Functions

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```
##
## Attaching package: 'dplyr'

## The following objects are masked from 'package:stats':
##
##     filter, lag

## The following objects are masked from 'package:base':
##
##     intersect, setdiff, setequal, union
```

Exponents and Logarithms

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  - Othertimes, it means $\log_e(n) = ln(n)$

Properties of exponents

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$$a^0 = 1$$

$$a^{1/n} = \sqrt[n]{a}$$

Properties of exponents (continued)

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$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n} = a^n b^{-n} \qquad \forall b \neq 0$$

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$$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n \qquad \forall a, b \neq 0$$

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$$\frac{\log_x n}{\log_x m} = \log_m n$$

You try!

$$2^4$$

$$\log(100)$$

$$a^3 \times a^7$$

$$\log_{10}(10z)$$

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    - y is the *output* from the function

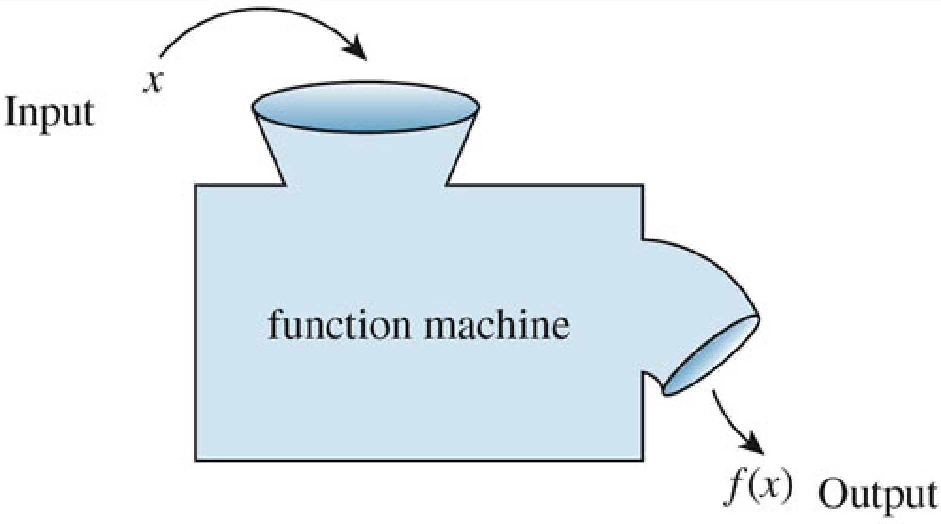

Figure 1:

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Linear functions

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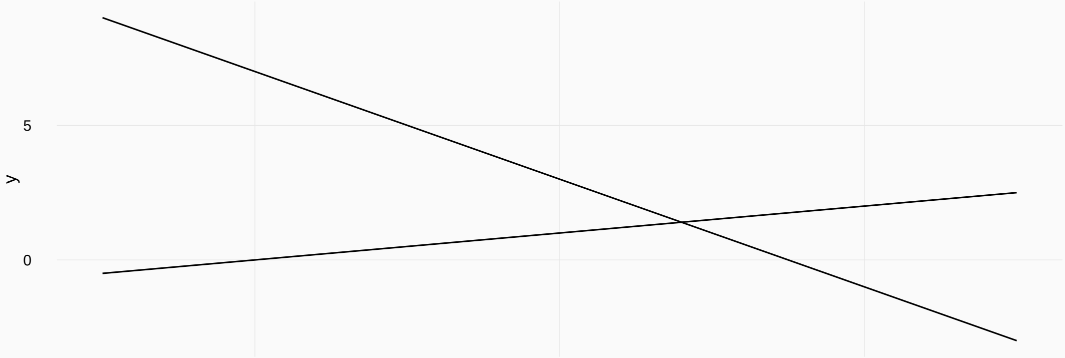
    - m is the slope (for every one unit increase in x, y increases m units)
    - b is the y-intercept: the value of y when $x = 0$

