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Optimal Police Patrol

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1. Abstract

Although there is an abundance of data regarding the spread of crimes and criminal behaviors in various major cities in the world, there seems to be minimal use of this data by the law enforcement agencies to prevent or predict crimes. Motivated by a request to maximize police patrol in Johannesburg, South Africa, the MathCCES department built a mathematical model based on particle interactions of police officers and criminals. Coupled with genetic algorithm, this mathematical model has provided useful insights into what is important in minimizing the number of crimes that happen over a certain period of time.

This paper discusses the mathematical model used in this experiment, how the genetic algorithm is used to optimize the police patrol on a regular grid and some patterns observed throughout the simulation. In the future, we hope that the result of this research can be extended and applied in our daily life to minimize the number of crimes in various major cities.

2. Background

One unfortunate bi-product of urban civilization is the presence of crime everywhere. Although crime itself is ubiquitous, it does not appear to be distributed uniformly. Crime “hotspots” are often observed in the same area of various major cities. While modern mapping technology allows scientists and mathematicians to track the evolution and formation of hotspots, it seems that the efforts of law enforcement agencies to utilize this understanding to reduce crimes have been hampered by the unpredictability of the patterns. Past papers have looked into developing mathematical models to simulate the interaction of criminals and law enforcement agents to observe any regularity or pattern in hotspots formation (Short, 2008)(Jones, 2010).

After being approached by the Police Department of Johannesburg in South Africa, RWTH Mathematics Department is motivated to use mathematical models and simulations to optimize police patrols based on known parameters and set-ups. Theoretically, by characterizing the simulation to represent distinct criminal trait in different cities, law enforcement agents should be able to extend the use of similar method to analyze and optimize police patrols in various locations around the world.

3. Research Objective

The goal of this project is to minimize the number of crimes occurring over a period of time using a proposed mathematical model through finding the optimum police patrol strategy.

4. Methodology

4.1 Mathematical Model

The mathematical model is built on a regular grid, in which police and criminal agents can occupy the vertices and move from one vertex to another. The police and criminal agents follow a particle model which is known to provide a realistic result (Jones, 2010). In this model, the police officers are represented by red dots, while the criminals, not visible to the viewer, appear at random vertices across the grid (Fig. 1). Both the criminals and the law enforcement agents have the same movement capability of one grid point per iteration.

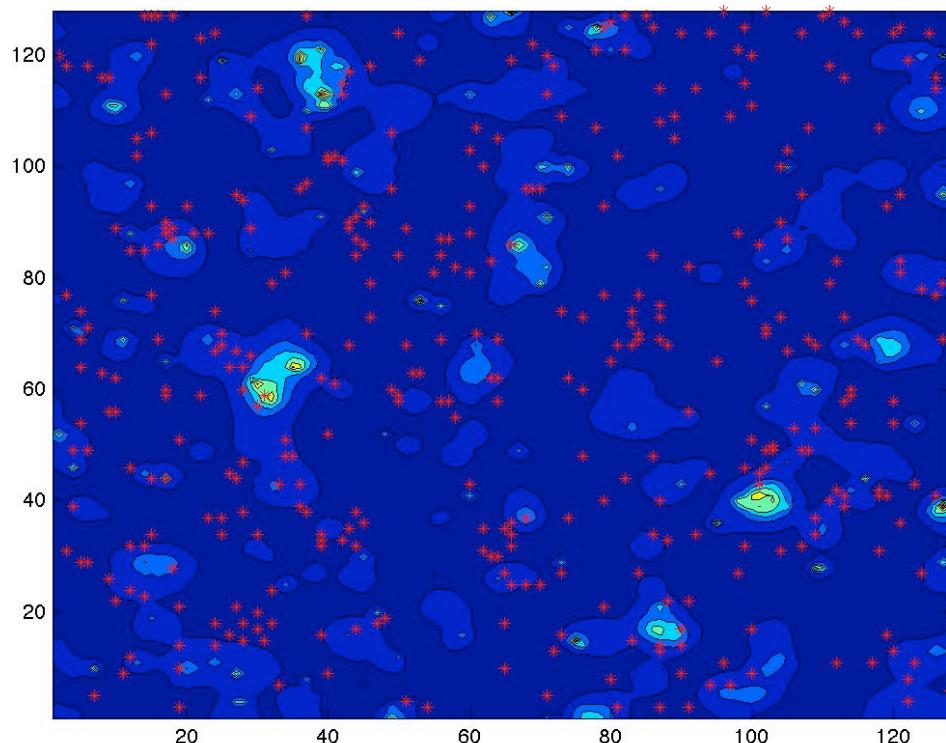


Figure 1: The mathematical model from the user perspective

At every iteration, the interaction between policemen and the criminals, how it affects the crime rate / the attractiveness level of a certain vertex in the grid and whether a burglary happen or not are calculated through specific algorithms that follows the discrete model (Fig. 2).

