Functions

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Exponents and Logarithms

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  - Othertimes, it means $\log_e(n) = ln(n)$

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$$\frac{\log_x n}{\log_x m} = \log_m n$$

You try!

$$2^4$$

$$\log(100)$$

$$a^3 \times a^7$$

$$\log_{10}(10z)$$

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    - x and z are the *arguments* that the function takes
    - y is the *output* from the function

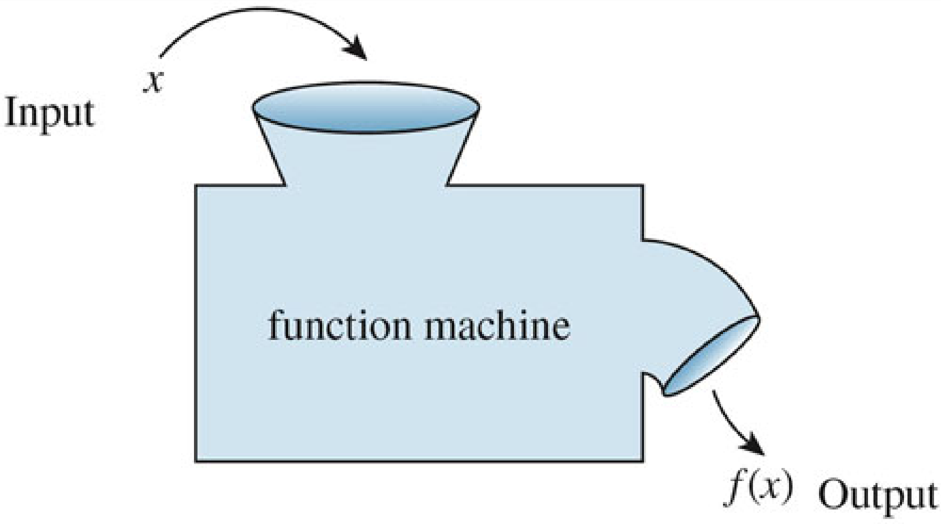

Figure 1:

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- $y = mx + b$
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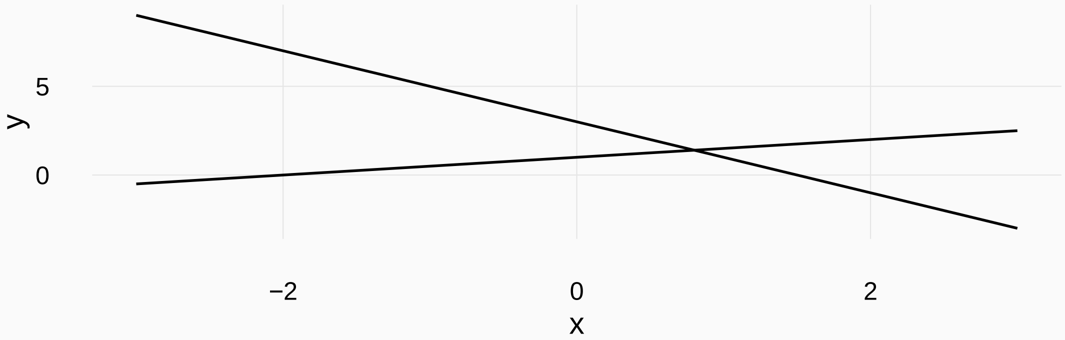
  - b is the y-intercept: the value of y when $x = 0$

```
ggplot(data.frame(x = c(-3, 3)), aes(x)) +
  stat_function(fun = function(x) -2 * x + 3, geom = "line")  +
  stat_function(fun = function(x) (1 / 2) * x + 1)
```



