

FUNCTIONS

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 - Othertimes, it means $\log_e(n) = \ln(n)$

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$$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n \quad \forall a, b \neq 0$$

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PROPERTIES OF LOGS

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$$m^{\log_m(a)} = a$$

$$\frac{\log_x n}{\log_x m} = \log_m n$$

YOU TRY!

$$2^4$$

$$\log(100)$$

$$a^3 \times a^7$$

$$\log_{10}(10z)$$

FUNCTIONS

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 - y is the *output* from the function

FUNCTION MACHINE

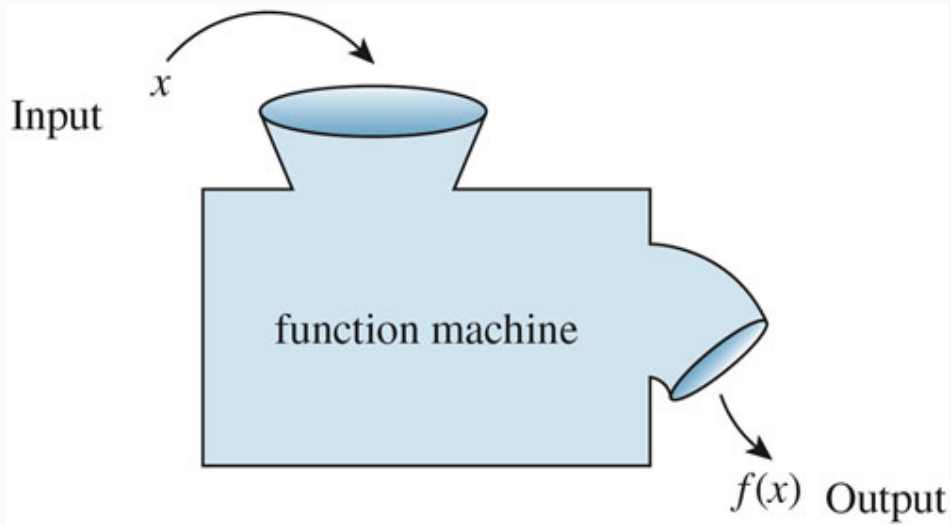


Figure 1:

- We can make a function that describes a line pretty easily

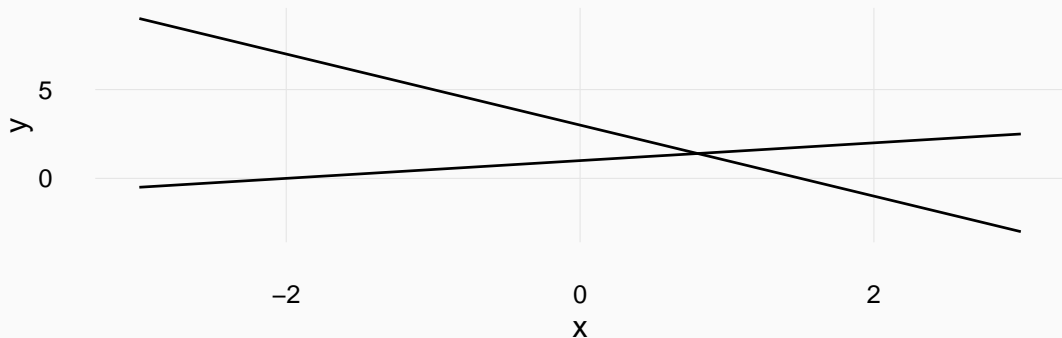
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 - m is the slope (for every one unit increase in x , y increases m units)
 - b is the y-intercept: the value of y when $x = 0$

LINEAR FUNCTIONS

```
ggplot(data.frame(x = c(-3, 3)), aes(x)) +  
  stat_function(fun = function(x) -2 * x + 3, geom = "line") +  
  stat_function(fun = function(x) (1 / 2) * x + 1)
```

