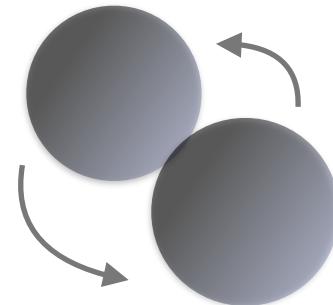


# MBH Binary Population Synthesis: NANOGrav's holodeck framework

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Luke Zoltan Kelley

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Lindheimer & Cottrell Postdoctoral Fellow  
CIERA | Northwestern University

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✉ LZK www.lzkelley.com



# Calculating the GW Background

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$$h_c^2(f) = \int_0^\infty dz \frac{d^2N}{dz d\ln f_r} h_s^2(f_r),$$

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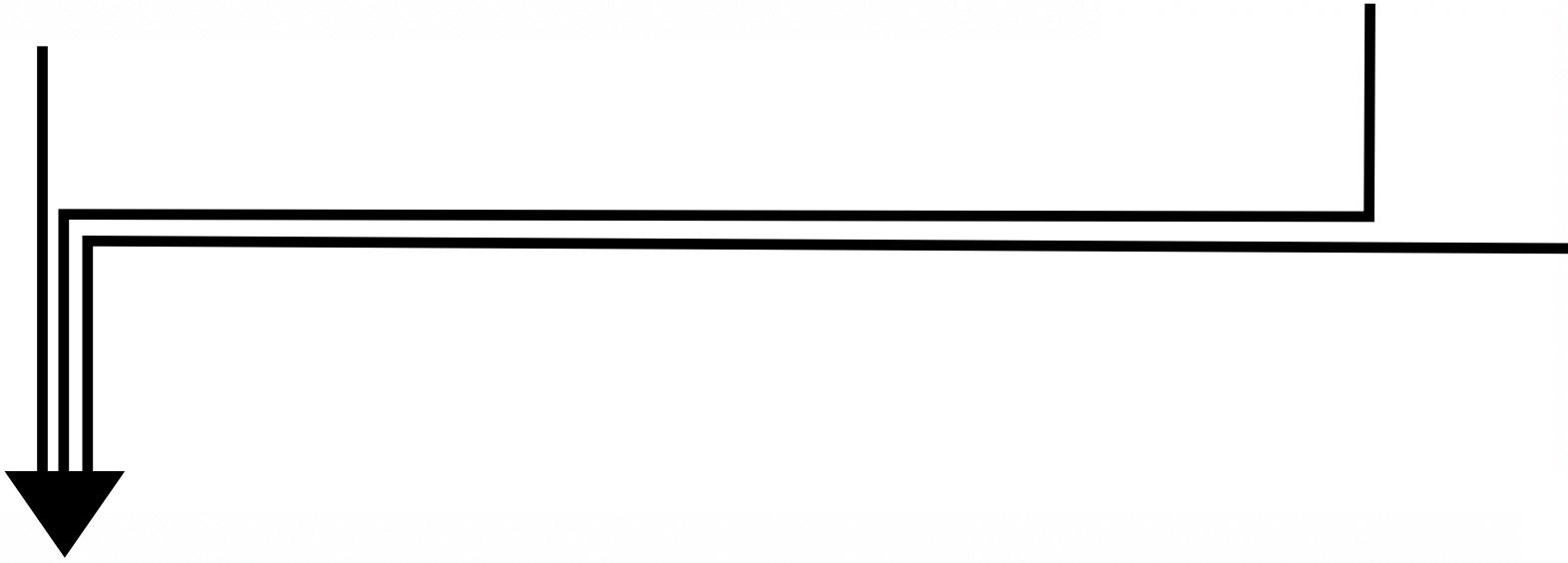
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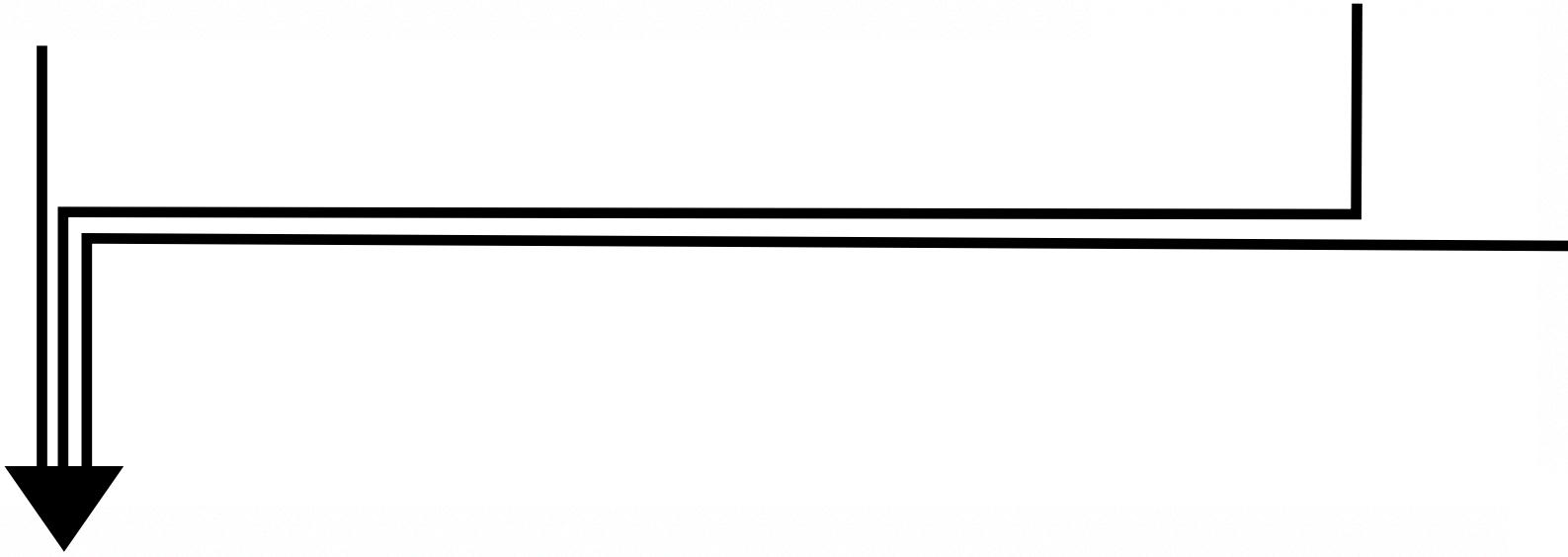


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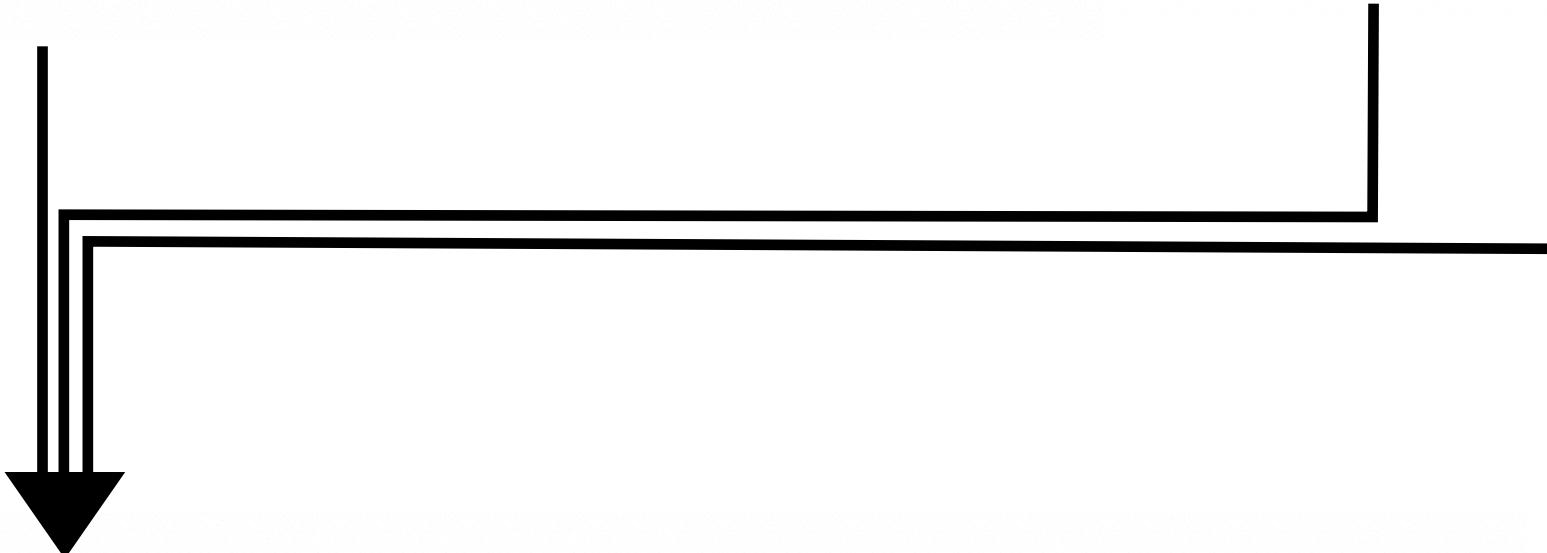
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A diagram illustrating the derivation of the equations. A vertical arrow points downwards from the second equation to the third. A horizontal bracket is placed under the second equation, spanning the terms  $\frac{dn_c}{dz}$  and  $\frac{dV_c}{dz}$ .