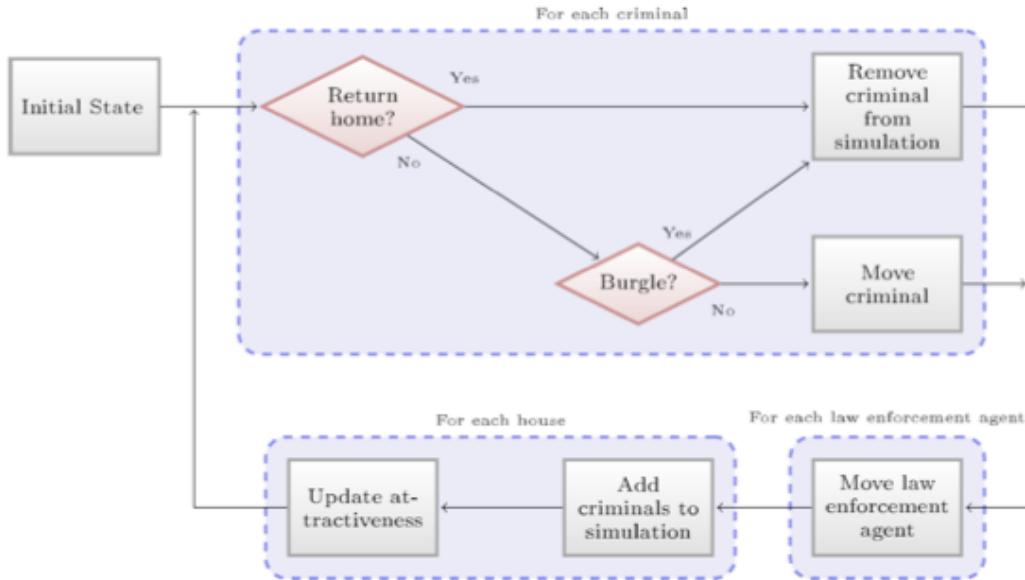


Figure 2: Flowchart summarizing the discrete simulation (Jones, 2010)

Each step involved is governed by a formula that describes the interaction between the criminals and the law enforcement agents. As discussed in past papers (Short, 2008), the determining factor whether a robber commits a crime or not is the attractiveness level of the vertex where the robber is located. The attractiveness level of a particular vertex has two different parts, the static and dynamic attractiveness level. The static attractiveness level (A_s) represents the attractiveness level of the vertex according to the previous iteration. On the other hand, the dynamic attractiveness level (B_s) counts for how the attractiveness level is affected by the history of crime in a specific vertex. The dynamic attractiveness level takes into account near-repeat or repeated burglary and the broken windows effect. Therefore the attractiveness level of a vertex is the sum of both the static and the dynamic attractiveness level of that point.

$$A_s(t) = A_s^0 + B_s(t) \quad (4.1.1)$$

The probability of a robber breaking into a house (p_s) in any given vertex is calculated with an exponential function dependent on the attractiveness level of that vertex.

$$p_s(t) = 1 - e^{-A_s(t)\delta t} \quad (4.1.2)$$

The higher the attractiveness level of a vertex is, the more likely it is for the robber to commit a crime there. The movement of the criminal and law enforcement agents is governed by the attractiveness level of the vertices surrounding the location of the agent. Assuming that it is possible for a robber (R) to move to four different vertices after an iteration and each of the vertices has a certain attractiveness level as shown in the diagram below (Fig. 3), there is a 45% probability that the robber moves down, 10% that the robber moves left, 25% that the robber moves up and 20% that the robber moves right.

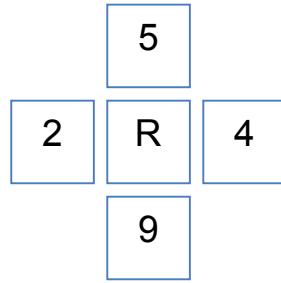


Figure 3: A diagram describing the possible movement of a robber with different attractiveness level surrounding its location

The dynamic attractiveness (B_s) depends on several factors, which are the spacing between each of the grid point (ℓ), the degree of spreading of the broken windows effect (η), how far the vertex is from the location of the crime (z), the time scale over which repeated burglaries are most likely to occur (ω), the number of burglaries on that location so far (E_s) and how much each of the burglaries increases the attractiveness level of that particular vertex (θ). The effect of each of these factors follow the formula described below (Short, 2008).

$$B_s(t + \delta t) = \left[B_s(t) + \frac{\eta \ell^2}{z} \Delta B_s(t) \right] (1 - \omega \delta t) + \theta E_s(t) \quad (4.1.3)$$

On the other hand, the presence of policemen in a certain location reduces the overall attractiveness level exponentially. This reduction depends only on the number of policemen on a particular vertex (κ_s) (Jones, 2010).

$$\tilde{A}(t) = e^{-\chi \kappa_s(t)} A_s(t) \quad (4.1.4)$$

χ in this formula is any positive constant. According to this formula, the more policemen there are on a particular vertex, the lower the attractiveness level on that vertex is.

4.2 Police Strategies

There are four different police strategies implemented in this simulation:

