Sun's position for navigation with DM15/HP-15c Manual

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Overview

The handheld calculator DM15 (a HP-15c look-a-like with more memory) can be used for determining the sun's position with precision enough for celestial navigation purposes. The accompanying program, listed in the Appendix, constitutes a handy tool for either finding the *Nautical Almanac*'s entries GHA (*Greenwich Hour Angle*) and Declination, or — with AP (*Assumed Position*) — directly calculate the sun's Altitude *Hc* and Azimuth *Az* for this position.

The algorithm relies on pure Keplerian motion of the sun. No planetary perturbations are taken into account. Resulting angular accuracy is about 1 minute of arc, which is adequate for general navigation at sea.

General notes

Before use, notice that:

- All times entered are UT ("GMT") even if observer's longitude is not the prime meridian. Of course, local hour angles take longitude into consideration, but all times are still UT. Time format is *hh.mmss*, where *mm* and *ss* must be two-digit numbers.
- The program makes use of the calculator's internal decimal to degrees, minutes and seconds routines both for **entry** and **displayed result**. In navigation a more common format of degrees, minutes and tenths of a minute is used. That conversion, if needed, is readily done by dividing the arc-seconds number or multiplying the minute's decimal by 6.

  Example

  Convert angle in *ddd.mmss* to *ddd.mm·t*

  $98^\circ\,26'\,12''$, entered as `98.2612`, is $98^\circ\,26{\cdot}2'$ where $12''/6 = 2$
  $98^\circ\,26'\,43''$, entered as `98.2643`, is $98^\circ\,26{\cdot}7'$ where $43''/6 \approx 7$

  ■

  Example

  Convert angle in *ddd.mm·t* to *ddd.mmss*

  $14^\circ\,7{\cdot}3'$ is $14^\circ\,7'\,18''$ where $3 \cdot 6 = 18$
  $277^\circ\,4{\cdot}5'$ is $277^\circ\,4{\cdot}30'$ where $5 \cdot 6 = 30$

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Usage

Functional buttons are

Button	Function
A	Date for Aries angle at UT=0h
B	Time for Sun Altitude and Azimuth
C	SHA and declination for *own object*
D	Time for *own object*'s Altitude and Azimuth
E	Time for GHA Aries and LHA Aries
.5	After **B** for GHA and declination (as a Nautical Almanac entry)

An AP (Assumed Position) is entered in registers `8` and `.8` before any calculations can be peformed.

Example

Entering AP.

A location of Lat N58° 34′, Long E14° 34′ 12″ is entered into registers `8` and `.8`:

Data	Format	Key		Display shows
`58.3400`	±*dd.mmss*	g ->H	`STO 8`	`58.5667`
`14.3412`	±*dd.mmss*	g ->H	`STO .8`	`14.5700`

East and North are positive, West and South are negative.
The position is permanently stored until manually changed and need only be set once.

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Every day has its own parameters that require the `A`-routine to be run each new day.

Example

Entering the date.

Enter June 12th 2022, i.e. year 2022, month 6 and day 12

Data	Format	Key	Display shows
`2022`	*YYYY*	ENTER	`2022.0000`
`6`	*mm*	ENTER	`6.0000`
`12`	*dd*	f `A`	`260.1816` (260° 18′ 17″ = 260° 18·3′, GHA Aries at 0h)

This is only necessary once for each day.

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Next the sun's coordinates for time of date UT/GMT can be calculated.

Example

Find sun's Hc and Az for UT 09h 54m 48s. Date entered as above.

Enter time in format *hh.mmss* then use routine `B`.

Data	Format	Key	Display shows
`9.5448`	*hh.mmss*	f `B`	`52.3845` (Hc = 52° 38′ 45″)
		`x<>y`	`154.1140` (Az = 154° 11′ 40″)

Result: Hc = 52° 38·7′, Az = 154°

A new time can be entered directly. For example, also find sun's Hc and Az a few minutes later at UT 10h 02m 30s.