$$h_c^2(f) = \sum_{\text{redshift}} \sum_{\text{binaries}} h_s^2 \frac{4\pi c d_c^2 \cdot (1+z)}{V_{\text{sim}}} \tau_f.$$

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$$h_c^2(f) = \sum_{\text{redshift}} \sum_{\text{binaries}} h_s^2 \cdot \mathcal{P}(\Lambda),$$
$$\Lambda \equiv \frac{4\pi c d_c^2 \cdot (1+z)}{V_{\text{sim}}} \tau_f.$$

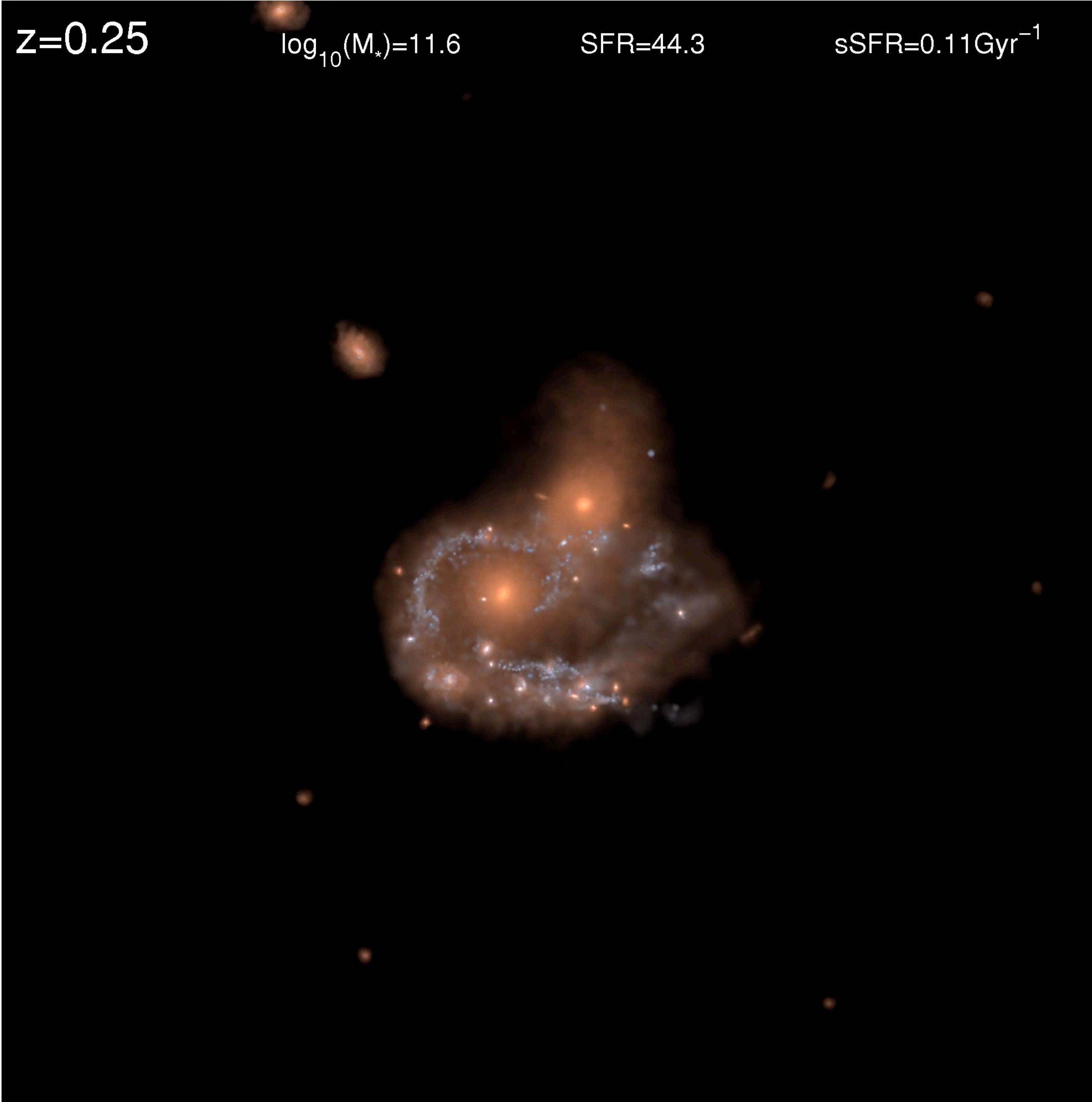


$z=0.25$

$\log_{10}(M_*)=11.6$

SFR=44.3

sSFR= $0.11\text{Gyr}^{-1}$



Illustris / IllustrisTNG

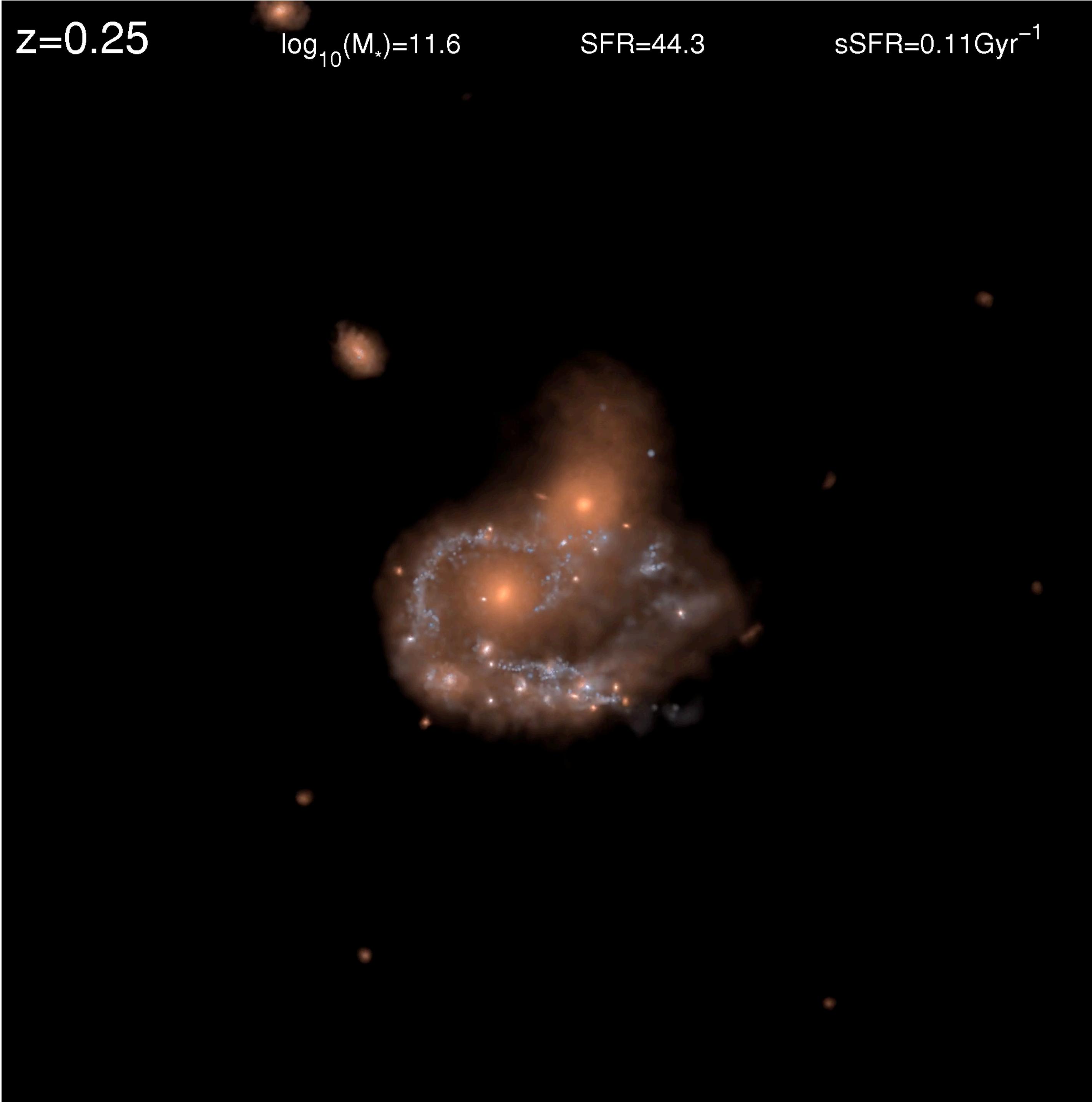
Vogelsberger+(2014a/b); Genel+(2014);  
