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A stochastic equilibrium approach to the reserve capacity market under the risk-aversion paradigm

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Abstract

Variations in a two settlement market that trades both energy and reserve, first in day-ahead and then in real-time, are analysed through a stochastic equilibrium framework. This framework takes into account the real-time uncertainty and the introduction of the risk-aversion paradigm. Variations in the markets take the form of design choices, namely (i) allowing or not virtual trading of energy in day ahead and (ii) holding or not a real time market for energy. We proposed a decomposition heuristic that iterates until convergence is reached between (i) the resolvent of the modified risk-neutral market and (ii) the aversion problem from the day-ahead point of view. The technique is then tested on a simple market to extract general trends.

List of symbols

Sets

G	Set of possible generators.
L	Set of possible loads.
RL	Set of possible reserve loads.
Ω	Set of possible scenarios.

Variables

$p_{g,\omega}^{RT}$ Real time production of generator g in the scenario ω.

$d_{l,\omega}^{RT}$ Real time demand of load l in the scenario ω.

$r_{g,\omega}^{RT}$ Real time reserve of generator g in the scenario ω.

$d_{l,\omega}^{R,RT}$ Real time demand of reserve for the reserve load l in the scenario ω.

$\mu_{g,\omega}^{G,RT}$ Marginal benefit of generator g in the scenario ω. Dual variable relative to the real-time physical constraint on production.

$\mu_{l,\omega}^{L,RT}$ Marginal benefit of load l in the scenario ω. Dual variable relative to the real-time physical demand.

$\mu_{g,\omega}^{G,R,RT}$ Marginal benefit for reserve of generator g in the scenario ω. Dual variable relative to the real-time ramp constraint for reserve.

$\mu_{l,\omega}^{R,RT}$ Marginal benefit of reserve load l in the scenario ω. Dual variable relative to the real-time physical constraint for reserve.

λ_{ω}^{RT} Real-time price of energy in the scenario ω. Dual variable relative to the real-time market clearing constraint on energy.

$\lambda_{\omega}^{R,RT}$ Real-time price of reserve in the scenario ω. Dual variable relative to the real-time market clearing constraint on reserve.

p_{g}^{DA} Day-ahead production of generator g.

d_{l}^{DA} Day-ahead demand of load l.

r_{g}^{DA} Day-ahead reserve of generator g.

$d_{l}^{R,DA}$ Day-ahead demand of reserve for the reserve load l.

$\mu_{g}^{G,DA}$ Dual variable relative to the Day-ahead physical constraint on production. Active when virtual trading is not allowed.

$\mu_{l}^{L,DA}$ Dual variable relative to the real-time physical demand. Active when virtual trading is not allowed.

$\mu_{g}^{G,R,DA}$ Marginal benefit for reserve of generator g. Dual variable relative to the day-ahead ramp constraint for reserve.

$\mu_{l}^{R,DA}$ Marginal benefit of reserve load l. Dual variable relative to the day-ahead physical constraint for reserve.

λ^{DA} Day-ahead price of energy. Dual variable relative to the day-ahead market clearing constraint on energy.

$\lambda^{R,DA}$ Day-ahead price of reserve. Dual variable relative to the day-ahead market clearing constraint on reserve.

y_g Unit commitment value of the generator g.

The day-ahead position of reserve and energy are the quantity of forward contract bought. The prices of those forward contract are the day-ahead prices.

Profit maximisation problems

$\mathcal{G}_{g,\omega}^{RT,US}$ The generator g real-time profit maximisation problem in the scenario ω for the US model.

$\mathcal{L}_{l,\omega}^{RT,US}$ The load l real-time profit maximisation problem in the scenario ω for the US model.

$\mathcal{S}_{\omega}^{RT,US}$ The system operator real-time profit maximisation problem in the scenario ω for the US model.

$\mathcal{G}_{g}^{DA,US}$ The generator g day-ahead profit maximisation problem for the US model.

$\mathcal{L}_{l}^{DA,US}$ The load l day-ahead profit maximisation problem for the US model.

$\mathcal{S}^{DA,US}$ The system operator day-ahead profit maximisation problem ω for the US model.

For other models, the notation remains identical except for the US that needs to be replaced with the appropriate name.

Parameters

C_g Marginal cost of production for generator g.

$P_{g,\omega}^{+}$ Maximal production of generator g in the scenario ω.

R_g Reserve ramp of generator g.

V_l Marginal value of energy for load l.

$D_{l,\omega}^{+}$ Maximal demand of load l in the scenario ω.

V_l^R Real-time marginal value of reserve for reserve load l.

D_l^R Maximal demand of reserve for reserve load l.

K_g Start-up cost of generator g.

$V_l^{R,DA}$ Day-ahead marginal value of reserve for reserve load l.

P_ω Probability of the scenario ω.

Risk-measure

$\mathcal{R}_a$	Risk-measure of agent a.
$\mathcal{M}_a$	Risk-set of agent a.
$\mathbb{E}_{q_a}(\cdot)$	Risk-adjusted expecation of agent a.
$CVaR_\alpha$	Conditional value at risk with risk-aversion α.
$MVaR_{\beta,\alpha}$	Mean value at risk with risk-aversion α and weight of the real expectation $1-\beta$.
Π_a	Profit function of agent a.

Others

ORDC	Operating reserve demand curve.
RA	Risk-averse.
RN	Risk-neutral.

Contents

CHAPTER 1

Introduction

Even if the 2020 objective of 20.9% of production of energy through renewable technology [3] will not be met, we should expect an increase of renewable electricity in order to meet the global european objective of 2030.

But the integration of renewable energy in our electric power system is more easily said than done. Renewable technology is inherently an unpredictable source of electricity and in our developed country, electricity is seen as a necessity that should be available at all time. There should be no uncertainty about whether the lamp will light, or the stove will heat when turned on.

In order to incorporate smoothly renewable technology in our electricity generation mix, flexibles generators are needed. The role of those flexibles generators is to ensure an appropriate supply of electricity at all time. They should be able to make up for the low renewable electricity production period cause by unfavorable weather by activating swiftly their production. The other way around, when the weather is beneficial and the market is full of renewable energy, they should be able to lower their production to balance the market.

As we could expect it, flexibility does not come at no cost. Flexible generators are usually more expensive than standard generators and even more than renewable generators which does not need to pay for fuel. There is a paradox in the fact that on the one hand the market needs more and more flexible generators to compensate the growing unpredictability of renewable production but on the other hand the expensive flexible generators are being pushed out of the merit order by cheap renewable generators, making it less and less worth it to invest in flexible technology.

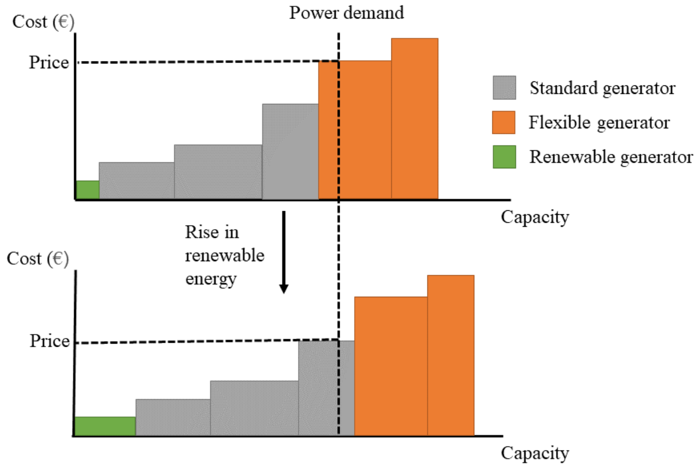


Figure 1.1: Illustration of merit order effet

The process of the flexible generators being pushed out of the merit order is shown in picture 1.1.

In this global discussion on the integration of renewable technology and the remuneration of flexible generators, this master thesis is comparing how different policies might affect the behaviors of a 2-stage market for energy and reserve. The thought process behind this master thesis is to extend the analysis from [1] under the risk-averse paradigm.

The novelty brought by risk aversion is that the weight of a scenario in the decision-making process of a risk-averse agent is not only established by its real probability of happening P_ω but also by whether the scenario is beneficial to the agent or not. A risk-averse agent will want to play it safe so if a scenario is unfavorable to him, the “risk-adjusted” probability of that scenario to happen will be higher than the real probability. In contrast, if a scenario is beneficial to the agent, the “risk-adjusted” probability of that scenario to happen will be lower than its real probability.

From there, the main questions we will address in this thesis are:

1. What is the impact of the no-virtual trading policy on the day ahead price and on the commitment of the generators?

2. To what extend does the risk aversion of an agent impact the back-propagation of real-time prices?

3. Who are the "losers" and "winner" of the different market?

This master thesis can be structured into three components: the modeling part, the algorithmic part and the interpretation of the results.

In the modeling part, the markets are formally defined. We start in chapter 2 with the introduction of the core model which is like a US-style market. The market is characterized by the profit maximization of its different agents. The risk-measures used to model risk aversion as well as the operating reserve demand curve (or ORDC) used to value reserve are defined in this chapter. We continue the modeling part with chapter 3, that introduce the two policies, namely the no-virtual trading policy and the no real time market for reserve policy, used to make the US model closer to a European style market.

The following component is the algorithmic part in chapter 4. The heuristic used to find equilibrium point of our markets is explained in this chapter. Because of the risk-aversion of our agents, the markets cannot be solved with classical solver and instead we used an algorithm called the forward-backward algorithm. This method is used to solve variational inequalities which are a unifying framework for classic optimization problem.

In the last part of the thesis, we will test the algorithm on a toy system and try to extract trends from the results obtained. The toy system is introduced in chapter 5. As for chapter 6, it will be dedicated to the interpretation of the result and the extraction of general trends.

CHAPTER 2

Core model or US model

The core model of our research is introduced in this chapter. This model is based on the core model of [1].

This chapter is built as followed. We begin with a bit of background on the problem as well as its definition. Then we will introduce the concept of risk measure and of coherent risk measure. We complete the explanation on the risk-measures with a series of examples. Then we will explicitly state the equilibrium by beginning with the real time optimization problems and ending with the day ahead ones. After that we will introduce the concept of ORDC whose goal is to valuate the reserve. We close this chapter with a brief conclusion.

2.1 Definition of the problem

Our model is a two settlement electricity market for energy and reserve. Energy is simply the electricity traded and reserve is the generating capacity available to the system operator in case of unforeseen event such as a plant breaking down.

The first stage of the market is the day ahead. The different agents take position in function of an uncertain future. They make forward contract to hedge their risk and the generators have to decide wheter to turn their plant on. The second stage of the market is the real time market. At that point in time, there is no more uncertainty as the state of the world is revealed.

The agents in our model can take two types of decision. The first stage decisions take place in the day-ahead when the future real-time state of the world is not known yet. Those first stage decisions are taken in function of the different possible scenario ω from the set of all possible scenario Ω that may happen. The second stage decision are the decision taken in real time to clear the market when the state of the world is known and finite.

The problem can be characterized as followed : a series of risk-averse or risk-neutral generators and loads wants to maximize their day ahead profit in function of the different scenarios that may happen in real time. Each of those agents' day ahead profit maximization problem will vary in function of their real time profit maximization problems. Next to them, there is a system operator whose goal is to ensure that the market's reserve is appropriate. This system operator also solves an optimization problem in the day ahead in function of what happen in real time. Finally, the market clearing constraints need to be fulfilled for both the real time and the day ahead market and for both energy and reserve.

Each agent's profit maximization problem as well as the market clearing constraints can be characterized by a set a KKT conditions. The union of all those sets of equations and inequations will form the full equilibrium problem that will define the market.

Given a finite set of scenarios ω belonging to the set of all possible scenarios Ω with known probability to happen P_ω, each generator g in the set of generator G will have to solve a real time profit maximization problem $\mathcal{G}_{g,\omega}^{RT,US}$ for each scenario. Each generator will also solve its day ahead profit maximization problem $\mathcal{G}_{g}^{DA,US}$. Both the real time and the day ahead profit maximization are linked because the real time profit will depend on the activation of the plant that is decided in day ahead, and the day ahead problem is defined in function of the real time profits.

Similarly, each load l in the set of load L will solve a day-ahead profit maximization problem $\mathcal{L}_{l}^{DA,US}$ that will be impacted by the solution to their respective real time profit maximization problem $\mathcal{L}_{l,\omega}^{RT,US}$.

We also have to introduce the system operator's day-ahead and real-time optimization problem $S^{DA,US}$ and $S_{\omega}^{RT,US}$.

We are looking for an equilibrium that would satisfy the union of the Karush-Kun-Tucker (KKT) condition of every agent's profit maximization problem, including the system operator, as well as the market clearing constraints.

2.2 Risk measure

For any risky situation where we have to take decision based on the probability of getting a profit for a given scenario, the concept of risk measure can be introduced. This is particularly true in a two stage settlement such as the electricity market we are working with. When an agent is *risk-neutral*, it will take decisions in function of the expected profit it would get from the different scenarios. On the other side when an agent is *risk-averse*, the decisions are based on its own *risk-adjusted* expectation of the profits.

An agent defines its *risk-adjusted* probabilities by modifying the real probabilities in order to put more emphasis on the disadvantageous scenarios and less emphasis on the beneficial scenario.

In this section, we will discuss the concepts of risk-measure and of coherent risk measures from [2] and [14] adapted to fit our problem. We will also introduce two different coherent risk measures and their optimization formulation: (i) The conditional value at risk and the (ii) a weighted sum of the conditional value at risk and the risk neutral expectation

2.2.1 Risk measure and coherent risk measure

A risk measure $\mathcal{R}$ is a function that maps uncertain outcomes $Z(\omega)$ to the extended real line $\bar{\mathbb{R}} = \mathbb{R} \cup \{+\infty\} \cup \{-\infty\}$. The space $\mathcal{Z}$ which $Z(\omega)$ belongs to is defined in [14].

If on top of that the risk-measure respects the four following axioms, the risk-measure is said to be coherent.

1. **Monotonicity**: If $Z_1, Z_2 \in \mathcal{Z}$ and $Z_1 \leq Z_2$ then $\mathcal{R}(Z_1) \geq \mathcal{R}(Z_2)$
2. **Sub-additivity**: If $Z_1, Z_2 \in \mathcal{Z}$ then $\mathcal{R}(Z_1 + Z_2) \leq \mathcal{R}(Z_1) + \mathcal{R}(Z_2)$
3. **Positive homogeneity**: If $\alpha \geq 0$ and $Z \in \mathcal{Z}$ then $\mathcal{R}(\alpha Z) = \alpha \mathcal{R}(Z)$
4. **Translation invariance**: If $Z \in \mathcal{Z}$ and $\alpha \in \mathbb{R}$ then $\mathcal{R}(Z + \alpha) = \mathcal{R}(Z) - \alpha$

The uncertain outcomes in our settings are the profit of the generators and loads in real time: $\Pi^{RT}_{g,\omega}$ and $\Pi^{RT}_{l,\omega}$. Those profits are the solutions to the real time profit maximization problems $\mathcal{G}^{RT}_{g,\omega}$ and $\mathcal{L}^{RT}_{l,\omega}$.

We can now define the coherent risk measure (CRM) $\mathcal{R}_a$ of an agent a as the minimum of the expectation of the profit Π_a over a risk-adjusted distribution q_a. This profit function is specific to each agent and depends on the decision of the agent that depends on the different scenarios ω.

$$\mathcal{R}_a(\Pi_a) = \min_{q_a \in \mathcal{M}_a} \mathbb{E}_{q_a}[\Pi_a] = \min_{q_a \in \mathcal{M}_a} \sum_{\omega \in \Omega} q_{a,\omega} \cdot \Pi_{a,\omega} \tag{2.1}$$

with $\mathcal{M}_a$ being a space of PDFs where the risk-adjusted probability distribution q_a belongs. This space is called the *risk-set*. This risk set is dependant on the risk measure used.

[14] defines the differential of a coherent risk measure as

$$\partial\mathcal{R}_a(\Pi_a) = \tilde{q}_a = \arg \min_{q_a \in \mathcal{M}_a} \mathbb{E}_{q_i}[\Pi_a] \tag{2.2}$$

The formula for the partial derivation in function of a variable x of a risk measure $\mathcal{R}_a$ can be obtained trough the chain rule:

$$\frac{\partial\mathcal{R}_a(\Pi_a)}{\partial x} = \sum_{\omega\in\Omega} \frac{\partial\mathcal{R}_a}{\partial\Pi_{a,\omega}} \frac{\partial\Pi_{a,\omega}}{\partial x} = \mathbb{E}_{\tilde{q}_a}[\frac{\partial\Pi_a}{\partial x}]$$

2.2.2 Conditional value at risk

The first coherent risk measure we want to introduce is the conditional value at risk. We based our research on [13] and modified the notation to accommodate positive random variable. An example of implementation applied to the capacity expansion problem can be found in [5].

In this thesis we are going to define the conditionnal value at risk with averse coefficient $\alpha \in [0, 1]$ as $CVaR_\alpha$. The value of α determines the risk aversion of an agent. A coefficient $\alpha = 0$ means a completely risk averse agent whereas a coefficient $\alpha = 1$ will represent a completely risk-neutral agent.

An intuitive interpretation about the value $CVaR_\alpha(\Pi)$, where Π is a positive continuous random variable, is to compute the expectation over the $100 \times \alpha\%$ worst scenario of Π.

The $CVaR$ is introduced in the continuous case even though it will be applied in the discrete case. This choice has been done to ease the explanation of the concept and to simplify the notation.

The value of $CVaR_\alpha$ is intricately linked to the value at risk with coefficient α. The VaR_α of a profit random variable is defined as

$$VaR_\alpha = \max\{z : F_\Pi(z) \leq \alpha\}$$

with $F_\Pi(z)$ being the cumulative distribution function of the profit. The VaR_α is a non-convex and discontinuous function that lacks coherence as a risk-measure. The $CVaR$ can then be defined as

$$CVaR_\alpha = \int_{-\infty}^{\infty} z \cdot dF_\Pi^\alpha(z)$$

with $F_{\Pi}^{\alpha}(z)$ defined as follow:

$$F_{\Pi}^{\alpha}(z) = \begin{cases} \frac{F_{\Pi}(z)}{\alpha}, & \text{if } z < VaR_{\alpha}(\Pi) \\ 0, & \text{if } z \geq VaR_{\alpha}(\Pi) \end{cases}$$

For the proof of coherence of $CVaR$ for negative random variables that would represent a loss and not a profit, the reader is referred to [11] .

The problem of finding the $CVaR_{\alpha}$ can then be defined as a linear program by using equation (2.1). The primal formulation of this problem can then be characterized as followed.

$$\min_{q \in \mathcal{M}_{CVaR_{\alpha}}} \quad \sum_{\omega \in \Omega} q_{\omega} \cdot \Pi_{\omega} \tag{2.3}$$

with the risk set of the conditional value at risk with aversion coefficient α defined as follow.

$$\mathcal{M}_{CVaR_{\alpha}} = \{q : \sum_{\omega \in \Omega} q_{\omega} = 1, q_{\omega} \leq \frac{P_{\omega}}{\alpha}, q_{\omega} \geq 0, \omega \in \Omega\} \tag{2.4}$$

We can convince ourselves of the formulation of (2.4) by looking at the following example. We want to compute $CVaR_{0.5}$ from the profit function from table 2.1.

i	1	2	3	4	5	6
π_i	100	200	400	800	900	1000
$P(\Pi = \pi_i)$	0.1	0.2	0.5	0.18	0.01	0.01

Table 2.1: Profit function example

We understand from (2.5) that we should put as much weight as possible on the smallest profits. We make q_1 as big as possible so $q_1 = 0.2$. We do the same for q_2 so $q_2 = 0.2$. We try to do the same for q_3 but the total would be too big so we reduce q_3 to 0.6. The value of the conditionnal value at risk can then be computed as follows:

$$CVaR_{0.5} = 0.2 \times 100 + 0.2 \times 200 + 0.6 \times 400 + 0 \times 800 + 0 \times 900 + 0 \times 1000 = 300$$

As $CVaR_{0.5}$ is the expectation over the 50% worst profit, the result is correct.