Data	Format	Key	Display shows
`10.0230`	*hh.mmss*	f `B`	`53.0338` (Hc = 53° 03′ 38″)
		`x<>y`	`157.0236` (Az = 157° 2′ 36″)

Result: Hc = 53° 03·6′, Az = 157°

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GHA and declination

The program can also produce values for GHA and declination imitating the *Nautical Almanac*.

Example

Find GHA and decl for 10h on June 12th 2023

The date must be set as above. Now enter time to get Hc and Az if wanted. Or ignore the output and use `GSB .5` to get GHA and decl:

Data	Format	Key	Display shows
`10.0000`	*hh.mmss*	f `B`	`52.5508` (Hc = 52° 55′ 08″, ignore)
		`GSB .5`	`330.0248` (GHA = 330° 2′ 48″)
		`x<>y`	`23.0901` (Decl = 23° 09′ 01″)

Result: GHA = 330° 2·8′, Decl = 23° 09·0′ (*Nautical Almanac* gives 330° 2·8′ and 23° 8·8′).

GHA and Decl can of course be calculated for any other time during the day in the same manner.

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Specify and calculate position for an object with known SHA and declination

The coordinates of a celestial object, for example a star, are given as SHA (*Sidereal Hour Angle*) and declination.[1]

Example

Coordinates of *Vega* (SHA 80° 34·3′, declination 38° 48·2′) are entered

Data	Format	Key	Display shows
`80.3418`	*ddd.mmss*	ENTER	`80.3418`
`38.4812`	*ddd.mmss*	f `C`	`279.4283` (RA in decimal degrees as a bonus)

Now find *Vega*'s calculated position for UT = 23h 02m 10s on June 12th 2023 already entered above.

Data	Format	Key	Display shows
`23.0210`	*hh.mmss*	f `D`	`67.0015` (Hc = 67° 00′ 15″)
		`x<>y`	`141.1224` (Az = 141° 12′ 24″)

Result: Vega can be expected at $Hc = 67° 0{\cdot}2'$ and $Zn = 141°$. Set the sextant for 67° and search for it in south-east.

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GHA Aries and LHA Aries

Find GHA Aries on 4 October 2022 at 7h 57m 20s. Also find LHA Aries longitude in `.8` (14° 34′ 12″ E as before).

Data	Format	Key	Display shows
`2022`	*YYYY*	ENTER	`2022.0000`
`10`	*mm*	ENTER	`10.0000`
`4`	*dd*	f `A`	`12.4006` (12° 40′ 6″, GHA Aries at 0h)
`7.5720`	*hh.mmss*	f `E`	`132.1942` (GHA Aries = 132° 19′ 42″)
		`x<>y`	`146.5354` (LHA Aries = 146° 53′ 54″)

Use as Sight Reduction Table

The program can also solve the navigational triangle and be used as a (Ho-214/Ho-229 etc) *Sight Reduction Table* replacement. To solve the triangle AP latitude, object's declination and hour angle needs to be entered.

[1] Right Ascension can be entered as $\alpha = 360 - SHA$ if needed.

AP latitude is entered in register `8` as before, declination is set via **C** and hour angle is entered into register `.2`. The hour angle is positive if westward.

Example

Find Hc and Az as in Ho-214

Assume longitude 58° N, Declination 8° 30′ and an hour angle of 54° (object to the west of observer).

Data	Format	Key	Display shows
`58`	*dd.mmss*	`STO 8`	`58.0000`
`8.3000`	*dd.mmss*	**C**	`8.5000`
`54`	*dd.mmss*	g ->H	`54.0000`
		`STO .2`	`54.0000`
		`GSB 7`	`25.4102` (Hc = 25° 41′ 02″)
		`x<>y`	`242.3616` (Az = 242° 36′ 16″ = 242.60°)

Ho-214 gives Alt. = 25° 41·0′ and Az. = 117.4°. Where true azimuth is 360 − 117.4 = 242.6°.

Program and information

Register usage

The lower registers `r0..r7` are used by the calculator's statistics functions and are not *permanently* used by this program. They *are* used however for intermediate results via the normal operating sequences **A**-**B** or **A**-**C**-**D** or **A**-**E**.