Quadratics

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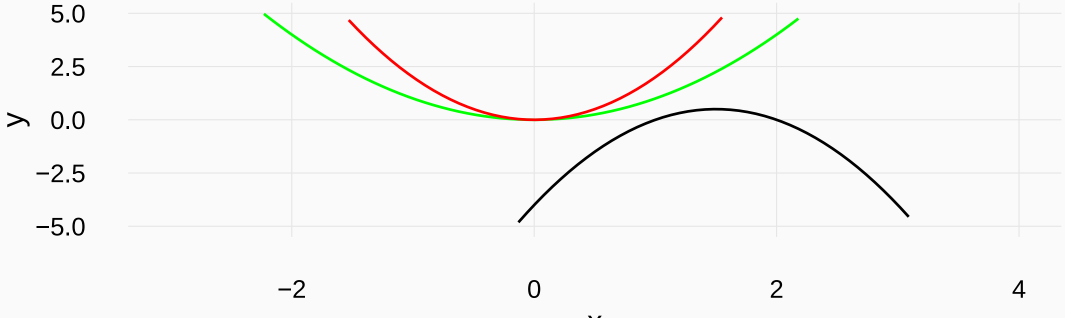
- $y = ax^2 + bx + c$

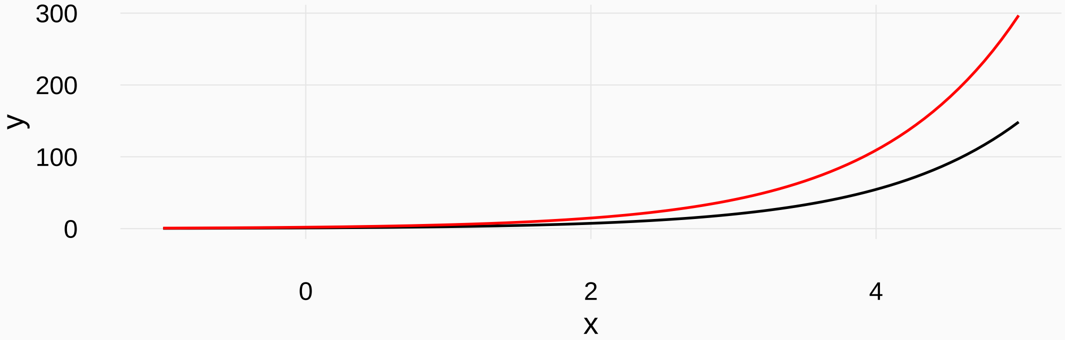
```
ggplot(data.frame(x = c(-3, 4)), aes(x)) +
  stat_function(fun = function(x) x ^ 2, color = "green") +
  stat_function(fun = function(x) 2 * x ^ 2, color = "red") +
  stat_function(fun = function(x) -2 * x ^ 2 + 6 * x - 4) +
  ylim(c(-5, 5))
```



Exponential

- General form: $y = a * b^{kx} + k$

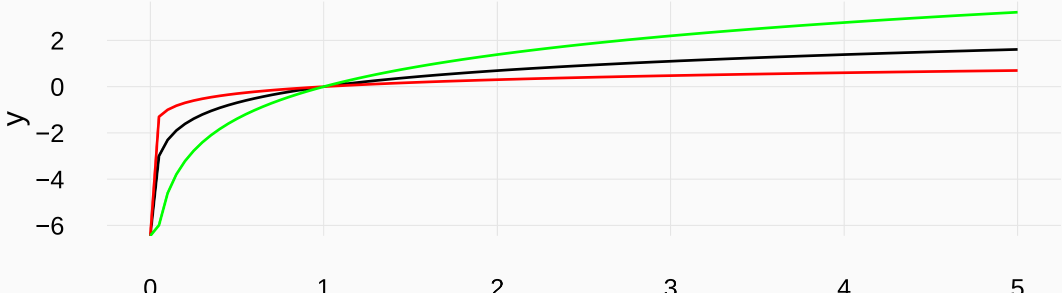
```
ggplot(data.frame(x = c(-1, 5)), aes(x)) +
  stat_function(fun = function(x) exp(x)) +
  stat_function(fun = function(x) 2 * exp(x), color = "red")
```



Logs

- General form: $y = a * \log(bx) + k$

```
ggplot(data.frame(x = c(0, 5)), aes(x)) +
  stat_function(fun = function(x) log(x)) +
  stat_function(fun = function(x) log10(x), color = "red") +
  stat_function(fun = function(x) 2 * log(x), color = "green")
```



Logs and Exponents

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- The log and exponent charts are obviously related
- In fact, these functions are each others “inverse” function
    - Plug in y to find x
- Exponents have vertical asymptote
- Logs have horizontal asymptote

You try!

What function describes this line?