The dual of this linear program can also be formulated. We need to apply the dual variable VaR to the constraint on the unicity of the risk adjusted probability and the dual variable u_ω to the constraint on the maximum value of q_ω.

$$\max_{CVaR,VaR,u_\omega} \quad CVaR \tag{2.5}$$

$$(\chi): \quad CVaR = VaR - \frac{1}{\alpha}\sum_{\omega\in\Omega} P_\omega \cdot u_\omega \tag{2.6}$$

$$(q_\omega): \quad u_\omega \geq VaR - \Pi_\omega$$

$$u_\omega \geq 0$$

Let's note that the variable $CVaR$ is introduced in an illustrative way and is not needed as such. We can replace it in (2.5) by its value in (2.6).

The KKT condition of the linear program can be computed as follows.

$$0 \leq q_\omega \perp u_\omega - VaR + \Pi_\omega \geq 0 \qquad \forall\omega \in \Omega$$

$$0 \leq u_\omega \perp \frac{P_\omega}{\alpha} - q_\omega \geq 0 \qquad \forall\omega \in \Omega$$

$$(VaR): \quad \sum_{\omega\in\Omega} q_\omega - 1 = 0$$

$$CVaR = VaR - \frac{1}{\alpha}\sum_{\omega\in\Omega} P_\omega \cdot u_\omega$$

The variable VaR represents the Value at risk.

2.2.3 Mean value at risk

The mean value at risk can be understood as a weighted sum of the real expectation and of $CVaR_\alpha$. Inspiration was taken from [12]. It can be defined as followed:

$$MVaR_{\beta,\alpha}(\Pi) = (1-\beta)\cdot \mathbb{E}_{P_\omega}[\Pi] + \beta \cdot CVaR_\alpha(\Pi)$$

The risk set of the mean value at risk can be defined in a similar way than the $CVaR_\alpha$:

$$\mathcal{M}_{MVaR_{\beta,\alpha}} = \{q : \sum_{\omega\in\Omega} r_\omega = 1, r_\omega \leq \frac{P_\omega}{\alpha}, r_\omega \geq 0, q_\omega = \beta \cdot r_\omega + (1-\beta)\cdot P_\omega, \omega \in \Omega\}$$

The risk-adjusted probability r are computed as if it was the risk adjusted probability of the $CVaR$ and the next step is to build the risk adjusted probability of the $MVaR$ with a weigthed sum.

Let's note that a value of β equal to 1 make the $MVaR$ equal to the $CVaR$.

The primal formulation of the linear program can be built exactly as in (2.3) by changing the risk-set. The dual can also be constructed similarly as in (2.5) by maximizing the intermediate variable $MVaR$. The dual variable u_ω is associated to the constraint relative to the maximal value of r_ω, the risk adjusted probability relative to the $CVaR$. The dual variable σ_ω is associated to the constraint relative to the weighted sum of the real probability and the $CVaR$ risk adjusted probability. This weighted sum build the $MVaR$ risk-adjusted probability.

$$\max_{VaR,u_\omega,\sigma_\omega} \quad (1-\beta)\cdot\sum_{\omega\in\Omega} P_\omega\cdot\sigma_\omega + VaR - \frac{1}{\alpha}\sum_{\omega\in\Omega} P_\omega\cdot u_\omega \tag{2.7}$$

$$(r_\omega): \quad u_\omega - VaR + \beta\cdot\sigma_\omega \geq 0 \tag{2.8}$$

$$(q_\omega): \quad \Pi_\omega - \sigma_\omega \geq 0 \tag{2.9}$$

$$u_\omega \geq 0$$

The KKT condition for a risk aversion problem with the mean value at risk can then be characterized.

$$
\begin{aligned}
&0 \leq r_\omega \perp u_\omega - VaR + \beta\cdot\sigma_\omega \geq 0 && \forall\omega\in\Omega\\
&0 \leq q_\omega \perp \Pi_\omega - \sigma_\omega \geq 0 && \forall\omega\in\Omega\\
&0 \leq u_\omega \perp \frac{P_\omega}{\alpha} - r_\omega \geq 0 && \forall\omega\in\Omega\\
&(VaR): \quad \sum_{\omega\in\Omega} r_\omega - 1 = 0 &&\\
&(\sigma_\omega): \quad q_\omega - (1-\beta)\cdot P_\omega - \beta\cdot r_\omega = 0 && \forall\omega\in\Omega\\
&MVaR = (1-\beta)\cdot\sum_{\omega\in\Omega} P_\omega\cdot\sigma_\omega + VaR - \frac{1}{\alpha}\sum_{\omega\in\Omega} P_\omega\cdot u_\omega &&
\end{aligned}
$$

2.2.4 Real time market

In real time, the agents know what is happening and the scenarios are independent from each other. Here we will present the optimization problem for one scenario but it can easily be extended to all scenarios.

We begin with the profit maximization problem of the generator g in scenario ω in real time:

Real time profit maximization of generator g in scenario ω : $\mathcal{G}_{g,\omega}^{RT,US}$

$$\max_{p^{RT},r^{RT}} \quad \lambda_{\omega}^{RT} \cdot p_{g,\omega}^{RT} + \lambda_{\omega}^{R,RT} \cdot r_{g,\omega}^{RT} - C_g \cdot p_{g,\omega}^{RT}$$

$$(\mu_{g,\omega}^{G,RT}) : \quad p_{g,\omega}^{RT} + r_{g,\omega}^{RT} \leq P_{g,\omega}^{+} \cdot y_g \tag{2.10}$$

$$(\mu_{g,\omega}^{G,R,RT}) : \quad r_{g,\omega}^{RT} \leq R_g \tag{2.11}$$

$$p_{g,\omega}^{RT}, r_{g,\omega}^{RT}, y_g \geq 0$$

Given the prices of energy and reserve λ_{ω}^{RT} and $\lambda_{\omega}^{R,RT}$ and the value of its first stage decision the unit commitment variable y_g, the generator will maximize its profit with its second stage decision: the real time production of energy and of reserve in scenario ω ($p_{g,\omega}^{RT}$ and $r_{g,\omega}^{RT}$).

The prices of energy and reserve are exogenous variable that the generator g cannot impact. Similarly, the value of the unit commitment variable is taken in the day ahead the generator cannot influence it anymore.

The generator needs to take into account two constraints. Constraint (2.10) states that the maximum production of energy and reserve in a particular scenario must be lower or equal to the maximum production of that generator in that particular scenario $P_{g,\omega}^{+}$ multiplied by the value of the unit commitment variable.

The constraint (2.11) expresses the condition that the maximum reserve the generator can allocate must be lower or equal than a ramp value R_g. As reserve must be available in a given time frame, the maximum value of reserve that a generator can commit is bound by its own physical limitation on the speed of activation of its reserve.

$\mathcal{G}_{g,\omega}^{RT,US}$ can also be characterized in function of its KKT condition then:

$$0 \leq p_{g,\omega}^{RT} \perp C_g - \lambda_{\omega}^{RT} + \mu_{g,\omega}^{G,RT} \geq 0 \tag{2.12}$$

$$0 \leq r_{g,\omega}^{RT} \perp -\lambda_{\omega}^{R,RT} + \mu_{g,\omega}^{G,RT} + \mu_{g,\omega}^{G,R,RT} \geq 0 \tag{2.13}$$

$$0 \leq \mu_{g,\omega}^{G,RT} \perp P_{g,w}^{+} \cdot y_g - p_{g,\omega}^{RT} - r_{g,\omega}^{RT} \geq 0 \tag{2.14}$$

$$0 \leq \mu_{g,\omega}^{G,R,RT} \perp R_g - r_{g,\omega}^{RT} \geq 0 \tag{2.15}$$

Equations (2.12) and (2.13) allow us to interpret the dual variables $\mu_{g,\omega}^{G,RT}$ and $\mu_{g,\omega}^{G,R,RT}$.

If a generator is not producing reserve, $\mu_{g,\omega}^{G,RT}$ is interpreted as the marginal benefit of producer g in scenario ω.

In the case where a generator g produces reserve but its production is not bounded by equation (2.11), $\mu_{g,\omega}^{G,RT}$ is equal to the price of reserve and can be seen as an adder on the energy price.

We can also characterize the profit maximization problem of the load l in scenario ω:

Real time profit maximization of load l in scenario ω : $\mathcal{L}_{l,\omega}^{RT,US}$

$$\begin{aligned} \max_{d^{RT}} \quad & V_l \cdot d_{l,\omega}^{RT} - \lambda_{\omega}^{RT} \cdot d_{l,\omega}^{RT} \\ (\mu_{l,\omega}^{L,RT}): \quad & d_{l,\omega}^{RT} \leq D_{l,\omega}^{+} \\ & d_{l,\omega}^{RT} \geq 0 \end{aligned} \tag{2.16}$$

Similarly to the generator, the load will maximize its profit with its second stage decision, the quantity of energy bought in real time $d_{l,\omega}^{RT}$, given the exogenous price of energy and its valuation of energy V_l.

The loads are not allowed to trade reserve in our setting.

As shown in constraint (2.16) the maximum quantity of energy bought is bounded by the maximum demand of energy for that load in that scenario $D_{l,\omega}^{+}$.

The KKT condition of the problem $\mathcal{L}_{l,\omega}^{RT,US}$ is then:

$$0 \leq d_{l,\omega}^{RT} \perp \lambda_{l,\omega}^{RT} - V_l + \mu_{l,\omega}^{L,RT} \geq 0 \tag{2.17}$$

$$0 \leq \mu_{l,\omega}^{L,RT} \perp D_{l,\omega}^{+} - d_{l,\omega}^{RT} \geq 0 \tag{2.18}$$

The last step in real time will be to define the optimization problem of the system operator for the scenario ω: $\mathcal{S}_{\omega}^{RT,US}$.

Real time profit maximization of the system operator in scenario ω : $\mathcal{S}_{\omega}^{RT,US}$

$$\begin{aligned} \max_{d^{R,RT}} \quad & \sum_{l \in RL} V_l^{R} \cdot d_{l,\omega}^{R,RT} - \lambda_{\omega}^{R,RT} \cdot d_{l,\omega}^{R,RT} & \\ (\mu_{l,\omega}^{R,RT}): \quad & d_{l,\omega}^{R,RT} \leq D_l^{R} & \forall l \in RL \\ & d_{l,\omega}^{R,RT} \geq 0 & \forall l \in RL \end{aligned} \tag{2.19}$$

The system operator maximizes its profit with its consumption of reserve in real time $d_{l,\omega}^{R,RT}$ in function of its valuation of reserve.

The value of the reserve for the system operator is defined with an operating reserve demand curve (ORDC). The demand in reserve is composed of different blocks that are valued in a declining importance. The first block of reserve will be valued very highly but on the contrary, the last block will have almost no value.

The blocks are valued this way because when there is no reserve, the system operator is willing to pay a lot for reserve in order to avoid shortage when facing perturbation in production. But on the other hand, when there is already a lot of reserve available, the system operator does not really need to buy more of it.

The constraint (2.19) sets the maximum demand of a block of reserve.

The system operator can be seen as an aggregation of reserve load l belonging to the set of reserve load RL. They value the reserve at different prices V_l^R and have a demand of D_l^R.

The readers are referred to section 2.3 for more information on how ORDCs are built.

The KKT condtion of the problem $\mathcal{S}_{\omega}^{RT,US}$ can be identified as follows:

$$0 \leq d_{l,\omega}^{R,RT} \perp -V_l^R + \lambda_{l,\omega}^{R,RT} + \mu_{l,\omega}^{R,RT} \geq 0 \qquad \forall l \in RL, \tag{2.20}$$

$$0 \leq \mu_{l,\omega}^{R,RT} \perp D_l^R - d_{l,\omega}^{R,RT} \geq 0 \qquad \forall l \in RL, \tag{2.21}$$

As for the generator's and load's problems, the KKT formulation allows us to interpret the dual variable $\mu_{l,\omega}^{R,RT}$ as a surplus for the reserve load l in scenario ω.

To those problems, we have to add the market clearing constraints for every scenario in order to complete the real time market. This means that the equation (2.22) and (2.23) need to be fulfilled so that the energy or reserve sold in a scenario is the same as the energy or reserve bought.

$$\sum_{g \in G} p_g^{RT} = \sum_{l \in L} d_l^{RT} \qquad \forall \omega \in \Omega \tag{2.22}$$

$$\sum_{g \in G} r_g^{RT} = \sum_{l \in RL} d_l^{R,RT} \qquad \forall \omega \in \Omega \tag{2.23}$$

2.2.5 Day ahead market

In the day ahead, the generators, the loads and the system operator have the possibility to trade forward contracts on their production or consumption. Those forward contract are a way for the agents to hedge their risk.

The profit maximization problem of the generator g in day ahead $\mathcal{G}_g^{DA,US}$ can be stated as follows.

Day ahead profit maximization of generator g : $\mathcal{G}_g^{DA,US}$

$$\begin{aligned}
\max_{y,p^{DA},r^{DA}} \quad & \lambda^{DA} \cdot p_g^{DA} + \lambda^{R,DA} \cdot r_g^{DA} - K_g \cdot y_g \\
& + \mathcal{R}_g(\Pi_{g,\omega}^{RT}(y_g) - \lambda_\omega^{RT} \cdot p_g^{DA} - \lambda_\omega^{R,RT} \cdot r_g^{DA}) && (2.24)\\
(\mu_g^{G,R,DA}): \quad & r_g^{DA} \leq R_g && (2.25)\\
(\delta_g): \quad & y_g \leq 1 && (2.26)\\
& y_g, r_g^{DA} \geq 0
\end{aligned}$$

The first stage decisions are the unit commitment variable y_g that are directly impacting the real time profit through equation (2.10) and the day ahead production of energy and reserve p_g^{DA} and r_g^{DA}. Those day ahead productions are used for the forward contract.

The forward contract works as follows. A price λ^{DA} is defined for a quantity p_g^{DA} of energy in the day ahead and is used in real time for that quantity instead of the real time price of the scenario $\lambda_{g,\omega}^{RT}$. For a generator that is only trading energy, the profit for a scenario ω with a forward contract $\Pi_{g,\omega}^{FC}$ is defined in equation (2.27).

$$\Pi_{g,\omega}^{FC} = \lambda^{DA} \cdot p_g^{DA} + \lambda_\omega^{RT} \cdot (p_{g,\omega}^{RT} - p_g^{DA}) - C_g \cdot p_{g,\omega}^{RT} \tag{2.27}$$

The day ahead price is fixed whatever happens in real time so this process allows the generator to hedge its risk. A similar reasoning can be applied for the trading of reserve.

In day ahead a generator g will want to maximize its earning from (i) the forward contract, (ii) minus the price of activating its plant $K_g \cdot y_g$, (iii) plus the risk measure of random outcomes over every real-time scenario. This random outcome for a scenario consists in the real time profit $\Pi_{g,\omega}^{RT}(y_g)$ for that scenario minus what was already traded in day ahead multiplied by the real-time prices of that scenario. The real time profit of a generator g in scenario ω $\Pi_{g,\omega}^{RT}$ is defined as the solution to the problem $\mathcal{G}_{g,\omega}^{RT,US}$. This random outcome is dependant on the real time scenario that is happening. Equation (2.24) describes the function to maximize.

Constraint (3.9) limits the trading of reserve in day ahead to the physical capacity of the generator. We notice that there is no constraint on the energy trading. This lack of constraint for energy in day ahead is a characteristic of the US day-ahead market and is called virtual trading.

The constraint (3.10) is a relaxation on the unit commitment variable. Instead of only allowing the generator to be ON or OFF, it is allowed to only be partially ON.

We can state the KKT condition for the problem $\mathcal{G}_g^{DA,US}$ of a generator g as follows:

$$0 \leq y_g \perp \delta_g + K_g - \sum_{\omega \in \Omega} q_{g,\omega} \cdot P^+_{g,\omega} \cdot \mu^{G,RT}_{g,\omega} \geq 0 \tag{2.28}$$

$$0 \leq \delta_g \perp 1 - y_g \geq 0 \tag{2.29}$$

$$0 \leq r_g^{DA} \perp -\lambda^{R,DA} + \sum_{\omega \in \Omega} q_{g,\omega} \cdot \lambda^{R,RT}_{\omega} + \mu^{G,R,DA}_g \geq 0 \tag{2.30}$$

$$0 \leq \mu^{G,R,DA}_g \perp R_g - r_g^{DA} \geq 0 \tag{2.31}$$

$$(p_g^{DA}): \quad \lambda^{DA} = \sum_{\omega \in \Omega} q_{g,\omega} \cdot \lambda^{RT}_{\omega} \tag{2.32}$$

$$q_g \in \partial \mathcal{R}_g[\Pi^{RT}_{g,\omega}(y_g) - \lambda^{RT}_{\omega} \cdot p_g^{DA} - \lambda^{R,RT}_{\omega} \cdot r_g^{DA}] \tag{2.33}$$

The inspection of those KKT conditions reveal some interesting facts.

First of all, equation (2.33) displays the fact that the risk adjusted probabilities must belong to the risk-set of the the risk measure that is defined by its subgradient.

Then, equations (2.28) and (2.29) give some hindsight about the behaviour of the unit commitment variable. The unit commitment variable is defined by the relation between (i) the start up cost K_g and (ii) the risk-adjusted expectation over the real-time scenarios of the product of the maximum possible production by the surplus from selling energy (or energy and reserve) $\sum_{\omega \in \Omega} q_{g,\omega} \cdot P^+_{g,\omega} \cdot \mu^{G,RT}_{g,\omega}$.

- If $K_g > \sum_{\omega \in \Omega} q_{g,\omega} \cdot P^+_{g,\omega} \cdot \mu^{G,RT}_{g,\omega}$ then the generator will not commit.
- If $K_g = \sum_{\omega \in \Omega} q_{g,\omega} \cdot P^+_{g,\omega} \cdot \mu^{G,RT}_{g,\omega}$ then the generator will commit between 0 and 1 excluded. The dual variable δ_g that can be interpreted as a surplus for each generator will be equal to 0.
- If $K_g < \sum_{\omega \in \Omega} q_{g,\omega} \cdot P^+_{g,\omega} \cdot \mu^{G,RT}_{g,\omega}$ then the generator will commit fully and the dual variabke δ_g will be greater than 0. This can be considered as a surplus from committing.

After that, equations (2.30) and (2.31) show us that the dual variable $\mu^{G,R,DA}_g$ is a scarcity surplus from selling reserve in day ahead when a generator trade up to its maximum reserve capacity.

Finally, the equation (2.32) displays the day ahead price as the risk-adjusted expectation of the real-times prices. This constraint shows us that the risk-adjusted probabilities of the different agents will be linked together by the day-ahead price.

The same kind of reasoning can be done for the profit maximization problem of the load in the day ahead $\mathcal{L}_l^{DA,US}$.