In short: Use `r0..r7` as you wish but they will be altered by **A**.

Program installation

For a fresh install of the program perform steps 1–6 below.

1. Make space on the DM15 for program and registers:
   - Enter `21 f DIM (i)`
   - Double check: `g MEM` should read `21.209`
2. In HP-15C/Preferences/DM15 menu: Select 229 as Number of registers.
3. File/Open Program: file.15c
4. Write program to DM15.
   - On device enable serial communication (hold C while pressing ON-button)
   - File/Write DM15

5. Before use a number of constants must be entered in the following registers:

Register	Constant
.3	279.4055638
.4	283.3328093
.5	1.016860112
.6	23.44188400
.7	0.002737909

6. That's it. Now the run samples in this document give expected results.

Program listing

Note: In the listing below some minor self explanatory key appearances have changed. SIN^{-1} is replaced with ASIN etc, x↔y is x<>y and R↓ is Rv.

```
000 {               }
001 { 42 21 48   8 } f LBL .8
002 {            3 } 3
003 {            6 } 6
004 {            0 } 0
005 {        43 32 } g RTN
006 {     42 21  4 } f LBL 4
007 {           23 } SIN
008 {           34 } x<>y
009 {           23 } SIN
010 {     22 48  6 } GTO .6
011 {     42 21  5 } f LBL 5
012 {           24 } COS
013 {           34 } x<>y
014 {           24 } COS
015 {     22 48  6 } GTO .6
016 {     42 21  2 } f LBL 2
017 {     32 48  8 } GSB .8
018 {           10 } /
019 {        42 44 } f FRAC
020 {     43 30  1 } g TEST x>0
021 {        22  3 } GTO 3
022 {            1 } 1
023 {           40 } +
024 {     42 21  3 } f LBL 3
025 {     32 48  8 } GSB .8
026 { 42 21 48  6 } f LBL .6
027 {           20 } *
028 {        43 32 } g RTN
029 { 42 21 48  2 } f LBL .2
030 {            1 } 1
031 {            5 } 5
032 {     22 48  6 } GTO .6
033 {     42 21 12 } f LBL B
034 {        32 15 } GSB E
035 {        45  6 } RCL 6
036 {            2 } 2
037 {            4 } 4
038 {            5 } 5
039 {            9 } 9
040 {            9 } 9
041 {            4 } 4
042 {            4 } 4
043 {           48 } .
044 {            5 } 5
045 {           30 } -
046 {        45  4 } RCL 4
047 {            2 } 2
048 {            4 } 4
049 {           10 } /
050 {           40 } +
051 {     32 48  0 } GSB .0
052 {           20 } *
053 {     45 48  3 } RCL .3
054 {           40 } +
055 {     45 48  4 } RCL .4
056 {           30 } -
057 {        42  3 } f -> RAD
058 {        44  9 } STO 9
059 {        43  8 } g RAD
060 {           36 } ENTER
061 {            1 } 1
062 {            0 } 0
063 {     42  7  9 } f FIX 9
064 {     42 10  8 } f SOLVE 8
065 {     42  7  4 } f FIX 4
066 {            2 } 2
067 {           10 } /
068 {           25 } TAN
069 {     45 48  5 } RCL .5
070 {           20 } *
071 {        43 25 } g ATAN
072 {            2 } 2
073 {           20 } *
074 {        43  3 } g ->DEG
075 {        43  7 } g DEG
```

```
076 {     45 48   4 } RCL .4
077 {           40 } +
078 {     44 48   0 } STO .0
079 {     45 48   0 } RCL .0