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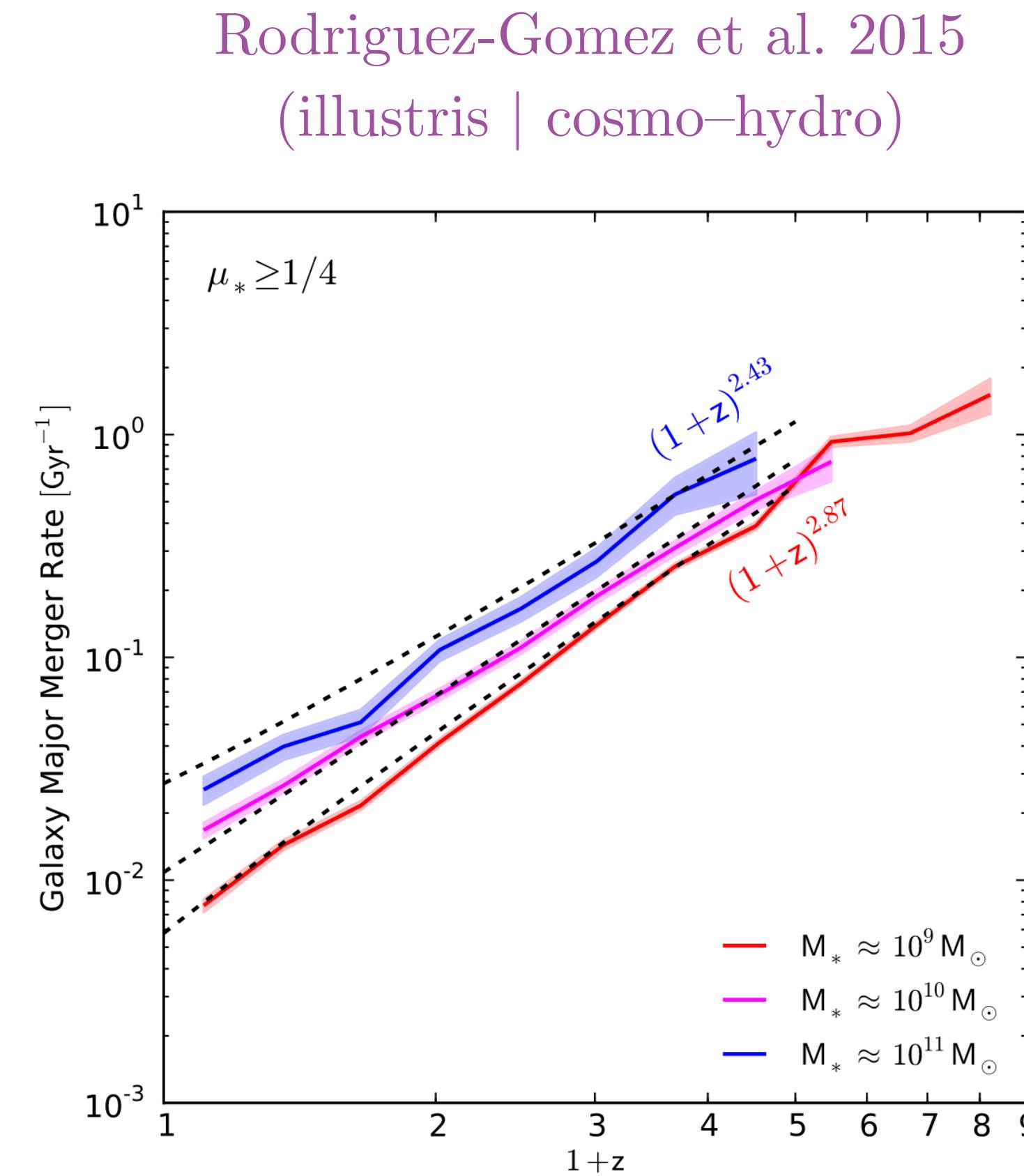
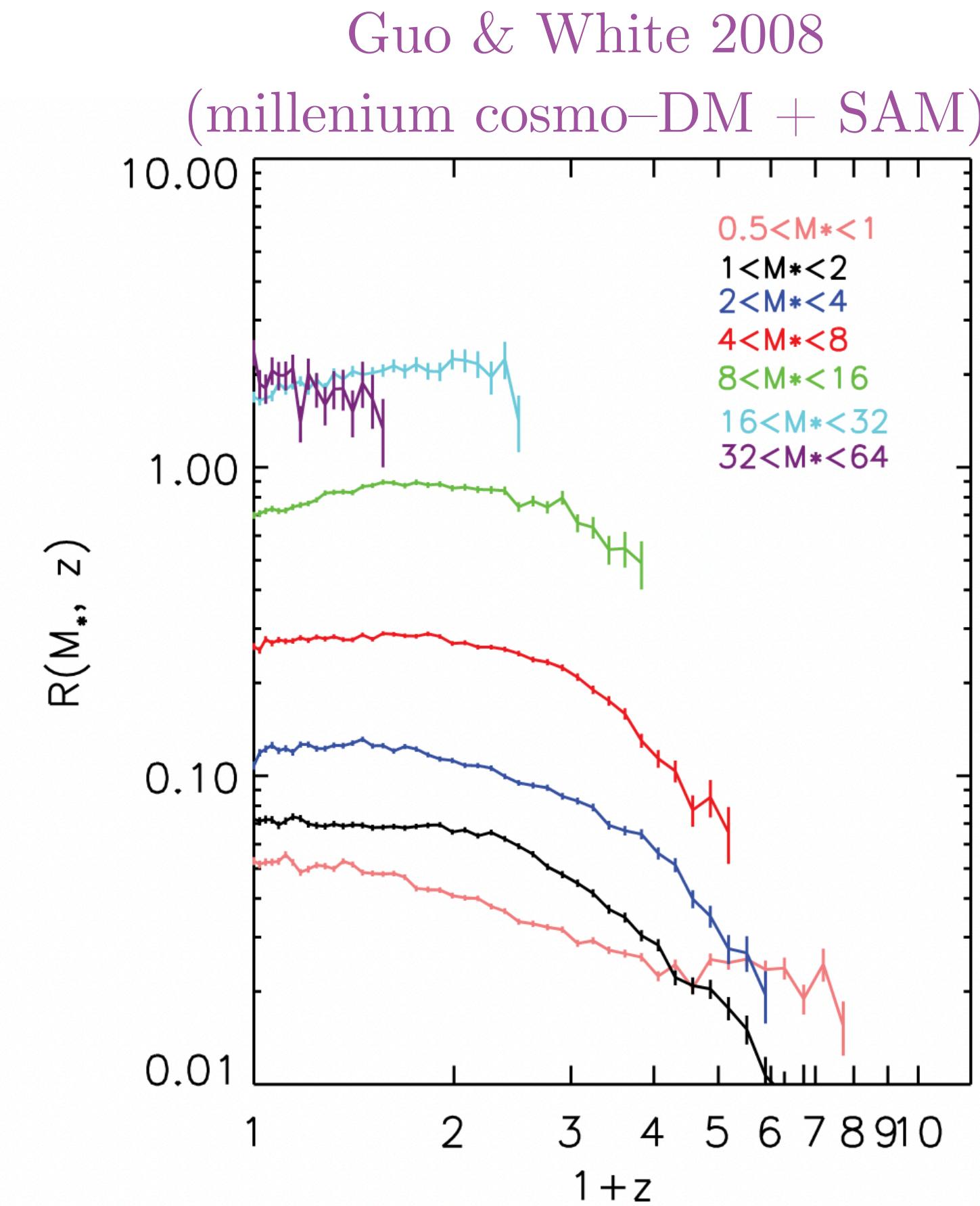
# “Cosmological Simulations”

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- Hydrodynamic: evolve fluids + particles explicitly,  
e.g. *Illustris* / *IllustrisTNG*, *Eagle*, *Romulus*, *Simba*, *Horizon*, *Gadget*, *Ramses*, *Enzo*, *Gasoline*...
- semi-analytic / semi-empirical methods
  - analytic models + empirical scaling relationships
  - observational catalogs

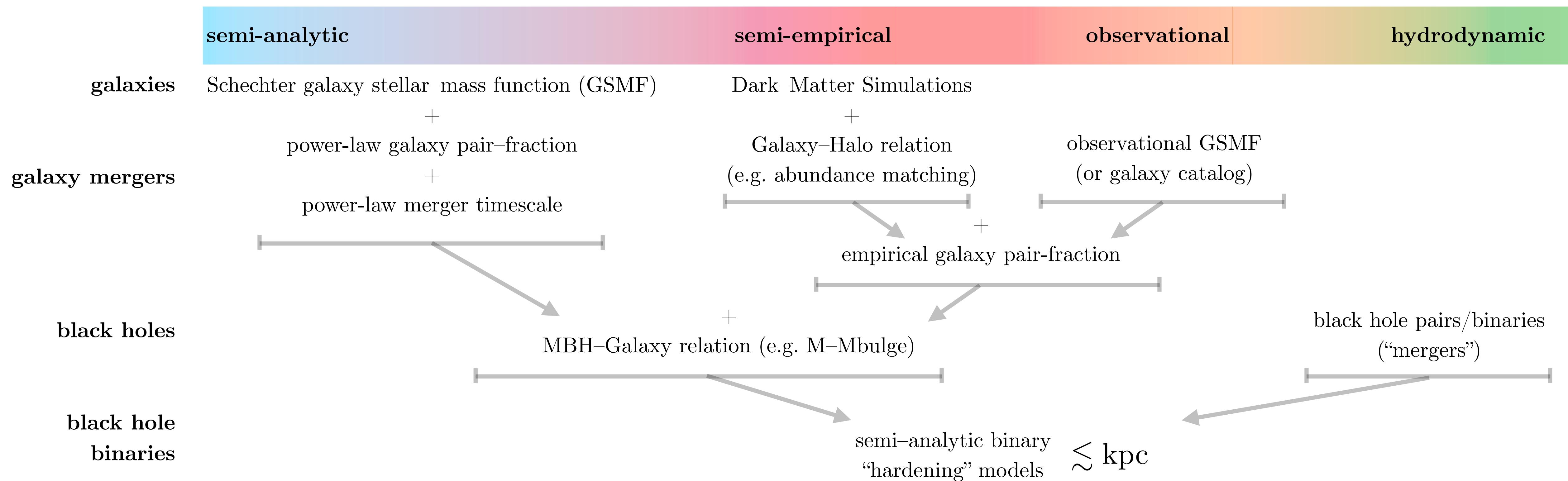
# systematics

- differences between modeling styles can be significant  
e.g. galaxy–galaxy merger rates