Day ahead profit maximization of load l : $\mathcal{L}_l^{DA,US}$

$$\max_{d^{DA}} \quad -\lambda^{DA} \cdot d_l^{DA} + \mathcal{R}(\Pi_{l,\omega}^{RT} + \lambda_\omega^{RT} \cdot d_l^{DA}) \tag{2.34}$$

In (2.34), there is also a forward contract but it is done the other way around than in (2.24). The first stage decision here is the quantity traded in day-ahead d_l^{DA}.

We can notice the lack of constraint of the quantity traded that is typical from virtual trading.

The KKT condition of the problem $\mathcal{L}_l^{DA,US}$ can be presented as followed:

$$(d_l^{DA}): \quad \lambda^{DA} = \sum_{\omega\in\Omega} q_{l,\omega} \cdot \lambda_\omega^{RT} \tag{2.35}$$

$$q_l \in \partial\mathcal{R}_l[\Pi_{l,\omega}^{RT} + \lambda_\omega^{RT} \cdot d_l^{DA}] \tag{2.36}$$

The last problem to characterize is $\mathcal{S}^{DA,US}$, the optimization problem of the System operator in day ahead,

Day ahead profit maximization of the system operator : $\mathcal{S}^{DA,US}$

$$\begin{aligned} \max_{d^{R,DA}} \quad & \sum_{l\in RL} V_l^{R,DA} \cdot d_l^{R,DA} - \lambda^{R,DA} \cdot d_l^{R,DA} \\ & + \sum_{\omega\in\Omega} P_\omega \left(\sum_{l\in RL} \Pi_{l,\omega}^{RT} + \lambda_\omega^{R,RT} \cdot d_l^{R,DA} - V_l^R \cdot d_l^{R,DA} \right) \end{aligned} \tag{2.37}$$

$$(\mu_l^{R,DA}): \quad d_l^{R,DA} \leq D_l^R \quad \forall l \in RL \tag{2.38}$$

$$d_l^{R,DA} \geq 0 \quad \forall l \in RL$$

In day ahead, the system operator values the reserve with a day ahead ORDC $V_l^{R,DA}$ that is different from the real time one. We assume that the system operator is risk neutral so that it considers the random outcomes from the real time with the real probability P_ω instead of risk-adjusted probability. $\Pi_{l,\omega}^{RT}$ is defined as the profit for each load of reserve l in every scenario ω.

Constraint 3.12 gives a bound on the day ahead reserve bought.

The KKT conditions of the problem $\mathcal{S}^{DA,US}$ are defined as follows:

$$0 \leq d_l^{R,DA} \perp \lambda^{R,DA} - V_l^{R,DA} + \mu_l^{R,DA} - \sum_{\omega \in \Omega} P_\omega \cdot (\lambda_\omega^{R,RT} - V_l^R) \geq 0 \quad \forall l \in RL \tag{2.39}$$

$$0 \leq \mu_l^{R,DA} \perp D_l^R - d_l^{R,DA} \geq 0 \quad \forall l \in RL \tag{2.40}$$

Those KKT conditions and particularly equation (2.39) allow us to interpret the dual variable $\mu_l^{R,DA}$. When the consumption of a block l $d_l^{R,DA}$ is at the limit D_l^R, $\mu_l^{R,DA}$ can be seen as the marginal benefit of selling in day ahead instead of selling in real time.

Equation (2.39) states that a load of the system operator will only buy in real time if its marginal profit in day ahead $V_l^{R,DA} - \lambda^{R,DA}$ is bigger than the expected marginal profit in real time $\sum_{\omega \in \Omega} P_\omega \cdot (V_l^R - \lambda_\omega^{R,RT})$.

As we expect our market to be balanced, we have to add the market clearing constraint in the day ahead market.

$$\sum_{g \in G} p_g^{DA} = \sum_{l \in L} d_l^{DA} \tag{2.41}$$

$$\sum_{g \in G} r_g^{DA} = \sum_{l \in RL} d_l^{R,DA} \tag{2.42}$$

2.3 Operating Reserve Demand Curve or ORDC

This section's aim is to define the operating reserve demand curve for the real time and day ahead system operator problem. The approach is based on [9].

The ORDC can be characterized as the marginal expected value of the unserved energy as written in (2.43).

$$ORDC(r) = (VoLL - C) \cdot Lolp(r) \tag{2.43}$$

Wherein $VoLL$ is the value of lost load defined as the money lost by a consumer if the energy is not served or as the maximum amount of money that a load is willing to pay for electricity. C would be the marginal cost of production and $Lolp(r)$ the loss of load probability so the probability of the shortage being bigger than the reserve.

We consider here a real time ORDC for a single product of reserve. Multiple products of reserve are studied in [9].

Random variables will be differentiated from other variables by wearing a tilde.

We will start by defining the real time ORDC and we will then move to the day ahead one.

2.3.1 Loss of load probability in Real time

The loss of load probability in real time for a particular scenario ω is the probability that the random shortage in that scenario, $\tilde{Z}_\omega^{RT}$, is greater than the total reserve available r_ω^{RT} as stated in equation (2.44).

$$Lolp_\omega^{RT}(r_\omega^{RT}) = \mathbb{P}\Big[\tilde{Z}_\omega^{RT} \geq r_\omega^{RT}\Big] \tag{2.44}$$

If the random variable $\tilde{Z}_\omega^{RT}$'s cumulative distribution function $F_{\tilde{Z}_\omega^{RT}}$ is known, it is possible to rewrite the loss of load probability as in equation (2.45).

$$\begin{aligned} Lolp_\omega^{RT}(r_\omega^{RT}) &= 1 - \mathbb{P}\Big[\tilde{Z}_\omega^{RT} \leq r_\omega^{RT}\Big] \\ &= 1 - F_{\tilde{Z}_\omega^{RT}}(r_\omega^{RT}) \end{aligned} \tag{2.45}$$

The random variable of real time shortage in a particular scenario is built as the difference between the random variable of the actual demand from the market in that scenario $\tilde{L}_\omega$ and the real time demand of the loads for that scenario $D_\omega^{RT,+}$.

$$\tilde{Z}_\omega^{RT} = \tilde{L}_\omega - D_\omega^{RT,+} \tag{2.46}$$

In general, a specific loss of load probability could be defined for each scenario. This specific loss of load probability would then result in specific real time ORDC for each scenario. However, in this thesis, we make the assumption that the real time loss of load probability for each scenario $Lolp_\omega^{RT}(r_\omega^{RT})$ can be assimilated to a global real time loss of load probability $Lolp^{RT}(r^{RT})$.

As the total demand of a given scenario is assumed to be dependant on this scenario, we can approximate the shortage of each scenario by a global shortage.

2.3.2 Loss of load probability in day ahead

In day-ahead, the loss of load probability in function of the day ahead reserve is the probability that the uncertain shortage in real time $\tilde{Z}^{RT}$ is greater than the uncertain real time reserve $\tilde{r}^{RT}$.

$$Lolp^{DA}(r^{DA}) = \mathbb{P}\Big[\tilde{Z}^{RT} \geq \tilde{r}^{RT}\Big] \tag{2.47}$$

The random variable of the real time shortage $\tilde{Z}^{RT}$ is taken as the difference between the random actual demand $\tilde{L}$ and the random total demand load $\tilde{D}^{RT,+}$.

$$\tilde{Z}^{RT} = \tilde{L} - \tilde{D}^{RT,+} \tag{2.48}$$

The difference between equation (2.46) and (2.48) is twofold. Firstly, we have less information on the actual demand in day ahead than in real time because we do not know yet which scenario is going to unfold. Secondly, we do not know yet the actual demand of the load in day ahead for the same reason.

We assume in our model that there is a strong interaction between the day ahead reserve schedule and the changes in the real time demand's load.

This strong interaction leads to equation (2.49). We see that the real time reserve schedule $\tilde{r}^{RT}$ change with the real time energy schedule $\tilde{D}^{RT,+}$.

$$\tilde{r}^{RT} = r^{DA} + (D^{DA,+} - \tilde{D}^{RT,+}) \tag{2.49}$$

We can then insert equations (2.49) and (2.48) into the day ahead loss of load probability from (2.47).

$$Lolp^{DA}(r^{DA}) = \mathbb{P}\Big[\tilde{L} - D^{DA,+} \geq r^{DA}\Big] \tag{2.50}$$

We can now introduce the random day ahead shortage $\tilde{Z}^{DA}$ as the difference between the random actual demand and the demand scheduled from the day ahead point of view.

$$\tilde{Z}^{DA} = \tilde{L} - D^{DA,+} \tag{2.51}$$

Equation (2.50) can then be written in function of the shortage from the day ahead point of view.

$$Lolp^{DA}(r^{DA}) = \mathbb{P}\Big[\tilde{Z}^{DA} \geq r^{DA}\Big] \tag{2.52}$$

If we know the cumulative distribution $F_{\tilde{Z}^{DA}}$ we can then rewrite the loss of load probability as follows in equation (2.53).

$$Lolp^{DA}(r^{DA}) = 1 - F_{\tilde{Z}^{DA}}(r^{DA}) \tag{2.53}$$

To conclude this discussion about the ORDC, we point out the equations (2.46) and (2.51). It can be deduced from them that the random variable of the shortage from the day ahead point of view $\tilde{Z}^{DA}$ has a greater variance than the one from the real time point of view $\tilde{Z}^{RT}$. This can be explained from the fact that in the day ahead we have less information about the actual demand so the prediction on the shortage is going to be less accurate.

2.4 Conclusion

Appendix A.1 summarises the core model with the KKT conditions extracted from the agent's profit maximization problem as well as the market clearing constraints. This equilibrium represents a two settlement reserve capacity market whose agent can be risk-averse. From now on the core model will be refferred to as the US market and the full equilibrium problem of the US model as the US equilibrium.

Our objective will be to find an equilibrium for the US market. The problem that we are facing is similar to the one studied in [5]. The heuristic used to find a solution to the US equilibrium will be described in chapter 4.

CHAPTER 3

Market design variants

In addition to the core model defined in chapter 2, a series of market design variants will be introduced in this chapter.

All the variants introduced in this chapter will keep a structure similar to the one the core model. There will be a day ahead and a real time market and every agents will try to maximize their profit in function of the state of the world. Sets of KKT condition will be extracted from the different agents' profit maximization problems to form the stochastic equilibrium that will constitute a specific model.

We will consider two types of variants:

1. Allowing or not virtual trading of energy in the day ahead. The opposite of virtual trading is physical trading.

2. Holding or not a real time market for reserve.

Table 3.1 summarises the different possible combinations of the policies.

On one side of the spectrum, the core model allows virtual trading of energy and holds a real time market for reserve. This model is the closest to a "US style" market. On the other side of the spectrum we have what we call the EU market that does not allow day-ahead virtual trading and does not hold a real time market for reserve. This model is closer to a "European style" market. Between the two edges, we have (i) the real time reserve model or RTR model that does not allow day-ahead virtual trading but holds a real time market for energy and the (ii) EUVT market that does not hold a real time market for reserve but allows virtual trading.

	Virtual trading	No-virtual trading
Real time and day ahead market for reserve	US	RTR
Day ahead only market for reserve	EUVT	EU

Table 3.1: Summary of the different models considered

3.1 No virtual trading policy

This section introduces the variant of forbidding virtual trading. Virtual trading will be defined as the action of trading energy in day ahead regardless of the real time physical production (or demand for the loads) capacity.

Whether or not allowing virtual trading is a question that is currently raised by the CREG [1]. The motivation behind allowing virtual trading is to make the day-ahead backpropagation of real time closer to the real-time reality. If virtual trading was allowed, it would give exterior bidders the opportunity to take advantage of day-ahead prices that would be under or over-valued compared to the expected real time prices.

The core model can be altered into the RTR model that forbids virtual trading. In this model, the generator g day ahead profit maximization problem $\mathcal{G}_g^{DA,RTR}$ can be built by adding constraints (3.1) and (3.2) to the profit maximization problem $\mathcal{G}_g^{DA,US}$.

Similarly the load l day ahead profit maximization problem $\mathcal{L}_l^{DA,RTR}$ is constructed by adding constraints (3.3) and (3.4) to $\mathcal{L}_l^{DA,US}$.

$$
\begin{aligned}
(\mu_g^{G,DA}): \quad & p_g^{DA} + r_g^{DA} \leq y_g \cdot P_g^{DA,+} && (3.1)\\
& p_g^{DA} \geq 0 && (3.2)\\
(\mu_l^{L,DA}): \quad & d_l^{DA} \leq D_l^{DA,+} && (3.3)\\
& d_l^{DA} \geq 0 && (3.4)
\end{aligned}
$$

In those constraints, the parameter $P_g^{DA,+}$ and $D_l^{DA,+}$ is defined as the maximum over $\omega \in \Omega$ of $P_{g,\omega}^{+}$ and $D_{l,\omega}$.

The others profit maximization problems of the RTR model are the same as for the US model.

The full equilibrium of the RTR model can be found in appendix A.2. As for the US model, the equilibrium is composed of the KKT condition of every agents' profit maximization problem both in day ahead and in real time as well as the market clearing constraints.

The most notable differences from the US equilibrium can be found in equations (A.37) and (A.40).

Those equations state that the day ahead price of energy when virtual trading is not allowed is not the risk adjusted expected price anymore. The marginal surplus relative to the no-virtual trading constraints $\mu_g^{G,DA}$ and $\mu_g^{L,DA}$ need to be taken into account. The day ahead price is now bounded by equation (3.5).

$$\sum_{\omega\in\Omega} q_{l,\omega}\cdot\lambda_\omega^{RT} - \mu_l^{L,DA} \leq \lambda^{DA} \leq \sum_{\omega\in\Omega} q_{g,\omega}\cdot\lambda_\omega^{RT} + \mu_g^{G,DA} \quad \forall l\in L, \forall g\in G \tag{3.5}$$

The bounds are tight when the generator g (or the load l) sell (or buy) energy in day ahead.

3.2 No real time market for reserve policy

In this section, we introduce two new models: the EUVT model and the EU model. The EUVT and EU models are two stage-market where energy is traded both in real time and in day ahead but where reserve is only traded in day ahead. The difference between the two models concerns the authorisation or not of virtual trading. The EUVT model allows virtual trading whereas the EU forbids it.

In those models, reserve is interpreted as the available capacity that should be kept available even after the activation of energy. If one megawatt of reserve was promised in day ahead, one megawatt of reserve should be kept available at all time even after the current interval.

Holding or not a real time market for energy is in fact one of the differences between a US style market and a more European style market. The difference of nodal and zonal pricing are here ignored.

The generator g real time profit maximization $\mathcal{G}_{g,\omega}^{RT,EUVT}$ can be stated as follows.

Real time profit maximization problem of generator g for scenario ω: $\mathcal{G}_{g,\omega}^{RT,EUVT}$

$$\begin{aligned} \max_{p^{RT}} \quad & \lambda_{\omega}^{RT} \cdot p_{g,\omega}^{RT} - C_g \cdot p_{g,\omega}^{RT} \\ (\mu_{g,\omega}^{G,RT}): \quad & p_{g,\omega}^{RT} + r_g^{DA} \leq P_{g,\omega}^{+} \cdot y_g \qquad (3.6) \\ & p_{g,\omega}^{RT} \geq 0 \end{aligned}$$

Equation (3.6) shows that the day ahead reserve should still be available after the activation of energy.

The load l profit maximization problem in real time of the EUVT market is similar to the one of the US market. As there is no real-time market for energy, there is no real-time system operator profit maximization problem for the EUVT market and the real time market clearing constraints are restricted to equation (3.7)

$$\sum_{g \in G} p_{g,\omega}^{RT} = \sum_{l \in L} d_{l,\omega}^{RT} \quad \forall \omega \in \Omega \qquad (3.7)$$

The generator g day ahead profit maximization problem $\mathcal{G}_g^{DA,EUVT}$ is defined as followed.

Day ahead profit maximization problem of generator g: $\mathcal{G}_g^{DA,EUVT}$

$$\begin{aligned} \max_{y,p^{DA},r^{DA}} \quad & \lambda^{DA} \cdot p_g^{DA} + \lambda^{R,DA} \cdot r_g^{DA} - K_g \cdot y_g \\ & + \mathcal{R}_g(\Pi_{g,\omega}^{RT}(y_g, r_g^{DA}) - \lambda_{\omega}^{RT} \cdot p_g^{DA}) \qquad (3.8) \\ (\mu_g^{G,R,DA}): \quad & r_g^{DA} \leq R_g \qquad (3.9) \\ (\delta_g): \quad & y_g \leq 1 \qquad (3.10) \\ & y_g, r_g^{DA} \geq 0 \end{aligned}$$

We can note that the real time profit of a generator is now not only in function of the unit commitment variable y_g but also in function of the day ahead reserve r_g^{DA} as seen in equation (3.8).

The day ahead profit maximization problem of the load is similar to the one of the US market. The day ahead profit maximization problem of the system operator in the EUVT model $\mathcal{S}^{DA,EUVT}$ can be defined as follows.

Day ahead profit maximization problem of the system operator: $\mathcal{S}^{DA,EUVT}$

$$\max_{d^{R,DA}} \quad \sum_{l \in RL} V_l^{R,DA} \cdot d_l^{R,DA} - \lambda^{R,DA} \cdot d_l^{R,DA} \tag{3.11}$$

$$(\mu_l^{R,DA}): \quad d_l^{R,DA} \leq D_l^R, l \in RL \tag{3.12}$$

$$d_l^{R,DA} \geq 0$$

The day ahead market clearing constraints are similar to the one from the US model. The full EUVT equilibrium formed from the agents' profit maximization problems as well as the market clearing constraints can be found in appendix A.3.

Forbidding virtual trading on the EUVT model can be done to build the EU model. Similarly as in the previous section, adding constraints (3.1) and (3.2) to the EUVT day-ahead profit maximization problem of the generators and constraints (3.3) and (3.4) to the EUVT day-ahead profit maximization problem of the load generators form the EU profit maximization problem of the generators and of the loads.

The EU equilibrium can be found in appendix A.4.

3.3 Conclusion

This chapter consisted of the introduction of three new models in addition to the US model. The difference of policies between those models is whether or not allowing day ahead virtual trading and whether or not holding a real-time market for reserve. Table 3.1 gives an overview of those models and their full equilibrium problems can be found in appendix A.

CHAPTER 4

The forward-backward Algorithm applied to a 2 stage market

In this chapter, we explain the algorithm we used to find an equilibrium for our different models. Finding risk averse equilibrium can be a tricky exercise. We decided to forego classic equilibrium finder such as the "path" solver that was used in [8] and to work with a more heuristic approach. The use of alternative technique to find risk-averse equilibrium in electricity market can be found in [10] and [7]. While the former used an ADMM-based method for solving the capacity problem, the latter used a decomposition similar to the one we will use.

We will start by introducing the theory on the variational inequality, which are a generalisation of our equilibrium problem. Then we will take a look at the risk neutral version of our equilibrium and finally we will explain the heuristic used to find the risk-averse equilibria. This heuristic is called the forward backward algorithm.

We emphasize on the fact that the method described in this chapter is a heuristic. We make no claim on the property that our markets should satisfies to fulfil the hypothesis of the technique introduced.

4.1 Theory of the variational inequalities

The equilibrium problems we are trying solve can be generalised as a variational inequality. An extensive theory about variational inequality and how to solve them can be found in [6]. For this master thesis we restricted ourselves to some basic definition.

Definition 1. Given a subset K of the Euclidea n-dimensional space $\mathbb{R}^n$n and a mapping $F : K \to \mathbb{R}^n$, the variational inequality denoted $VI(K, F)$ is to find a vector $x \in K$ such that:

$$(y - x)^T F(x) \geq 0, \quad \forall y \in K$$

The set of solution is denoted $SOL(K, F)$.