080 {           23 } SIN
081 {     45 48   6 } RCL .6
082 {           24 } COS
083 {           20 } *
084 {     45 48   0 } RCL .0
085 {           24 } COS
086 {        43  1 } g ->P
087 {           33 } Rv
088 {        44  9 } STO 9
089 {     45 48   0 } RCL .0
090 {           23 } SIN
091 {     45 48   6 } RCL .6
092 {           23 } SIN
093 {           20 } *
094 {        43 23 } g ASIN
095 {     44 48   0 } STO .0
096 {        45  9 } RCL 9
097 { 42 21 48   1 } f LBL .1
098 {        45  5 } RCL 5
099 {     32 48   8 } GSB .8
100 {        45  9 } RCL 9
101 {           30 } -
102 {           40 } +
103 {        32  2 } GSB 2
104 {     44 48   2 } STO .2
105 {        32  7 } GSB 7
106 {        43 32 } g RTN
107 {     42 21  7 } f LBL 7
108 {     45 48  2 } RCL .2
109 {           24 } COS
110 {        45  8 } RCL 8
111 {     45 48  0 } RCL .0
112 {        32  5 } GSB 5
113 {           20 } *
114 {     45 48  0 } RCL .0
115 {        45  8 } RCL 8
116 {        32  4 } GSB 4
117 {           40 } +
118 {        43 23 } g ASIN
119 {           36 } ENTER
120 {           36 } ENTER
121 {     44 48  1 } STO .1
122 {        45  8 } RCL 8
123 {        32  4 } GSB 4
124 {           16 } CHS
125 {     45 48  0 } RCL .0
126 {           23 } SIN
127 {           40 } +
128 {           34 } x<>y
129 {        45  8 } RCL 8
130 {        32  5 } GSB 5
131 {           10 } /
132 {        43 24 } g ACOS
133 { 42  4 48  2 } f x<> .2
134 {           23 } SIN
135 {     43 30  2 } g TEST x<0
136 {        22  6 } GTO 6
137 {     32 48  8 } GSB .8
138 {     45 48  2 } RCL .2
139 {           30 } -
140 {     44 48  2 } STO .2
141 {     42 21  6 } f LBL 6
142 {     45 48  1 } RCL .1
143 {     45 48  2 } RCL .2
144 {        22  9 } GTO 9
145 {     42 21  8 } f LBL 8
146 {           23 } SIN
147 {           48 } .
148 {            0 } 0
149 {            1 } 1
150 {            6 } 6
151 {            7 } 7
152 {            1 } 1
153 {            8 } 8
154 {           20 } *
155 {           16 } CHS
156 {           40 } +
157 {        45  9 } RCL 9
158 {           30 } -
159 {        43 32 } g RTN
160 {     42 21 13 } f LBL C
161 {        43  2 } g ->H
162 {     44 48  0 } STO .0
163 {           33 } Rv
164 {        43  2 } g ->H
165 {     32 48  8 } GSB .8
166 {           34 } x<>y
167 {           30 } -
168 {        44  9 } STO 9
169 {        43 32 } g RTN
170 {     42 21 15 } f LBL E
171 {        43  2 } g ->H
172 {        44  4 } STO 4
173 {     45 48  7 } RCL .7
174 {            1 } 1
175 {           40 } +
176 {           20 } *
177 {     32 48  2 } GSB .2
178 {        45  1 } RCL 1
179 {           40 } +
180 {           36 } ENTER
181 {           36 } ENTER
182 {     45 48  8 } RCL .8
183 {           40 } +
184 {        32  2 } GSB 2
185 {        44  5 } STO 5
186 {           34 } x<>y
187 {        32  2 } GSB 2
188 {           34 } x<>y
189 {     42 21  9 } f LBL 9
190 {        42  2 } f ->H.MS
191 {           34 } x<>y
192 {        42  2 } f ->H.MS
193 {        43 32 } g RTN
