Sun's position for navigation with DM15/HP-15c Manual

Michael Josefsson

January 12, 2024

1.1 Overview

The handheld calculator DM15 (a HP-15c look-a-like with more memory) can be used for determining the sun's position with precision enough for celestial navigation purposes. The accompanying program, listed in the Appendix, constitutes a handy tool for either finding the *Nautical Almanac*'s entries GHA (*Greenwich Hour Angle*) and Declination, or — with AP (*Assumed Position*) — directly calculate the sun's Altitude *Hc* and Azimuth *Az* for this position.

The algorithm relies on pure Keplerian motion of the sun. No planetary perturbations are taken into account. Resulting angular accuracy is about 1 minute of arc, which is adequate for general navigation at sea.

1.2 General notes

Before use, notice that:

- All times entered are UT ("GMT") even if observer's longitude is not the prime meridian. Of course, local hour angles take longitude into consideration, but all times are still UT. Time format is *hh.mmss*, where *mm* and *ss* must be two-digit numbers.
- The program makes use of the calculator's internal decimal to degrees, minutes and seconds routines both for **entry** and **displayed result**. In navigation a more common format of degrees, minutes and tenths of a minute is used. That conversion, if needed, is readily done by dividing the arc-seconds number or multiplying the minute's decimal by 6.

Example

```
Convert angle in ddd.mmss to ddd.mm \cdot t
```

```
98^{\circ} \ 26' \ 12'', entered as 98.2612, is 98^{\circ} \ 26 \cdot 2' where 12''/6 = 2 98^{\circ} \ 26' \ 43'', entered as 98.2643, is 98^{\circ} \ 26 \cdot 7' where 43''/6 \approx 7
```

Example

Convert angle in $ddd.mm \cdot t$ to ddd.mmss

```
14^{\circ} \ 7 \cdot 3' is 14^{\circ} \ 7' \ 18'' where 3 \cdot 6 = 18 277^{\circ} \ 4 \cdot 5' is 277^{\circ} \ 4 \cdot 30' where 5 \cdot 6 = 30
```

1.3 Public routines

Full use of the program entails the following routines where **A–E** are main functionality and .0, 3, 4, 5, 9 are helpers. Their use is described in this document.

Button	Function
Α	Date for Υ hour angle at UT=0h
В	Time for Sun Altitude and Azimuth
С	SHA and declination for own object
D	$GMT o \mathit{own} \ \mathit{object}$'s Altitude and Azimuth
E	$GMT \to GHA\ \Upsilon$ and $LHA\ \Upsilon$
.0	GHA, time and declination \rightarrow Altitude and Azimuth
.3	Calculate average or mid time between two times or angles
.4	Increments and Corrections for any object
	given data from Nautical Almanac
.5	After $\mathbf{B} \to \mathrm{GHA}$ and declination
	(mimicks a Nautical Almanac entry)
.9	True altitude \rightarrow Apparent Altitude

1.4 Usage for Sun

The program's main purpose is to give altitude and azimuth for the sun. This requires an *Assumed Position*, date and time.

1.4.1 First set AP (Assumed Position)

An AP is entered in registers 8 and .8 before any calculations can be peformed.

Example

Entering AP.

A location of Lat N58° 34', Long E14° 34' 12'' is entered into registers 8 and .8:

Data	Format	Key		Display shows
58.3400	$\pm dd.mmss$	g ->H	STO 8	58.5667
14.3412	$\pm dd.mmss$	g ->H	STO .8	14.5700

East and North are positive, West and South are negative.

The position is permanently stored until manually changed and need only be set once.

1.4.2 Then set the date

Each day has its own parameters and require the A-routine to be run each new day.

Example

Entering the date.

Enter June 12th 2022, i.e. year 2022, month 6 and day 12

Data	Format	Key	Display shows
2022	YYYY	ENTER	2022.0000
6	mm	ENTER	6.0000
12	dd	f A	260.1816 ($260^{\circ} 18' 17'' = 260^{\circ} 18 \cdot 3'$, GHA Υ at 0h)

This is only necessary once for each day.

1.4.3 Finally enter time in GMT for Altitude and Azimuth

Next the sun's coordinates for time of date UT/GMT can be calculated.