Definition 2. The normal cone to the set K at x' is defined as the following set.

$$\mathcal{N}(x'; K) = \{d \in \mathbb{R}^n : d^T(y - x') \leq 0, \forall y \in K\}$$

From those two definitions, we can state that a vector $x \in K$ solves VI(K, F) if and only if $-F(x)$ is a normal vector to K at x or equivalently in equation (4.1).

$$0 \in T(x) \quad \text{with} \quad T(x) = F(x) + \mathcal{N}(x; K) \tag{4.1}$$

If K is a cone, we can define the complementarity problem as an equivalent form of the variational inequality.

Definition 3. Given a cone K and a mapping $F : K \to \mathbb{R}^n$, the complementarity problem, denoted as $CP(K, N)$ is to find a vector x satisfying

$$K \ni x \perp F(x) \in K^*$$

With K^* being the dual cone of K.

The proof of the equivalence between the complementarity problem and the variational inequality can be found in proposition 1.1.3 from [6]. The equivalence between (i) finding a solution to our equilibrium problem and (ii) finding the zeros of the mapping of the corresponding variational inequality to which the normal cone was added results from this proof.

4.2 The risk neutral problem

A special algorithm is needed in order to find the equilibrium of the KKT conditions of the different problems because risk measure are involved. If the agents were risk neutral, the equilibrium problem would become a simple optimisation problem that would be solvable with a standard solver.

The risk neutral optimisation problem of our US model can be stated as follows.

The risk neutral problem:

$$
\begin{aligned}
\min_{\substack{p^{RT},d^{RT},r^{RT},d^{R,RT},\\ p^{DA},r^{DA},d^{DA},d^{R,DA},y}} \quad & \sum_{g\in G} K_g \cdot y_g - \sum_{l\in RL} V_l^{R,DA} \cdot d_l^{R,DA} + \sum_{\omega\in\Omega} P_\omega \cdot \Bigg(\sum_{g\in G} C_g \cdot p_{g,\omega}^{RT} \\
& - \sum_{l\in L} V_l \cdot d_{l,\omega}^{RT} - \sum_{l\in RL} V_l^R \cdot (d_{l,\omega}^{R,RT} - d_l^{R,DA}) \Bigg) \\
(\mu_{g,\omega}^{G,RT}): \quad & P_\omega(P_{g,\omega}^{+} \cdot y_g - p_{g,\omega}^{RT} - r_{g,\omega}^{RT}) \geq 0 && \forall g \in G, \forall \omega \in \Omega \\
(\mu_{g,\omega}^{G,RT}): \quad & P_\omega(R_g - r_{g,\omega}^{RT}) \geq 0 && \forall g \in G, \forall \omega \in \Omega \\
(\mu_{l,\omega}^{L,RT}): \quad & P_\omega(D_{l,\omega}^{+} - d_{l,\omega}^{RT}) \geq 0 && \forall l \in L, \forall \omega \in \Omega \\
(\mu_{l,\omega}^{R,RT}): \quad & P_\omega(D_l^{R} - d_{l,\omega}^{R,RT}) \geq 0 && \forall l \in RL, \forall \omega \in \Omega \\
(\mu_{g}^{G,R,DA}): \quad & R_g - r_g^{DA} \geq 0 && \forall g \in G \\
(\mu_{l}^{R,DA}): \quad & D_l^{R} - d_l^{R,DA} \geq 0 && \forall l \in RL \\
(\lambda_\omega^{RT}): \quad & P_\omega(\sum_{g\in G} p_{g,\omega}^{RT} - \sum_{l\in L} d_{l,\omega}^{RT}) \geq 0 && \forall \omega \in \Omega \\
(\lambda_\omega^{R,RT}): \quad & P_\omega(\sum_{g\in G} r_{g,\omega}^{RT} - \sum_{l\in RL} d_{l,\omega}^{R,RT}) \geq 0 && \forall \omega \in \Omega \\
(\lambda^{DA}): \quad & \sum_{g\in G} p_g^{DA} - \sum_{l\in L} d_l^{DA} \geq 0 \\
(\lambda^{R,DA}): \quad & \sum_{g\in G} r_g^{DA} - \sum_{l\in RL} d_l^{R,DA} \geq 0 \\
(\delta_g): \quad & 1 - y_g \geq 0 && \forall g \in G \\
& p_{g,\omega}^{RT}, r_{g,\omega}^{RT}, r_g^{DA}, y_g \geq 0 && \forall g \in G, \forall \omega \in \Omega \\
& d_{l,\omega}^{RT} \geq 0 && \forall l \in L, \forall \omega \in \Omega \\
& d_{l,\omega}^{R,RT}, d_l^{R,DA} \geq 0 && \forall l \in RL, \forall \omega \in \Omega
\end{aligned}
$$

The KKT conditions of this optimisation problem are equivalent to the KKT conditions of risk neutral agents' day ahead and real time profit maximisation problem as well as the market clearing constraints both in day ahead and real time.

4.3 Description of the backward forward algorithm

The idea behind the backward-forward algorithm is an extension of the one behind the proximal point algorithm. With the proximal point algorithm, we iterate over the resolvent operator of a mapping $T(x)$ maximal monotone in order to find the zeros of this mapping. The iteration is stated in (4.2).

$$x^{(l+1)} = J_{\rho T}(x^{(l)}) \tag{4.2}$$

Before defining the resolvent of F, we need to introduce the "relation" also called multifunction or a set valued map. We warn the reader that the intention of the explanation below is not to give solid and stable theory but to convince the reader of the mechanisms behind the proximal point algorithm.

Definition 4. A relation is a map F from $\mathbb{R}^n$ into the power set of $\mathbb{R}^n$. It is characterised by:

- its domain: $\text{dom } F \equiv \{x \in \mathbb{R}^n : F(x) \neq \emptyset\}$
- its range: $\text{ran } F \equiv \cup_{x \in \text{ dom } F} F(x)$
- its graph: $\text{gph } F \equiv \{(x, y) \in \mathbb{R}^n : y \in F(x)\}$.

We can define operator on those relation and one of such operator is the inverse. The inverse of a relation F is given hereunder.

$$F^{-1} = \{(y, x) | (x, y) \in \text{ gph } F\}$$

Definition 5. The resolvent of a mapping T is defined as

$$J_{\rho T} \equiv (I + \rho T)^{-1} \equiv \{(x + \lambda y, x) | (x, y) \in \text{ gph } F\}. \tag{4.3}$$

Equation (4.2) allows us to find the fixed point of its resolvent and, as stated in proposition 12.3.5 of [6], the fixed points of the resolvent of a monotone mapping are also the zeros of this mapping when this mapping has zeros.

To convince ourselve that the fixed points of the resolvent of a monotone mapping are also the zeros of this mapping we use equation (4.3) to obtain the following result.

$$0 \ni F(x) \iff (x, 0) \in \text{ gph } F \iff (x, x) \in \text{ gph } J_{\rho F}$$

The recursion on the resolvent of $T(x)$ is expected to reach its fixed point because resolvents of monotone maps are nonexpansive maps.

Definition 6. A mapping F with Lipschitz constant L such that

$$||F(x) - F(y)||_2 \leq L||x - y||_2 \quad \forall x, y \in \text{ dom } F \tag{4.4}$$

is said to be non-expansive if $L = 1$ and a contraction if $L < 1$.

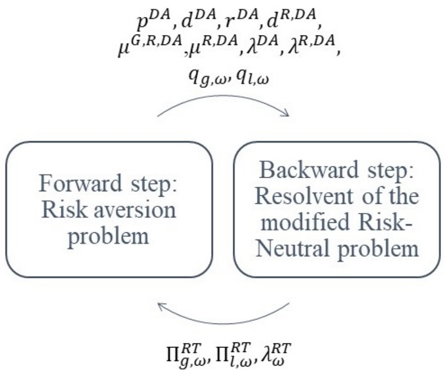


Figure 4.1: Information shared between the forward and the backward step of the algorithm

However, the complexity of computing the resolvent of an operator can make the proximal algorihtm impractical. The alternative solution is then to use the forward backward algorithm. Instead of computing the resolvent of the complete mapping T we decompose it into a mapping A maximal monotone and B, a single-valued function from the domain of A into $\mathbb{R}^n$. Instead of computing the resolvent of T, the resolvent of A is computed at each iteration. The iteration follows (4.5).

$$x^{(l+1)} = J_{\rho A}\Big((I - \rho B)(x^{(l)})\Big) \tag{4.5}$$

The complete formulation of the mappings T, A and B for our US model can be found in appendix B.1. T (see equation (B.1)) is decomposed into A (see equation (B.2)), being the risk neutral problem that can be seen as a pure optimisation problem and B (see equation (B.3)) as the deviation from our pure optimisation problem caused by the risk aversion of the different agents.

As stated earlier, the algorithm consists of two steps: the forward step and the backward step. The backward step needs the day ahead variables from the forward step whereas the forward step will need the real time profit computed in the backward step.

The figure 4.1 shows a schematic view of the algorithm.

4.3.1 The backward step

The backward step involves the computation of the resolvent of the mapping A. We know from proposition 1 in appendix B that the resolvent of a pure optimisation problem can be computed by solving a modified pure optimisation problem. The original idea as well as the proof from proposition 1 come from Professor Yves Smeers.

Applied to our setting, computing $J_{\rho A}\Big((I-\rho B)(x^{(k)})\Big)$ is equivalent to solving the following problem.

$$
\begin{aligned}
\min_{\substack{p^{RT},d^{RT},r^{RT},d^{R,RT},\\ p^{DA},r^{DA},d^{DA},d^{R,DA},y\\ v^{\mu},v^{\lambda},v^{\delta}}} \quad & \sum_{g\in G} K_g \cdot y_g - \sum_{l\in RL} V_l^{R,DA}\cdot d_l^{R,DA} + \sum_{\omega\in\Omega} P_\omega \cdot \Bigg(\sum_{g\in G} C_g\cdot p_{g,\omega}^{RT} \\
& - \sum_{l\in L} V_l \cdot d_{l,\omega}^{RT} - \sum_{l\in RL} V_l^R\cdot (d_{l,\omega}^{R,RT} - d_l^{R,DA})\Bigg) \\
& + \frac{1}{2\cdot\rho}\Bigg(\sum_{g\in G}\Big((p_g^{DA} - p_g^{DA,(k)} + \rho\cdot\eta_g^{(k)})^2 + (r_g^{DA} - r_g^{DA,(k)} + \rho\cdot\eta_g^{R,(k)})^2\Big) \\
& + \sum_{l\in L}(d_l^{DA} - d_l^{DA,(k)} + \rho\cdot\eta_l^{(k)})^2 + \sum_{l\in RL}(d_l^{R,DA} - d_l^{R,DA,(k)})^2 \\
& + \sum_{g\in G}(y_g - y_g^{(k)} - \rho\cdot\gamma_g^{(k)})^2 + \sum_{g\in G,\omega\in\Omega}\Big((p_{g,\omega}^{RT} - p_{g,\omega}^{RT,(k)})^2 + (r_{g,\omega}^{RT} - r_{g,\omega}^{RT,(k)})^2\Big) \\
& + \sum_{l\in L,\omega\in\Omega}(d_{l,\omega}^{RT} - d_{l,\omega}^{RT,(k)})^2 + \sum_{l\in RL,\omega\in\Omega}(d_{l,\omega}^{R,RT} - d_{l,\omega}^{R,RT,(k)})^2\Bigg) \\
& + \frac{\rho}{2}\cdot\Bigg(\sum_{g\in G,\omega\in\Omega}\Big((v_{g,\omega}^{\mu^{G,RT}})^2 + (v_{g,\omega}^{\mu^{G,R,RT}})^2\Big) + \sum_{l\in L,\omega\in\Omega}(v_{l,\omega}^{\mu^{L,RT}})^2 \\
& + \sum_{l\in RL,\omega\in\Omega}(v_{l,\omega}^{\mu^{R,RT}})^2 + \sum_{\omega\in\Omega}\Big((v_\omega^{\lambda^{RT}})^2 + (v_\omega^{\lambda^{R,RT}})^2\Big) + \sum_{g\in G}(v_g^{\delta})^2 \\
& + \sum_{g\in G}(v_g^{\mu^{G,R,DA}})^2\Big) + \sum_{l\in RL}(v_l^{\mu^{R,DA}})^2 + (v^{\lambda^{DA}})^2 + (v^{\lambda^{R,DA}})^2\Bigg) \\
& + \sum_{g\in G,\omega\in\Omega} v_{g,\omega}^{\mu^{G,RT}}\cdot\mu_{g,\omega}^{G,RT,(k)} + \sum_{g\in G,\omega\in\Omega} v_{g,\omega}^{\mu^{G,R,RT}}\cdot\mu_{g,\omega}^{G,R,RT,(k)} \\
& + \sum_{l\in L,\omega\in\Omega} v_{l,\omega}^{\mu^{L,RT}}\cdot\mu_{l,\omega}^{L,RT,(k)} + \sum_{l\in RL,\omega\in\Omega} v_{l,\omega}^{\mu^{R,RT}}\cdot\mu_{l,\omega}^{R,RT,(k)} \\
& + \sum_{\omega\in\Omega} v_\omega^{\lambda^{RT}}\cdot\lambda_\omega^{RT,(k)} + \sum_{\omega\in\Omega} v_\omega^{\lambda^{R,RT}}\cdot\lambda_\omega^{R,RT,(k)} + \sum_{\omega\in\Omega} v_g^{\delta}\cdot\delta_g^{(k)} \\
& + \sum_{g\in G} v_g^{\mu^{G,R,DA}}\cdot\mu_g^{G,R,DA,(k)} + \sum_{l\in RL} v_l^{\mu^{R,DA}}\cdot\mu_l^{R,DA,(k)} \\
& + v^{\lambda^{DA}}\cdot\lambda^{DA,(k)} + v^{\lambda^{R,DA}}\cdot\lambda^{R,DA,(k)}
\end{aligned}
$$

$$
\begin{aligned}
(\mu^{G,RT}_{g,\omega}): &\quad P_\omega(P^+_{g,w}\cdot y_g - p^{RT}_{g,\omega} - r^{RT}_{g,\omega}) + v^{\mu^{G,RT}}_{g,\omega} \geq 0 && \forall g \in G, \forall \omega \in \Omega\\
(\mu^{G,R,RT}_{g,\omega}): &\quad P_\omega(R_g - r^{RT}_{g,\omega}) + v^{\mu^{G,RT}}_{g,\omega} \geq 0 && \forall g \in G, \forall \omega \in \Omega\\
(\mu^{L,RT}_{l,\omega}): &\quad P_\omega(D^+_{l,\omega} - d^{RT}_{l,\omega}) + v^{\mu^{L,RT}}_{l,\omega} \geq 0 && \forall l \in L, \forall \omega \in \Omega\\
(\mu^{R,RT}_{l,\omega}): &\quad P_\omega(D^R_l - d^{R,RT}_{l,\omega}) + v^{\mu^{R,RT}}_{l,\omega} \geq 0 && \forall l \in RL, \forall \omega \in \Omega\\
(\lambda^{RT}_\omega): &\quad P_\omega(\sum_{g\in G} p^{RT}_{g,\omega} - \sum_{l\in L} d^{RT}_{l,\omega}) + v^{\lambda^{RT}}_\omega \geq 0 && \forall \omega \in \Omega\\
(\lambda^{R,RT}_\omega): &\quad P_\omega(\sum_{g\in G} r^{RT}_{g,\omega} - \sum_{l\in L} d^{R,RT}_{l,\omega}) + v^{\lambda^{RT}}_\omega \geq 0 && \forall \omega \in \Omega\\
(\mu^{G,R,DA}_g): &\quad (R_g - r^{DA}_g) + v^{\mu^{G,DA}}_g \geq 0 && \forall g \in G\\
(\mu^{R,DA}_{l,\omega}): &\quad (D^R_l - d^{R,DA}_l) + v^{\mu^{R,DA}}_l \geq 0 && \forall l \in RL\\
(\lambda^{DA}): &\quad (\sum_{g\in G} p^{DA}_g - \sum_{l\in L} d^{DA}_l) + v^{\lambda^{DA}} \geq 0 &&\\
(\lambda^{R,DA}): &\quad (\sum_{g\in G} r^{DA}_g - \sum_{l\in L} d^{R,DA}_l) + v^{\lambda^{DA}} \geq 0 &&\\
(\delta_g): &\quad 1 - y_g + v^\delta_g \geq 0 && \forall g \in G\\
&\quad p^{RT}_{g,\omega}, r^{RT}_{g,\omega}, r^{DA}_g, y_g \geq 0 && \forall g \in G, \forall \omega \in \Omega\\
&\quad d^{RT}_{l,\omega} \geq 0 && \forall l \in L, \forall \omega \in \Omega\\
&\quad d^{R,RT}_{l,\omega}, d^{R,DA}_l \geq 0 && \forall l \in RL, \forall \omega \in \Omega
\end{aligned}
$$

4.3.2 The forward step

In the forward step, we solve the risk aversion problem with the new iteration of the real time profit of the generators and load, $\Pi^{RT,(k+1)}_{g,\omega}$ and $\Pi^{RT,(k+1)}_{l,\omega}$, characterized below. From this risk aversion problem, we can compute the risk adjusted probabilities of the different agents as well as the values of the primal and dual variables relating to the day ahead market.

$$
\begin{aligned}
&\Pi^{RT,(k+1)}_{g,\omega} = p^{RT,(k+1)}_{g,\omega}\cdot(\lambda^{RT,(k+1)}_\omega - C_g) + r^{RT,(k+1)}_{g,\omega}\cdot\lambda^{R,RT,(k+1)}_\omega && \forall g \in G, \forall \omega \in \Omega\\
&\Pi^{RT,(k+1)}_{l,\omega} d^{RT,(k+1)}_{l,\omega}\cdot(V_l - \lambda^{RT,(k+1)}_\omega) && \forall l \in L, \forall \omega \in \Omega
\end{aligned}
$$

The forward step can be stated as the following optimisation problem.

$$
\min_{\substack{q_g\in\mathcal{M}_{\mathcal{R}_g}\forall g\in G\cup L\\ \mu^{G,R,DA},\mu^{R,DA},\lambda^{DA},\lambda^{R,DA}}} \quad \sum_{g\in G\cup L,\omega\in\Omega} q_{g,\omega}\cdot\Pi^{RT,(k+1)}_{g,\omega} + \sum_{g\in G}\mu^{G,R,DA}_g\cdot R_g + \sum_{l\in RL}\mu^{R,DA}_l\cdot D^R_l
$$

$$
\begin{aligned}
(p_g^{DA}): \quad & -\lambda^{DA} + \sum_{\omega\in\Omega} q_{g,\omega} \cdot \lambda_\omega^{RT,(k+1)} = 0 && \forall g \in G \\
(d_l^{DA}): \quad & \lambda^{DA} - \sum_{\omega\in\Omega} q_{l,\omega} \cdot \lambda_\omega^{RT,(k+1)} = 0 && \forall l \in L \\
(r_g^{DA}): \quad & -\lambda^{R,DA} + \sum_{\omega\in\Omega} q_{g,\omega} \cdot \lambda_\omega^{R,RT,(k+1)} + \mu_g^{G,R,DA} \geq 0 && \forall g \in G \\
(d_l^{R,DA}): \quad & \lambda^{R,DA} - V_l^{R,DA} + \mu_l^{R,DA} - \sum_{\omega\in\Omega} P_\omega \cdot (\lambda_\omega^{R,RT,(k+1)} - V_l^R) \geq 0 && \forall l \in L \\
& \mu_g^{G,R,DA}, \mu_l^{R,DA} \geq 0 && \forall g \in G, \forall l \in RL
\end{aligned}
$$

After solving this optimisation problem, it is possible to compute the deviation from the risk neutral optimisation (as defined in equation (B.3)) problem as follows.

$$
\begin{aligned}
\gamma_g^{(k+1)} &= \sum_{\omega\in\Omega} (P_\omega - q_{g,\omega}^{(k+1)}) \cdot P_{g,\omega}^{+} \cdot \mu_{g,\omega}^{G,RT,(k+1)} && \forall g \in G \\
\eta_g^{R,(k+1)} &= \sum_{\omega\in\Omega} q_{g,\omega}^{(k+1)} \cdot \lambda_\omega^{R,RT,(k+1)} && \forall g \in G \\
\eta_g^{(k+1)} &= \sum_{\omega\in\Omega} q_{g,\omega}^{(k+1)} \cdot \lambda_\omega^{RT,(k+1)} && \forall g \in G \\
\eta_l^{(k+1)} &= -\sum_{\omega\in\Omega} q_{l,\omega}^{(k+1)} \cdot \lambda_\omega^{RT,(k+1)} && \forall l \in L
\end{aligned}
$$

4.3.3 Application to other models

The algorithm needs some adjustments to be applied to other models.