1. Active Response Strategy (Tactic 1)

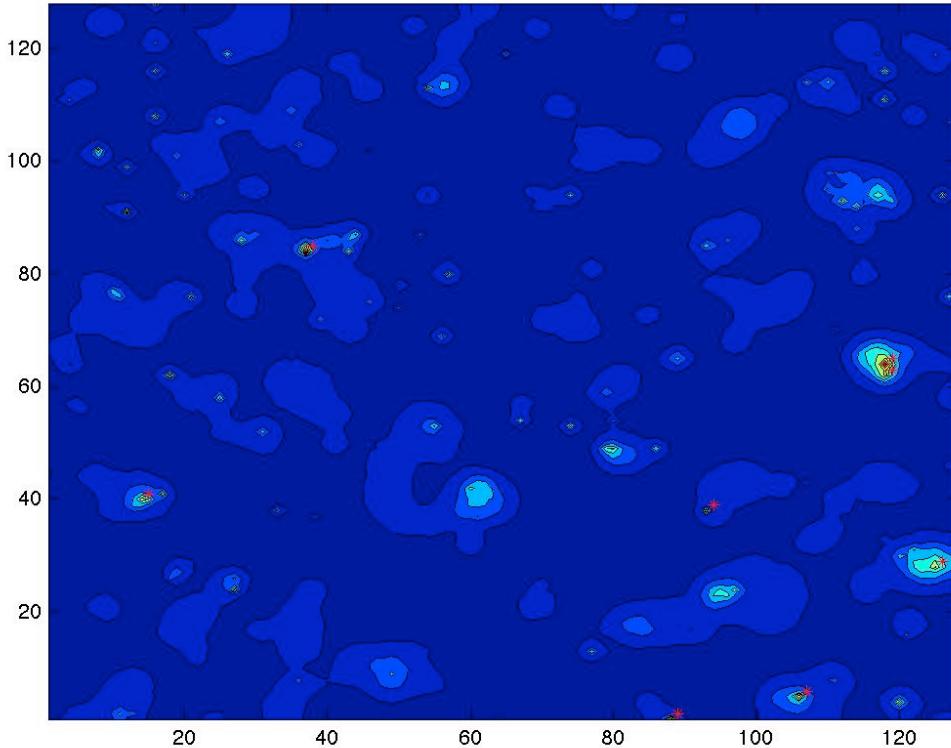


Figure 4: Active Response Police Strategy (Tactic 1)

As shown in the above figure (Fig. 4), tactic 1 simulates an active response police force. In this case, policemen are placed in response to the previous state of crime or attractiveness level. All of the policemen are placed in area with high attractiveness level, simulating area of high crime rate.

2. Stationary Policemen (Tactic 2)

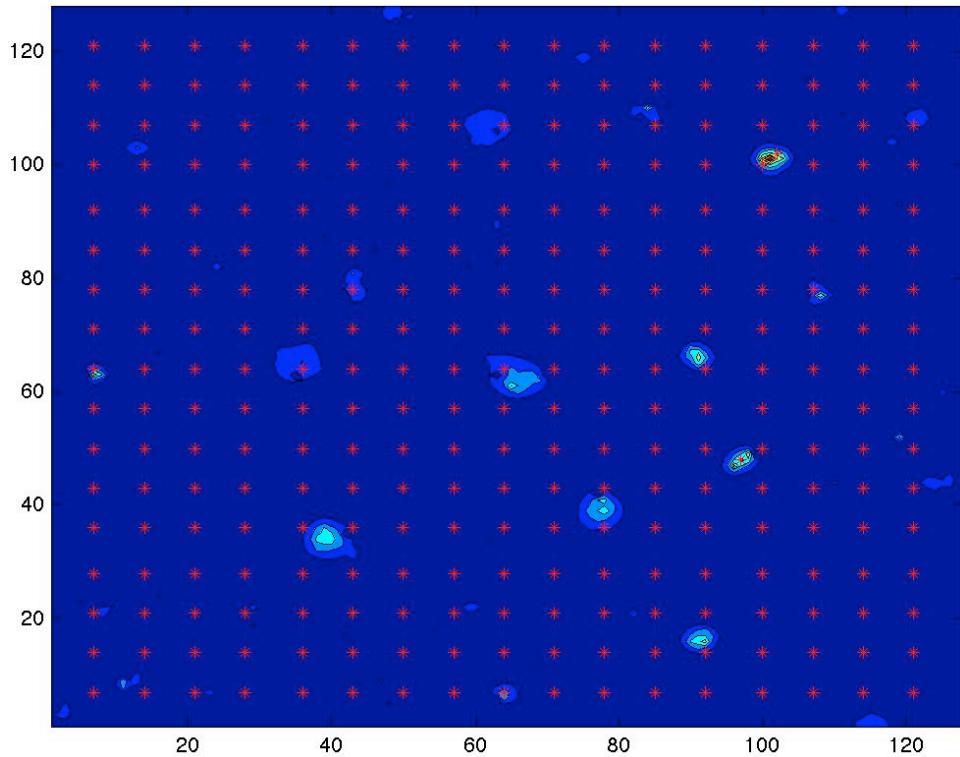


Figure 5: Stationary policemen (Tactic 2)

The problem with the first tactic is that the crime happens on the opposite side of the map whenever the police forces are concentrated on a particular location. Therefore, it might be better to distribute the policemen equally across the grid as shown in the above figure (Fig. 5). In this tactic, a certain portion of the policemen are stationary and distributed across the grid while the other portion of the policemen move the same way as described in tactic 1.