Example

Find sun's Hc and Az for UT 09h 54m 48s. Date entered as above.

Enter time in format hh.mmss then use routine **B**.

Result: Hc = $52^{\circ} 38.7'$, Az = 154°

A new time can be entered directly. For example, also find sun's Hc and Az a few minutes later at UT 10h 02m 30s.

```
Data Format Key Display shows

10.0230 hh.mmss f B 53.0338 ( Hc = 53^{\circ} 03' 38'' )

x<>y 157.0236 ( Az = 157^{\circ} 2' 36'' )
```

Result: Hc = $53^{\circ} 03.6'$, Az = 157°

1.4.4 Calculate Sun's GHA and Declination

If wanted the program can also produce values for GHA and Declination imitating the *Nautical Almanac*.

Example

Find GHA and decl for 10h on June 12th 2023

The date must be set as above. Now enter time to get Hc and Az if wanted. Or ignore the output and use GSB . 5 to get GHA and decl:

Result: GHA = $330^{\circ} \cdot 2.8'$, Decl = $N23^{\circ} \cdot 09.0'$ (Nautical Almanac gives $330^{\circ} \cdot 2.8'$ and $N23^{\circ} \cdot 8.8'$).

GHA and Decl can of course be calculated for any other time during the day in the same manner.

1.5 SHA and Declination -> Altitude and Azimuth

The coordinates of a celestial object, for example a star, are given as SHA (Sidereal Hour Angle) and Declination.¹

Example

Coordinates of Vega (SHA 80° 34·3′, declination 38° 48·2′) are entered

Data	Format	Key	Display shows
80.3418	ddd.mmss	ENTER	80.3418
38.4812	ddd.mmss	f C	279.4283 (RA in decimal degrees as a bonus)

Now find *Vega*'s calculated position for UT = 23h 02m 10s on June 12th 2023 already entered above.

```
Data Format Key Display shows 23.0210 hh.mmss f D 67.0015 ( Hc = 67^{\circ} 00' 15'' ) x<>y 141.1224 ( Az = 141^{\circ} 12' 24'' )
```

Result: Vega can be expected at $Hc = 67^{\circ} \, 0.2'$ and $Zn = 141^{\circ}$. Set the sextant for 67° and search for it in south-east.

1.6 Date and time \rightarrow GHA Υ and LHA Υ

Find GHA Υ on 4 October 2022 at 7h 57m 20s. Also find LHA Υ longitude in .8 (E14° $34'\,12''$ as before).

Data	Format	Key	Display shows
2022	YYYY	ENTER	2022.0000
10	mm	ENTER	10.0000
4	dd		12.4006 (12° 40′ 6″, GHA Υ at 0h)
7.5720	hh.mmss	f E	132.1942 (GHA Υ = $132^{\circ} 19' 42''$)
		х<>у	146.5354 (LHA Υ = 146° 53′ 54″)

¹Right Ascension can be entered as $\alpha = 360 - SHA$ if needed.

1.7 .3, Average of two times or angles

The average of two times is useful for finding the middle time of a series of shots taken with a bubble sextant.

Example

A bubble sextant's first mark was at 21h18m22s and the last at 21h19m15s. When was halftime?

Data	Format	Key	Display shows	
21.1822	hh.mmss	ENTER	21.1822	First time
21.1915	hh.mmss	GSB .3	21.1849	Halftime was at 21h 18m 49s.

The routine also works for angles ddd.mmss.

Data	Format	Key	Display shows		
225.0954	ddd.mmss	ENTER	225.0954	GHA at 17h	
239.3742	ddd.mmss	ENTER	239.3742	GHA at 18h	
0.1829	0.mmss	GSB.4	229.3714	GHA Moon 17h 18m 29s:	= 229° 37′ 14″

1.8 .4, INCREMENTS AND CORRECTIONS

This routine eliminates the need for Increments and Corrections table in the *Nautical Almanac*. It performs a linear interpolation between two hourly values given the number of minutes and seconds after the first one. All entries are in sexagesimal form ddd.mmss and 0.mmss. The ddd.mmss-values can be both GHA or both Declinations and works for all objects.