The RTR model: The no-virtual trading constraints are added both in the backward and forward optimisation problems. The deviations from the pure optimisation problem need to be modified to fit the RTR version of the mapping B.

The EUVT model: The backward step needs to be modified to account for the risk neutral problem of a market that does not hold a real time market for reserve.

For the forward step, it is easier to formulate the dual of the risk averse problem than the primal. This is caused by constraints that mix both real time and day ahead variables. The dual formulation of the forward step is defined below.

$$
\begin{aligned}
\max_{\substack{p^{DA}, r^{DA}, \\ d^{DA}, d^{R,DA}}} \quad & \sum_{g\in G} \mathcal{R}_g(\Pi_{g,\omega}^{RT,(k+1)} - \lambda_\omega^{RT,(k+1)} \cdot p_g^{DA}) \\
& + \sum_{l\in L} \mathcal{R}_l(\Pi_{l,\omega}^{RT,(k+1)} + \lambda_\omega^{RT,(k+1)} \cdot d_l^{DA})
\end{aligned}
$$

$$
\begin{aligned}
& \quad + \sum_{l \in RL} V_l^{R,DA} \cdot d_l^{R,DA} - \sum_{g \in G, \omega \in \Omega} \phi \cdot e_{g,\omega}^2 && \text{(4.6)} \\
(\lambda^{DA}): \quad & \sum_{g \in G} p_g^{DA} = \sum_{l \in L} d_l^{DA} \\
(\lambda^{R,DA}): \quad & \sum_{g \in G} r_g^{R,DA} = \sum_{l \in L} d_l^{R,DA} \\
(\mu_{g,\omega}^{G,RT}): \quad & P_g^{DA,+} \cdot y_g^{(k+1)} - p_{g,\omega}^{RT,(k+1)} - r_g^{DA} + e_{g,\omega} \geq 0 \quad \forall g \in G, \forall \omega \in \Omega && \text{(4.7)} \\
(\mu_g^{G,R,DA}): \quad & R_g - r_g^{DA} \geq 0 \quad \forall g \in G \\
(\mu_l^{R,DA}): \quad & D_l^{R,DA} - d_l^{R,DA} \geq 0 \quad \forall l \in RL \\
& r^{DA}, d^{R,DA} \geq 0
\end{aligned}
$$

We have to intoduce a slack variable $e_{g,\omega}$ in the constraint (4.7) to make the problem feasible in the first iterations of the algorithm. There is a regularisation term in equation (4.6) to make the slack variable drop to zero when the algorithm has done some iterations and is running.

The EU model: The process is similar to the modification from the US model to the RTR model but from the EUVT model.

4.3.4 Limitation of the algorithm

The algorithm as it is now has some limitations when applied to practical problems.

Firstly, the equilibria obtained do not strictly respect some of the equilibrium constraints. This is particularly true for the perpendicularity constraints where a slackness of up to 0.15 was allowed. This means that for a constraint $x \perp y$, that either $x \leq 0.15$ or $y \leq 0.15$ was considered good enough for the constraint to be satisfied. Obviously this slackness was only allowed for the constraints with big numbers. The equilibrium constraints regarding the risk-adjusted expectation needs to be tight to be considered as an equilibrium.

Secondly, some day-ahead variables are both computed in the backward and the forward step. The way those variables should be updated is unclear as of now. The most elegant solution would consist in using the day ahead variables of the forward step as the new value for the next iteration of the backward step. Unfortunately, the algorithm does not converge when using this technique. The solution used consists in separating the day-ahead variables from the forward and the backward step and using the values from either the backward of the forward step in the equilibrium. The choice of the forward or the backward step's value needs to be done on a case-by-base basis.

Thirdly, the method cannot be applied to every risk-measures. Convergence was relatively easily reached for the mean value at risk in most test situations but the conditional value at risk was another story. The method would only work under relatively controlled parameters.

4.4 Conclusion

This chapter explained a heuristic to solve risk-averse 2-settlements reserve capacity markets. The technique used has the potential to be extended to a wide range of 2-settlement markets with risk-averse agents and to find solutions for the broader framework of risk-averse stochastic equilibrium.

CHAPTER 5

Description of the test system

In order to test the different policies, we built a toy system. We consider four generators in this system: nuclear, coal, gas and wind. Their characteristics are shown in the table 5.1.

	C	P^+	K	R
Nuclear	6.5	7100	1000	1000
Coal	25	2000	1000	1000
Gas	80	2200	1000	1000
Wind	0	1000 or 0	0	0

Table 5.1: Characteristic of the generators

The wind generator is particular in two ways. Firstly, its maximum production is variable. It can either produce 1000 units of energy or 0 with equal probability. This represent the probability of a day being windless or not. Secondly, the wind generator cannot trade reserve.

We then have three "classic" generators. The nuclear one which would be the base production, the coal one which would be the shoulder and the gas one which would generate during peak demand.

The production functions of the two different scenarios can be found in **??**.

We consider that the demand is aggregated by one big load whose size depends on the scenario. We consider six scenarios for the demand whose characteristics can be found in the following table.

	D	Probability
Scenario 1	10000	0.5
Scenario 2	10050	0.1
Scenario 3	10104	0.1
Scenario 4	10168	0.1
Scenario 5	10256	0.1
Scenario 6	10618	0.1

Those numbers are a discretization of the normal law. The figure 5.1 shows this operation.

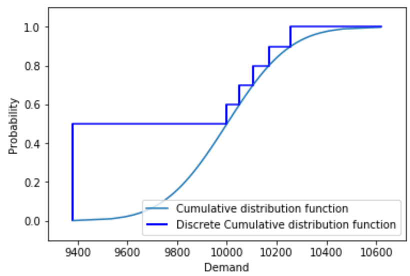


Figure 5.1: Discretization of the normal law with mean $\mu = 10000$ and standard deviation $\sigma = 200$

The value of lost load is set at 8300 and is in accordance with the Belgium Federal Planning Bureau [4].

We deem the probability of a day being windy or windless and the probability of the different quantity of demand as independant. As such we can define 12 scenarios, 6 for windy day and 6 for windless day, with the probability of each of them being the product of the probability of the quantity of wind and of the quantity of demand for the day.

The ORDC for both the day ahead and real time reserve market has been computed in accordance with section 2.3. The loss of load probability is defined by a normal law and the parameter for the real time ORDC can be found in table 5.2.

The same parameters are used for the day ahead ORDC excepted that $\sigma^{DA} = 210$. The numerical value of both the day ahead and real time ORDC can be found in table C.1.

VoLL	8300
C	50
$Lolp^{RT}$	$1 - \mathcal{N}(\mu, \sigma)$
μ	0
σ	200

Table 5.2: Parameters of the real time ORDC

CHAPTER 6

Comparison of designs under different risk aversion sets and the mean value at risk risk-measure

This chapter will present the results obtained by our algorithm on different market designs. They will replicate the toy system defined in chapter 5.

The purpose of this analysis is twofold: firstly we want to compare the different market designs and secondly we want to understand how the behaviours of the agents are influenced by their risk aversion in a given design.

We will focus our comparison on the back-propagation of the prices, the value of the unit commitment variables, the goods traded in the day ahead and the profit generated by the different agents.

We will define four different sets of risk aversion. The first set is for a risk neutral market and can be used as a benchmark. The second one is for a risk averse market where all the agents share the same risk aversion. The third and fourth sets will represent a market where the load will be more risk averse than the generators and the contrary. Those risk aversion sets can be found in table 6.1.

It is important to remind the reader of the simplicity of the system used. The conclusions drawn from the equilibria may not necessarily apply to more realistic systems. Nevertheless, the general trends of the models can give interesting insight to better understand the markets.

This chapter is built as follows: we will begin by explaining our choice of risk measure. Then, we will give some insight on the equilibria of the different markets under the different risk aversion sets. For this section we will focus on the day ahead

	α_N	α_C	α_G	α_W	α_L
Risk neutral	1	1	1	1	1
Similar risk aversion	0.9	0.9	0.9	0.9	0.9
Load more risk averse	0.9	0.9	0.9	0.9	0.8
Load less risk averse	0.9	0.9	0.9	0.9	1

Table 6.1: Set of risk aversion of the different agents

profits of the agents, the day-ahead position of the agents, the unit commitment value of the generators and the back-propagation of prices. Finally, we will report the reaction of the US and the RTR markets when we modify the risk aversion of a specific agent.

6.1 Choice of risk measure

In order to make a comparison of the different policies, we have to commit ourselves to a single risk-measure. We settled on the mean Value at risk over the conditional value at risk because of stability issues.

Even if the mean value at risk is not as well known as the conditional value at risk, the MVaR can be seen as a generalisation of the CVaR. We noticed more stability as soon as $1-\beta$, the parameter relative to the weight of the real expectation, was different than 0.

An intuitive reasoning behind the better behaviour of the MVaR lies in its attenuation of the risk aversion. The lowest the risk-adjusted probability of a scenario could go is $P_\omega/(1-\beta)$ compared to the 0 of the CVaR.

In order to reduce this attenuation of the risk aversion but at the same time keep the result as close as possible to the CVaR, the choice was made to set β equal to 0.99.

6.2 Analysis of the agents' day ahead profit

The profits of the agents for the different markets can be analysed to begin this study. The tables D.1, D.2, D.3 and D.4 from appendix D list the profits of the agents in function of the risk aversion set. Table 6.2 reminds the reader on how the different profits of the models are computed.

	US and RTR market	EUVT and EU market
$\Pi_{g,\omega}^{RT}$	$p_{g,\omega}^{RT} \cdot (\lambda_\omega^{RT} - C_g) + r_{g,\omega}^{RT} \cdot \lambda_\omega^{R,RT}$	$p_{g,\omega}^{RT} \cdot (\lambda_\omega^{RT} - C_g)$
$\Pi_{l,\omega}^{RT}$	$d_{l,\omega}^{RT} \cdot (V_l - \lambda_\omega^{RT})$	$d_{l,\omega}^{RT} \cdot (V_l - \lambda_\omega^{RT})$
Π_g^{DA}	$p_g^{DA} \cdot \lambda^{DA} + r_g^{DA} \cdot \lambda^{R,DA} + \sum_{\omega\in\Omega} q_{g,\omega} \cdot \left(\Pi_{g,\omega}^{RT} - p_g^{DA} \cdot \lambda_\omega^{RT} - r_g^{DA} \cdot \lambda_\omega^{R,RT}\right)$	$p_g^{DA} \cdot \lambda^{DA} + r_g^{DA} \cdot \lambda^{R,DA} + \sum_{\omega\in\Omega} q_{g,\omega} \cdot \left(\Pi_{g,\omega}^{RT} - p_g^{DA} \cdot \lambda_\omega^{RT}\right)$
Π_l^{DA}	$-d_l^{DA} \cdot \lambda^{DA} + \sum_{\omega\in\Omega} q_{l,\omega} \cdot \left(\Pi_{l,\omega}^{RT} + d_l^{DA} \cdot \lambda_\omega^{RT}\right)$	$-d_l^{DA} \cdot \lambda^{DA} + \sum_{\omega\in\Omega} q_{l,\omega} \cdot \left(\Pi_{l,\omega}^{RT} + d_l^{DA} \cdot \lambda_\omega^{RT}\right)$

Table 6.2: Definition of day ahead and real time profit for a generator $g \in G$ and a load $l \in L$.

Global comments: Figure 6.1 with the legend in figure 6.2 represents the day-ahead profit of the agents in function of (i) the market used and (ii) the risk-aversion set used. We see from that figure that the EU and EUVT market are beneficial to the generators. The wind generator is kind of a outlier because its day ahead profit is more impacted by the virtual trading policy than by the holding of a real time market for reserve policy. The US and RTR markets are more beneficial to the load.

We also notice that the day ahead profit of the gas generator for the US and RTR markets is equal to zero.

Impact of virtual trading: the effect of virtual trading on the day ahead price is minimal. When there is a real-time market for reserve, forbidding virtual trading is detrimental to the nuclear and coal generator in the "load less risk averse" and the "load more risk averse". The impact on the load is too small to be seen on the figure. Similarly, when there is no real-time market for reserve, the nuclear and coal generators are making more day ahead profit when virtual trading is allowed with the "load less risk averse" aversion set than in the EU market.

6.3 Analysis of the agents' day ahead position

We are also interested in the day ahead positions of the agents. Tables 6.3 shows the amount of energy traded in day-ahead and 6.4 the amount of reserve traded in day ahead.

Day ahead trading of energy: the first thing to notice is that no generator

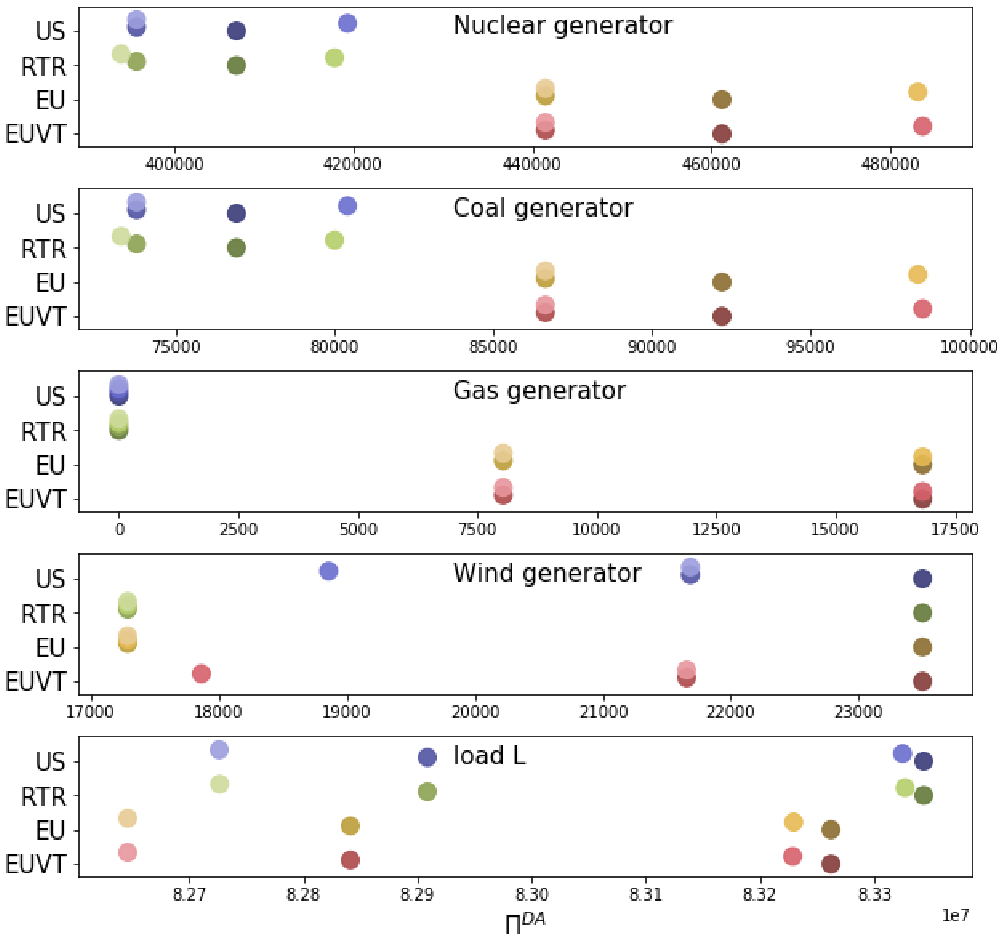


Figure 6.1: Comparison of the day ahead profits of the different agents

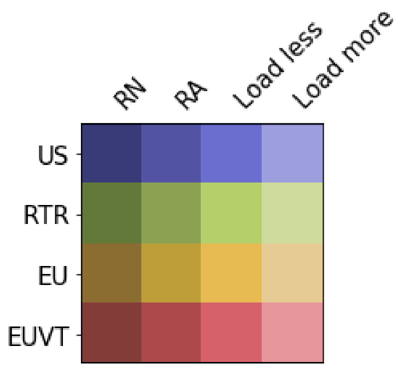


Figure 6.2: Legend for figure 6.1

Aversion set	Model	p_N^{DA}	p_C^{DA}	p_G^{DA}	p_W^{DA}	d_L^{DA}
RN	US	0	0	0	0	0
	RTR	0	2000	1418	0	3418
	EUVT	602,92	-1479,55	-1831,06	-2449,58	-5157,27
	EU	2390,48	890,55	687,75	461,92	4430,7
RA	US	7100	2000	85,73	-121,22	9064,5
	RTR	7100	2000	85,73	0	9185,73
	EUVT	7100	2000	1165,73	-9,73	10256
	EU	7100	2000	1156	0	10256
Load less risk averse	US	7100	2000	586,39	1000	10686,39
	RTR	7100	2000	704,45	813,55	10618
	EUVT	7100	2000	1220,06	1000	11320,06
	EU	7100	2000	1229,35	288,65	10618
Load more risk averse	US	7100	2000	85,73	-4172,79	5012,94
	RTR	5331,06	0	0	0	5331,06
	EUVT	7100	2000	1077,73	-9,73	10168
	EU	7100	2000	1068	0	10168

Table 6.3: Day ahead production under the MVaR risk measure

Aversion set	Model	r_N^{DA}	r_C^{DA}	r_G^{DA}	r_W^{DA}
RN	US	0	750	0	0
	RTR	0	0	750	0
	EUVT	0	0	682	0
	EU	0	0	682,01	0
RA	US	0	0	750	0
	RTR	0	0	750	0
	EUVT	0	0	700,03	0
	EU	0	0	700,03	0
Load less risk averse	US	0	0	750	0
	RTR	0	0	750	0
	EUVT	0	0	682	0
	EU	0	0	682	0
Load more risk averse	US	0	0	750	0
	RTR	750	0	0	0
	EUVT	0	0	700,03	0
	EU	0	0	700,03	0

Table 6.4: Day ahead reserve production under the MVaR risk measure

ever trades higher than its maximum possible physical production. We also notice that most of the time, the nuclear and the gas generators trade up to their maximum physical capacity. The stability of their real time production allows them to hedge most of their risk by trading fully in day ahead.

The wind generator trades differently in day-ahead compared to the nuclear and coal generators. It is the only generator that might buy energy in day ahead instead of selling it when virtual trading is allowed.