```

```
194 {     42 21   0 } f LBL 0
195 {             1 } 1
196 {            36 } ENTER
197 {             0 } 0
198 {        32   1 } GSB 1
199 {             2 } 2
200 {             4 } 4
201 {             1 } 1
202 {             5 } 5
203 {             0 } 0
204 {             2 } 2
205 {             0 } 0
206 {            30 } -
207 {     32 48   0 } GSB .0
208 {            20 } *
209 {             9 } 9
210 {             9 } 9
211 {            48 } .
212 {             4 } 4
213 {             1 } 1
214 {             3 } 3
215 {        43   2 } g ->H
216 {            40 } +
217 {        32   2 } GSB 2
218 {        44   3 } STO 3
219 {        45   6 } RCL 6
220 {        44   0 } STO 0
221 {        43  32 } g RTN
222 {     42 21  11 } f LBL A
223 {        44   4 } STO 4
224 {            33 } Rv
225 {        44   5 } STO 5
226 {            33 } Rv
227 {        32   0 } GSB 0
228 {        45   1 } RCL 1
229 {        45   5 } RCL 5
230 {        45   4 } RCL 4
231 {        32   1 } GSB 1
232 {        45   0 } RCL 0
233 {            30 } -
234 {     32 48   0 } GSB .0
235 {            20 } *
236 {        45   3 } RCL 3
237 {            40 } +
238 {        32   2 } GSB 2
239 {        44   1 } STO 1
240 {        42   2 } f ->H.MS
241 {        43  32 } g RTN
242 {     42 21   1 } f LBL 1
243 {             1 } 1
244 {             7 } 7
245 {             2 } 2
246 {             1 } 1
247 {             0 } 0
248 {             1 } 1
249 {             3 } 3
250 {            48 } .
251 {             5 } 5
252 {            40 } +
253 {            34 } x<>y
254 {        44   1 } STO 1
255 {             2 } 2
256 {             7 } 7
257 {             5 } 5
258 {            20 } *
259 {             9 } 9
260 {            10 } /
261 {        43  44 } g INT
262 {            40 } +
263 {            34 } x<>y
264 {            36 } ENTER
265 {     42   4   1 } f x<> 1
266 {             9 } 9
267 {            40 } +
268 {             1 } 1
269 {             2 } 2
270 {            10 } /
271 {        43  44 } g INT
272 {            40 } +
273 {             7 } 7
274 {            20 } *
275 {             4 } 4
276 {            10 } /
277 {        43  44 } g INT
278 {            16 } CHS
279 {            40 } +
280 {        45   1 } RCL 1
281 {             3 } 3
282 {             6 } 6
283 {             7 } 7
284 {            20 } *
285 {            40 } +
286 {        44   6 } STO 6
287 {        43  32 } g RTN
288 {     42 21  14 } f LBL D
289 {        32  15 } GSB E
290 {     32 48   1 } GSB .1
291 {        43  32 } g RTN
292 { 42 21 48   5 } f LBL .5
293 {        45   5 } RCL 5
294 {     45 48   8 } RCL .8
295 {            30 } -
296 {     32 48   8 } GSB .8
297 {        45   9 } RCL 9
298 {            30 } -
299 {            40 } +
300 {        32   2 } GSB 2
301 {     45 48   0 } RCL .0
302 {        22   9 } GTO 9
303 { 42 21 48   0 } f LBL .0
304 {     45 48   7 } RCL .7
305 {     32 48   8 } GSB .8
306 {            20 } *
307 {        43  32 } g RTN
```