Example

GHA Moon is at 17h $225^{\circ} 09.9'$ and at 18h $239^{\circ} 37.7'$. What is GHA at 17h 18m 29s?

Data	Format	Key	Display shows		
225.0954	ddd.mmss	ENTER	225.0954	GHA at 17h	
239.3742	ddd.mmss	ENTER	239.3742	GHA at 18h	
0.1829	0.mmss	GSB.4	229.3714	GHA Moon 17h 18m 29s:	= 229° 37′ 14″

Interpolated GHA Moon is $229^{\circ}\,37\cdot2'$. Use of Increments and Corrections gives exactly same answer.

1.9 .0, GHA and Declination \rightarrow Altitude and Azimuth

When coordinates are given as GHA (*Greenwich Hour Angle*) and Declination, as is the case for the planets and Moon listed in NA, proceed as follows. As GHA changes continuously during the day a time is also required.

Example

Coordinates of Sun as from Nautical Almanac above are GHA $330^{\circ}~2\cdot8'$, declination N23° $8\cdot8'$ at 10h.

Data	Format	Key	Display shows
330.0248	ddd.mmss	ENTER	330.0248
10.0000	ddd.mmss	GSB .0	279.3413 , ignore
23.0848	ddd.mmss	R/S	52.5455 (Hc = 52° 54′ 55″)
		x<>y	156.0821 (Az = $156^{\circ} 08' 21''$)

Result: The Sun is expected at a height of $52^{\circ} 54.9'$ and in a direction of 156° .²

1.10 .9, True Altitude → Apparent Altitude

The calculated altitude of any object is the True Altitude. Due to refraction the object will appear at a slightly higher altitude, its Apparent Altitude. This routine adds the correct small amount of refraction correction.³

Example

Calculated altitude of a star is $17^{\circ} 45' 00''$. What is the star's Apparent Altitude?

```
Data Format Key | Display shows 
17.4500 ddd.mmss GSB .9 | 17.4806 H_{app} = 17° 48′ 06″ 
 x<>y | 0.0306, the increment added, 03′ 06″
```

Note: With negligible error this routine can also be used to find the refraction correction as in ALTITUDE CORRECTION TABLES in *Nautical Almanac*.

Example

Measured altitude of a star is $17^{\circ} 48' 06''$. What is the star's True Altitude? (Inverse example of above)

```
Data Format Key Display shows 17.4806 ddd.mmss GSB .9 17.5112, ignore result x <> y 0.0306, the increment to subtract, 03'06''
```

This value matches closely the Nautical Almanac's value of -03'.

1.11 Use as Sight Reduction Table

The program can also solve the navigational triangle and be used as a (Ho-214/Ho-229 etc) *Sight Reduction Table* replacement. To solve the triangle AP latitude, object's declination and hour angle need to be entered.

AP latitude is entered in register 8 as before, declination is set via \boldsymbol{C} and hour angle is entered into register .2. The hour angle is positive if westward.

Example

Find Hc and Az as in Ho-214

 $^{^2\}text{Compare}$ these results with the direct method above (52° 54′ 55″ vs 52° 55′ 08″ and 156° 08′ 16″ vs 156° 08′ 21″). Pretty close but not exactly the same.

 $^{^3}$ Sæmann's formula is used.

Assume longitude 58° N, Declination $8^{\circ} 30'$ and an hour angle of 54° (object to the west of observer).

Data	Format	Key	Display shows
58	dd.mmss	STO 8	58.0000
8.3000	dd.mmss	С	8.5000
54	dd.mmss	g ->H	54.0000
		STO .2	54.0000
		GSB 7	25.4102 (He = $25^{\circ} 41' 02''$)
		х<>у	242.3616 (Az = $242^{\circ} 36' 16'' = 242.60^{\circ}$)

Ho-214 gives Alt. = 25° 41.0′ and Az. = 117.4° . Where true azimuth is $360-117.4=242.6^{\circ}$.

1.12 Program and information

1.12.1 Register usage

The lower registers ${\tt r0..r7}$ are used by the calculator's statistics functions and are not permanently used by this program. They are used however for intermediate results via the normal operating sequences **A-B** or **A-C-D** or **A-E**.

In short: Use r0..r7 as you wish but they will be altered by A.

