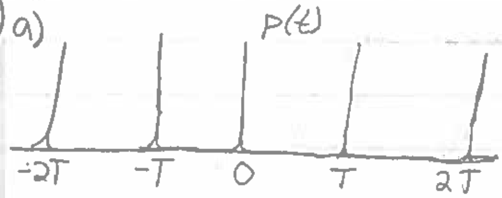
Jacob
Riedel

Sig Sys PS 07

1) a)



b) $p(t) = \sum_{k=-\infty}^{\infty} C_k e^{-j\frac{2\pi}{T}kt} = \sum_{k=-\infty}^{\infty} \frac{1}{T} e^{jk\omega_0 t}$

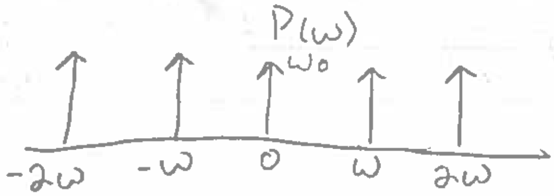
c) $x(t) = \sum_{k=-\infty}^{\infty} C_k e^{j\frac{2\pi}{T}kt}$, $e^{j\omega_0 kt} \rightarrow \text{Transform} = 2\pi\delta(\omega - k\omega_0)$

$X(\omega) = \sum_{k=-\infty}^{\infty} x(t) e^{-j\omega t} dt = \sum_{k=-\infty}^{\infty} C_k 2\pi\delta(\omega - k\omega_0)$

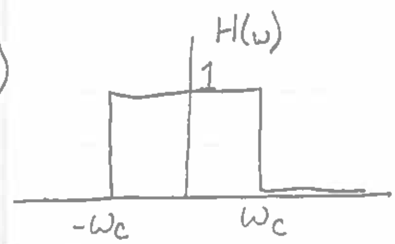
$X(\omega) = 2\pi \sum_{k=-\infty}^{\infty} C_k \delta(\omega - k\omega_0)$

d) $P(\omega) = \sum_{k=-\infty}^{\infty} \frac{1}{T} 2\pi\delta(\omega - k\omega_0) = \frac{2\pi}{T}\sum_{k=-\infty}^{\infty} 2\pi\delta(\omega-\omega_0) = \omega_0 p(\omega)$

e)



2a)

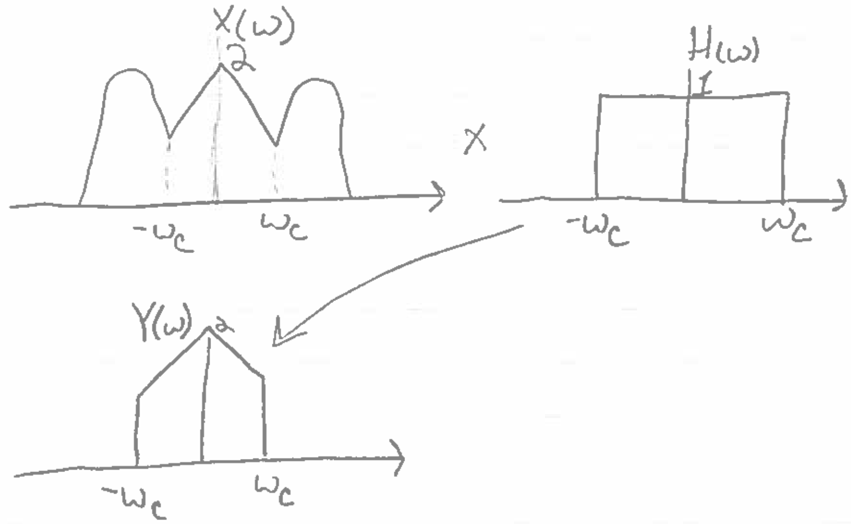


a) $h(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega) e^{j\omega t} d\omega = \frac{1}{2\pi}\Big(\cancel{\int_{-\infty}^{-\omega_c} 0 e^{j\omega t} d\omega} \xrightarrow{} 0 + \cancel{\int_{\omega_c}^{\infty} 0 e^{j\omega t}} \xrightarrow{} 0 +$

$\int_{-\omega_c}^{\omega_c} 1 e^{j\omega t} d\omega\Big) = \frac{1}{2\pi}\int_{-\omega_c}^{\omega_c} e^{j\omega t} d\omega = \frac{1}{2\pi}\left(\frac{1}{jt}e^{j\omega_c t} - \frac{1}{jt}e^{-j\omega_c t}\right)$