3. Patrolling Specific Area (Tactic 3)

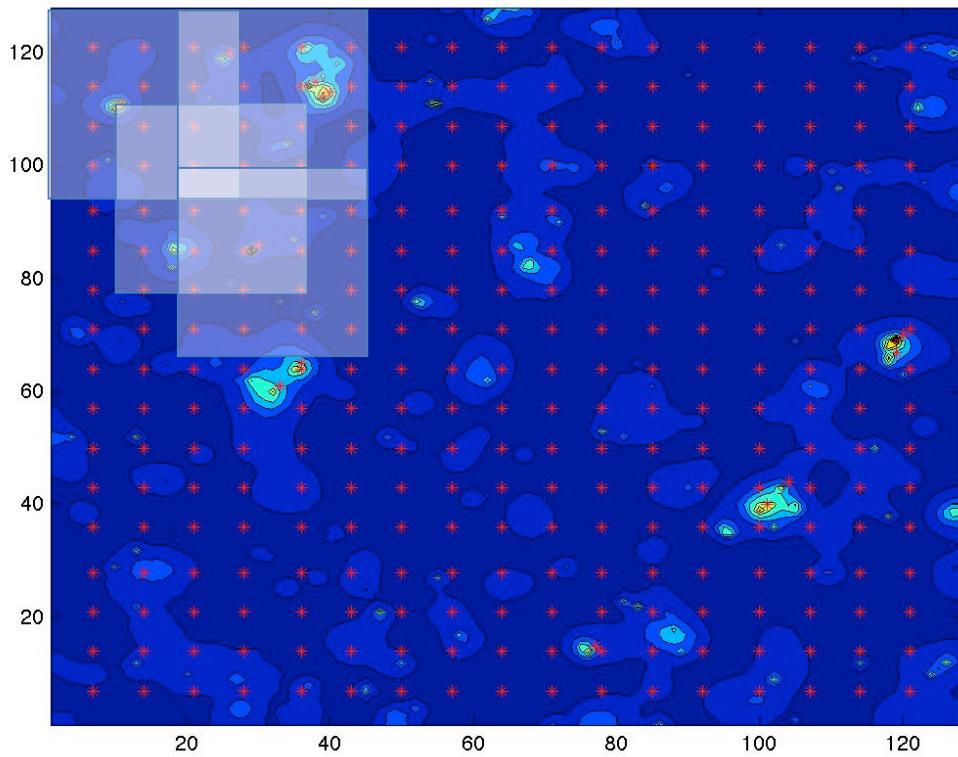


Figure 6: Patrolling Specific Area (Tactic 3)

The problem with stationary police strategy is that the crime happens in between the grid points where the policemen are stationed. Therefore, it might be a better idea to have an area for this distributed police force to patrol. That is how the third tactic came about.

4. Concentrated Police Patrol (Tactic 4)

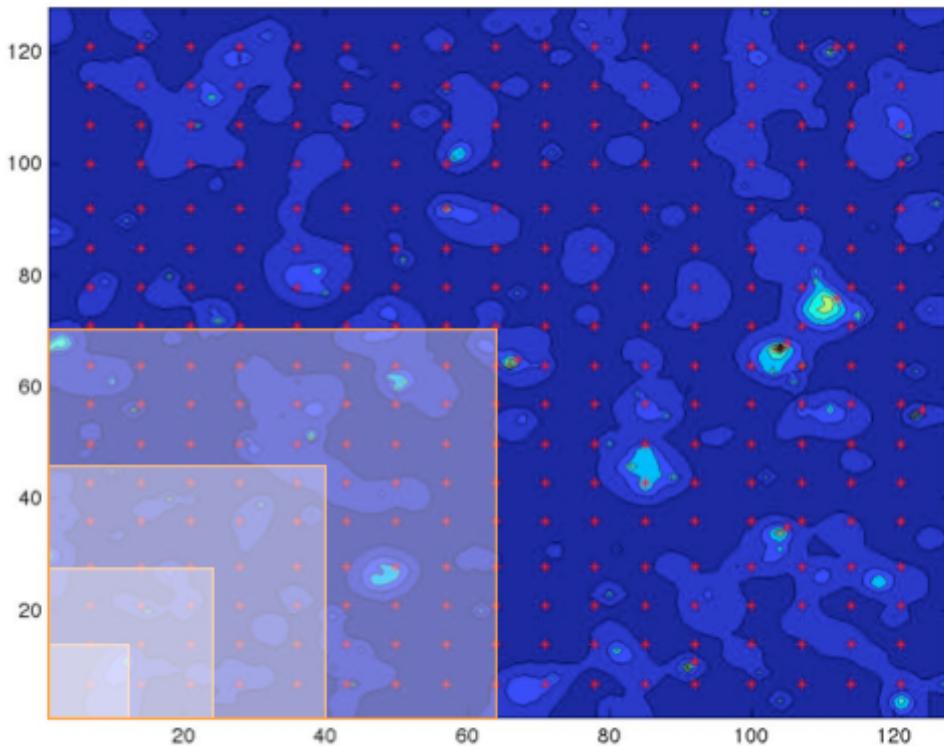


Figure 7: Concentrated Police Patrol (Tactic 4)

The idea behind the last strategy is that there are always certain areas with much higher crime rate compared to any other area in a city. Therefore, more police forces need to be concentrated in that particular area compared to other areas. As shown above (Fig. 7), the lower left corner is covered by the most policemen while the upper right corner is covered by the least policemen. This strategy is used to simulate concentrated police forces in a specific location.

4.3 Optimization Methods

There are two main optimization methods used throughout the experiment. First is the linear search. By incrementing each of the variable bit by bit across the entire possible range of values, this method is the most obvious method to find the optimal strategy to be used given any scenario. However, the problem with linear search in this simulation is that it takes too long to conduct as there are more than 20,000 different combinations of variables to simulate and each of the simulation takes about an hour to complete. Nevertheless, this “brute-force” method provides a standard of comparison to the values obtained through the second method.

The second method used is known as the genetic algorithm. It is a searching / optimization method that mimics the process of natural evolution. This method relies on the survival of the fittest concept. It may not find the true optimum but the result gets better as the number of simulation increases (Strelow, 2007). It consists of four main stages as shown in the diagram below (Fig. 8).