1.12.2 Program installation

For a fresh install of the program perform steps 1-6 below.

- 1. Make space on the DM15 for program and registers:
 - Enter 21 f DIM (i)
 - Double check: g MEM should read 21.209
- 2. In HP-15C/Preferences/DM15 menu: Select 229 as Number of registers.
- 3. File/Open Program: file.15c
- 4. Write program to DM15.
 - On device enable serial communication (hold C while pressing ON-button)
 - File/Write DM15
- 5. Before use a number of constants must be entered in the following registers:

Register	Constant
. 3	279.4055638
. 4	283.3328093
. 5	1.016860112
. 6	23.44188400
.7	0.002737909, (1/365.2422)

6. That's it. Now the samples in this document should give the expected results.

1.12.3 Program listing

Note: In the listing below some minor self explanatory key appearances have changed. SIN^{-1} is replaced with ASIN etc, $x \leftrightarrow y$ is x <> y and $R \downarrow$ is Rv.

```
# -----45-48--7-1-RCL-.7------
                        089 { 33 } RV

090 { 44 9 } STO 9

091 { 45 48 0 } RCL .0

092 { 23 } SIN

093 { 45 48 6 } RCL .6
 093 { 40

094 { 23 } 00

095 { 20 } *

096 { 43 23 } g ASIN

097 { 44 48 0 } STO .0

098 { 45 9 } RCL 9

099 { 42 21 48 1 } f LBL .1
```

100 { 101 {		RCL 5 GSB .8	159 { 160 {	45 9 <u>.</u> 30 .	RCL 9
102 {	45 9	RCL 9	161 { 162 {	43 32	g RTN
104 {	40		163 {		} f LBL C } g ->H
105 { 106 {			164 { 165 {		} STO .0 } Rv
107 {	32 7	GSB 7	166 {	43 2	} g ->H
108 { 109 {		der RTN de la facilitation de la f de la facilitation de la facilitati	167 { 168 {		} GSB .8 } x<>y
110 {	45 48 2	RCL .2	169 {	30	} –
111 { 112 {		COS RCL 8	170 { 171 {		STO 9 g RTN
113 {	45 48 0		172 {	42 21 15	} f LBL E
114 { 115 {		} GSB 5 } *	173 { 174 {		} g ->H } STO 4
116 { 117 {	45 48 0]	RCL .0	175 { 176 {	45 48 7 1	RCL .7
118 {	32 4		177 {	40	
119 { 120 {	40] 43 23]	+ La ASIN	178 { 179 {	20 j 32 48 2	} * } GSB .2
121 {	36	g ASIN ENTER	180 {	45 1	RCL 1
122 { 123 {			181 { 182 {	40 36	} + } ENTER
124 {	[45 8]	RCL 8	183 {	36	ENTER
125 { 126 {		GSB 4 CHS	184 { 185 {	45 48 8 40	
127 {	45 48 0	DCT O	106 (32 2	GSB 2
128 { 129 {	23] 40]	RCL .U SIN + + x<>y RCL 8 GSB 5	187 { 188 {	44 5 34	} STO 5 } x<>y
130 { 131 {	34	x<>y	189 { 190 {	32 2	GSB 2
132 {	32 5	GSB 5	190 { 191 {	42 21 9	} x<>y } f LBL 9
133 { 134 {		} /	192 { 193 {		f ->H.MS x<>y
135 {	42 4 48 2	f X<> .2	194 {	42 2	$f \rightarrow H.MS$
136 { 137 {		SIN	195 {	43 32 1 42 21 0	} g RTN } f LBL 0
138 {	22 6	L GTO 6	197 {	1	} 1
139 { 140 {		GSB .8 RCL .2	198 { 199 {	36 <u> </u>	
141 {	30) — GEO 0	200 {	32 1	GSB 1
142 { 143 {		+ - + STO .2 + f LBL 6 + RCL .1	201 { 202 {	2 4	
144 {	45 48 1				} 1
145 { 146 {	22 9]	RCL .2 GTO 9	204 { 205 {	0	} 0
147 { 148 {		f LBL 8 SIN	206 { 207 {	2 0	} 2 } 0
149 {	48		208 {	30	} –
150 { 151 {		} 0 } 1	209 { 210 {	3 6	
152 {	[6]	ł 6	211 {	5	} 5
153 { 154 {			212 { 213 {	2 5	
155 {	[8]	8	214 {	10	} /
156 { 157 {		* CHS	215 { 216 {	36	} ENTER } ENTER
158 {			217 {	48	