```r
ggplot(data.frame(x = c(-3, 3)), aes(x)) +
  stat_function(fun = function(x) -2 * x + 3, geom = "line")  +
  stat_function(fun = function(x) (1 / 2) * x + 1)
```



- These lines have one curve

Quadratics

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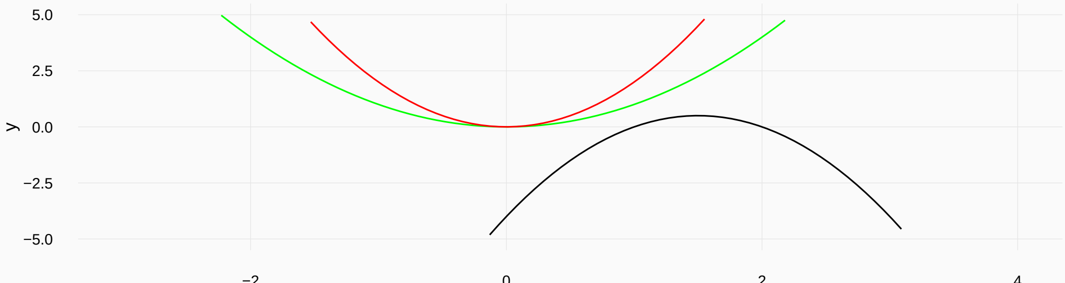
- $y = ax^2 + bx + c$

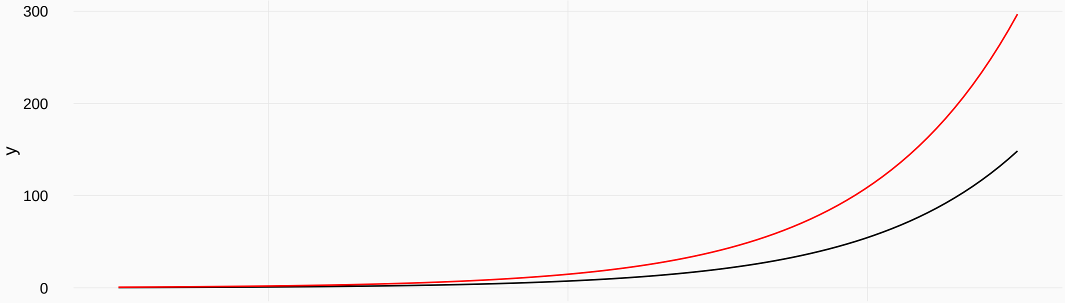
```
ggplot(data.frame(x = c(-3, 4)), aes(x)) +
  stat_function(fun = function(x) x ^ 2, color = "green") +
  stat_function(fun = function(x) 2 * x ^ 2, color = "red") +
  stat_function(fun = function(x) -2 * x ^ 2 + 6 * x - 4) +
  ylim(c(-5, 5))
```



Exponential

- General form: $y = a * b^{kx} + k$

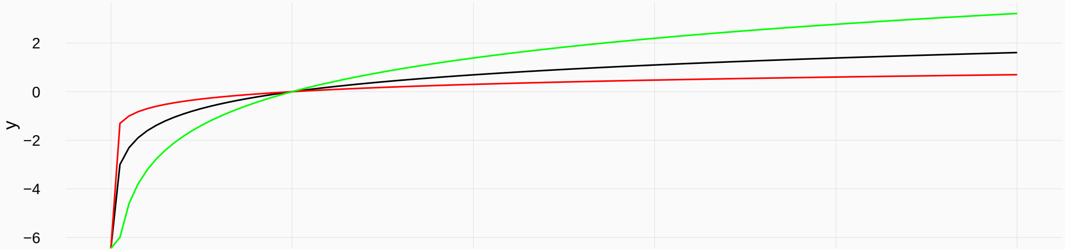
```
ggplot(data.frame(x = c(-1, 5)), aes(x)) +
  stat_function(fun = function(x) exp(x)) +
  stat_function(fun = function(x) 2 * exp(x), color = "red")
```



Logs

- General form: $y = a * \log(bx) + k$

```r
ggplot(data.frame(x = c(0, 5)), aes(x)) +
  stat_function(fun = function(x) log(x)) +
  stat_function(fun = function(x) log10(x), color = "red") +
  stat_function(fun = function(x) 2 * log(x), color = "green")
```



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- In fact, these functions are each others “inverse” function
  - Plug in y to find x
- Exponents have horizontal asymptote
- Logs have vertical asymptote

You try!

What function describes this line?