The load usually trades more energy in day ahead in the EU and EUVT markets than the US and RTR markets. We also see that in the "load less risk averse" risk-aversion set, the load tries to take advantage of its lower risk-aversion compared to the generator. When virtual trading is allowed, it trades more energy than when it is not allowed.

Day-ahead trading of reserve: not much can be said about the day ahead trading of reserve for the US and the RTR markets excepts that the entire reserve asked in day ahead (750 units) is served.

On the other hand, we see that two possible equilibria can be reached for the EU and EUVT markets. Either 682 units of reserve is served for the "RN" and "Load less risk averse" risk aversion set, either 700 units of reserve are served for the "RA" and "Load more risk averse" risk aversion set. The difference between those equilibria is that in the latter case, there will be a shortage of energy in one scenario because too much reserve was sold in day-ahead by the gas generator.

This is due to the risk aversion of the generators and more specifically to the risk aversion of the gas generator. The more the gas generator is risk averse, the more it will be tempted to sell reserve in the day ahead to make sure it will earn something. But when the gas generator is risk neutral, it takes more risks and decides to keep some production available to sell energy in the extreme scenario instead of reserve. We can see that the risk neutrality of the load in the "load less risk averse" risk aversion set achieves the same result. This illustrates the fact that the market is lead by the most risk-neutral agent. Its risk-neutrality influences all the other agents and makes them behave with a less aversion-driven mindset.

6.4 Analysis of the unit commitment variable of the generators

The unit commitment variables can be seen in table 6.5.

Global comments: every generator decides to fully commit with every risk aversion set and in every model excepted for the gas generator in the US and RTR markets.

The intuition we can get from the table is that the more risk averse the gas generator is, the less it will decide to commit. In the "load risk averse" aversion set, this outcome is counterbalanced by the risk neutrality of the load. Allowing virtual trading helps to increase the unit commitment for the "load less risk averse" risk aversion set. We could also extrapolate from those data that virtual trading is good for the unit-commitment of marginal generator.

6.5 Analysis of the back-propagation of price

The back-propagation of price can be seen in table 6.6. This table represents four different quantities: the day ahead price of energy and of reserve as well as the

Aversion set	Model	y_N	y_C	y_G	y_W
RN	US	1	1	0,9855	1
	RTR	1	1	0,9855	1
	EUVT	1	1	1	1
	EU	1	1	1	1
RA	US	1	1	0,8264	1
	RTR	1	1	0,8264	1
	EUVT	1	1	1	1
	EU	1	1	1	1
Load less risk averse	US	1	1	0,94	1
	RTR	1	1	0,9173	1
	EUVT	1	1	1	1
	EU	1	1	1	1
Load more risk averse	US	1	1	0,8264	1
	RTR	1	1	0,8264	1
	EUVT	1	1	1	1
	EU	1	1	1	1

Table 6.5: Unit commitment variable under the MVaR risk measure

expected real time price of energy and of reserve with the real probability P_ω. We can note that because there is no real time market for reserve for the EUVT and EU model, there is no expected real time price for reserve for those models.

To understand this table, we firstly need to understand how the day-ahead prices behave. There are two possible scenarios: either virtual trading is allowed, either virtual trading is not allowed. When virtual trading is allowed, the day ahead price for energy is the risk adjusted expectation of the real-time prices. When virtual trading is forbidden, the marginal surplus relative to the no-virtual trading constraints $\mu_g^{G,DA}$ and $\mu_l^{L,DA}$ needs to be taken into account. The day ahead prices of energy are then bounded by the equation (6.1).

$$\sum_{\omega \in \Omega} q_{l,\omega} \cdot \lambda_\omega^{RT} - \mu_l^{L,DA} \leq \lambda^{DA} \leq \sum_{\omega \in \Omega} q_{g,\omega} \cdot \lambda_\omega^{RT} + \mu_g^{G,DA} \quad \forall l \in L, \forall g \in G \tag{6.1}$$

The tightness of the bound is determined by the day-ahead position of the agents concerning energy.

Global comments: the first comment we can make on the back-propagated prices is that adding a real time market for reserve (as in the US and RTR models) helps to control the real time prices of energy. The explosion of the expected real time prices that we see under the "risk averse" and "load more risk averse" aversion sets for the EU and EUVT model is avoided. This explosion of the prices is caused by the shortage explained previously.

We also notice that the day-ahead prices are higher for the EU and EUVT markets. We see that the more risk averse the agents are, the higher the expected real time prices of energy and reserve. In the the "risk-neutral" and the "load less risk averse" risk aversion sets, the risk neutrality of at least one agent allows the real time prices to stay low.

Risk-neutrality and virtual trading: One of the conclusion of [1] was that virtual trading has no real impact on the back-propagation of price under risk-neutrality. Our results are in agreement with that. If we examine (6.1) under risk neutrality we obtained the following bounds.

$$\sum_{\omega \in \Omega} P_\omega \cdot \lambda_\omega^{RT} - \mu_l^{L,DA} \leq \lambda^{DA} \leq \sum_{\omega \in \Omega} P_\omega \cdot \lambda_\omega^{RT} + \mu_g^{G,DA} \quad \forall l \in L, \forall g \in G \tag{6.2}$$

The day-ahead price without virtual trading is equal to the day-ahead price with virtual trading. Those prices are equal to the expectation of the real-time prices because of (i) the tightness of the bounds from (6.2) when the generators and loads trade in day-ahead and of (ii) the positivity of $\mu_l^{L,DA}$ and $\mu_g^{G,DA}$.

Virtual trading effect: the virtual trading effect is most easily interpreted on the "load less risk averse" aversion set. In this set, the load is risk neutral so the day ahead prices of the models where virtual trading is allowed (US and EUVT) are equal to the the real expectation of the real time prices.

When virtual trading is not allowed (RTR and EU models), the day ahead prices are lower than the expected real time prices. This behaviour is explained by the equation (6.1). Because the load trades up to its maximal physical constraint (see table 6.3) its marginal surplus $\mu_L^{L,DA}$ can be greater than zero. This leads to the lower bound on λ^{DA} from equation (6.1) to be lower than the real expected prices.

The economic interpretation is that the load is taking advantage of the risk aversion of the generators to push the price down in day-ahead. A risk averse agent is more vulnerable to the action of a less risk averse agent in a market without virtual trading than in a market with virtual trading.

Day ahead price of reserve for US and RTR models: another interesting result concerns the day ahead price for reserve for the US and RTR models. This prices remains the same at 0.455 in every setting used and does not reflect the real-time prices of reserves.

This is explained by two facts for the US model. Firslty the real time price for reserve $\lambda_\omega^{R,RT}$ will be equal to the surplus $\mu_{Gas,\omega}^{G,RT}$ with gas being the marginal generator for the corresponding scenario ω. Secondly, δ_{Gas} will be equal to zeros because y_{Gas} is lower than 1 in every setting we tried. Because of that $K_{Gas} = \sum_{\omega\in\Omega} q_{Gas,\omega} \cdot P_{Gas,\omega}^{+} \cdot \mu_{Gas,\omega}^{G,RT}$. The day ahead price of reserve can then be computed from equation (6.3).

$$\begin{aligned}\lambda^{R,DA} &= \sum_{\omega\in\Omega} q_{g,\omega} \cdot \lambda_\omega^{R,RT} \quad \forall g \in G \cup L \\ &= \sum_{\omega\in\Omega} q_{g,\omega} \cdot \mu_{Gas,\omega}^{G,RT} \\ &= \frac{K_{Gas}}{P_{Gas}^{+}} \end{aligned} \tag{6.3}$$

We note that the physical constraint of the gas generator $P_{Gas,\omega}^{+}$ remains constant throughout the scenarii.

When there is no-virtual trading, the results are similar because the gas generator never trades reserve up to its limits in day ahead (see table 6.4).

Aversion set	Model	λ^{DA}	$\lambda^{R,DA}$	$\mathbb{E}_{P_\omega}[\lambda_\omega^{RT}]$	$\mathbb{E}_{P_\omega}[\lambda_\omega^{R,RT}]$
RN	US	63,955	0,455	63,955	0,455
	RTR	63,955	0,455	63,955	0,455
	EUVT	71,601	8,101	71,601	
	EU	71,601	8,101	71,601	
RA	US	62,382	0,455	96,828	33,328
	RTR	62,382	0,455	96,595	33,095
	EUVT	68,82	4,11	474,5	
	EU	68,82	4,11	474,5	
Load less risk averse	US	65,704	0,455	65,704	2,204
	RTR	65,501	0,455	66,938	3,438
	EUVT	74,761	8,101	74,761	
	EU	74,685	8,101	75,212	
Load more risk averse	US	62,382	0,455	96,828	33,328
	RTR	62,14	0,455	98,815	35,315
	EUVT	68,82	4,11	474,5	
	EU	68,82	4,11	474,5	

Table 6.6: Price back propagation under the MVaR risk measure

6.6 Analysis of the gas generator in the US and RTR models

Through the previous sections, we saw that the risk aversion of the gas generator and, by extension, its unit commitment have a huge impact especially for the US and the RTR markets. To study this effect, we investigated the effect of varying risk-aversion for the gas generator in the US and RTR market. In this experiment, the other agents follow the "risk averse" and "load less risk averse" aversion sets.

Figures 6.3, 6.4 and 6.5 show the effect of varying the risk aversion of the gas generator on its unit commitment value, on the expected real-time price and on the day ahead price. The figures on the right display markets with the "load less risk averse" aversion set and the figures on the left with the "risk-averse" aversion set.

Impact on the unit commitment: figures 6.3a and 6.3b show that as the risk aversion of the gas generator increases, its unit commitment decreases. This result is in line with our intuition from section 6.4. This makes sense because the less a generator commits the less risk it takes.

On a mathematical point of view, this behaviour can be explained by the KKT condition (A.11) for the US market. This equation states that when a generator is not committing fully, equation (6.4) is true.

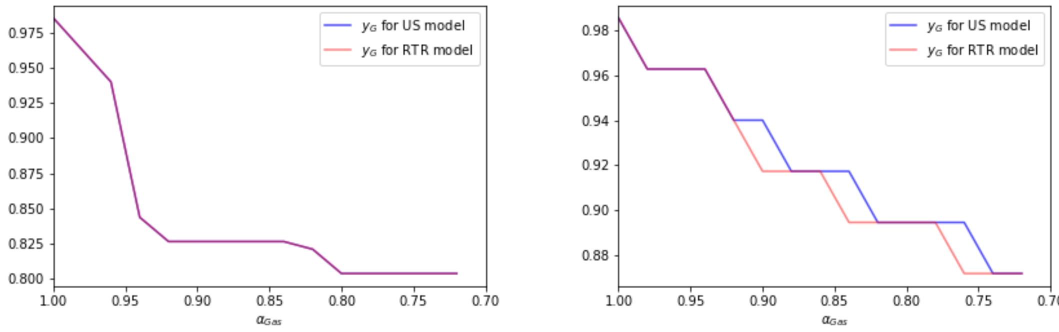
$$K_g = \sum_{\omega \in \Omega} q_{g,\omega} \cdot P^{+}_{g,\omega} \cdot \mu^{G,RT}_{g,\omega} \tag{6.4}$$

As the risk aversion of a generator increases, its risk adjusted probabilities of the beneficial marginal scenarios will decrease. For the following equation to keep being satisfied, the marginal benefit $\mu^{G,RT}_{g,\omega}$ of those scenarios will automatically increase. In the case of the gas generator, the marginal benefit is equal to the real time price of reserve. If the real time price of reserve increases too much, some of the reserve loads might not be valued enough anymore to be traded. This reduction in reserve traded leads to the gas generator committing less.

As figure 6.3b shows it, virtual trading is beneficial to the unit commitment value.

Impact on the expectation of the real time prices: the risk aversion of the gas generator has also an impact on the real time prices as it is shown in figures 6.4a and 6.4b. The increase of the expectation of the real time prices can be directly explained by equation (6.4) and the previous discussion on the increase on the marginal benefit $\mu^{G,RT}_{Gas,\omega}$. We remind that this marginal benefit is here equivalent to the real-time price of reserve that directly increases the real-time price of energy.

Virtual trading has no impact when the other agents follow the "risk averse" aversion set as shown in figure 6.4a but things are different when they follow the



(a) "Risk averse" aversion set

(b) "Load less risk averse" aversion set

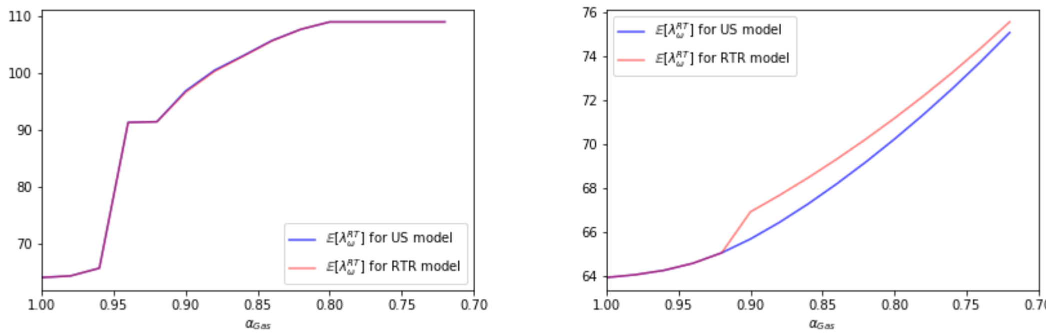
Figure 6.3: The unit commitment value of the gas generator in function of its risk aversion α_{Gas} under the mean value at risk risk-measure

"load less risk-averse" aversion set as shown in figure 6.4b. With that risk aversion set, the price increases more slowly when there is virtual trading. This effect is similar to the slower decrease of unit commitment noticed in 6.3b.

Impact on the day ahead prices: figures 6.5a and 6.5b show the impact of the risk aversion of the gas generator on the day ahead prices. We see that globally the day ahead prices increase with the risk aversion.

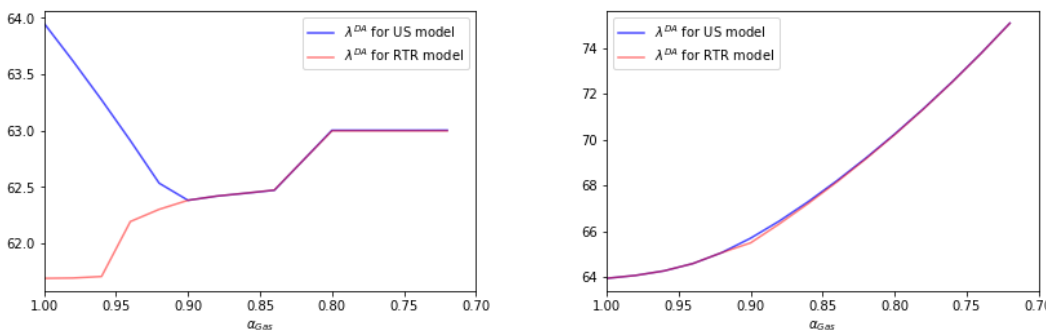
In the "load less risk averse" aversion set, the load is risk neutral so the day ahead price follows closely the real time price as seen in figure 6.4b. This is especially true when virtual trading is allowed. When virtual trading is not allowed, the day ahead price is lower than the expected real time price.

In the risk averse case (see figure 6.5a), the day ahead price does not follow closely the expectation of the real time price even if there is an increase due to an overall increase in the expected real time prices. Virtual trading has an impact on the day ahead price when α_{Gas} is between 1 and 0.9. The US day-ahead price follows closely the expected real time price whereas the RTR day-ahead price is lower. No-virtual trading allosw the price to be different than the risk-adjusted expectation and (as stated in equation (6.4)) lower than the risk-adjusted expectation of the generators.



(a) "Risk averse" aversion set (b) "Load less risk averse" aversion set

Figure 6.4: The expectation of the real-time price in function of the risk aversion of the gas generator α_{Gas} under the mean value at risk risk-measure



(a) "Risk averse" aversion set (b) "Load less risk averse" aversion set

Figure 6.5: The day ahead price in function of the risk aversion of the gas generator α_{Gas} under the mean value at risk risk-measure

CHAPTER 7

Conclusion

In this thesis, we developed a series of models to represent the impact that different policies had on the reserve capacity market under the risk-averse paradigm. The two policies studied were (i) allowing or not virtual trading for energy in day-ahead and (ii) holding or not a real-time market for reserve. The combination of those policies allowed us to build four markets: (i) the US market that allows virtual trading and holds a real-time market for reserve, (ii) the RTR market that holds a real-market for reserve but does not allow virtual trading, (iii) the EUVT market that does not hold a real time market for reserve but allows virtual trading and (iv) the EU market that does not allow virtual trading and does not hold a real time market for reserve.

Those markets were characterized by a stochastic equilibrium under the variational inequality unifying framework. A heuristic inspired by the forward-backward algorithm was proposed to solve those equilibria. The proposed algorithm will iterate until convergence was reached between (i) the backward step that would solve the resolvent of the modified risk-neutral version of the market and (ii) the forward step that would solve the risk-aversion problem from the day ahead point of view.

Finally, a simple system was introduced in order to test the algorithm and to analyse general trends regarding the application of the policies. This allowed us to answer the questions raised in the introduction.

1. **What is the impact of the no-virtual trading policy on the day ahead price and on the commitment of the generators?**

Unlike risk-neutral markets, risk-averse markets are impacted by the no-virtual trading policy. Differences in back-propagation of price appeared and allowing virtual trading had the effect of improving the unit commitment of marginal generators.

2. **To what extend does the risk aversion of an agent impact the back-propagation of real-time prices?**

   The risk-aversion of marginal generators can cause the real-time prices of extreme scenarios to increase sharply. This increase in price is back-propagated on the day-ahead price but the back-propagation was kept minimal if no risk-neutral agent was present in the market. We add that back-propagation is more efficient when virtual trading is allowed.

3. **Who are the "losers" and the "winners" of the different markets?**

   Under the risk-averse paradigm, virtual trading was shown to impact beneficially the generators. We noticed that holding a real time market for reserve was good for the generators but harmful to the load because it may cause shortage in real-time.

Below are listed a number of improvements to go further on the topic.

On the modeling part: the four models studied can be refined to better reflect the real European and US markets. Models where deviation from day ahead position were considered in [1] and could be an example of a direction to pursue. The discussion of creating specific ORDC for every real-time scenario was raised in [9] and could also improve the models.

On the algorithmic part: the heuristic is in its infancy stage. The extension of the algorithm to more complex market has not been implemented yet and might prove challenging. The discussion of the link between the backward and the forward steps remains open and is a key component to better understand the mechanisms of the heuristic. Some works are needed to steady the theory behind the algorithm. Finally, the extension of the heuristic to other risk-measure would be interesting.

On the result part: under the condition that the algorithm finds equilibrium to more extensive example, extending the discussion to more realistic model might prove insightful.

To conclude this thesis, I would say that modeling electricity market with risk-averse agents broadens the policy discussion but, at the same, raises a series of technical difficulty.