1.12 Program and information

```
218 {
                0 } 0
                                      277 {
                                                   43 32 } g RTN
                                                       1 } f LBL 1
1 } 1
219 {
                 0 } 0
                                      278 {
                                                42 21
                0 } 0
220 {
                                      279 {
                                                        7 } 7
221 {
                                      280 {
                2 } 2
                                                       2 } 2
222 {
                                      281 {
223 {
                5 } 5
                                      282 {
                                                       1 } 1
224 {
                8 } 8
                                      283 {
                                                       0 } 0
225 {
                 1 } 1
                                      284 {
                                                       1 } 1
226 {
                20 } *
                                      285 {
                                                       3 } 3
                2 } 2
                                      286 {
                                                      48 } .
227 {
                 4 } 4
228 {
                                      287 {
                                                       5 }
                                                            5
                0 } 0
229
                                      288 {
                                                      40 } +
230 {
                0 } 0
                                     289 {
                                                      34 > x <> y
                                     290 {
                                                   44 1 } STO 1
231 {
                48 } .
                0 } 0
                                      291 {
232 {
                                                       2 } 2
                                                        7 }
233 {
                5 } 5
                                      292 {
234 {
                 1 } 1
                                     293 {
                                                       5 } 5
235 {
                2 } 2
                                     294 {
                                                      20 } *
                                      295 {
                6 } 6
                                                       9 } 9
236 {
                2 } 2
40 } +
237
                                      296 {
                                                      10 } /
                                     297 {
                                                   43 44 } g INT
238 {
239 {
                20 } *
                                     298 {
                                                      40 } +
                                      299 {
                6 } 6
240 {
                                                      34 \} x <> y
                48 } .
                                      300 {
241 {
                                                      36 } ENTER
                                     301 {
242
                6 } 6
                                                42 4 1 } f X<> 1
                                                       9 } 9
243 {
                4 } 4
                                      302 {
244 {
                6 } 6
                                      303 {
                                                       40 } +
                                                       1 } 1
2 } 2
245 {
                                      304 {
                0 } 0
246
                 6 } 6
                                      305 {
247 {
                 5 } 5
                                      306 {
                                                      10 } /
                                                   43 44 } g INT
248 {
                 6 } 6
                                      307 {
                                      308 {
                                                      40 } +
249 {
                40 } +
                2 } GSB .2
2 } GSB 2
250 {
         32 48
                                    310 {
                                      309 {
                                                       7
                                                         } 7
                                                      20 } *
251 {
             32
                                     311 {
312 {
252 {
                3 } STO 3
             44
                                                       4 } 4
            45 6 } RCL 6
253 {
                                                      10 } /
                                     313 {
314 {
254 {
            44
                0 } STO 0
                                                   43 44 } g INT
255 {
         43 32 } g RTN
42 21 11 } f LBL A
                                                      16 } CHS
                                     315 {
256 {
                                                      40 } +
257 {
            44
                4 } STO 4
                                     316 {
                                                   45 1 } RCL 1
                33 } Rv
                                     317 {
                                                       3 } 3
258 {
