Police Patrol Service Optimization Based on the Spatial Pattern of Hotspots

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Abstract—**Many algorithms have been developed to optimize police patrol services. Previous studies have mainly focused on determining the important locations (e.g., crime hotspots) and identifying important routes based on the topology of road networks. However, the impact of the patterns of hotspots on patrol route selections and the collective performance of patrol activities were rarely considered in these studies. In addition, some algorithms lack a mechanism to ensure the randomness in patrolling and some are not efficient enough for real-time applications. In this study, we propose an approach to determine sets of patrol routes, which can help to optimize the collective efforts of multiple patrol activity. In our approach, we integrate the Getis-Ord Gi* with the Cross Entropy approach to produce randomized optimal patrol routes. We also address how to ensure the randomness in the route identification using the Cross Entropy approach. Our results indicate that our approach can help to improve the collective performance of police patrol service and it is also efficient enough for real-time applications.**

I. Introduction

Criminal activities are a major risk factor for the well-being of society in many countries. Police patrol service is a critical instrument for the combating of criminal activities with violent aspects [1], [2]. Police officials generally regard the patrol function as having two major law enforcement objectives. First, it provides, through sector concentration, rapid responses to emergency calls. Second, it involves a randomized street surveillance to deter crimes [3]. To achieve these two objectives, limited police resources need to be carefully allocated. The police resource allocation for street patrolling is one of the most important tactical management activities.

Hot-spot patrolling, which focuses patrolling resources to the locations with clusters of crimes, is one of the widely accepted patrolling strategies. The underline reason for this patrolling strategy is that crime is not spread evenly across city landscape and there are usually clusters of crime, or "hot spots", that generate half of all criminal events [4]. Because of these observations, focusing limited resources on places with clusters of crime seems to be a rational choice. Some police departments have identified a list of "hot spots" and used that as a guidance for patrol patrol activities. It was relatively easy for the experts to spot those points and then to allocate the teams to patrol mostly around them. However, it is unclear how did patrol officers get to these points and whether these points were indeed visited more often than other locations.

Algorithms have developed to plan patrol routes to cover hotspots [5], [6]. Previous work mainly focuses on determining the importance scores of locations based on hotspots analysis and identifying important routes based on the topology of road networks [6]. However, there are three problems in patrol route planning based solely on the locations of hotspots. First, although the impact of the distribution of hotspots on patrol route selections was highlighted, the patterns of such distributions was rarely considered in patrol planning [7]. Second, some existing patrol optimization algorithms that maximize hotspots coverage result in a single (fixed) patrol recommendation [6]. Fixed patrol routes can obviously lead to a predictable patroller, which may encourage potential criminals to predict the arrival patterns of patrol vehicles. The importance of randomized patrols has been recognized in law enforcement for some time, but not the nature of the randomization [8]. Nevertheless, patrol activities are repeated every day. It is important to identify an approach that can optimize the collective efforts of patrolling, instead of focusing on one single optimal patrol activity. Planning single patrol route might not be able to produce an optimal and effective allocation plan. Finally, some algorithms proposed to comprehensively list all possible routes and calculate the weights of these routes based on the scores of locations. However, these solutions only work for small jurisdiction. For typical jurisdictions consisting of hundreds or thousands of street-segments, more efficient algorithms are needed.

To solve these problems, the goal of patrol route planning should be to select a set of suitable routes to optimize the overall performance patrol activities. To achieve this goal, a through understanding of the nature of hotspots and its implications on patrol route optimization are required. For example, hotspots might not be spatially independent to each other and some hotspots are surrounded by other hotspots. The impact of this knowledge on the patrol route optimization needs to investigated and algorithms that can efficiently identify suitable routes should be used.

This paper aims to integrate a spatial pattern identification approach with an efficient route optimization algorithm to produce randomized optimal patrol routes. We note that our goal is not to provide a solution for the entire process of planning patrol routes. Rather, we propose an approach to determine sets of patrol routes, which can help to achieve the optimization of the collective efforts of multiple patrol

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activities. We will also address how to ensure the randomness in the optimal route identification with an efficient algorithm.