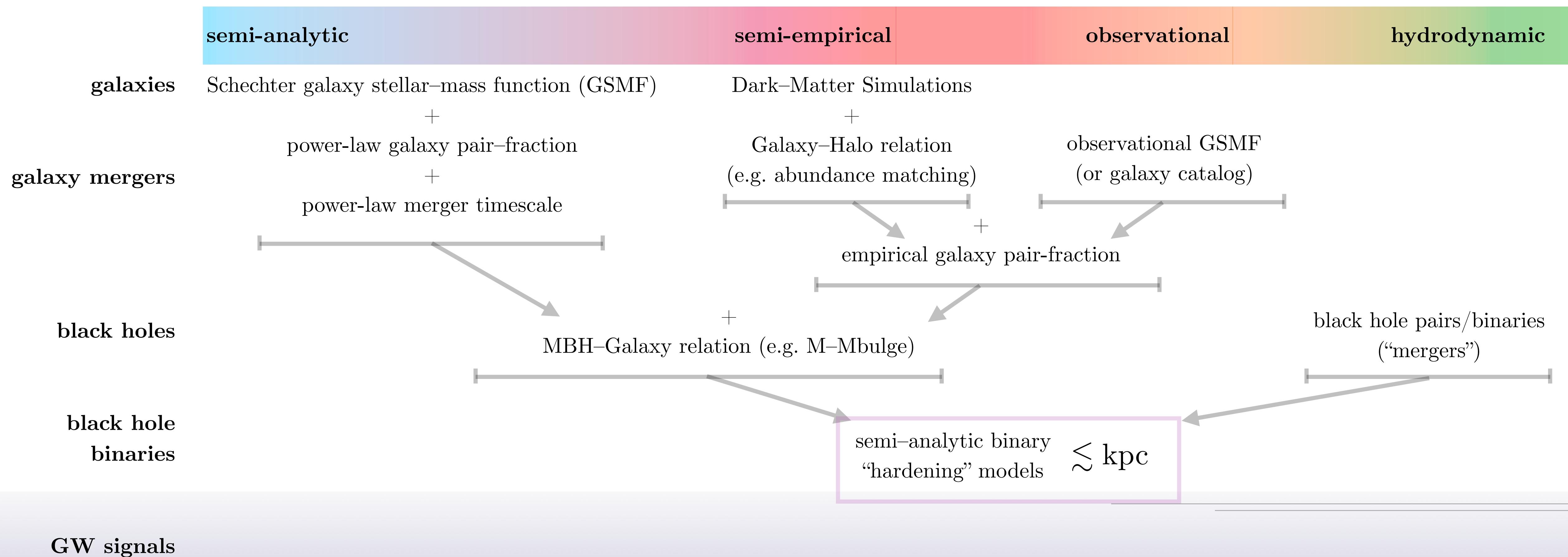
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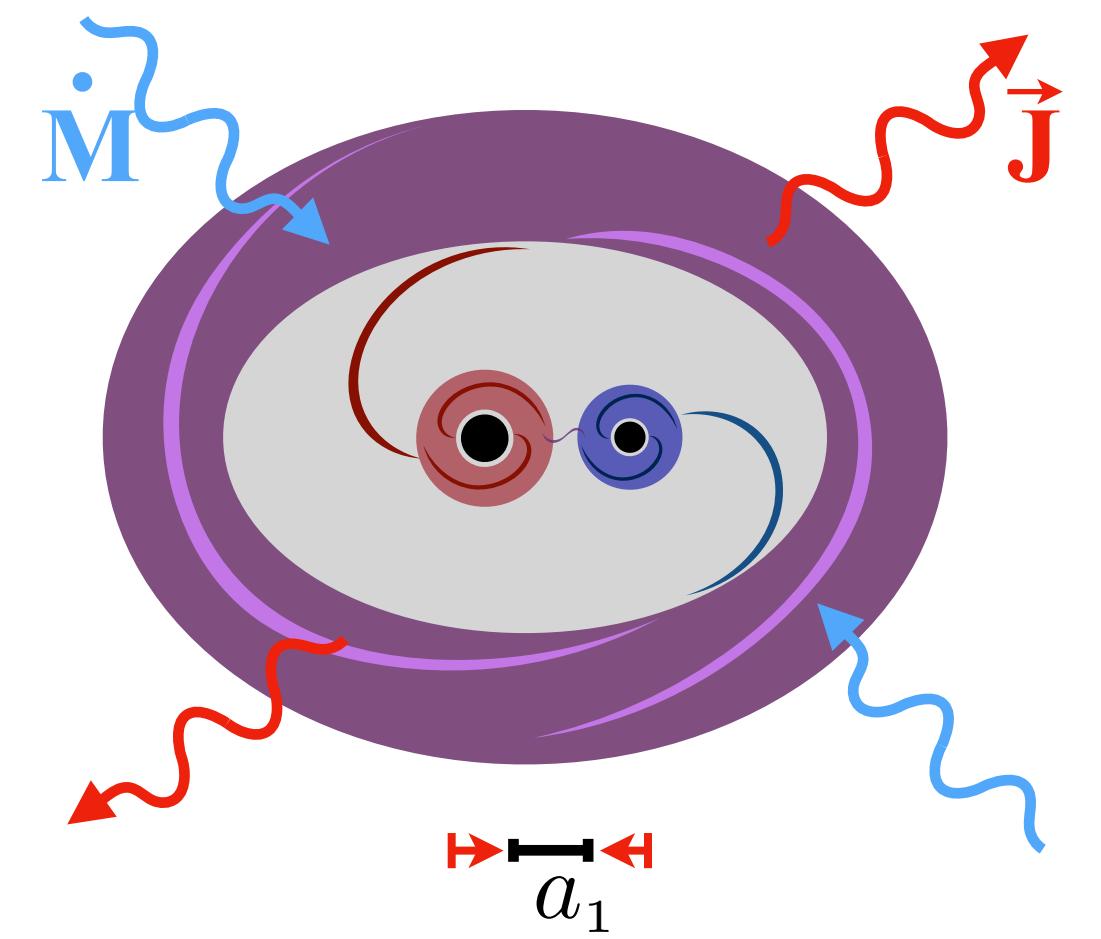
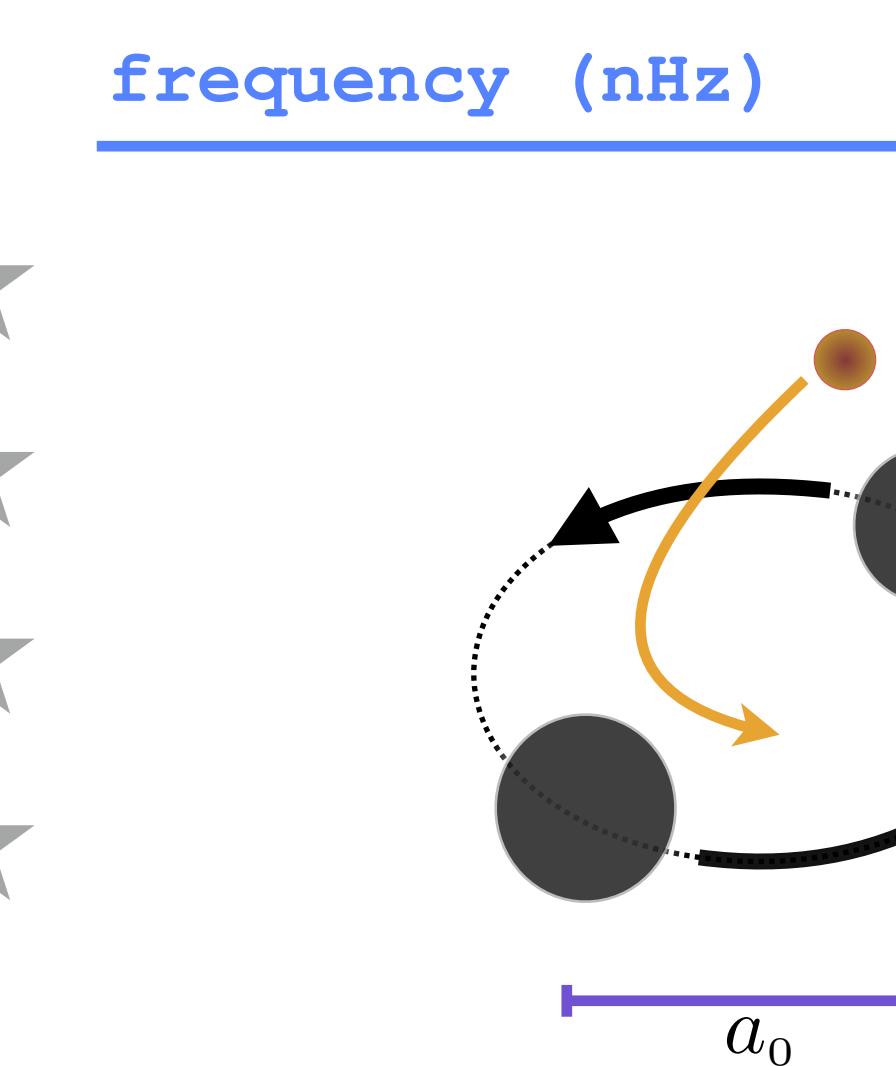
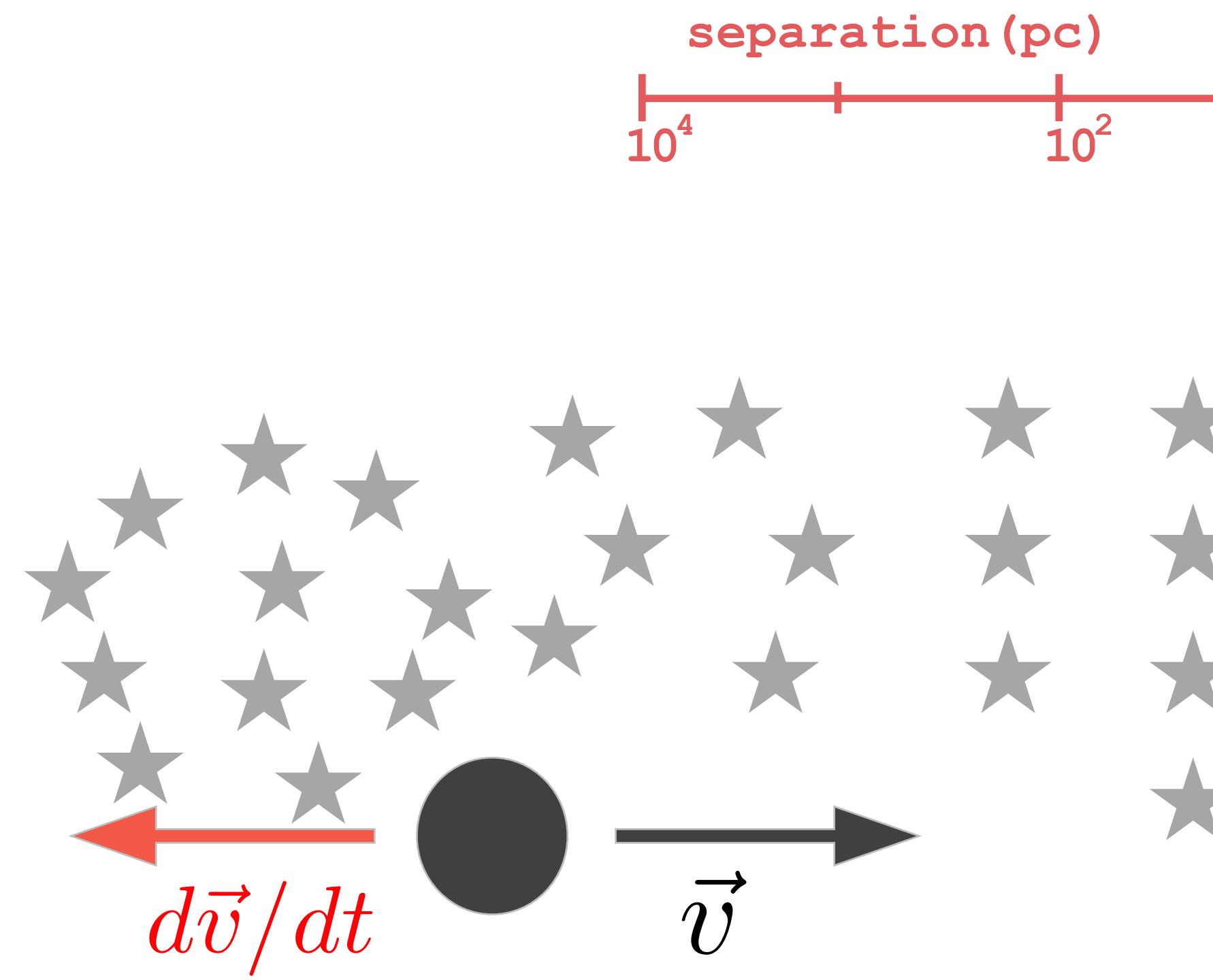
