

# STATISTICS

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# STATISTICS

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*Statistics* allow us to learn from data.

Observations of a *variable*:

$$x = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

# UNIVARIATE STATISTICS

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A **statistic** summarizes data. You're already familiar with some common statistics, like averages.

We oftentimes want to find the “center” of the data — this describes typical values

$$x = [1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10]$$

The *mean* ( $\bar{x}$ ) is calculated by summing the data, then dividing by the number of observations:

$$\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i$$

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The *median* is found by ordering the observations from highest to lowest and finding the one in the middle:



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- The median balances the number of observations on either side
- Which is a better measure?

## MEAN VS MEDIAN

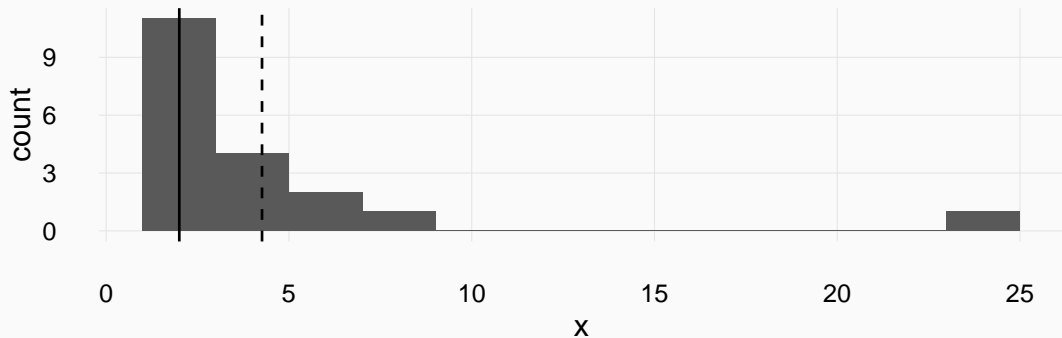
```
x <- c(1, 1, 2, 1, 1, 3, 2, 4, 2, 1,  
       1, 1, 5, 7, 9, 4, 5, 6, 25)  
mean(x)
```

```
## [1] 4.263158
```

```
median(x)
```

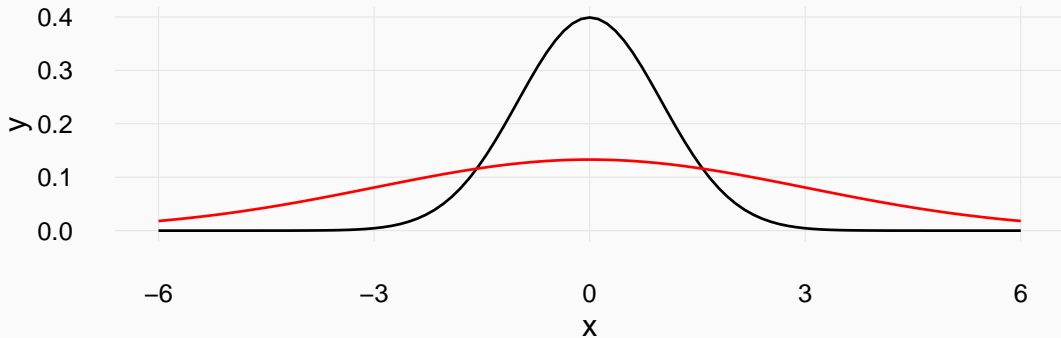
```
## [1] 2
```

## MEAN VS MEDIAN



## VARIANCE

Finding central tendency is good, but we might go a step further. Consider these two distributions:



## YOU TRY!

Here are the average high's from a previous year's math camp:

Day	M	Tu	W	Th	F	M	Tu
High	95	103	100	97	39	108	112

Find the mean, median, and mode.



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Here are the average high's from a previous year's math camp:

Day	M	Tu	W	Th	F	M	Tu
High	95	103	100	97	39	108	112

Find the mean, median, and mode.

What's weird with this data?

## YOU TRY (ANSWERS)

```
x <- c(95, 103, 100, 97, 39, 108, 112)  
mean(x)
```

```
## [1] 93.42857
```

```
median(x)
```

```
## [1] 100
```

*Variance* measures how spread out a distribution is. One way to calculate it is like so:

$$s_x^2 = \frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2$$

That measures the sum of the squared average deviation from the mean

“Squared average deviation from the mean” is a bit weird, though, so oftentimes we use standard deviations instead, which is just the squared root of the variance:

$$s_x = \sqrt{s_x^2}$$

## YOU TRY!

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I flipped a coin 4 times, one of which was a heads. What's the mean of the data? What's the variance? The standard deviation?