                                      318 {
259 {
            44
                5 } STO 5
                                                       6 } 6
                33 } Rv
                                                       7 }
                                      319 {
                                                            7
260 {
            32
                0 } GSB 0
                                     320 {
                                                      20 } *
261 {
                                     321 {
262 {
            45 1 } RCL 1
                                                      40 } +
                5 } RCL 5
4 } RCL 4
                                      322 {
263 {
                                                   44 6 } STO 6
            45
                                                43 32 } g RTN
42 21 14 } f LBL D
264 {
            45
                                      323 {
                1 } GSB 1
                                     324 {
265 {
            32
266 {
             45
                0 } RCL 0
                                     325 {
                                                   32 15 } GSB E
                30 } -
267 {
                                     326 {
                                                32 48 1 } GSB .1
                                      327 { 43
328 { 42 21 48
268 {
         45 48
                7 } RCL .7
                                                   43 32 } g RTN
               8 } GSB .8
269 {
         32 48
                                                      5 } f LBL .5
270 {
                20 } *
                                      329 {
                                                 45
                                                      5 } RCL 5
271 {
                                      330 {
                                                45 48 8 } RCL .8
                20 } *
                                      331 {
                3 } RCL 3
                                                      30 } -
272 {
             45
                40 } +
                                      332 {
                                                32 48
                                                       8 } GSB .8
273 {
                2 } GSB 2
274 {
                                     333 {
                                                 45 9 RCL 9
            32
275 {
                                                      30 } -
             44
                1 } STO 1
                                      334 {
276 {
            42 2 } f ->H.MS
                                     335 {
                                                      40 } +
```

```
336 { 32 2 } GSB 2 370 { 25 } TAN

337 { 45 48 0 } RCL .0 371 { 48 } .

338 { 22 9 } GTO 9 372 { 0 } 0

339 { 42 21 48 0 } f LBL .0 373 { 1 } 1

340 { 32 15 } GSB E 374 { 7 } 7

341 { 43 2 } g ->H 375 { 34 } x<>y

342 { 43 33 } g R^ 376 { 10 } /

343 { 43 2 } g ->H 377 { 40 } +

344 { 34 } x<>y 378 { 42 21 48 4 } f LBL .4

347 { 31 } R/S 381 { 43 2 } g ->H

348 { 32 13 } GSB C 382 { 22 48 7 } GTO .7

349 { 45 4 } RCL 4 383 { 42 21 48 3 } f LBL .3

350 { 42 2 } f ->H.MS 384 { 48 } .

351 { 32 14 } GSB D 385 { 5 } 5

352 { 43 32 } g RTN 386 { 42 21 48 7 } f LBL .7
                           32 13 } GSB C

45 4 } RCL 4

42 2 } f ->H.MS

32 14 } GSB D

43 32 } g RTN
 351 {
352 {
                                                                                                       385 { 5 } 5
386 { 42 21 48 7 } f LBL .7
387 { 33 } RV
388 { 43 2 } g ->H
389 { 34 } x<>y
390 { 43 2 } g ->H
391 { 30 } -
392 { 43 33 } g R^
393 { 43 36 } g LSTX
394 { 33 } RV
395 { 20 } *
 353 { 42 21 48 9 } f LBL .9

354 { 43 2 } g ->H

355 { 36 } ENTER

356 { 36 } ENTER
 357 {
                                              36 } ENTER
                                           5 } 5
48 } .
1 } 1
1 } 1
 358 {
 359 {
                                                                                                                                                33 } RV
20 } *
43 33 } g R^
 360 {
                                                                                                            394 {
395 {
20 } *
396 {
43 33 } g R^
397 {
40 } +
398 {
42 2 } f ->H.
399 {
43 32 } g RTN
 361 {
 362 {
                                            40 } +
                                            1 } 1 0 } 0
                                                                                                                                                    40 } +
42 2 } f ->H.MS
 363 {
 364 {
                                            48 } .
 365 {
 366 {
                                                                                                    # -----
                                            34 } x<>y
 367 {
                                               10 } /
 368 {
 369 {
```

Short summary

Format	Button	Format Button Function
YYYY MM DD	∢	Initialize with date
hh.mmss	മ	Get Sun's Altitude for Time, x<>y Azimuth
$ddd.mmss \pm decl$	ပ	Enter own objects coordinates
hh.mmss	Δ	Own object's Altitude for Time, x<>y Azimuth
hh.mmss	ш	GHA \(\text{T} \), x<>y for LHA \(\text{T} \)
ddd.mmss hh.mmss	GSB .0	GSB.0 GHA, x<>y for Azimuth of Sun
dd.mmss	R/S	Object's Altitude for Time, x<>y Azimuth
h.mmss hh.mmss	GSB .3	GSB.3 Average of two times
$ddd.mmss\ ddd.mmss\ 0.mmss$	GSB .4	Increments and Corrections
After A	GSB .5	Sun's GHA, x<>y Declination
dd.mmss	GSB .9	GSB.9 Apparent altitude, x<>y refr corr ⁿ used (in ')