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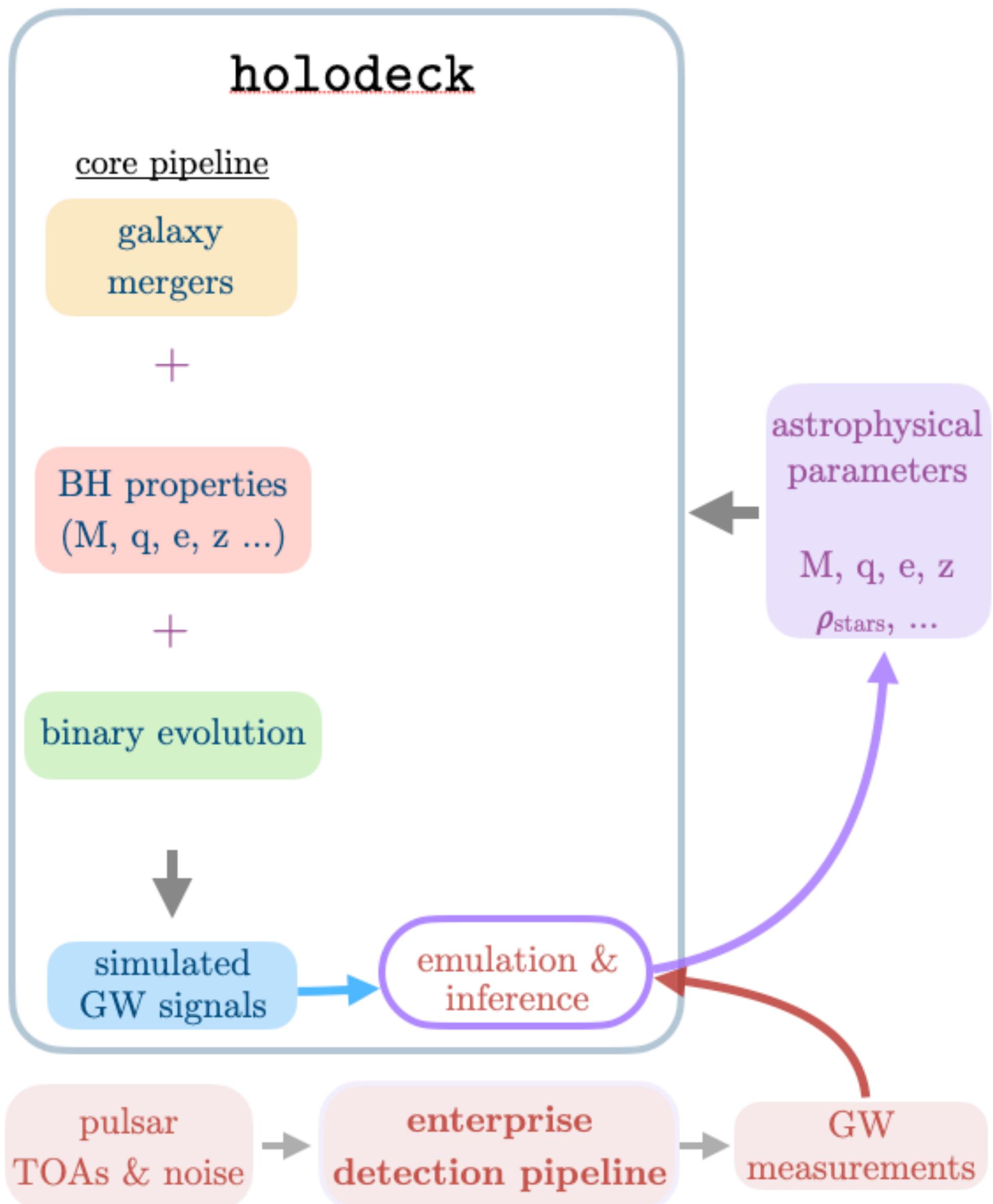
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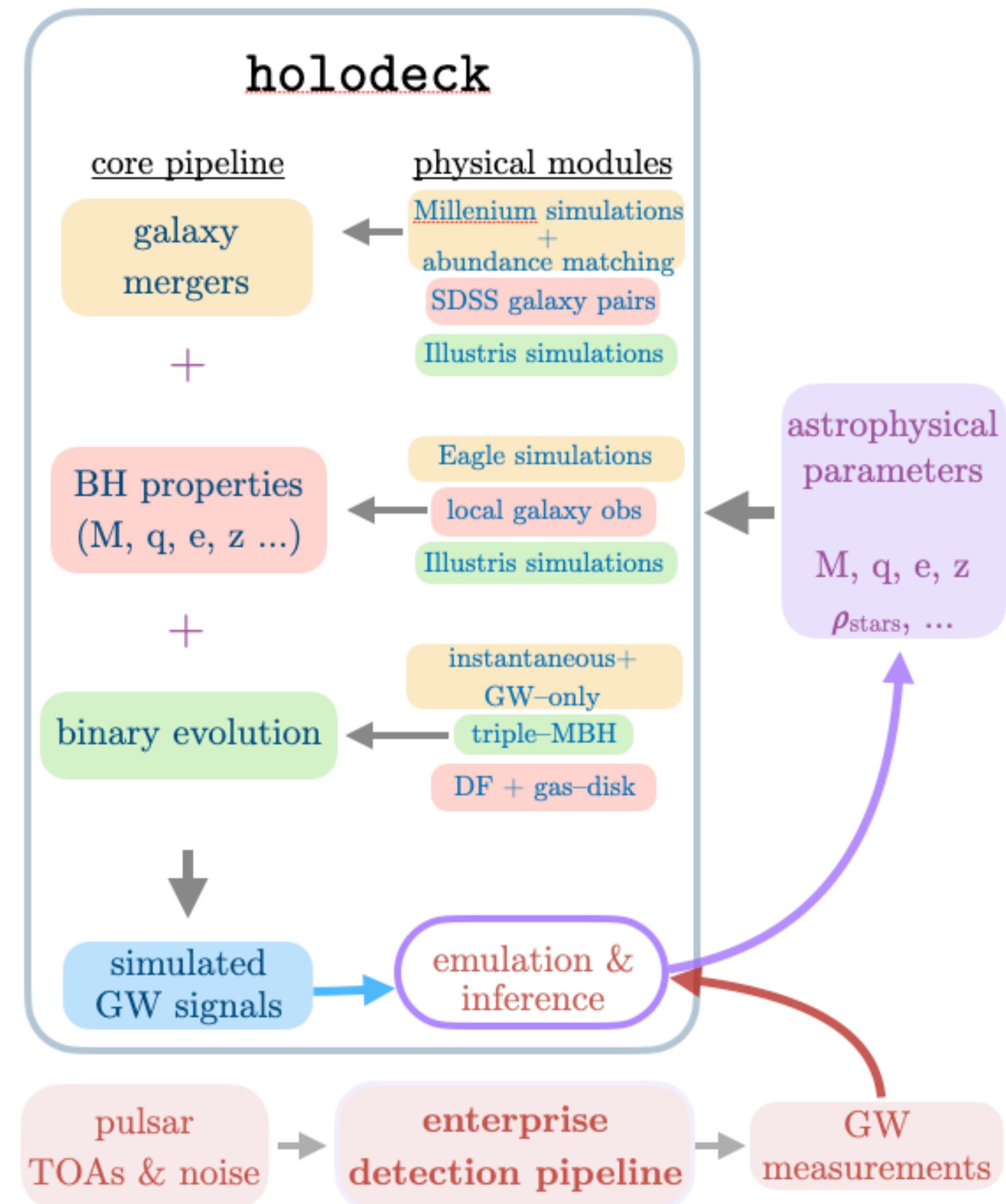




