Functions

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  - Othertimes, it means $log_e(n) = ln(n)$

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- $\left(\dfrac{a}{b}\right)^{-n} = \left(\dfrac{b}{a}\right)^n \qquad \forall a, b \neq 0$

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- $\frac{log_x n}{log_x m} = log_m n$

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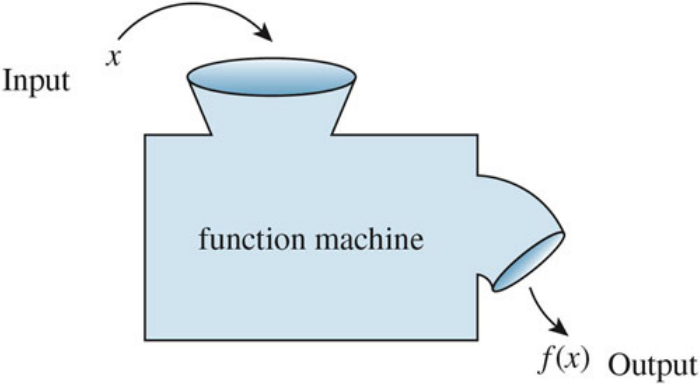

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- $y = mx + b$
    - m is the slope (for every one unit increase in x, y increases m units)
    - b is the y-intercept: the value of y when $x = 0$

Linear functions

```
ggplot(data.frame(x=c(-3, 3)), aes(x)) +
  stat_function(fun=function(x)-2*x+3, geom="line")  +
  stat_function(fun=function(x)(1/2)*x+1)
```

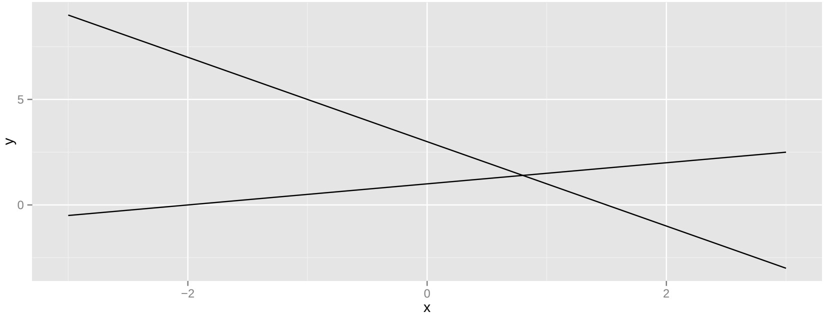

Figure 1:

Quadratics

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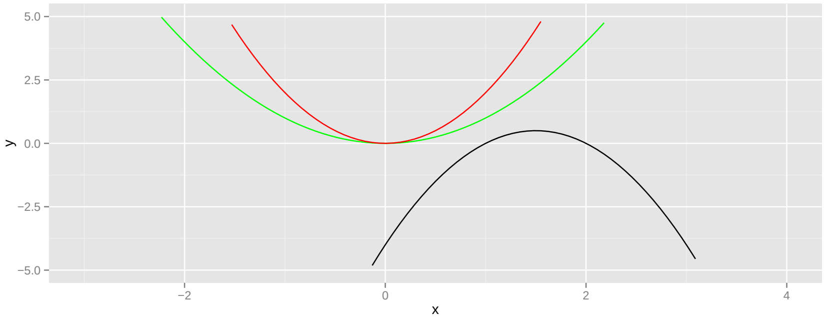
```
ggplot(data.frame(x=c(-3,4)), aes(x)) +
  stat_function(fun=function(x)x^2, color="green") +
  stat_function(fun=function(x)2*x^2, color="red") +
  stat_function(fun=function(x)-2*x^2 + 6*x -4) +
  ylim(c(-5, 5))
```



Exponential

- General form: $y = a * b^{kx} + k$

```
ggplot(data.frame(x=c(-1, 5)), aes(x)) +
  stat_function(fun=function(x)exp(x)) +
  stat_function(fun=function(x)2*exp(x), color="red")
```

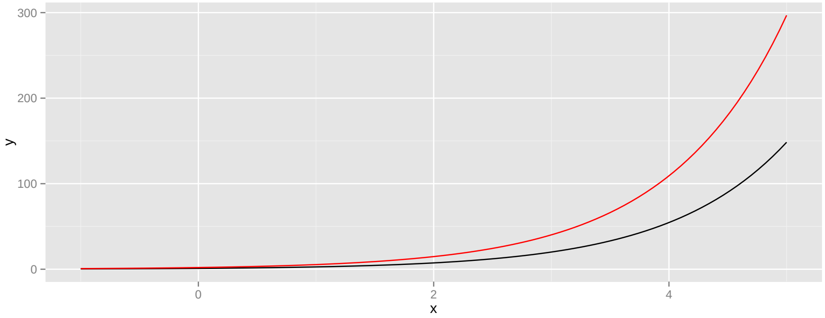
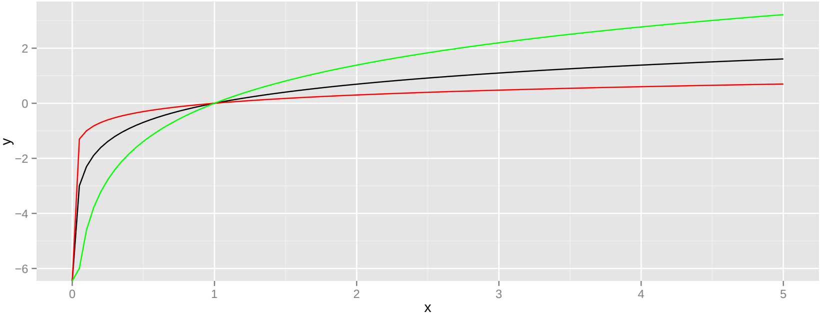


Figure 3:

Logs

- General form: $y = a * log(bx) + k$

```
ggplot(data.frame(x=c(0,5)), aes(x)) +
  stat_function(fun=function(x)log(x)) +
  stat_function(fun=function(x)log10(x), color="red") +
  stat_function(fun=function(x)2*log(x), color="green")
```



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    - Plug in y to find x
- Exponents have horizontal asymptote
- Logs have vertical asymptote