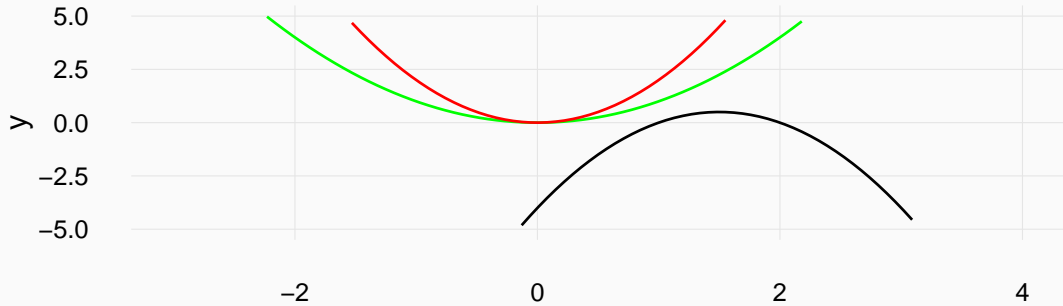


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- $y = ax^2 + bx + c$

QUADRATICS

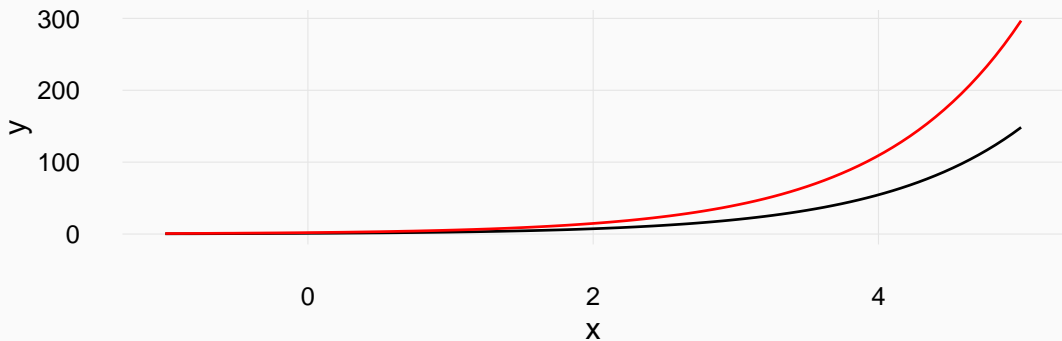
```
ggplot(data.frame(x = c(-3, 4)), aes(x)) +  
  stat_function(fun = function(x) x ^ 2, color = "green") +  
  stat_function(fun = function(x) 2 * x ^ 2, color = "red") +  
  stat_function(fun = function(x) -2 * x ^ 2 + 6 * x - 4) +  
  ylim(c(-5, 5))
```



EXPONENTIAL

- General form: $y = a * b^{kx} + k$

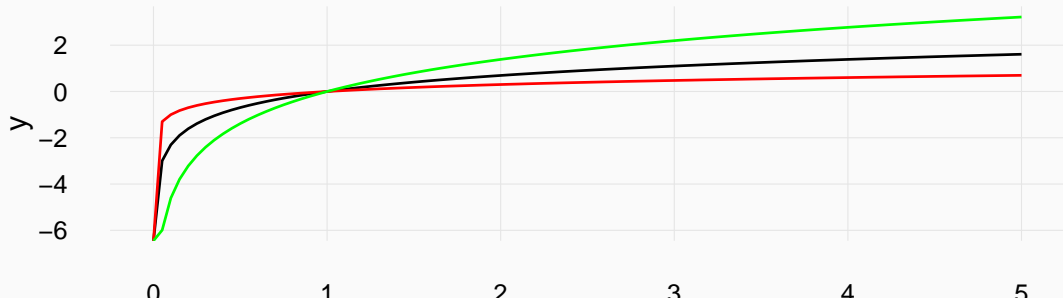
```
ggplot(data.frame(x = c(-1, 5)), aes(x)) +  
  stat_function(fun = function(x) exp(x)) +  
  stat_function(fun = function(x) 2 * exp(x), color = "red")
```



LOGS

- General form: $y = a * \log(bx) + k$

```
ggplot(data.frame(x = c(0, 5)), aes(x)) +  
  stat_function(fun = function(x) log(x)) +  
  stat_function(fun = function(x) log10(x), color = "red") +  
  stat_function(fun = function(x) 2 * log(x), color = "green")
```



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- In fact, these functions are each others “inverse” function
 - Plug in y to find x
- Exponents have vertical asymptote
- Logs have horizontal asymptote

YOU TRY!

What function describes this line?