# structure



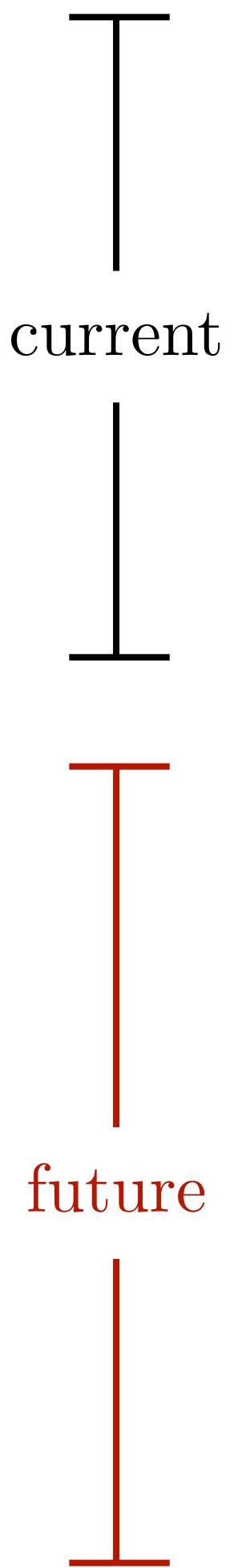
# structure



# motivation

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- NANOGrav's 15yr astrophysics interpretation paper
  - **unified**: collaboration-wide effort (instead of individual's binary models)
  - **robust**: marginalization over *systematic* uncertainties
  - **reproducible**: comprehensive review, maintenance & upgrades; open source\*
- Expansion to LISA
  - plug-and-play with alternative simulations or parametrizations
- Multimessenger GW+EM constraints
  - GW measurements
  - EM binary candidates (e.g. periodic variables, Graham+2015, Charisi+2015)
  - AGN properties (e.g. luminosity function, galaxy scaling relations)



# requirements

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- **rapidly map: astrophysical parameters => GW signals**  
(e.g. M–Mbulge, Mdot, stellar scattering efficiencies, ...)
  - [cosmo sims =>] binary evo => GW library => emulation
- **systematic characterization/inclusion of uncertainties**  
(e.g. galaxy-galaxy merger rates, BH–host relations, ...)
  - both parametric/statistical variations & numerous model classes  
(e.g. hydro sims; semi-analytic models)