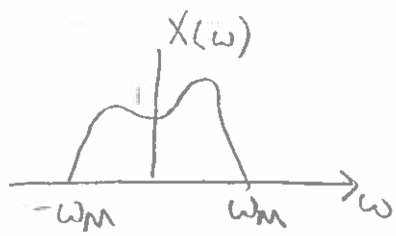
$= \frac{1}{\pi t}\left(\frac{1}{2j}e^{j\omega_c t} - \frac{1}{2j}e^{-j\omega_c t}\right) = \frac{1}{\pi t}\sin(\omega_c t)$

b)

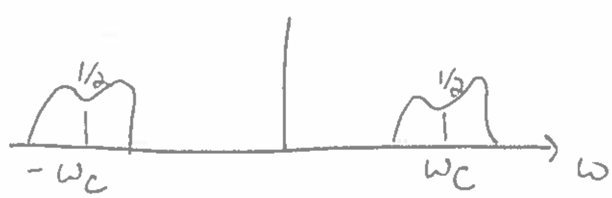


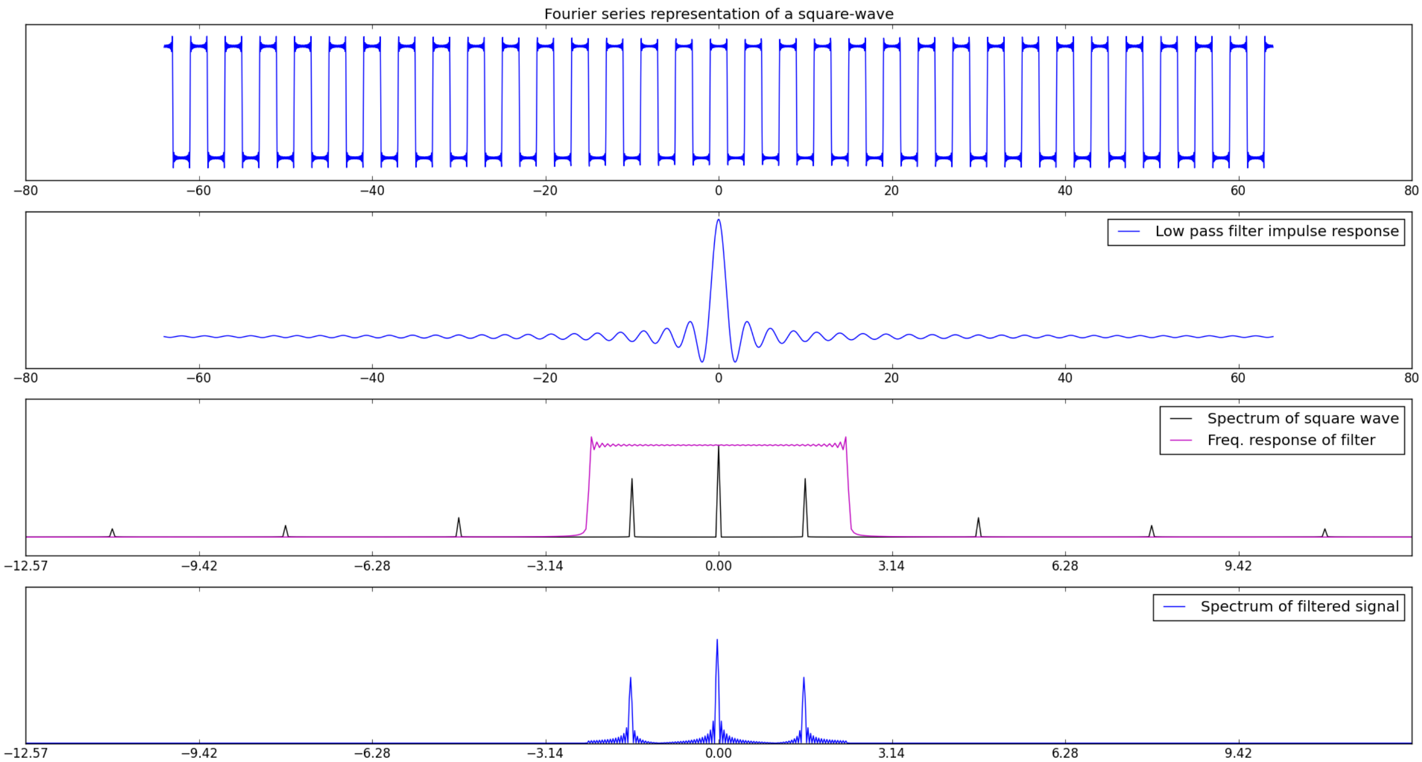
c) This LTI system clips all frequencies above ω_c and $-\omega_c$. Because it has a magnitude of 1, the wave is perfectly preserved between $-\omega_c$ and ω_c