## YOU TRY (ANSWERS)

```
x <- c(1, 0, 0, 0)
```

```
mean(x)
```

```
## [1] 0.25
```

```
var(x)
```

```
## [1] 0.25
```

```
sd(x)
```

```
## [1] 0.5
```

# BIVARIATE STATISTICS

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Thus far, we've focused on statistics that summarize just one variable. But we're oftentimes interested in relationships between different variables. This can be hard to see with the raw data, though:



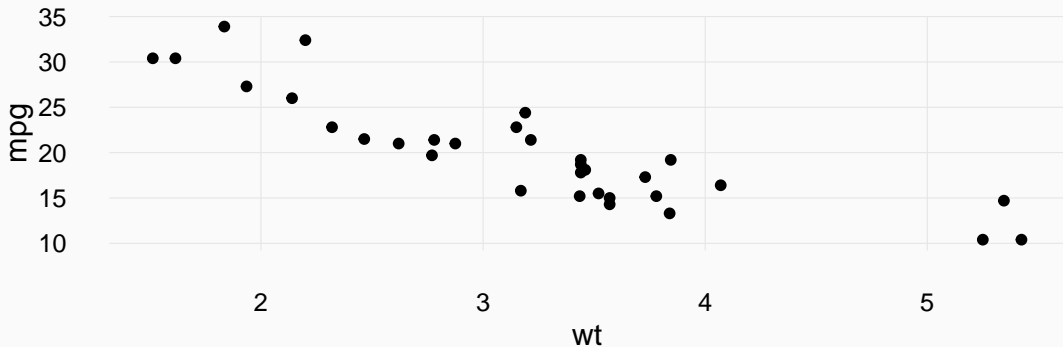
What's the relationship between mpg and wt?

```
head(mtcars, 9)
```

##		mpg	cyl	disp	hp	drat	wt	qsec	vs	am	gear	carb
##	Mazda RX4	21.0	6	160.0	110	3.90	2.620	16.46	0	1	4	4
##	Mazda RX4 Wag	21.0	6	160.0	110	3.90	2.875	17.02	0	1	4	4
##	Datsun 710	22.8	4	108.0	93	3.85	2.320	18.61	1	1	4	1
##	Hornet 4 Drive	21.4	6	258.0	110	3.08	3.215	19.44	1	0	3	1
##	Hornet Sportabout	18.7	8	360.0	175	3.15	3.440	17.02	0	0	3	2
##	Valiant	18.1	6	225.0	105	2.76	3.460	20.22	1	0	3	1
##	Duster 360	14.3	8	360.0	245	3.21	3.570	15.84	0	0	3	4
##	Merc 240D	24.4	4	146.7	62	3.69	3.190	20.00	1	0	4	2
##	Merc 230	22.8	4	140.8	95	3.92	3.150	22.90	1	0	4	2

## ALWAYS PLOT YOUR DATA!!!

```
ggplot(mtcars) +  
  geom_point(aes(wt, mpg))
```



*Covariance* measures the direction of a relationship:

```
with(mtcars, cov(wt, mpg))
```

```
## [1] -5.116685
```

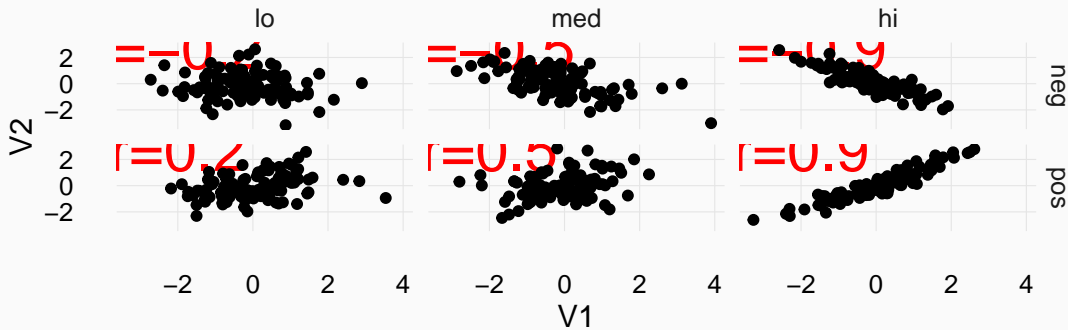
*Correlation* (pearson's  $r$ ) captures the direction and strength of a linear relationship between two variables.

Ranges from  $-1$  to  $1$

```
with(mtcars, cor(wt, mpg))
```

```
## [1] -0.8676594
```

# CORRELATION



# CORRELATION

ALWAYS PLOT YOUR DATA