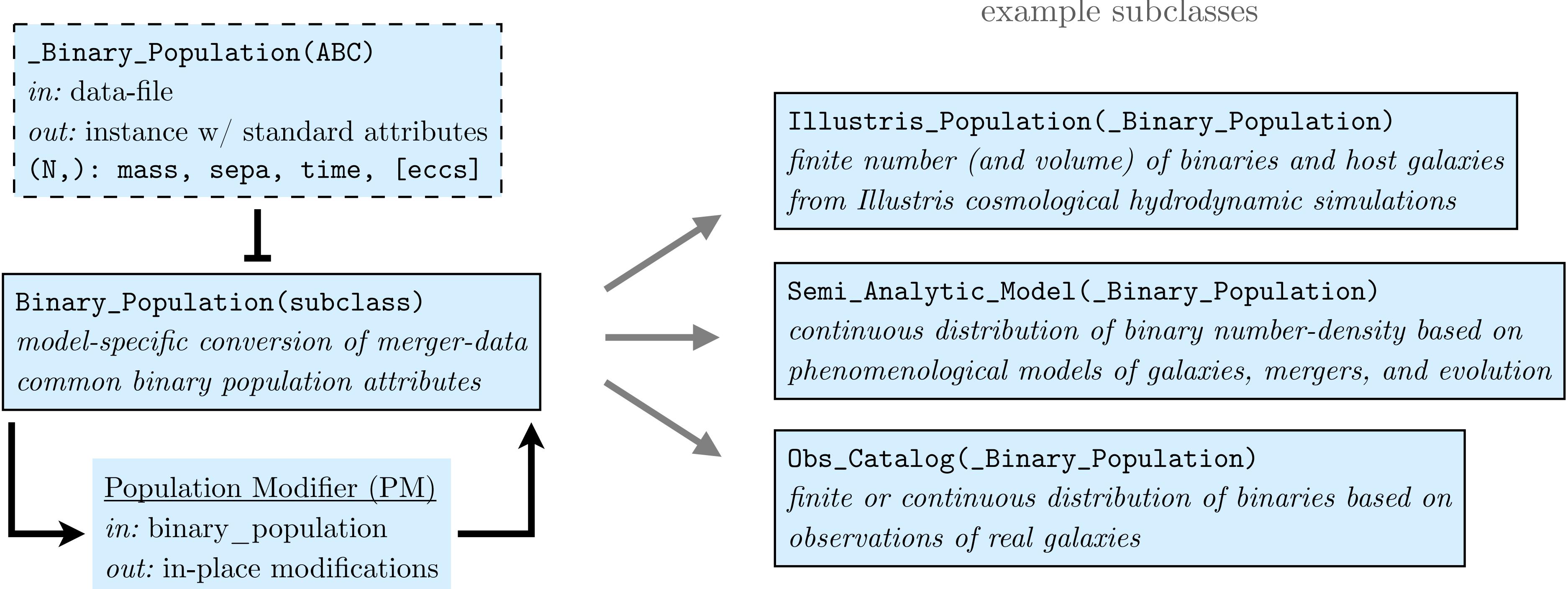
# implementation

- python, class-based framework with numba accelerated modules

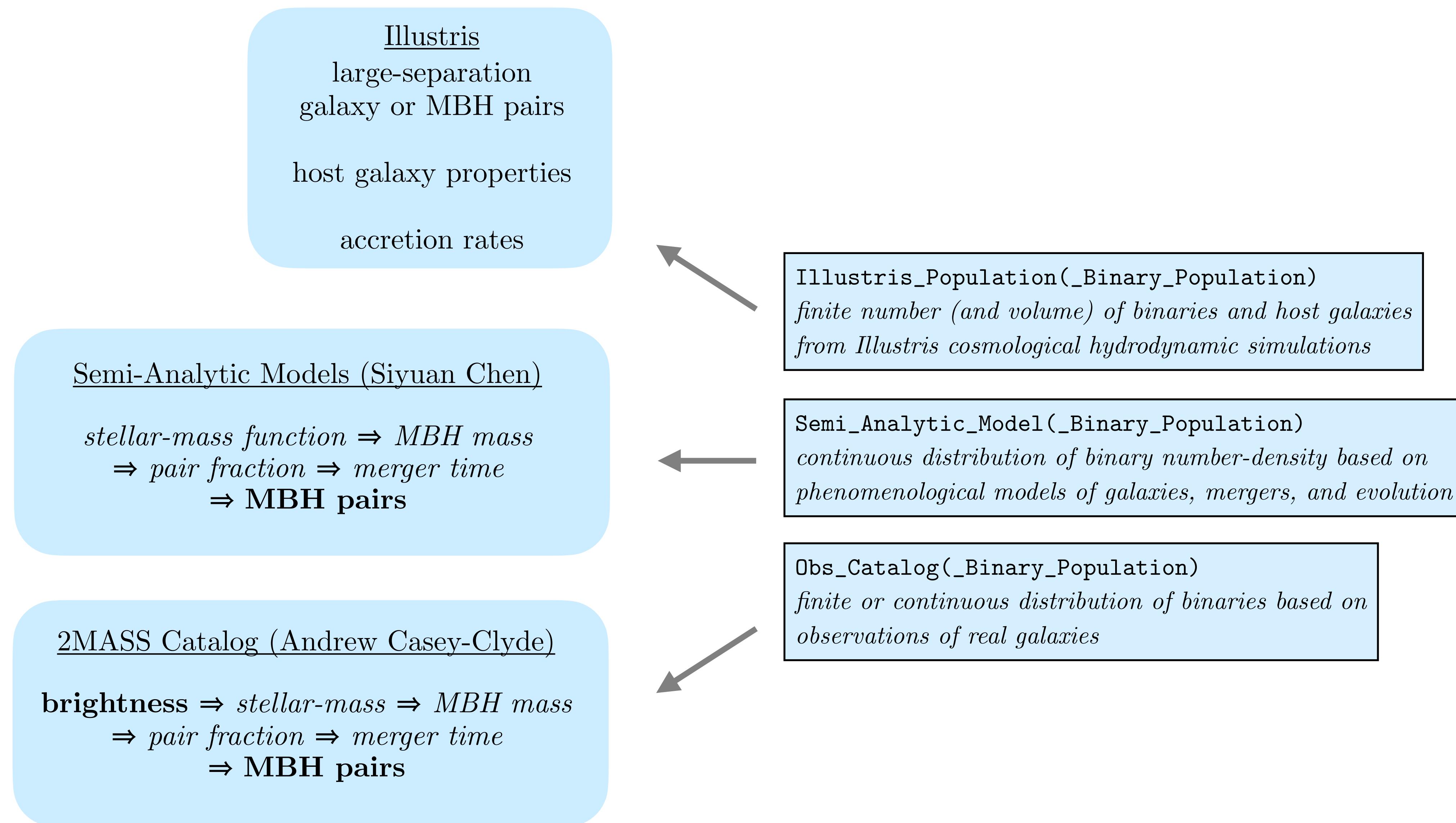
general/abstract  
base-class

model-specific  
subclassed  
implementation

theoretical or logistical  
post-processing modifier



# holodeck: core methodologies



# implementation

- python, class-based framework with numba accelerated modules

general/abstract  
base-class

model-specific  
subclassed  
implementation

theoretical or logistical  
post-processing modifier

```
_Binary_Evolution(ABC)
in: Binary_Population
out: instance w/ standard attributes
(N,T): mass, sepa, time, dadt ...
```

```
Binary_Evolution(subclass)
model-specific binary evolution,
formation to coalescence
```

```
Evolution Modifier (EM)
in: binary_evolution
out: in-place modifications
```

example subclasses

```
Chandra_Dyn_Fric(_Binary_Evolution)
Chandrasekhar dynamical friction
```

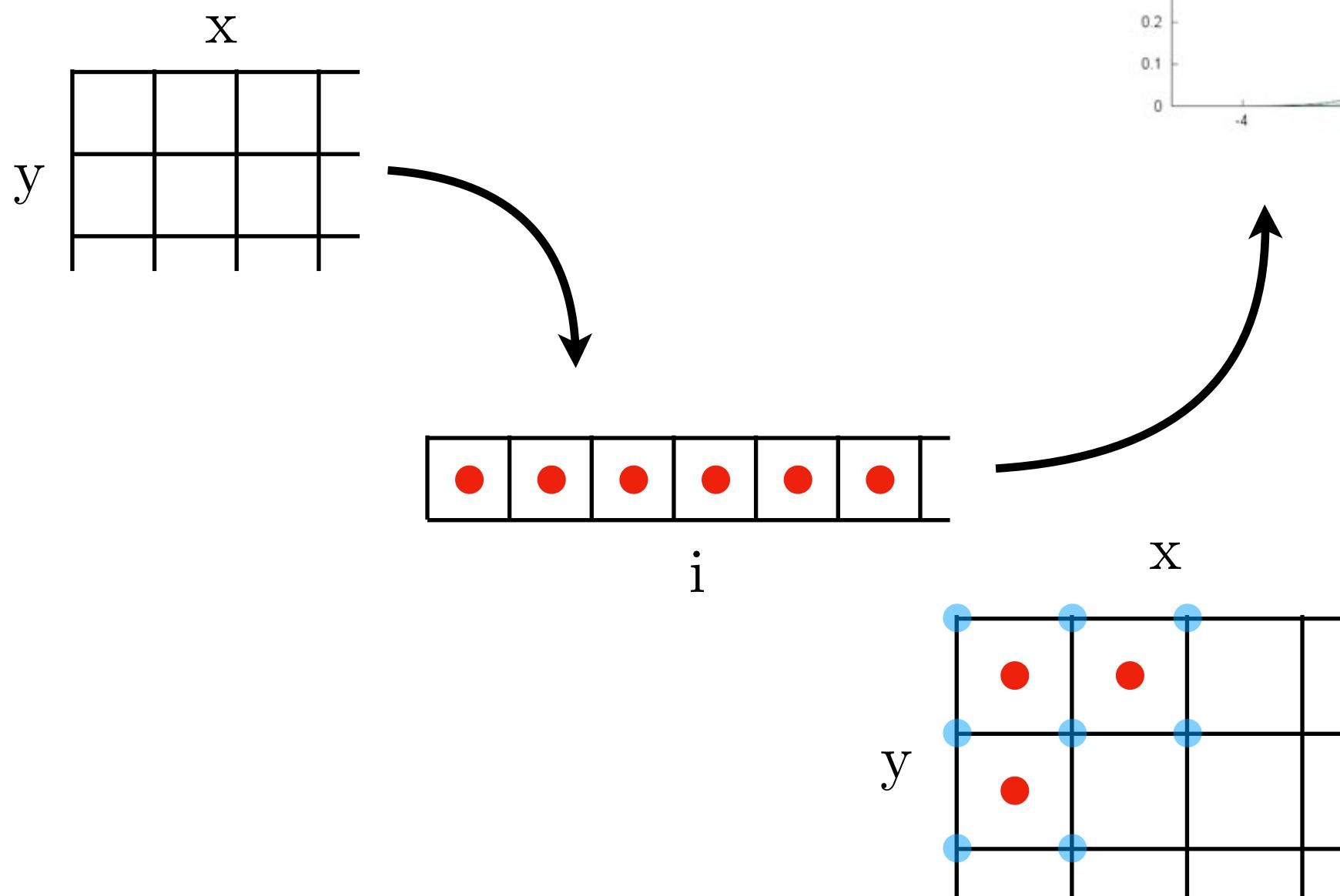
```
Sesana_Stellar_Scattering(_Binary_Evolution)
Quinlan et al formalism for stellar scattering with
coefficients from interpolated numerical scattering experiments
```

```
Double_Power_Law(_Binary_Evolution)
two-power-law hardening rate with
arbitrary indices and normalizations
```

