Set Theory & Combinations

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Set Theory
Combinatorics

What is it?
Subsets
Set universes
Graphical representation
Properties of Unions, Intersections

Intro to Set Theory

- What is set theory?

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- What is set theory?
- A branch of mathematics

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Intro to Set Theory

- What is set theory?
- A branch of mathematics
- Collects objects into *sets* and studies the properties

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What's a set?

- A **set** is a collection of objects

$$S = \{s_1, s_2, s_3, ...s_n\}$$

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- The objects can be anything

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What's a set?

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- The objects can be anything
- We usually use variables or units of observation

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Elements in or not

- We can say whether an object is in a set or not:

$$s_{13} \in S$$

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$$s_{13} \in S$$

- Or not:

$$q_1 \notin S$$

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Subsets

- We can divide sets into *subsets*

$$M \subset S$$

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Subsets

- We can divide sets into *subsets*

$$M \subset S$$

- M is a *proper subset* of S iff all elements of M are in S but not all elements of S are in M.
- We can also define non-proper subsets:

$$L \subseteq S$$

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Empty sets

- Some sets contain no elements

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Empty sets

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- For example, cities on Earth with average temperatures of greater than 1000 degrees

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Empty sets

- Some sets contain no elements
- For example, cities on Earth with average temperatures of greater than 1000 degrees
- Though Austin might be close. . .

$$Z = \{\emptyset\}$$

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Set Universes

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$$R = 1, 2, 3, 4, 5, 6$$

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- R represents all the possibilities of a (single) roll of a die

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$$R = 1, 2, 3, 4, 5, 6$$

- R represents all the possibilities of a (single) roll of a die
- We can define sets for the even possibilities and the odd possibilities

$$E = \{2, 4, 6\} \qquad O = \{1, 3, 5\}$$

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Compliments

- A *compliment* is that together, they contain all the elements of the relevant universe

$$E = O^C \qquad ; \qquad O = E^C$$

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Universe

- Board examples of how to draw sets

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Properties

- $A \cup B = B \cup A$

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- $(A \cup B) \cup C = A \cup (B \cup C)$

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- $A \cup (C \cap C) = (A \cup B) \cap (A \cup C)$

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- Countabl *discrete* structures
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$$R = \{1, 2, 3, 4, 5, 6\}$$

- How many different combinations of 3 dice rolls are there?

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- think "permutation" = "position"

Repitition?

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- more generally, n^r
  - n is number of things to choose from
  - choose r

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- How many different orderings of 9 horses in a race??
  - $9 * 8 * 7 \ldots = 9! = 362,880$
- Alternatively, maybe we just care about the first three:
  - $9 * 8 * 7 = 504$
- More generally:

$$\frac{n!}{(n-r)!}$$

Repition not allowed

- Easiest way: do permutation without repitition, then get rid of all the "same" outcomes

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- We've already figured out hte permutation part, so need to figure out the second part

$$\frac{n!}{(n-r)!} * \frac{1}{r!} = \frac{n!}{r!(n-r)!} = \binom{n}{r}$$

Repitition allowed

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- Also the most difficult to calculate

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- Formula:

$$\binom{n+r-1}{r} = \frac{(n+r-1)!}{r!(n-1)!}$$