In the second section, we introduce techniques on crime hotspots identification and the Cross Entropy approach, which is an fast algorithm that has been applied in patrol route optimization. In third section, we propose an approach to incorporate the spatial pattern of hotspots and randomness in the patrol route optimization process. In the end, we present our results and conclusions.

II. Background

A. Crime hotspots and their spatial patterns

Many versions of definitions exist for the term "hotspots" and there is no consensus on the best definition for crime hotspots. The major disagreement in the definition of "hotspot" is on whether to aggregate data or on how to aggregate them using different spatial units. In some studies, crime hotspots refer to the clusters of criminal events where the inter-distances between these events are shorter than it would be expected in a random pattern and data aggregation is not necessary. Crime hotspot studies often have different contexts and data aggregation is often necessary for taking the contextual information into account. For strategic planning of law enforcement, which uses zoning as a tool, hotspots often referred to as areas with higher level of criminal events than other areas. Within the context of police patrol, the hotspot identification using street segment as basic unit seems to be the most relevant as more than 80% of patrol activities are motor patrol which is constraint by the street networks [9]. Street segments have been used in cluster analysis as basic units. Shekhar et al. proposed "mean street" approach to identify streets or routes that have highest level of crimes based on spatially aggregated crime data [10]. A benefit of spatially aggregating data is that it may reduce the calculation difficulties. However, the success of this approach requires sophisticated approaches to define the basic unit, which has a great influence on the outcomes of aggregation. For point level data, Yamada and Thill proposed an approach to identify network constrained clusters. To apply this approach, we have to assume that crimes are located on streets, since this approach uses network distances to calculate the interactions between points. Li modified Kulldorff's spatial scan approach to identify clusters for points influenced by linear features [11]. In this approach, ellipses with varying sizes and orientations aligned with linear features are used to search for locations with higher than expected number of points at multiple scales. Note that this approach is different from elliptical cluster analysis which does not consider the influences of other features on points. This approach, although has not been applied to crime data, complies the true nature of crimes, of which the distribution is influenced, not necessarily constrained, by the distribution of streets. Please refer to [11] for the details on this approach.

B. Spatial patterns of hotspots

Spatial autocorrelation is a fundamental concept in spatial analysis [12]. Spatial autocorrelation in crime data has often been observed [13]. Spatial autocorrelation analysis consists of a set of statistics describing how a variable (e.g., crime likelihood) is autocorrelated though space [14]. This variable can also be an attribute of street segments. The autocorrelation in attributes of street network implies that the attributes of each edge in a network can be predicted in part from knowledge of the attributes of related edges. For patrol route planning, inclusion of an edge in a route many lead to impacts that are disproportional to the crime likelihood of this edge, since the neighbors of this edge that are unavoidable to be added in the route may also have high/low values [15]. Getis-Ord Gi*, known as Hot Spot Analysis, is commonly used to detect spatial clusters of the magnitude of an attribute [16]. It is a multiplicative measure of overall spatial association of values which fall within a critical distance of each other. The following equation is for general Getis-Ord Gi* test [16].

$$G(d) = \frac{\sum_{i=1}^{K}\sum_{j=1}^{K} w_{i,j}(d)x_i x_j}{\sum_{i=1}^{K}\sum_{j=1}^{K} x_i x_j} \tag{1}$$

where x_i is the value of ith point, $w_{i,j}(d)$ is the weight for point i and j for distance d. The general Getis-Ord Gi* only measures the spatial clusters and does not test the statistical significance of the obtained measurements. The Z-score of Getis-Ord Gi* provides an estimate of the statistical significance of Gi* score. The length equation for the z-score is omitted and can be found in [11].