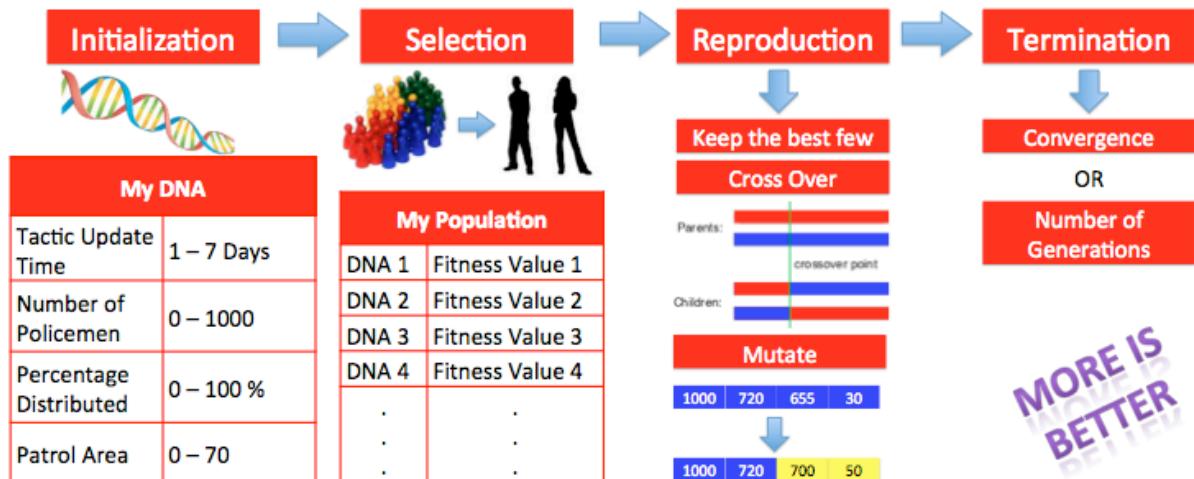


Figure 8: Schematic of the Genetic Algorithm for this project

The genetic algorithm relies on the existence of populations consisting of individuals characterized by different input variables for the simulation. There are four variables used in this simulation: tactic update time, number of policemen, percentage of distributed policemen and patrol area. Each of these variables has different range as shown above (Fig. 8). The more individuals a population has, the better the coverage of the range would be. Initialization is the stage, in which these individuals are created.

Then, during the selection process, through a weighted random process, two individuals are chosen to create a new individual for the next generation. The weighted random process ensures that individuals with more desirable fitness value have a better chance to be chosen as a parent individual for the next generation. In the reproduction stage, using an elitist strategy, the best few of the previous generation are kept and new individuals are created by crossing over the parents' DNA elements (the different variables, in this case). Each of the new individual is then given a certain chance to mutate. Mutation basically changes the value of one or more element of the individual's DNA (variable value). The process is then repeated until a termination criterion is achieved.

There are two different approaches to terminate the genetic algorithm: convergence or after a certain number of generations. Convergence is achieved when the optimum result from previous generation becomes close enough to the result obtained in the current generation. However, due to the nature of the simulation, achieving convergence does not mean that the optimum solution has been reached. Therefore, in this simulation, the termination is set to be after a specific number of generations. However, note that the more generations and the more individuals there are, the closer the end result would be to the true optimum (Strelow, 2007).

5. Results and Discussion

Since a major part of the model is based on random numbers (e.g. locations of robbers), a distribution of results is observed (about $\pm 5\%$ from the average) (Fig. 9). This is why the convergence criteria do not work for the genetic algorithm as it might result from the same set of variables.

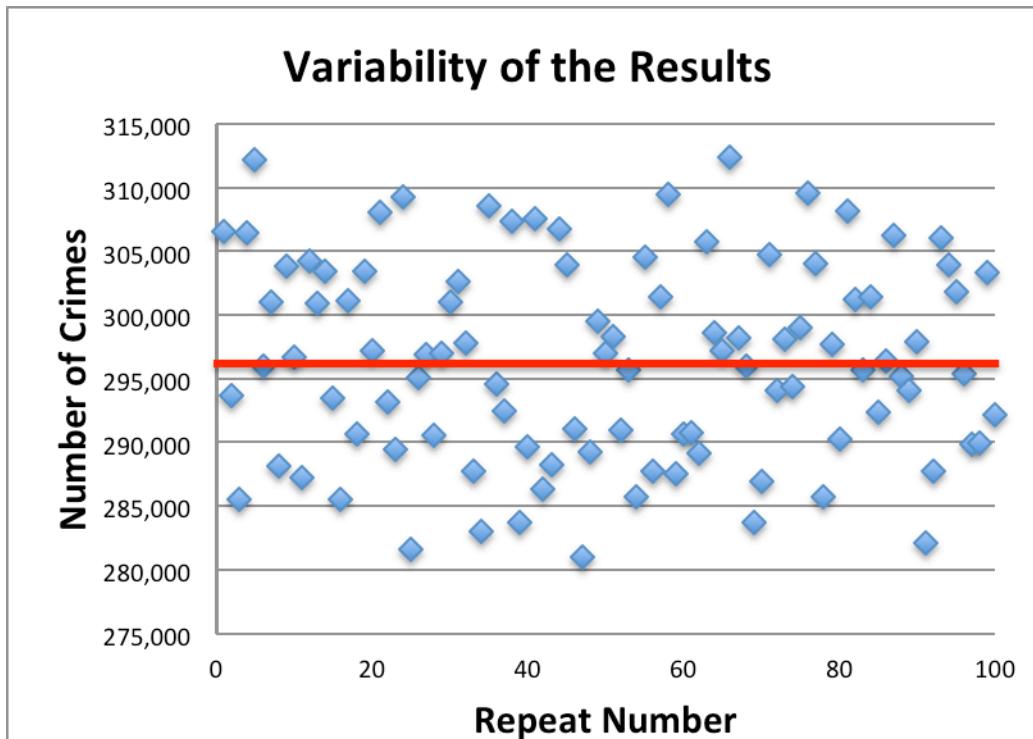


Figure 9: Variability of result from simulation_main (5000,965,851,50)

There are several other trends observed from changing the variables in the simulation. Firstly, due to the dependence of the attractiveness level to the number of policemen in a given vertex (formula 4.1.4), and the dependence of probability of burglary to the attractiveness level of a vertex (formula 4.1.2), the more policemen there are in a simulation, the less number of crimes observed throughout the same period of time, regardless which strategy is used as shown in the chart below (Fig. 10).