Program Resources

Labels

Name	Description
A	
B	
C	
D	
E	
0	
1	

Name	Description
2	
3	
4	
5	
6	
7	Lat -> r8, LHA -> r12, decl -> r10 ==>> Hc, Zn
8	

Name	Description
9	
11	
12	
15	After B: GHA and Declination
16	
18	

Storage Registers

Name	Description
0	JD of start of year
1	LHA 0h
3	GMST yearly constant
4	UT entered 0..24, decimal
5	LHA Aries
6	JD of date

Name	Description
8	Observer's latitude, degrees (N/S=+/-)
9	Objects Right Ascension, degrees
10	Object's declination, degrees (N/S=+/-)
11	Hc, calculated altitude, degrees
12	LHA of object -> Zn, calculated azimuth
13	Constant, L of epoch 279.4055638 for JD=2459944.5

Name	Description
14	Constant, Long of perigee, 283.3328090 fo JD above
15	1.016860112 [sqrt((1+e)/(1-e)]
16	Constant, Obliquity, 23.4382144
17	1/365.2422
18	Observer's longitude (E/W=+/-)

Program

Line	Display	Key Sequence
000		
001	42,21, .8	f LBL . 8
002	3	3
003	6	6
004	0	0
005	43 32	g RTN
006	42,21, 4	f LBL 4
007	23	SIN
008	34	x↔y
009	23	SIN

Line	Display	Key Sequence
113	45 .0	RCL . 0
114	32 5	GSB 5
115	20	×
116	45 .0	RCL . 0
117	45 8	RCL 8
118	32 4	GSB 4
119	40	+
120	43 23	g SIN-1
121	36	ENTER
122	36	ENTER

Line	Display	Key Sequence
226	20	×
227	2	2
228	4	4
229	0	0
230	0	0
231	48	.
232	0	0
233	5	5
234	1	1
235	2	2

010	22 .6	GTO . 6	123	44 .1	STO . 1	236	6	6
011	42,21, 5	f LBL 5	124	45 8	RCL 8	237	2	2
012	24	COS	125	32 4	GSB 4	238	40	+
013	34	x↔y	126	16	CHS	239	20	×
014	24	COS	127	45 .0	RCL . 0	240	6	6
015	22 .6	GTO . 6	128	23	SIN	241	48	.
016	42,21, 2	f LBL 2	129	40	+	242	6	6
017	32 .8	GSB . 8	130	34	x↔y	243	4	4
018	10	÷	131	45 8	RCL 8	244	6	6
019	42 44	f FRAC	132	32 5	GSB 5	245	0	0
020	43,30, 1	g TEST x>0	133	10	÷	246	6	6
021	22 3	GTO 3	134	43 24	g COS^{-1}	247	5	5
022	1	1	135	42, 4, .2	f X↔ . 2	248	6	6
023	40	+	136	23	SIN	249	40	+
024	42,21, 3	f LBL 3	137	43,30, 2	g TEST x<0	250	32 .2	GSB . 2
025	32 .8	GSB . 8	138	22 6	GTO 6	251	32 2	GSB 2
026	42,21, .6	f LBL . 6	139	32 .8	GSB . 8	252	44 3	STO 3
027	20	×	140	45 .2	RCL . 2	253	45 6	RCL 6
028	43 32	g RTN	141	30	−	254	44 0	STO 0
029	42,21, .2	f LBL . 2	142	44 .2	STO . 2	255	43 32	g RTN
030	1	1	143	42,21, 6	f LBL 6	256	42,21,11	f LBL A
031	5	5	144	45 .1	RCL . 1	257	44 4	STO 4
032	22 .6	GTO . 6	145	45 .2	RCL . 2	258	33	R↓
033	42,21,12	f LBL B	146	22 9	GTO 9	259	44 5	STO 5
034	32 15	GSB E	147	42,21, 8	f LBL 8	260	33	R↓
035	45 6	RCL 6	148	23	SIN	261	32 0	GSB 0
036	2	2	149	48	.	262	45 1	RCL 1
037	4	4	150	0	0	263	45 5	RCL 5
038	5	5	151	1	1	264	45 4	RCL 4
039	9	9	152	6	6	265	32 1	GSB 1
040	9	9	153	7	7	266	45 0	RCL 0