C. Route Optimization

Route optimization problems, such as the shortest path problem, are often dealt with using markov decision processes models. A route is defined as a series of connected edges and whether to travel from one edge to another is defined by transition probability. When the transition probability is unknown, the markov decision processes model are often solved using reinforcement learning, through which an agent learns the behavior of the system through trial-and-error with an unknown dynamic environment [17]. One problem of reinforcement learning is that the majority of algorithms developed based on it are computational inefficient. For police patrol planning, fast computational means are needed. Recently, some fast learning algorithms were developed based on the Cross Entropy method, which has become a standard tool in Monte Carlo estimation [18]. The following section introduces Cross Entropy method.

D. Cross Entropy Method

Cross Entropy (CE) method provides a simple, efficient and general method for solving complicated optimization problems. It has been used to solve traveling salesman problem, combinatorial optimization problem, as well as patrol route optimization [19]–[23]. CE method translates the "the deterministic" optimization problem into a related "stochastic" one. It has been applied to optimization problem concerning a weighted graph and introduces randomness in either the nodes or the edges of the graph [22]. In the case of patrol optimization, street networks are often simplified as a weighted graph.

In general, CE method consists of two phases: (1) Generate a random data sample according to a specific mechanism. (2) Update the parameters of the random mechanism to produce a "better" sample in the next iteration. For patrol route estimation, the first phase of CE method involves randomly generate patrol routes by selecting edges based on an initial transition matrix. This matrix defines the benefits of traveling between any two nodes. The initial matrix often assumes that traveling between different edges render equal benefits. In the second phase, the transition matrix will be updated based on the calculated true benefits of traveling between the selected edges [23]. This approach can consider the dynamic nature of criminal events. However, it uses aggregated crime data and does not consider the distribution of hotspots and the spatial pattern of them. The transition matrix was updated only based on the simulation process and it does not consider the spatial relationship between hotspots.

III. Strategy for Randomized Patrol Routes Planning Using Getis-Ord Gi* and Cross Entropy Approach

In this section, we introduce our approach to incorporating the spatial autocorrelation scores of hotspots in a patrol route optimization procedure. In our approach, the CE approach is used for the route optimization and randomization. The procedure of our approach is illustrated in Fig. 1. In our approach, we first calculated crime likelihood value for each basic unit in a patrol route model (the details of this model will be given in the next subsection). Crime likelihood values are typically used to calculate patrol rewards for a patrol route. We are adding a critical step which is to calculated the spatial autocorrelation values for the likelihood values. As illustrated in Fig. 1, we used Gi* values as patrol rewards to take into account the spatial patterns of crime hotspots. Using assigned patrol rewards, the Cross Entropy approach helps to update transition matrix, which determining the identification of optimal routes. To ensure the randomness of the generated patrol routes, we randomize the starting points of routes and the convergence rule of the CE approach is relaxed. The updating procedure for the transition matrix is stopped before it reaches it optimal state. Finally, a set of near-optimal routes are obtained. For comparison purpose, the original crime likelihood values are also used for the same procedure and another set of near-optimal routes is obtained. To examine if these two sets of near-optimal routes help to achieve a collective optimal solution for patrol service, R_2 score (proportion of variability in a data set that is accounted for by a statistical model) is used to test whether the frequency of each basic unit in these routes is proportion to the crime likelihood of this unit.

A. Patrol route model

Currently, patrol planning activities, including foot and motor patrol, are mainly planned using street-network model,

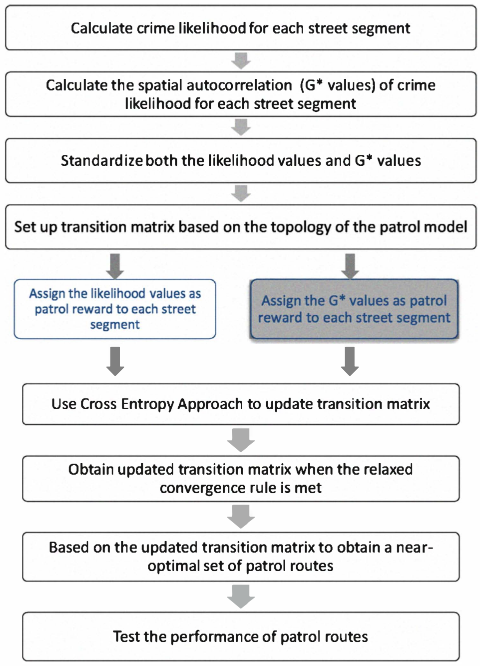

Fig. 1. Flowchart explaining the procedures in the proposed approach

such as street-central line maps. To ensure the usability of the proposed algorithms, we first introduce street-network based patrol route model that enables the patrol routes planning as followings:

- $\{1, 2, ..., N\}$ denotes edges in a street-network model for an area to be patrolled. In street-data, an edge $e_{(n)}$ usually represents the smallest street segment connecting two neighboring intersections.
- Tn denotes the traveling time for the patrol unit on edge n.
- C_n denotes the crime likelihood of edge n.
- G_n denotes the spatial autocorrelation value of the crime likelihood for edge n.
- P_n denotes transition probability of traveling on edge n. The initial value of weight for an edge is zero and it is updated using the Cross Entropy method, which will be introduced in the third subsection of this section.

For this route model, the patrol reward for a route r is:

$$r = \sum_{n=1}^{M} CnI_{n \in r}$$
$$I_{Cn \in r} \in \{0, 1\} \tag{2}$$

where M is the total number of edges in a route.

Given the patrol reward equation, the patrol planning problem is formulated as:

$$\max \sum_{n=1}^{N} CnI_{n \in r}$$
$$r \in \Omega \tag{3}$$

where Ω is a complete set of all possible suitable routes

Based on this route model, a random route (R_n) that can be completed with time T can be generated using the following algorithm:

- select a random edge e_0;
- stop until $(t_0 + t_1 + ... + t_n)\ T$
  - select the neighboring edge of e_k, which has highest transition probability;
  - check if $(t_0 + t_1 + ... + t_n)\ T$;
- Calculate the patrol rewords.

B. Cross Entropy Method

The CE algorithm is the following:

- Generate M number of random routes by the algorithm described in the above subsection;
- Sort these routes by patrol rewards;
- Select top $g\%$ of routes with highest importance values;
- Update transition matrix;
- Repeat the above steps until convergence.

The transition matrix can be updated using the following equation [18]. Using the street network model defined above, the transition probability of edge i is:

$$P_i = \frac{\sum_{n=1}^{NR} N_{\{E_i \in R_n\}}}{\sum_{m=1}^{ER} N_{\{E_i \in R_m\}}} \quad (4)$$

where NR is the total number of randomly generated routes, ER is the total number of selected routes with highest importance values. E_i is Edge i. $N_{\{E_i \in Rn\}}$ denotes number of times E_i appeared in randomly generated routes. $N_{\{E_i \in Rm\}}$ denotes number of times E_i appeared in selected routes.

The convergence rule is usually defined based on the patrol reward of the top performing route. In each iteration, the patrol reward of the generated route is recorded. Using cross-entropy minimization, this procedure stops when the patrol reward of the newly generated routes continuously equal to the highest record of patrol rewards. This procedure would favor the best solution obtained over all previous iterations, up to and including the current iteration. In this way, Cross Entropy algorithm ensures the global optimal solution and it stops until the results converge to the optimal one.

In the context of patrol planning, it is important that a near-optimal set of solutions (here, a solution is refereed as a route) is found, since single solution for patrolling would lead to blind points which should be avoided in patrol plans. The convergence rule can be relaxed. For example, the procedure can stop when an iteration reached a near optimal solution and its several subsequent iterations produce similar optimal solutions. The similarity of patrol reward can be defined using a threshold value. Using this convergence rule, the algorithm can recommend different patrol routes and these routes could be local optimal solutions. It is important to select an appropriate threshold value to ensure both the identification of near-optimal solutions and the near-comprehensive coverage of patrol plans.