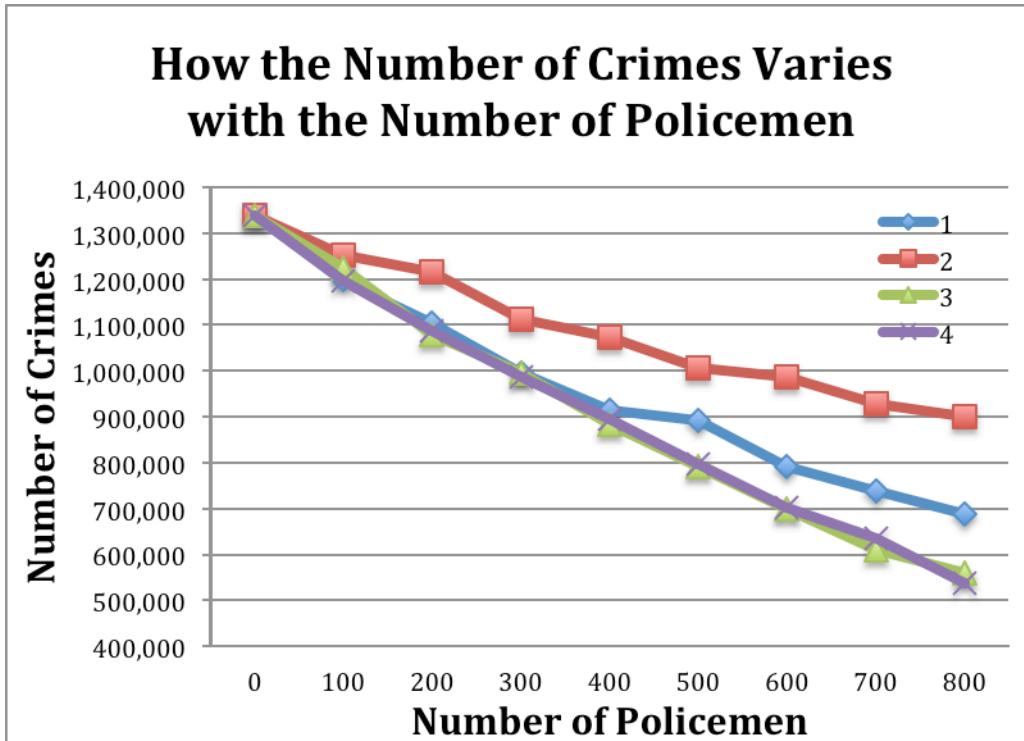


Figure 10: How the number of crimes varies with the number of policemen in different strategies

Secondly, the area of patrol does not seem to make significant difference after the limit becomes larger than 20 units (Fig. 11). The size of the area of patrol does not change anything in tactic 2 since the distributed policemen are stationary then (area of patrol = 0)

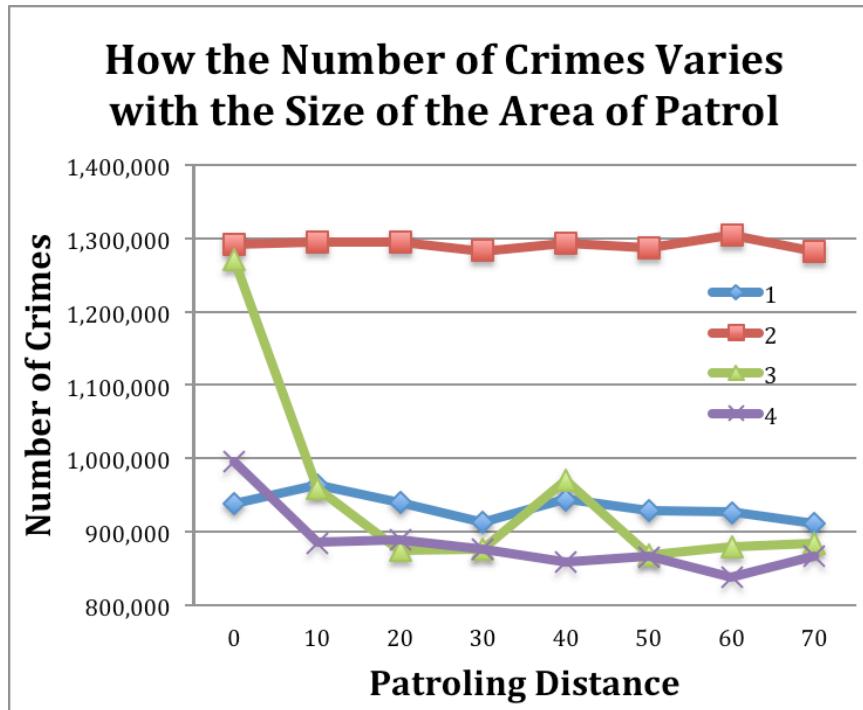


Figure 11: How the number of crimes varies with the size of the area of patrol in different strategies

Thirdly, the number of distributed policemen does not make any significant difference in tactic 1, 3 or 4. However, in tactic 2, the more policemen distributed across the map, the more stationary policemen there are, the higher the number of crimes become at the end of the simulation (Fig. 12). The second and third results show that strategies with stationary policemen do not work effectively in minimizing the number of crimes.