041	4	4	154	1	1	267	30	–
042	4	4	155	8	8	268	45 .7	RCL . 7
043	48	.	156	20	×	269	32 .8	GSB . 8
044	5	5	157	16	CHS	270	20	×
045	30	–	158	40	+	271	20	×
046	45 4	RCL 4	159	45 9	RCL 9	272	45 3	RCL 3
047	2	2	160	30	–	273	40	+
048	4	4	161	43 32	g RTN	274	32 2	GSB 2
049	10	÷	162	42,21,13	f LBL C	275	44 1	STO 1
050	40	+	163	43 2	g →H	276	42 2	f →H.MS
051	45 .7	RCL . 7	164	44 .0	STO . 0	277	43 32	g RTN
052	32 .8	GSB . 8	165	33	R↓	278	42,21, 1	f LBL 1
053	20	×	166	43 2	g →H	279	1	1
054	20	×	167	32 .8	GSB . 8	280	7	7
055	45 .3	RCL . 3	168	34	x↔y	281	2	2
056	40	+	169	30	–	282	1	1
057	45 .4	RCL . 4	170	44 9	STO 9	283	0	0
058	30	–	171	43 32	g RTN	284	1	1
059	42 3	f → RAD	172	42,21,15	f LBL E	285	3	3
060	44 9	STO 9	173	43 2	g →H	286	48	.
061	43 8	g RAD	174	44 4	STO 4	287	5	5
062	36	ENTER	175	45 .7	RCL . 7	288	40	+
063	1	1	176	1	1	289	34	x↔y
064	0	0	177	40	+	290	44 1	STO 1
065	42, 7, 9	f FIX 9	178	20	×	291	2	2
066	42,10, 8	f SOLVE 8	179	32 .2	GSB . 2	292	7	7
067	42, 7, 4	f FIX 4	180	45 1	RCL 1	293	5	5
068	2	2	181	40	+	294	20	×
069	10	÷	182	36	ENTER	295	9	9
070	25	TAN	183	36	ENTER	296	10	÷
071	45 .5	RCL . 5	184	45 .8	RCL . 8	297	43 44	g INT
072	20	×	185	40	+	298	40	+

073	43 25	g TAN⁻¹	186	32 2	GSB 2	299	34	x↔y
074	2	2	187	44 5	STO 5	300	36	ENTER
075	20	×	188	34	x↔y	301	42, 4, 1	f X↔ 1
076	43 3	g →DEG	189	32 2	GSB 2	302	9	9
077	43 7	g DEG	190	34	x↔y	303	40	+
078	45 .4	RCL . 4	191	42,21, 9	f LBL 9	304	1	1
079	40	+	192	42 2	f →H.MS	305	2	2
080	44 .0	STO . 0	193	34	x↔y	306	10	÷
081	45 .0	RCL . 0	194	42 2	f →H.MS	307	43 44	g INT
082	23	SIN	195	43 32	g RTN	308	40	+
083	45 .6	RCL . 6	196	42,21, 0	f LBL 0	309	7	7
084	24	COS	197	1	1	310	20	×
085	20	×	198	36	ENTER	311	4	4
086	45 .0	RCL . 0	199	0	0	312	10	÷
087	24	COS	200	32 1	GSB 1	313	43 44	g INT
088	43 1	g →P	201	2	2	314	16	CHS
089	33	R⬇	202	4	4	315	40	+
090	44 9	STO 9	203	1	1	316	45 1	RCL 1
091	45 .0	RCL . 0	204	5	5	317	3	3
092	23	SIN	205	0	0	318	6	6
093	45 .6	RCL . 6	206	2	2	319	7	7
094	23	SIN	207	0	0	320	20	×
095	20	×	208	30	–	321	40	+
096	43 23	g SIN⁻¹	209	3	3	322	44 6	STO 6
097	44 .0	STO . 0	210	6	6	323	43 32	g RTN
098	45 9	RCL 9	211	5	5	324	42,21,14	f LBL D
099	42,21, .1	f LBL . 1	212	2	2	325	32 15	GSB E
100	45 5	RCL 5	213	5	5	326	32 .1	GSB . 1
101	32 .8	GSB . 8	214	10	÷	327	43 32	g RTN
102	45 9	RCL 9	215	36	ENTER	328	42,21, .5	f LBL . 5
103	30	–	216	36	ENTER	329	45 5	RCL 5

104	40	+	217	48	.	330	45 .8	RCL . 8
105	32 2	GSB 2	218	0	0	331	30	–
106	44 .2	STO . 2	219	0	0	332	32 .8	GSB . 8
107	32 7	GSB 7	220	0	0	333	45 9	RCL 9
108	43 32	g RTN	221	0	0	334	30	–
109	42,21, 7	f LBL 7	222	2	2	335	40	+
110	45 .2	RCL . 2	223	5	5	336	32 2	GSB 2
111	24	COS	224	8	8	337	45 .0	RCL . 0
112	45 8	RCL 8	225	1	1	338	22 9	GTO 9