Fig. 2. Downtown area (dashed line), crime incidents in 2008 (black dots) and street-network (grey line)

IV. Applications

A. Study Site

The method defined above was demonstrated using crime data from the Spokane Police Department along with patrol boundaries for the city of Spokane (http://www.spokanecity.org/services/gis/). The Spokane Police Department deploys 16 patrol teams working 11 hour days. Among these teams, 8 teams each consisting of 8 patrol officers are on duty each day. The city is divided into 8 precincts and each patrol team is assigned to one of the precincts. Each precinct is further divided into two patrol neighborhoods. A patrol group usually consists of two police officers. Therefore, at any time, at least four police officers would be assigned to a precinct and a patrol group would be assigned to a patrol neighborhood. To demonstrate the proposed method, Downtown area, the 0.9 x 1.1 miles patrol neighborhood with the highest crime rate, was selected to be the study site. The boundary of the downtown area is shown in Fig. 2.

According to Corder and Kenney's study (1999), patrol activities are often interrupted and patrol officers spent 60% of their time on other duties than patrolling [24]. Patrol planning should select small time units to accommodate this problem. In this study, the length of recommended routes is controlled and a recommended patrol route should be finished within 10 minutes. Assuming patrol officers spend 40% of their shift time on patrolling, which is 5 hours, a patrol group can at least complete 30 patrol routes in each shift.

In the context of police patrol planning, selecting an appropriate time range to aggregate crime data ensure the reliability of crime likelihood estimation. The frequency and type of crimes varies in different time in a day or year [13]. However, the temporal aspect is not within the scope of this study. It is a common practice to use yearly data to estimate crime likelihood. To test the efficiency of the proposed approach, crime data from the entire 2008 (black dots in Fig. 2) which include 2175 crime cases are used. The types of the crimes include Arson, Assault, Burglary, Drugs, Malicious Mischief, Robbery, Theft and Murder. Li's approach is applied to the crime data to estimate crime likelihood for each basic patrol unit [11]. In Li's approach, centers of clusters were selected from locations with observed point events. To fit into the context of patrol route optimization, the centers of clusters in this study are centroid of basic patrol units. The Gi* value of the crime likelihood values is calculated and an inverse distance approach is used to determine the spatial proximity of the basic patrol units. To evaluate the temporal generality of crime data in 2008, the crime likelihood and and Gi* values are calculated for crime data obtained in 2009 and the relationship between these two sets of data are examined using R_2 score.

Also shown in Fig. 1, street network data are also obtained from the GIS unit of the city of Spokane and they are further broken into basic patrol units using ArcGIS (http://www.esri.com). The street-network data consists of 320 basic patrol units. The average length of the basic unit is 250 feet and the total length is 15 miles.

B. Empirical Results

Using crime likelihood method explained in section 3, the crime likelihood ratio for each basic patrol unit is calculated using crime data in 2008 and displayed as the size of patrol units in Fig. 3. The crime likelihood ratio values range from 0 to 1.51 with an average of 0.03. Based on the calculated crime likelihood ration, Gi* score is calculated for each edge using ArcGIS with Spatial Analysis extension. Fig. 3 displays Gi* z-scores as the size of grey dots. The range of Gi* z-score is from -0.860 to 6.91 with an average of -0.01. The relationships between the crime likelihood values/Gi* z-score in 2008 and 2009 are evaluated using R_2 score. The R_2 score for the crime likelihood is 0.785 and is 0.797 for the Gi* z-scores.

The z-score rescaling is applied to both the crime likelihood ratio and the Gi* z-score values. After rescaling, the crime likelihood ratio values range from 0 to 1 with an average of 0.3 and the Gi* z-score values range from 0 to 1 with an average of 0.6.

For comparison purposes, the CE Method is applied to both the rescaled crime likelihood and Gi* z-score using the process described in section . The random routes are generated using a python program and the igraph module of python is used to construct patrol route model. The CE algorithms are implemented in python code. Each process includes 500 iterations. This number of iteration is considered sufficient, since the total length of the simulated patrol routes in these iterations is 20 times of the total length of the entire street-network. The average convergence time increases and the possibility for each edge to appear in a set of patrol routes decreases when the converge threshold is reduced. To select the based threshold that ensure a reasonable converge time and an ideal patrol coverage, multiple thresholds ranging from 5% of the average value of the rescaled crime likelihood and Gi* z-score to 25% of such values with 5% increment are tested.