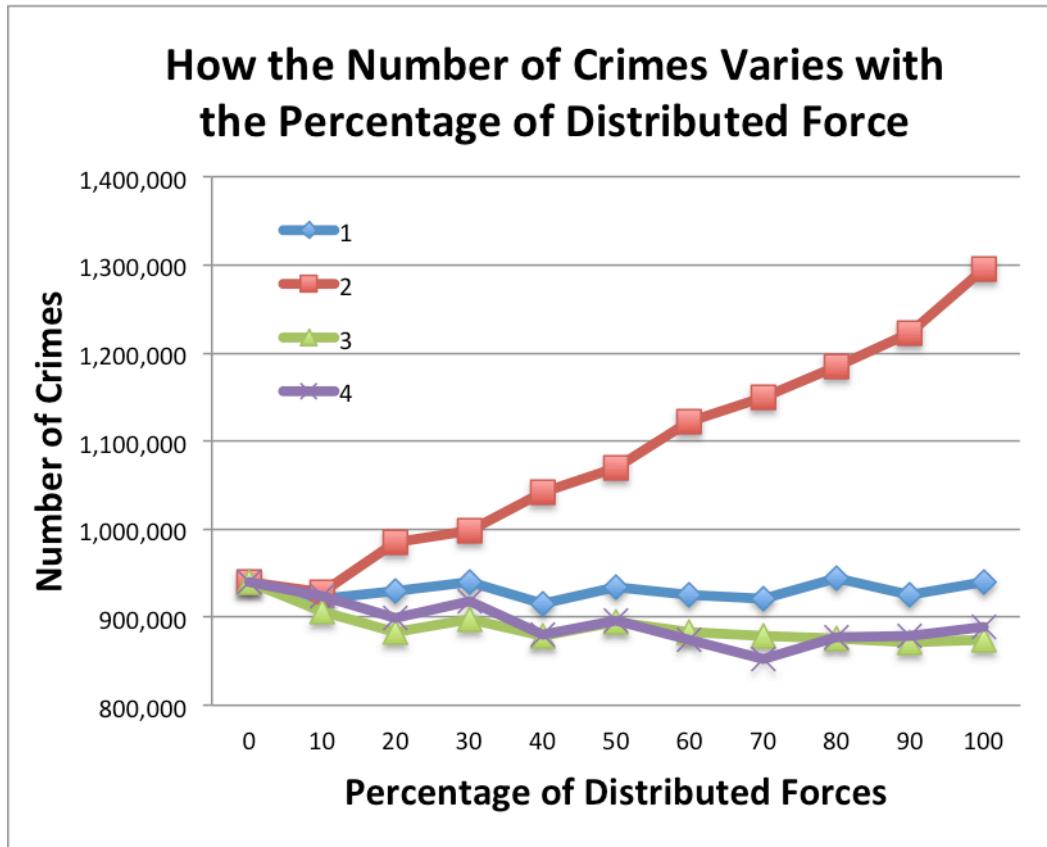


Figure 12: How the number of crimes varies with the percentage of distributed force in different strategies

An interesting point to notice is that if the tactic is applied less than once every 4 days, tactic 1 yields similar result with tactic 3 and 4 (Fig. 13). This might be due to the time needed for hot spots formation. In other strategies, other than tactic 1, the frequency of the tactic being applied does not seem to make a significant difference.

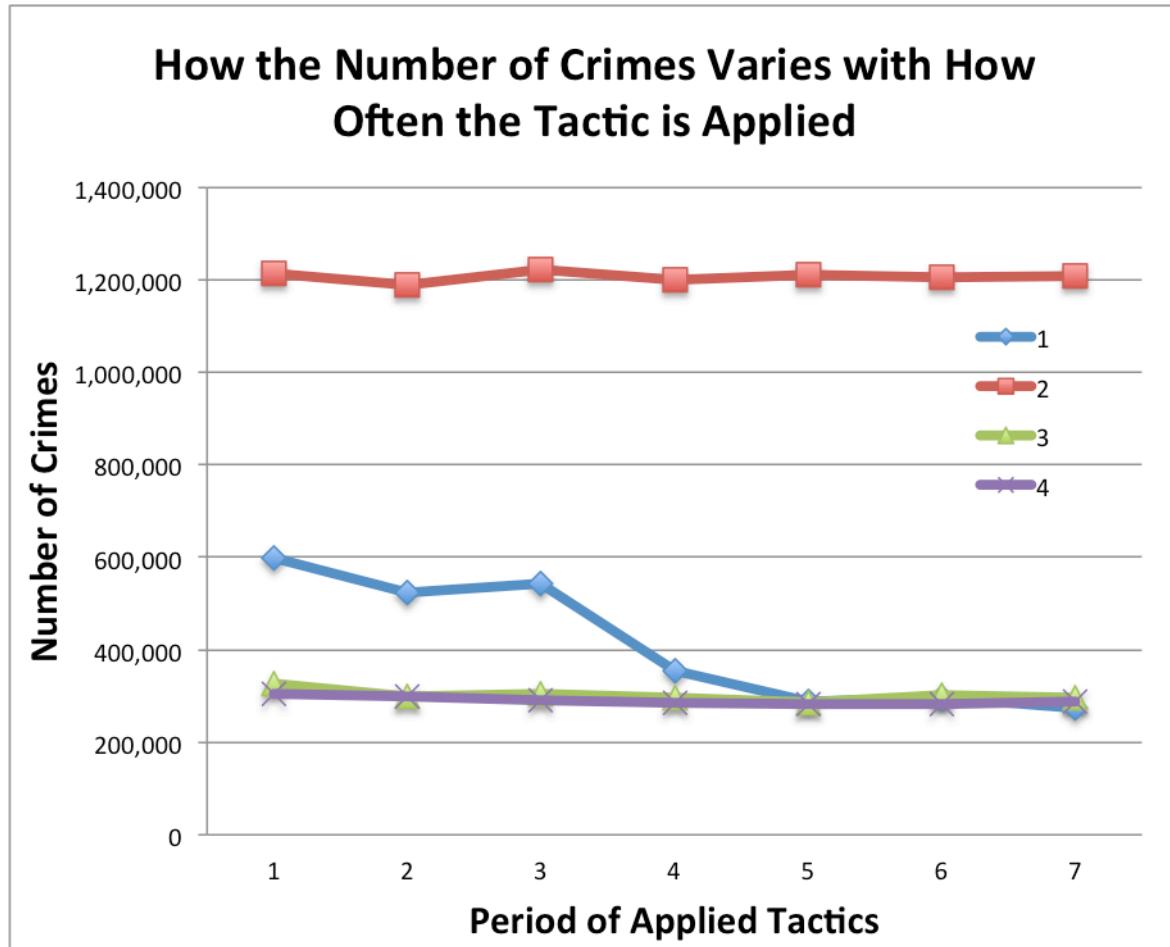


Figure 13: How the number of crimes varies with how often the tactic is applied in different strategies