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APPENDIX A

Equilibrium of the different model

A.1 The equilibrium of the US market

The US market represent a 2-stage market with reserve and energy traded both in real time and in the day ahead. Virtual trading is allowed for energy but not for reserve in day ahead.

$$0 \leq p_{g,\omega}^{RT} \perp C_g - \lambda_{\omega}^{RT} + \mu_{g,\omega}^{G,RT} \geq 0 \qquad \forall g \in G, \forall \omega \in \Omega \quad \text{(A.1)}$$

$$0 \leq r_{g,\omega}^{RT} \perp -\lambda_{\omega}^{R,RT} + \mu_{g,\omega}^{G,RT} + \mu_{g,\omega}^{G,R,RT} \geq 0 \qquad \forall g \in G, \forall \omega \in \Omega \quad \text{(A.2)}$$

$$0 \leq \mu_{g,\omega}^{G,RT} \perp P_{g,w}^{+} \cdot y_g - p_{g,\omega}^{RT} - r_{g,\omega}^{RT} \geq 0 \qquad \forall g \in G, \forall \omega \in \Omega \quad \text{(A.3)}$$

$$0 \leq \mu_{g,\omega}^{G,R,RT} \perp R_g - r_{g,\omega}^{RT} \geq 0 \qquad \forall g \in G, \forall \omega \in \Omega \quad \text{(A.4)}$$

$$0 \leq d_{l,\omega}^{RT} \perp \lambda_{l,\omega}^{RT} - V_l + \mu_{l,\omega}^{L,RT} \geq 0 \qquad \forall l \in L, \forall \omega \in \Omega \quad \text{(A.5)}$$

$$0 \leq \mu_{l,\omega}^{L,RT} \perp D_{l,\omega}^{+} - d_{l,\omega}^{RT} \geq 0 \qquad \forall l \in L, \forall \omega \in \Omega \quad \text{(A.6)}$$

$$0 \leq d_{l,\omega}^{R,RT} \perp -V_l^R + \lambda_{l,\omega}^{R,RT} + \mu_{l,\omega}^{R,RT} \geq 0 \qquad \forall l \in RL, \forall \omega \in \Omega \quad \text{(A.7)}$$

$$0 \leq \mu_{l,\omega}^{R,RT} \perp D_l^R - d_{l,\omega}^{R,RT} \geq 0 \qquad \forall l \in RL, \forall \omega \in \Omega \quad \text{(A.8)}$$

$$(\lambda_{\omega}^{RT}): \quad \sum_{g \in G} p_g^{RT} = \sum_{l \in L} d_l^{RT} \qquad \forall \omega \in \Omega \quad \text{(A.9)}$$

$$(\lambda_{\omega}^{R,RT}): \quad \sum_{g \in G} r_g^{RT} = \sum_{l \in RL} d_l^{R,RT} \qquad \forall \omega \in \Omega \quad \text{(A.10)}$$

$$0 \leq y_g \perp \delta_g + K_g - \sum_{\omega \in \Omega} q_{g,\omega} \cdot P_{g,\omega}^{+} \cdot \mu_{g,\omega}^{G,RT} \geq 0 \qquad \forall g \in G \quad \text{(A.11)}$$

$$0 \leq \delta_g \perp 1 - y_g \geq 0 \qquad \forall g \in G \quad \text{(A.12)}$$

$$0 \leq r_g^{DA} \perp -\lambda^{R,DA} + \sum_{\omega \in \Omega} q_{g,\omega} \cdot \lambda_\omega^{R,RT} + \mu_g^{G,R,DA} \geq 0 \qquad \forall g \in G \quad \text{(A.13)}$$

$$0 \leq \mu_g^{G,R,DA} \perp R_g - r_g^{DA} \geq 0 \qquad \forall g \in G \quad \text{(A.14)}$$

$$(p_g^{DA}): \quad -\lambda^{DA} + \sum_{\omega \in \Omega} q_{g,\omega} \cdot \lambda_\omega^{RT} = 0 \qquad \forall g \in G \quad \text{(A.15)}$$

$$q_g \in \partial \mathcal{R}_g[\Pi_{g,\omega}^{RT}(y_g) - \lambda_\omega^{RT} \cdot p_g^{DA} - \lambda_\omega^{R,RT} \cdot r_g^{DA}] \qquad \forall g \in G \quad \text{(A.16)}$$

$$(d_l^{DA}): \quad \lambda^{DA} - \sum_{\omega \in \Omega} q_{l,\omega} \cdot \lambda_\omega^{RT} = 0 \qquad \forall l \in L \quad \text{(A.17)}$$

$$q_l \in \partial \mathcal{R}_l[\Pi_{l,\omega}^{RT} + \lambda_\omega^{RT} \cdot d_l^{DA}] \qquad \forall l \in L \quad \text{(A.18)}$$

$$0 \leq d_l^{R,DA} \perp$$
$$\lambda^{R,DA} - V_l^{R,DA} + V_l^R + \mu_l^{R,DA} - \sum_{\omega \in \Omega} P_\omega \cdot \lambda_\omega^{R,RT} \geq 0 \qquad \forall l \in RL \quad \text{(A.19)}$$

$$0 \leq \mu_l^{R,DA} \perp D_l^R - d_l^{R,DA} \geq 0 \qquad \forall l \in RL \quad \text{(A.20)}$$

$$(\lambda^{DA}): \quad \sum_{g \in G} p_g^{DA} = \sum_{l \in L} d_l^{DA} \qquad \text{(A.21)}$$

$$(\lambda^{R,DA}): \quad \sum_{g \in G} r_g^{DA} = \sum_{l \in RL} d_l^{R,DA} \qquad \text{(A.22)}$$

A.2 The equilibrium of the real time reserve market or RTR market

The RTR market represent a 2-stage market with reserve and energy traded both in real time and in the day ahead. Virtual trading is neither allowed for energy nor for reserve in day ahead.

$$0 \leq p_{g,\omega}^{RT} \perp C_g - \lambda_\omega^{RT} + \mu_{g,\omega}^{G,RT} \geq 0 \qquad \forall g \in G, \forall \omega \in \Omega \quad \text{(A.23)}$$

$$0 \leq r_{g,\omega}^{RT} \perp -\lambda_\omega^{R,RT} + \mu_{g,\omega}^{G,RT} + \mu_{g,\omega}^{G,R,RT} \geq 0 \qquad \forall g \in G, \forall \omega \in \Omega \quad \text{(A.24)}$$

$$0 \leq \mu_{g,\omega}^{G,RT} \perp P_{g,w}^{+} \cdot y_g - p_{g,\omega}^{RT} - r_{g,\omega}^{RT} \geq 0 \qquad \forall g \in G, \forall \omega \in \Omega \quad \text{(A.25)}$$

$$0 \leq \mu_{g,\omega}^{G,R,RT} \perp R_g - r_{g,\omega}^{RT} \geq 0 \qquad \forall g \in G, \forall \omega \in \Omega \quad \text{(A.26)}$$

$$0 \leq d_{l,\omega}^{RT} \perp \lambda_{l,\omega}^{RT} - V_l + \mu_{l,\omega}^{L,RT} \geq 0 \qquad \forall l \in L, \forall \omega \in \Omega \quad \text{(A.27)}$$

$$0 \leq \mu_{l,\omega}^{L,RT} \perp D_{l,\omega}^{+} - d_{l,\omega}^{RT} \geq 0 \qquad \forall l \in L, \forall \omega \in \Omega \quad \text{(A.28)}$$

$$0 \leq d_{l,\omega}^{R,RT} \perp -V_l^R + \lambda_{l,\omega}^{R,RT} + \mu_{l,\omega}^{R,RT} \geq 0 \qquad \forall l \in RL, \forall \omega \in \Omega \quad \text{(A.29)}$$

$$0 \leq \mu_{l,\omega}^{R,RT} \perp D_l^R - d_{l,\omega}^{R,RT} \geq 0 \qquad \forall l \in RL, \forall \omega \in \Omega \quad \text{(A.30)}$$

$$(\lambda_\omega^{RT}): \quad \sum_{g\in G} p_g^{RT} = \sum_{l\in L} d_l^{RT} \qquad \forall \omega \in \Omega \quad (A.31)$$

$$(\lambda_\omega^{R,RT}): \quad \sum_{g\in G} r_g^{RT} = \sum_{l\in RL} d_l^{R,RT} \qquad \forall \omega \in \Omega \quad (A.32)$$

$$0 \le y_g \perp$$
$$\delta_g + K_g - \sum_{\omega\in\Omega} q_{g,\omega}\cdot P_{g,\omega}^{+}\cdot \mu_{g,\omega}^{G,RT} + P_g^{DA,+}\cdot \mu_{g,\omega}^{G,DA} \ge 0 \qquad \forall g \in G \quad (A.33)$$

$$0 \le \delta_g \perp 1 - y_g \ge 0 \qquad \forall g \in G \quad (A.34)$$

$$0 \le r_g^{DA} \perp$$
$$-\lambda^{R,DA} + \sum_{\omega\in\Omega} q_{g,\omega}\cdot \lambda_\omega^{R,RT} + \mu_g^{G,R,DA} + \mu_g^{G,DA} \ge 0 \qquad \forall g \in G \quad (A.35)$$

$$0 \le \mu_g^{G,R,DA} \perp R_g - r_g^{DA} \ge 0 \qquad \forall g \in G \quad (A.36)$$

$$0 \le p_g^{DA} \perp -\lambda^{DA} + \sum_{\omega\in\Omega} q_{g,\omega}\cdot \lambda_\omega^{RT} + \mu_{g,\omega}^{G,DA} \ge 0 \qquad \forall g \in G \quad (A.37)$$

$$0 \le \mu_g^{G,DA} \perp P_g^{DA,+}\cdot y_g - p_g^{DA} - r_g^{DA} \ge 0 \qquad \forall g \in G \quad (A.38)$$

$$q_g \in \partial \mathcal{R}_g[\Pi_{g,\omega}^{RT}(y_g) - \lambda_\omega^{RT}\cdot p_g^{DA} - \lambda_\omega^{R,RT}\cdot r_g^{DA}] \qquad \forall g \in G \quad (A.39)$$

$$0 \le d_l^{DA} \perp \lambda^{DA} - \sum_{\omega\in\Omega} q_{l,\omega}\cdot \lambda_\omega^{RT} + \mu_{l,\omega}^{L,DA} \ge 0 \qquad \forall l \in L \quad (A.40)$$

$$0 \le \mu_l^{L,DA} \perp D_l^{DA,+} - d_l^{DA} \ge 0 \qquad \forall l \in L \quad (A.41)$$

$$q_l \in \partial \mathcal{R}_l[\Pi_{l,\omega}^{RT} + \lambda_\omega^{RT}\cdot d_l^{DA}] \qquad \forall l \in L \quad (A.42)$$

$$0 \le d_l^{R,DA} \perp$$
$$\lambda^{R,DA} - V_l^{R,DA} + V_l^{R} + \mu_l^{R,DA} - \sum_{\omega\in\Omega} P_\omega \cdot \lambda_\omega^{R,RT} \ge 0 \qquad \forall l \in RL \quad (A.43)$$

$$0 \le \mu_l^{R,DA} \perp D_l^{R} - d_l^{R,DA} \ge 0 \qquad \forall l \in RL \quad (A.44)$$

$$(\lambda^{DA}): \quad \sum_{g\in G} p_g^{DA} = \sum_{l\in L} d_l^{DA} \qquad (A.45)$$

$$(\lambda^{R,DA}): \quad \sum_{g\in G} r_g^{DA} = \sum_{l\in RL} d_l^{R,DA} \qquad (A.46)$$

A.3 The EU market with virtual trading or EUVT market

The EUVT market represents a 2-stage market with energy traded both in real time and in the day ahead and reserve only in the day ahead. Virtual trading is allowed for energy but not for reserve in day ahead.

$$0 \le p_{g,\omega}^{RT} \perp C_g - \lambda_\omega^{RT} + \mu_{g,\omega}^{G,RT} \ge 0 \qquad \forall g \in G, \forall \omega \in \Omega \quad (A.47)$$

$$0 \leq \mu_{g,\omega}^{G,RT} \perp P_{g,w}^{+} \cdot y_g - p_{g,\omega}^{RT} - r_g^{DA} \geq 0 \qquad \forall g \in G, \forall \omega \in \Omega \quad \text{(A.48)}$$
$$0 \leq d_{l,\omega}^{RT} \perp \lambda_{l,\omega}^{RT} - V_l + \mu_{l,\omega}^{L,RT} \geq 0 \qquad \forall l \in L, \forall \omega \in \Omega \quad \text{(A.49)}$$
$$0 \leq \mu_{l,\omega}^{L,RT} \perp D_{l,\omega}^{+} - d_{l,\omega}^{RT} \geq 0 \qquad \forall l \in L, \forall \omega \in \Omega \quad \text{(A.50)}$$
$$(\lambda_{\omega}^{RT}): \quad \sum_{g\in G} p_g^{RT} = \sum_{l\in L} d_l^{RT} \qquad \forall \omega \in \Omega \quad \text{(A.51)}$$
$$0 \leq y_g \perp \delta_g + K_g - \sum_{\omega\in\Omega} q_{g,\omega} \cdot P_{g,\omega}^{+} \cdot \mu_{g,\omega}^{G,RT} \geq 0 \qquad \forall g \in G \quad \text{(A.52)}$$
$$0 \leq \delta_g \perp 1 - y_g \geq 0 \qquad \forall g \in G \quad \text{(A.53)}$$
$$0 \leq r_g^{DA} \perp -\lambda^{R,DA} + \sum_{\omega\in\Omega} q_{g,\omega} \cdot \mu_{g,\omega}^{G,RT} + \mu_g^{G,R,DA} \geq 0 \qquad \forall g \in G \quad \text{(A.54)}$$
$$0 \leq \mu_g^{G,R,DA} \perp R_g - r_g^{DA} \geq 0 \qquad \forall g \in G \quad \text{(A.55)}$$
$$(p_g^{DA}): \quad -\lambda^{DA} + \sum_{\omega\in\Omega} q_{g,\omega} \cdot \lambda_{\omega}^{RT} = 0 \qquad \forall g \in G \quad \text{(A.56)}$$
$$q_g \in \partial\mathcal{R}_g[\Pi_{g,\omega}^{RT}(y_g, r_g^{DA}) - \lambda_{\omega}^{RT} \cdot p_g^{DA}] \qquad \forall g \in G \quad \text{(A.57)}$$
$$(d_l^{DA}): \quad \lambda^{DA} - \sum_{\omega\in\Omega} q_{l,\omega} \cdot \lambda_{\omega}^{RT} = 0 \qquad \forall l \in L \quad \text{(A.58)}$$
$$q_l \in \partial\mathcal{R}_l[\Pi_{l,\omega}^{RT} + \lambda_{\omega}^{RT} \cdot d_l^{DA}] \qquad \forall l \in L \quad \text{(A.59)}$$
$$0 \leq d_l^{R,DA} \perp \lambda^{R,DA} - V_l^{R,DA} + \mu_l^{R,DA} \geq 0 \qquad \forall l \in RL \quad \text{(A.60)}$$
$$0 \leq \mu_l^{R,DA} \perp D_l^{R} - d_l^{R,DA} \geq 0 \qquad \forall l \in RL \quad \text{(A.61)}$$
$$(\lambda^{DA}): \quad \sum_{g\in G} p_g^{DA} = \sum_{l\in L} d_l^{DA} \qquad \text{(A.62)}$$
$$(\lambda^{R,DA}): \quad \sum_{g\in G} r_g^{DA} = \sum_{l\in RL} d_l^{R,DA} \qquad \text{(A.63)}$$

A.4 The EU market

The EUVT market represents a 2-stage market with energy traded both in real time and in the day ahead and reserve only in the day ahead. Virtual trading is neither allowed for energy nor for reserve in day ahead.

$$0 \leq p_{g,\omega}^{RT} \perp C_g - \lambda_{\omega}^{RT} + \mu_{g,\omega}^{G,RT} \geq 0 \qquad \forall g \in G, \forall \omega \in \Omega \quad \text{(A.64)}$$
$$0 \leq \mu_{g,\omega}^{G,RT} \perp P_{g,w}^{+} \cdot y_g - p_{g,\omega}^{RT} - r_g^{DA} \geq 0 \qquad \forall g \in G, \forall \omega \in \Omega \quad \text{(A.65)}$$
$$0 \leq d_{l,\omega}^{RT} \perp \lambda_{l,\omega}^{RT} - V_l + \mu_{l,\omega}^{L,RT} \geq 0 \qquad \forall l \in L, \forall \omega \in \Omega \quad \text{(A.66)}$$
$$0 \leq \mu_{l,\omega}^{L,RT} \perp D_{l,\omega}^{+} - d_{l,\omega}^{RT} \geq 0 \qquad \forall l \in L, \forall \omega \in \Omega \quad \text{(A.67)}$$
$$(\lambda_{\omega}^{RT}): \quad \sum_{g\in G} p_g^{RT} = \sum_{l\in L} d_l^{RT} \qquad \forall \omega \in \Omega \quad \text{(A.68)}$$

$$0 \leq y_g \perp$$
$$\delta_g + K_g - \sum_{\omega\in\Omega} q_{g,\omega} \cdot P^{+}_{g,\omega} \cdot \mu^{G,RT}_{g,\omega} + \mu^{G,DA}_g \cdot P^{DA,+}_g \geq 0 \qquad \forall g \in G \quad (A.69)$$
$$0 \leq \delta_g \perp 1 - y_g \geq 0 \qquad \forall g \in G \quad (A.70)$$
$$0 \leq r^{DA}_g \perp$$
$$-\lambda^{R,DA} + \sum_{\omega\in\Omega} q_{g,\omega} \cdot \mu^{G,RT}_{g,\omega} + \mu^{G,R,DA}_g + \mu^{G,DA}_g \geq 0 \qquad \forall g \in G \quad (A.71)$$
$$0 \leq \mu^{G,R,DA}_g \perp R_g - r^{DA}_g \geq 0 \qquad \forall g \in G \quad (A.72)$$
$$0 \leq p^{DA}_g \perp -\lambda^{DA} + \sum_{\omega\in\Omega} q_{g,\omega} \cdot \lambda^{RT}_\omega + \mu^{G,DA}_g \geq 0 \qquad \forall g \in G \quad (A.73)$$
$$0 \leq \mu^{G,DA}_g \perp P^{DA,+}_g \cdot y_g - p^{DA}_g - r^{DA}_g \geq 0 \qquad \forall g \in G \quad (A.74)$$
$$q_g \in \partial\mathcal{R}_g[\Pi^{RT}_{g,\omega}(y_g, r^{DA}_g) - \lambda^{RT}_\omega \cdot p^{DA}_g] \qquad \forall g \in G \quad (A.75)$$
$$0 \leq d^{DA}_l \perp \lambda^{DA} - \sum_{\omega\in\Omega} q_{l,\omega} \cdot \lambda^{RT}_\omega + \mu^{L,DA}_l \geq 0 \qquad \forall l \in L \quad (A.76)$$
$$0 \leq \mu^{L,DA}_l \perp D^{DA,+}_l - d^{DA}_l \geq 0 \qquad \forall l \in L \quad (A.77)$$
$$q_l \in \partial\mathcal{R}_l[\Pi^{RT}_{l,\omega} + \lambda^{RT}_\omega \cdot d^{DA}_l] \qquad \forall l \in L \quad (A.78)$$
$$0 \leq d^{R,DA}_l \perp \lambda^{R,DA} - V^{R,DA}_l + \mu^{R,DA}_l \geq 0 \qquad \forall l \in RL \quad (A.79)$$
$$0 \leq \mu^{R,DA}_l \perp D^R_l - d^{R,DA}_l \geq 0 \qquad \forall l \in RL \quad (A.80)$$
$$(\lambda^{DA}): \quad \sum_{g\in G} p^{DA}_g = \sum_{l\in L} d^{DA}_l \qquad (A.81)$$
$$(\lambda^{R,DA}): \quad \sum_{g\in G} r^{DA}_g = \sum_{l\in RL} d^{R,DA}_l \qquad (A.82)$$