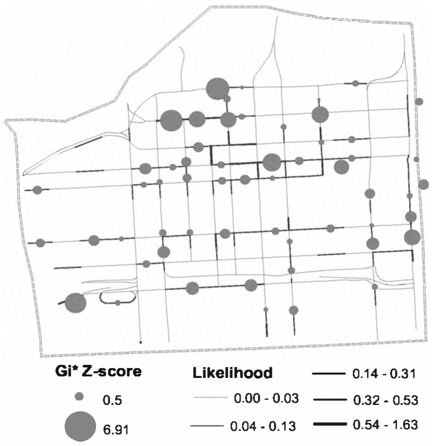

Fig. 3. Crime likelihood and Gi* scores

TABLE I
RUNNING TIME OF CROSS ENTROPY ALGORITHM ON THE CRIME LIKELIHOOD VALUES AND THE GI* VALUES OF THE CRIME LIKELIHOOD VALUES

Approach	Average running time (second)	R-Score
Crime Likelihood	13.02	0.1
Gi* value	12.1	0.3

Finally, converge thresholds are set to 5% of the average values of the rescaled crime likelihood and Gi* z-score. Using these thresholds, the Cross Entropy methods are run for 50 times on a computer with Dual-Core 2.53 GHz. and average running times for both the likelihood values and Gi* values were recorded in Table 1.

Ideally, in a collective optimal patrol plan, the frequency that an edge is visited is proportion to the crime likelihood of this edge. This is however difficult to ensure since each patrol activity is a chain of actions. To test the closeness of the recommended routes to the collective optimal plan, we record the top 30 routes in each run, which resulted in 1500 routes. We calculated the frequency of the appearance of each edge in these routes and regressed the frequency against the crime likelihood. We used R-score to evaluate the performance of a set of routes. R-score ranges from 0 to 1. The bigger the R-score, the better the collective performance of a set of routes. The R-scores of the regression are shown in Table 1.

V. CONCLUSION

In this manuscript, we described an approach to plan optimal randomized patrol routes. This approach is based on the Getis-ord Gi* and the CE algorithms. In this approach, basic street unit was used as basic analysis unit instead of hotspots, which are often used in patrol planning. We also examined the spatial autocorrelation of the crime likelihood defined using hotpsot identification approach.

Suggesting routes instead of point locations of crime hotspots may be technically difficult. As most of patrol cars are now equipped with sophisticated navigation equipments such as GPS, planning patrol activities at route level is feasible. In terms of running time, our approach can have a similar performance with patrol route planning strategies proposed by other studies. For example, in Chen and Yum's study (2010), the running times for several major patrol route planning approaches were around 20 second, which is close to the average running time of our approach (13 second).

In addition, the results show that the patrol routes suggested by our approach can help to achieve a comprehensive and near-optimal coverage of the patrol area. The advantage of using Getis-Ord Gi* values for patrol rewards is that it considers the spatial similarity of crime likelihood of nearby street units and it has a good temporal generality. Most importantly, it helps to identify routes that lead to the improvement of collective patrol rewords of multiple routes. If this approach is used continuously, the collective performance of patrol activities can be improved. Results also show that our approach is effective enough and can be used for real-time solutions.

In real application, our approach can be improved in several ways. First, when calculating the crime likelihood, different crime types can be assigned with different weights based on their emergency levels which is usually defined by local authorities. It can also be improved by better defining an appropriate time range to calculate crime likelihood. For example, burglary tends to clusters in the temporal domain and a shorter time range might provide more insights on the crime likelihood identification. Finally, the police patrol shift and disturbances in their patrol services are often needed to be considered in the implementation of the recommended patrol plans.

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Will be provided later

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