6. Conclusion

From these results, it seems that to minimize the number of crimes in this simulation, the number of policemen need to be maximized. However, there should not be any stationary policemen at all and an active response strategy (tactic 1) will only be effective if it is applied less than once every 4 days.

The genetic algorithm yields similar result. Due to the elitist strategy and the survival of the fittest concept, after 15 – 20 generations, the number of police forces used is always maximized, and the frequency of the tactics being applied always become less than once every four days.

7. Future Research

In the future, shortening the simulation time may prove to be useful as it takes about an hour to finish a simulation. However, as seen below (Fig. 14), the average number of crimes per day plateau off after 40,000 iterations and the number of crimes continues to increase linearly. Therefore, the system might have reached an equilibrium state after the first 40,000 iterations. Similar patterns

might be observed had the simulation been stopped at 40,000 iterations rather than 100,000 iterations.

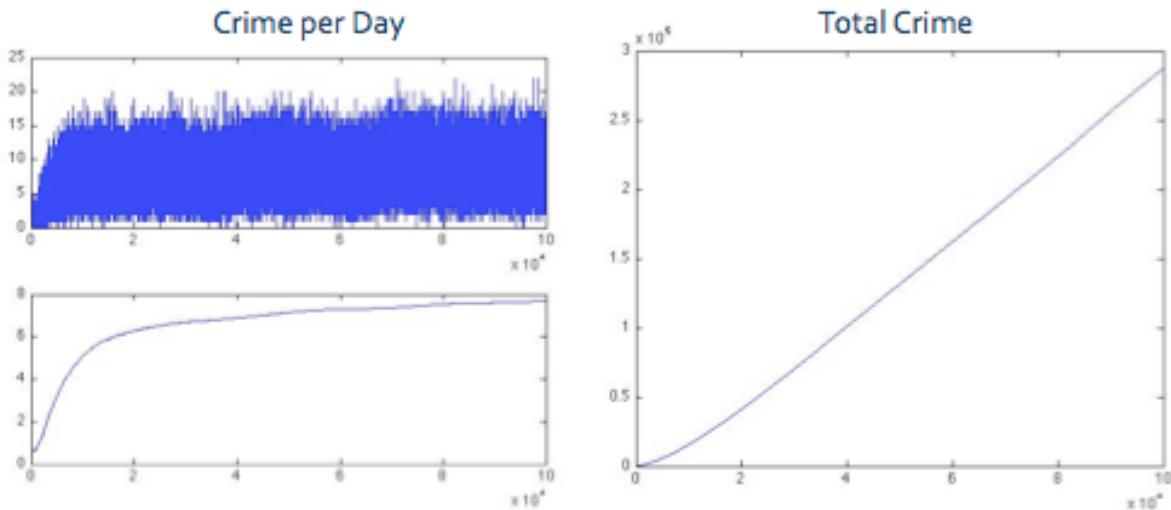


Figure 14: Average crime per day and the total crime chart

As discussed in other papers, there are other aspects to be considered to determine the optimal police patrol of a certain area. Some papers mention about the psychological and social effects of a police presence in the neighborhood, although it may not necessarily lead to lower crime rate (Kelling, 1974). Other papers mention about the cost of moving policemen or assigning a patrolling area for different police forces (Braga, 2008). The ability of the criminals to learn and adapt to a particular tactic should also be considered (Reis, 2004). Furthermore, applying different optimization methods, like bracketing method or binary search, may provide more data as well. In addition, comparing the result of the simulation with a known set of past data and applying it on a real map rather than a regular grid may prove to be interesting.

8. Acknowledgements

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9. Bibliography

1. Short, M. B., M. R. D'Orsogna, V. B. Pasour, G. E. Tita, P. J. Brantingham, A. L. Bertozzi, and L. B. Chayes. "A Statistical Model of Criminal Behavior." *Mathematical Models and Methods in Applied Sciences* Vol. 18 (2008): 1249–1267. Print.
2. Jones, Paul A., P. Jeffrey Brantingham, and Lincoln R. Chayes. "Statistical Models of Criminal Behavior: The Effects of Law Enforcement Actions." *Mathematical Models and Methods in Applied Sciences* Vol. 20 (2010): 1397 - 1423. Print.
3. Strelow, Martin. "Einsatz numerischer Optimierungs-verfahren zur Wirkungsgrad-Maximierung des Dampfkraftprozesses solarthermischer Kraftwerke." (2007) Print.
4. Reis, Danilo, Adriano Melo, Andre L. V. Coelho, and Vasco Furtado. "Towards Optimal Police Patrol Routes with Genetic Algorithm." *University of Fortaleza* (2004). Print.
5. Kelling, George L.. *The Kansas City preventive patrol experiment: a summary report*. Washington: Police Foundation, 1974. Print.
6. Braga, Anthony A.. "Police Enforcement Strategies to Prevent Crime in Hot Spot Areas." *Crime Prevention Research Review* 2 (2008): 1-36. Print.