APPENDIX B

Variationnal inequality

Proposition 1. *Suppose that the operator A corresponds to the following optimisation problem:*

$$\begin{aligned} \min_{x} \quad & c^T x \\ (u): \quad & Ax \geq b \\ & x \geq 0 \end{aligned}$$

The result of the operator $J_{\rho \cdot A} \begin{pmatrix} \overline{x} \\ \overline{u} \end{pmatrix}$ is the solution to the following optimisation problem:

$$\begin{aligned} \min_{x,v} \quad & c^T x + \frac{1}{2 \cdot \rho}(x - \overline{x})^2 + \frac{\rho}{2} v^2 + \overline{u}^T v \\ & Ax + v \geq b \\ & x \geq 0 \end{aligned}$$

Proof. The KKT conditions of the optimisation problem corresponding to operator A can be written as

$$\begin{aligned} 0 \leq x \perp c - A^T u \geq 0 \\ 0 \leq u \perp Ax - b \geq 0 \end{aligned}$$

We can then define the operator F as followed.

$$F\begin{pmatrix}x\\u\end{pmatrix} = \begin{pmatrix}c\\-b\end{pmatrix} + \begin{pmatrix}0 & -A^T\\A & 0\end{pmatrix}\begin{pmatrix}x\\u\end{pmatrix}$$

We notice that the operator A can be defined as $F + \mathcal{N}$. Now, let's consider the following optimisation problem.

$$\begin{aligned}\min_{x,v} \quad & c^T x + \frac{1}{2\cdot\rho}(x-\overline{x})^2 + \frac{\rho}{2}v^2 + \overline{u}^T v\\ & Ax + v \geq b\\ & x \geq 0\end{aligned}$$

The KKT condition of this regularised optimisation can be defined as followed.

$$\begin{aligned}0 \leq x \perp \rho\cdot c + x - \overline{x} - \rho\cdot A^T u \geq 0\\ 0 \leq u \perp \rho\cdot Ax + u - \overline{u} - \rho\cdot b \geq 0\end{aligned}$$

We can then define the operator $F_{\rho,(\overline{x},\overline{u})}$ as followed.

$$\begin{aligned}F_{\rho,(\overline{x},\overline{u})} &= \begin{pmatrix}\rho\cdot c\\-\rho\cdot b\end{pmatrix} + \begin{pmatrix}I & -\rho\cdot A^T\\\rho\cdot A & I\end{pmatrix}\begin{pmatrix}x\\u\end{pmatrix} - \begin{pmatrix}\overline{x}\\\overline{u}\end{pmatrix}\\ &= \rho\cdot F\begin{pmatrix}x\\u\end{pmatrix} + \begin{pmatrix}x\\u\end{pmatrix} - \begin{pmatrix}\overline{x}\\\overline{u}\end{pmatrix}\end{aligned}$$

The proposition 12.3.6 from [6] conclude the proof. It states that for K being nonempty closed and convex and $F : K \to \mathbb{R}^n$ continuous, the resolvent of a map $T = F + \mathcal{N}(\cdot; K)$ can be computed as followed as followed.

$$J_{\rho\cdot T}(\overline{x}) = SOL(K, F_{\rho,\overline{x}}), \text{ where } F_{\rho,\overline{x}}(x) \equiv x - \overline{x} + \rho\cdot F(x)$$

$\square$

B.1 Formulation of decomposed mapping

$$
T\begin{pmatrix}
p_{g,\omega}^{RT}\\
r_{g,\omega}^{RT}\\
\mu_{g,\omega}^{G,RT}\\
\mu_{g,\omega}^{G,R,RT}\\
d_{l,\omega}^{RT}\\
\mu_{l,\omega}^{L,RT}\\
d_{l,\omega}^{R,RT}\\
\mu_{l,\omega}^{R,RT}\\
y_g\\
\delta_g\\
r_g^{DA}\\
\mu_g^{G,R,DA}\\
d_l^{R,DA}\\
\mu_l^{R,DA}\\
p_g^{DA}\\
d_l^{DA}\\
\lambda_\omega^{RT}\\
\lambda_\omega^{R,RT}\\
\lambda^{DA}\\
\lambda^{R,DA}
\end{pmatrix}
=
\begin{pmatrix}
C_g-\lambda_\omega^{RT}+\mu_{g,\omega}^{G,RT}\\
-\lambda_\omega^{R,RT}+\mu_{g,\omega}^{G,RT}+\mu_{g,\omega}^{G,R,RT}\\
P_{g,w}^{+}\cdot y_g-p_{g,\omega}^{RT}-r_{g,\omega}^{RT}\\
R_g-r_{g,\omega}^{RT}\\
\lambda_{l,\omega}^{RT}-V_l+\mu_{l,\omega}^{L,RT}\\
D_{l,\omega}^{+}-d_{l,\omega}^{RT}\\
-V_l^R+\lambda_{l,\omega}^{R,RT}+\mu_{l,\omega}^{R,RT}\\
D_l^R-d_{l,\omega}^{R,RT}\\
\delta_g+K_g-\sum_{\omega\in\Omega}q_{g,\omega}\cdot P_{g,\omega}^{+}\cdot\mu_{g,\omega}^{G,RT}\\
1-y_g\\
-\lambda^{R,DA}+\sum_{\omega\in\Omega}q_{g,\omega}\cdot\lambda_\omega^{R,RT}+\mu_g^{G,R,DA}\\
R_g-r_g^{DA}\\
\lambda^{R,DA}-V_l^{R,DA}+\mu_l^{R,DA}-\sum_{\omega\in\Omega}P_\omega\cdot(\lambda_\omega^{R,RT}-V_l^R)\\
D_l^R-d_l^{R,DA}\\
-\lambda^{DA}+\sum_{\omega\in\Omega}q_{g,\omega}\cdot\lambda_\omega^{RT}\\
\lambda^{DA}-\sum_{\omega\in\Omega}q_{l,\omega}\cdot\lambda_\omega^{RT}\\
\sum_{g\in G}p_g^{RT}-\sum_{l\in L}d_l^{RT}\\
\sum_{g\in G}r_g^{RT}-\sum_{l\in RL}d_l^{R,RT}\\
\sum_{g\in G}p_g^{DA}-\sum_{l\in L}d_l^{DA}\\
\sum_{g\in G}r_g^{DA}-\sum_{l\in RL}d_l^{R,DA}
\end{pmatrix}
+\mathcal{N}\begin{pmatrix}
p_{g,\omega}^{RT}\\
r_{g,\omega}^{RT}\\
\mu_{g,\omega}^{G,RT}\\
\mu_{g,\omega}^{G,R,RT}\\
d_{l,\omega}^{RT}\\
\mu_{l,\omega}^{L,RT}\\
d_{l,\omega}^{R,RT}\\
\mu_{l,\omega}^{R,RT}\\
y_g\\
\delta_g\\
r_g^{DA}\\
\mu_g^{G,R,DA}\\
d_l^{R,DA}\\
\mu_l^{R,DA}\\
p_g^{DA}\\
d_l^{DA}\\
\lambda_\omega^{RT}\\
\lambda_\omega^{R,RT}\\
\lambda^{DA}\\
\lambda^{R,DA}
\end{pmatrix}
\tag{B.1}
$$

$$A \begin{pmatrix} p_{g,\omega}^{RT} \\ r_{g,\omega}^{RT} \\ \mu_{g,\omega}^{G,RT} \\ \mu_{g,\omega}^{G,R,RT} \\ d_{l,\omega}^{RT} \\ \mu_{l,\omega}^{L,RT} \\ d_{l,\omega}^{R,RT} \\ \mu_{l,\omega}^{R,RT} \\ y_g \\ \delta_g \\ r_g^{DA} \\ \mu_g^{G,R,DA} \\ d_l^{R,DA} \\ \mu_l^{R,DA} \\ p_g^{DA} \\ d_l^{DA} \\ \lambda_\omega^{RT} \\ \lambda_\omega^{R,RT} \\ \lambda^{DA} \\ \lambda^{R,DA} \end{pmatrix} = \begin{pmatrix} C_g - \lambda_\omega^{RT} + \mu_{g,\omega}^{G,RT} \\ -\lambda_\omega^{R,RT} + \mu_{g,\omega}^{G,RT} + \mu_{g,\omega}^{G,R,RT} \\ P_{g,w}^{+} \cdot y_g - p_{g,\omega}^{RT} - r_{g,\omega}^{RT} \\ R_g - r_{g,\omega}^{RT} \\ \lambda_{l,\omega}^{RT} - V_l + \mu_{l,\omega}^{L,RT} \\ D_{l,\omega}^{+} - d_{l,\omega}^{RT} \\ -V_l^R + \lambda_{l,\omega}^{R,RT} + \mu_{l,\omega}^{R,RT} \\ D_l^R - d_{l,\omega}^{R,RT} \\ \delta_g + K_g - \sum_{\omega\in\Omega} P_\omega \cdot P_{g,\omega}^{+} \cdot \mu_{g,\omega}^{G,RT} \\ 1 - y_g - \lambda^{R,DA} + \mu_g^{G,R,DA} \\ R_g - r_g^{DA} \\ \lambda^{R,DA} - V_l^{R,DA} + \mu_l^{R,DA} - \sum_{\omega\in\Omega} P_\omega \cdot (\lambda_\omega^{R,RT} - V_l^R) \\ D_l^R - d_l^{R,DA} \\ -\lambda^{DA} \\ \lambda^{DA} \\ \sum_{g\in G} p_g^{RT} - \sum_{l\in L} d_l^{RT} \\ \sum_{g\in G} r_g^{RT} - \sum_{l\in RL} d_l^{R,RT} \\ \sum_{g\in G} p_g^{DA} - \sum_{l\in L} d_l^{DA} \\ \sum_{g\in G} r_g^{DA} - \sum_{l\in RL} d_l^{R,DA} \end{pmatrix} + \mathcal{N} \begin{pmatrix} p_{g,\omega}^{RT} \\ r_{g,\omega}^{RT} \\ \mu_{g,\omega}^{G,RT} \\ \mu_{g,\omega}^{G,R,RT} \\ d_{l,\omega}^{RT} \\ \mu_{l,\omega}^{L,RT} \\ d_{l,\omega}^{R,RT} \\ \mu_{l,\omega}^{R,RT} \\ y_g \\ \delta_g \\ r_g^{DA} \\ \mu_g^{G,R,DA} \\ d_l^{R,DA} \\ \mu_l^{R,DA} \\ p_g^{DA} \\ d_l^{DA} \\ \lambda_\omega^{RT} \\ \lambda_\omega^{R,RT} \\ \lambda^{DA} \\ \lambda^{R,DA} \end{pmatrix} \tag{B.2}$$

$$B \begin{pmatrix} p_{g,\omega}^{RT} \\ r_{g,\omega}^{RT} \\ \mu_{g,\omega}^{G,RT} \\ \mu_{g,\omega}^{G,R,RT} \\ d_{l,\omega}^{RT} \\ \mu_{l,\omega}^{L,RT} \\ d_{l,\omega}^{R,RT} \\ \mu_{l,\omega}^{R,RT} \\ y_g \\ \delta_g \\ r_g^{DA} \\ \mu_g^{G,R,DA} \\ d_l^{R,DA} \\ \mu_l^{R,DA} \\ p_g^{DA} \\ d_l^{DA} \\ \lambda_\omega^{RT} \\ \lambda_\omega^{R,RT} \\ \lambda^{DA} \\ \lambda^{R,DA} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \sum_{\omega\in\Omega}(P_\omega - q_{g,\omega}) \cdot P_{g,\omega}^{+} \cdot \mu_{g,\omega}^{G,RT} \\ 0 \\ \sum_{\omega\in\Omega} q_{g,\omega} \cdot \lambda_\omega^{R,RT} \\ 0 \\ 0 \\ 0 \\ \sum_{\omega\in\Omega} q_{g,\omega} \cdot \lambda_\omega^{RT} \\ -\sum_{\omega\in\Omega} q_{l,\omega} \cdot \lambda_\omega^{RT} \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \gamma_g \\ 0 \\ \eta_g^R \\ 0 \\ 0 \\ 0 \\ \eta_g \\ \eta_l \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \tag{B.3}$$

APPENDIX C

ORDC

	V_l^R	$V_l^{R,DA}$
$0-50$	4125	4125
$50-100$	3310,673	3348,705
$100-150$	2545,435	2614,997
$150-200$	1869,676	1959,583
$200-250$	1308,906	1406,228
$250-300$	871,6106	964,6695
$300-350$	551,1594	631,6507
$350-400$	330,488	394,2704
$400-450$	187,6886	234,3455
$450-500$	100,8519	132,5139
$500-550$	51,22974	71,23027
$550-600$	24,58305	36,37245
$600-650$	11,13666	17,63328
$650-700$	4,760457	8,112017
$700-750$	1,91919	3,539748

Table C.1: Day ahead and real tiem ORDC

APPENDIX D

Result tables

Model	Profit	Nuclear	Coal	Gas	Wind	L	Total
US	Π_g^{DA}	406927	76909	0	23500	83343425	83850761
	$\mathbb{E}_{P_\omega}[\Pi_{g,\omega}^{RT}]$	407927	77909	985	23500	83343425	83853746
	$\mathbb{E}_{q_{g,\omega}}[\Pi_{g,\omega}^{RT}]$	407927	77909	985	23500	83343425	83853746
RTR	Π_g^{DA}	406927	76909	0	23500	83343425	83850761
	$\mathbb{E}_{P_\omega}[\Pi_{g,\omega}^{RT}]$	407927	77909	985	23500	83343425	83853746
	$\mathbb{E}_{q_{g,\omega}}[\Pi_{g,\omega}^{RT}]$	407927	77909	985	23500	83343425	83853746
EUVT	Π_g^{DA}	461215	92201	16822	23500	83262240	83855978
	$\mathbb{E}_{P_\omega}[\Pi_{g,\omega}^{RT}]$	462215	93201	12297	23500	83262240	83853453
	$\mathbb{E}_{q_{g,\omega}}[\Pi_{g,\omega}^{RT}]$	462215	93201	12297	23500	83262240	83853453
EU	Π_g^{DA}	461216	92202	16822	23500	83262246	83855986
	$\mathbb{E}_{P_\omega}[\Pi_{g,\omega}^{RT}]$	462216	93202	12297	23500	83262246	83853461
	$\mathbb{E}_{q_{g,\omega}}[\Pi_{g,\omega}^{RT}]$	462216	93202	12297	23500	83262246	83853461

Table D.1: Profit of the different agent under the MVaR risk measure and the risk neutral aversion set

Model	Profit	Nuclear	Coal	Gas	Wind	L	Total
US	Π_g^{DA}	395765	73765	0	21685	82908502	83399717
	$\mathbb{E}_{P_\omega}[\Pi_{g,\omega}^{RT}]$	641331	143657	60591	23500	82994501	83863580
	$\mathbb{E}_{q_{g,\omega}}[\Pi_{g,\omega}^{RT}]$	396765	74765	826	21685	82908502	83402543
RTR	Π_g^{DA}	395765	73765	0	17285	82908501	83395316
	$\mathbb{E}_{P_\omega}[\Pi_{g,\omega}^{RT}]$	639673	143189	60166	23500	82996980	83863508
	$\mathbb{E}_{q_{g,\omega}}[\Pi_{g,\omega}^{RT}]$	394890	74237	826	17285	82908501	83395739
EUVT	Π_g^{DA}	441472	86640	8042	21656	82841109	83398919
	$\mathbb{E}_{P_\omega}[\Pi_{g,\omega}^{RT}]$	3322801	899001	616488	23500	78984264	83846054
	$\mathbb{E}_{q_{g,\omega}}[\Pi_{g,\omega}^{RT}]$	442472	87640	6165	21656	82841109	83399042
EU	Π_g^{DA}	441472	86640	8042	17285	82841105	83394544
	$\mathbb{E}_{P_\omega}[\Pi_{g,\omega}^{RT}]$	3322801	899001	616488	23500	78984260	83846050
	$\mathbb{E}_{q_{g,\omega}}[\Pi_{g,\omega}^{RT}]$	424413	82442	6165	17285	82841105	83371410

Table D.2: Profit of the different agent under the MVaR risk measure and the risk averse aversion set

Model	Profit	Nuclear	Coal	Gas	Wind	L	Total
US	Π_g^{DA}	419350	80408	0	18858	83324848	83843464
	$\mathbb{E}_{P_\omega}[\Pi_{g,\omega}^{RT}]$	420350	81408	4558	23500	83324848	83854664
	$\mathbb{E}_{q_{g,\omega}}[\Pi_{g,\omega}^{RT}]$	420350	81408	940	18858	83324848	83846404
RTR	Π_g^{DA}	417905	80001	0	17285	83327012	83842203
	$\mathbb{E}_{P_\omega}[\Pi_{g,\omega}^{RT}]$	429107	83875	6937	23500	83311755	83855174
	$\mathbb{E}_{q_{g,\omega}}[\Pi_{g,\omega}^{RT}]$	392058	73439	917	17285	83311755	83795454
EUVT	Π_g^{DA}	483656	98523	16822	17861	83228680	83845542
	$\mathbb{E}_{P_\omega}[\Pi_{g,\omega}^{RT}]$	484656	99523	17095	23500	83228680	83853454
	$\mathbb{E}_{q_{g,\omega}}[\Pi_{g,\omega}^{RT}]$	484656	99523	12297	17861	83228680	83843017
EU	Π_g^{DA}	483116	98371	16822	17285	83229486	83845080
	$\mathbb{E}_{P_\omega}[\Pi_{g,\omega}^{RT}]$	487855	100424	17779	23500	83223894	83853452
	$\mathbb{E}_{q_{g,\omega}}[\Pi_{g,\omega}^{RT}]$	396107	75111	12297	17285	83223894	83724694

Table D.3: Profit of the different agent under the MVaR risk measure and the load less risk averse aversion set

Model	Profit	Nuclear	Coal	Gas	Wind	L	Total
US	Π_g^{DA}	395765	73765	0	21685	82726073	83217288
	$\mathbb{E}_{P_\omega}[\Pi_{g,\omega}^{RT}]$	641328	143656	60590	23500	82994504	83863578
	$\mathbb{E}_{q_{g,\omega}}[\Pi_{g,\omega}^{RT}]$	396765	74765	826	21685	82726073	83220114
RTR	Π_g^{DA}	394041	73279	0	17285	82726453	83211058
	$\mathbb{E}_{P_\omega}[\Pi_{g,\omega}^{RT}]$	655434	147629	64202	23500	82973409	83864174
	$\mathbb{E}_{q_{g,\omega}}[\Pi_{g,\omega}^{RT}]$	395041	74279	826	17285	82726453	83213884
EUVT	Π_g^{DA}	441472	86640	8042	21656	82645931	83203741
	$\mathbb{E}_{P_\omega}[\Pi_{g,\omega}^{RT}]$	3322801	899001	616488	23500	78984264	83846054
	$\mathbb{E}_{q_{g,\omega}}[\Pi_{g,\omega}^{RT}]$	442472	87640	6165	21656	82645931	83203864
EU	Π_g^{DA}	441472	86640	8042	17285	82645927	83199366
	$\mathbb{E}_{P_\omega}[\Pi_{g,\omega}^{RT}]$	3322801	899001	616488	23500	78984260	83846050
	$\mathbb{E}_{q_{g,\omega}}[\Pi_{g,\omega}^{RT}]$	425242	82430	6165	17285	82645927	83177049

Table D.4: Profit of the different agent under the MVaR risk measure and the load more risk averse aversion set

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