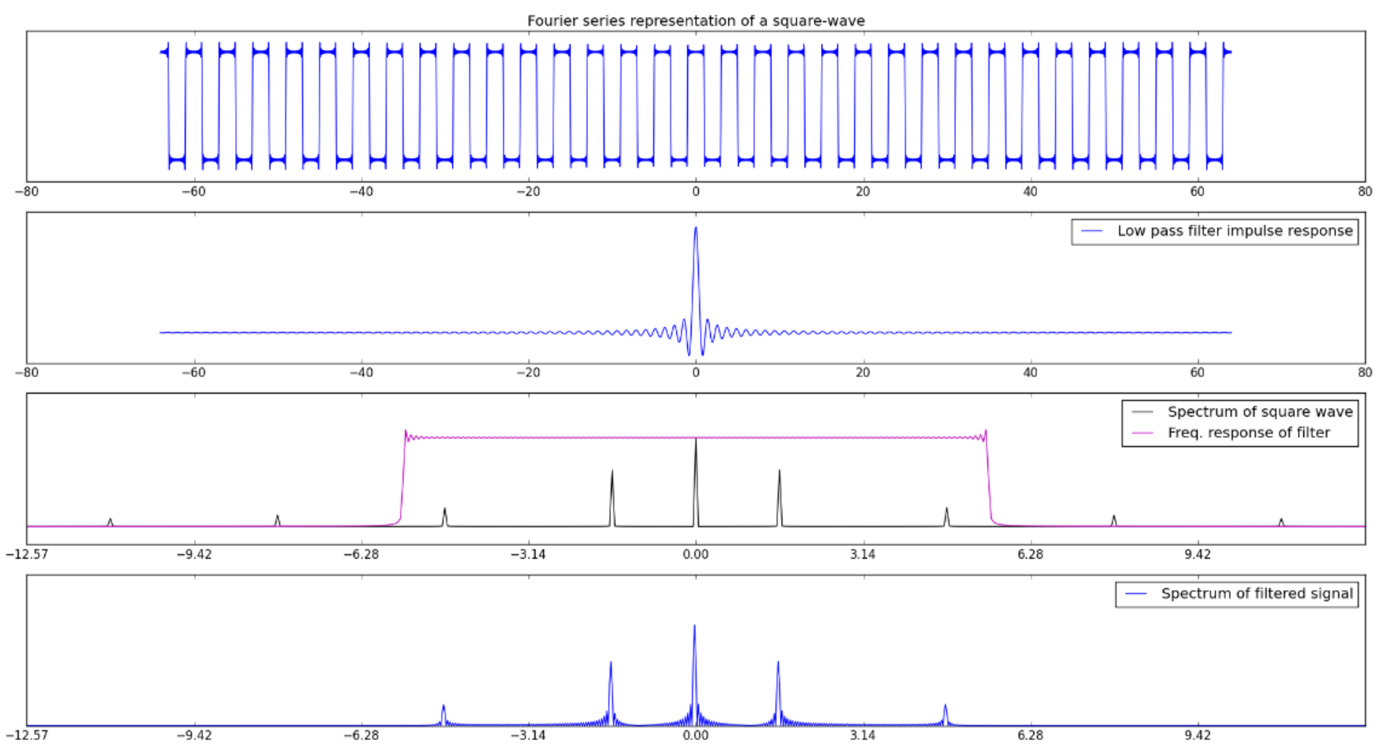
3)



$$y(t) = x(t)\cos(\omega_c t)$$
$$y(t) = \tfrac{1}{2}\left(e^{j\omega_c t} + e^{-j\omega_c t}\right)x(t)$$
$$Y(\omega) = \tfrac{1}{2}\left(X(\omega - \omega_c) + X(\omega + \omega_c)\right)$$




Fourier series representation of a square-wave
Low pass filter impulse response
Spectrum of square wave
Freq. response of filter
Spectrum of filtered signal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