# kalepy sampling

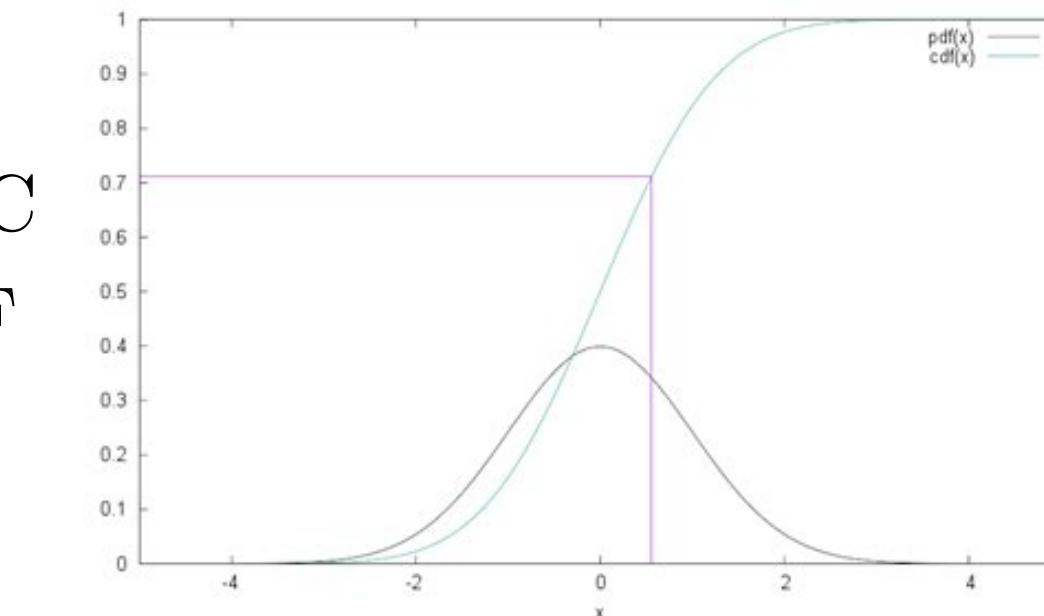
$$\frac{d^4 N_{\text{BH}}}{dM \, dq \, dz \, d \ln f_o}$$

4D “continuous” (discretized)  
distribution of binaries

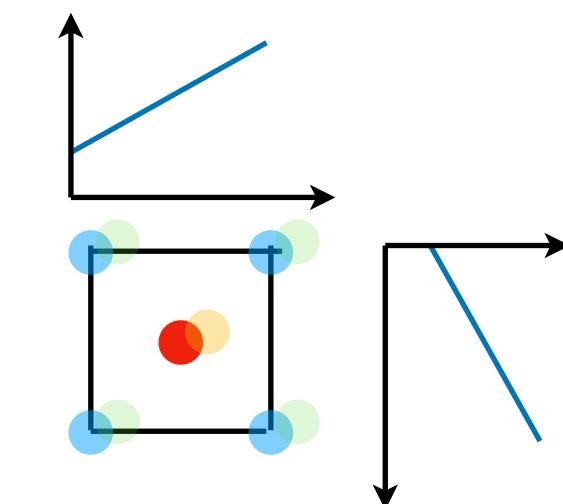


P/C  
DF

inversion sampling



variable



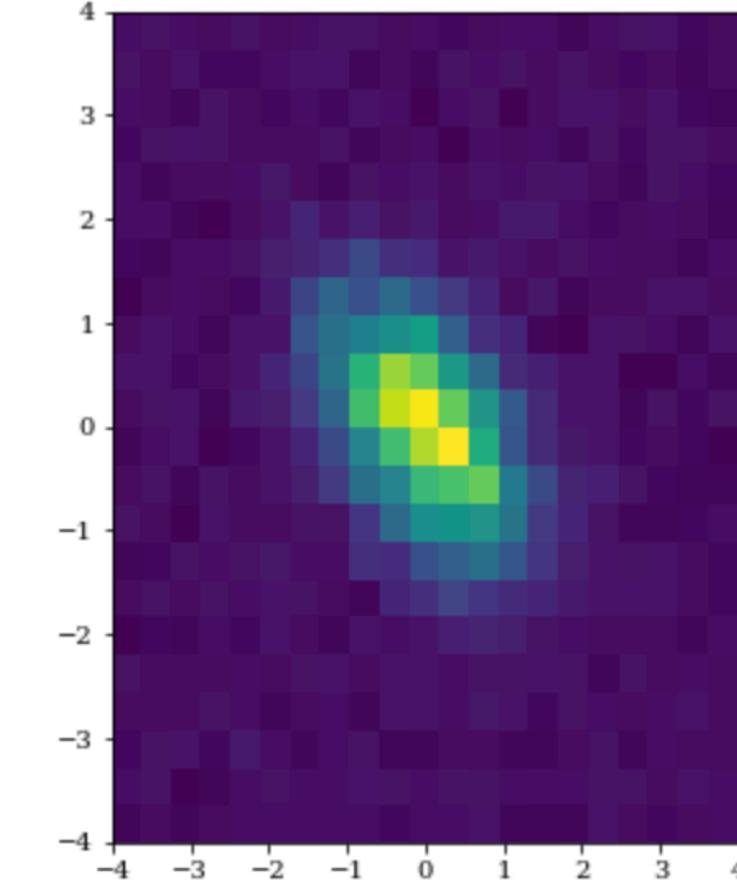
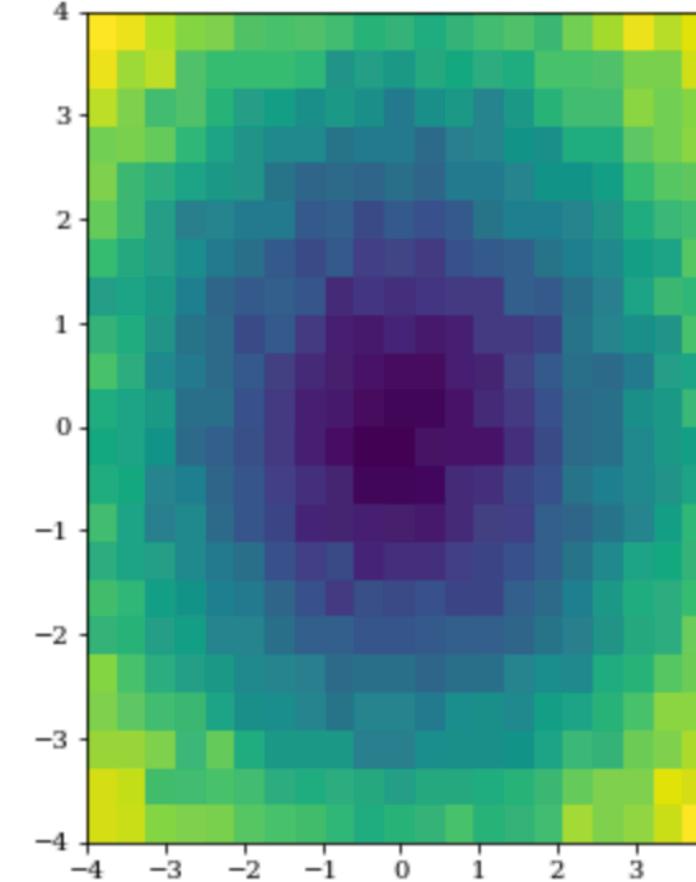
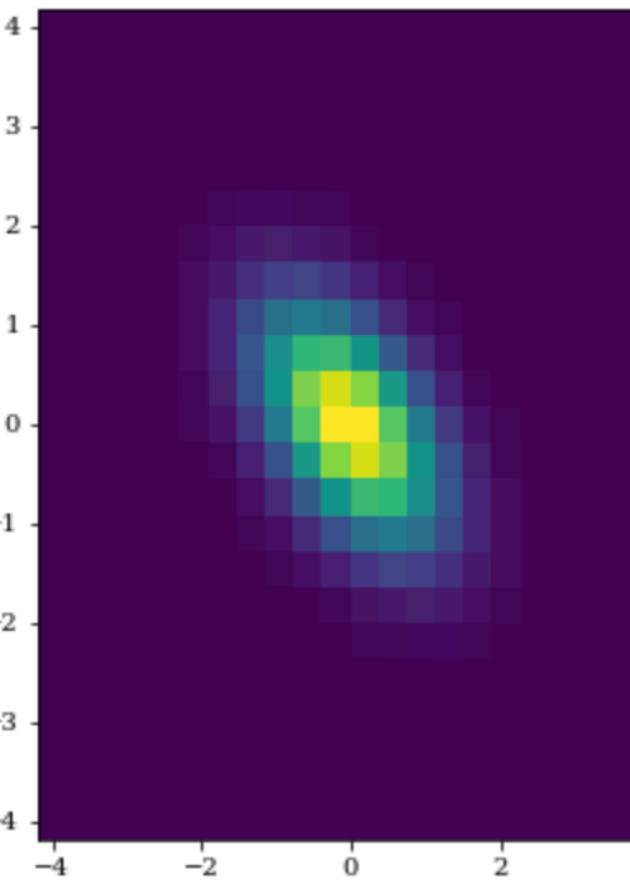
# kalepy sampling

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proportional  
sampling

$$E[f(x)] = \int f(x)p(x)dx = \int f(x)\frac{p(x)}{q(x)}q(x)dx \approx \frac{1}{n} \sum_i f(x_i) \frac{p(x_i)}{q(x_i)}$$

distribution (p)    samples (q)    reconstruction (p/q)

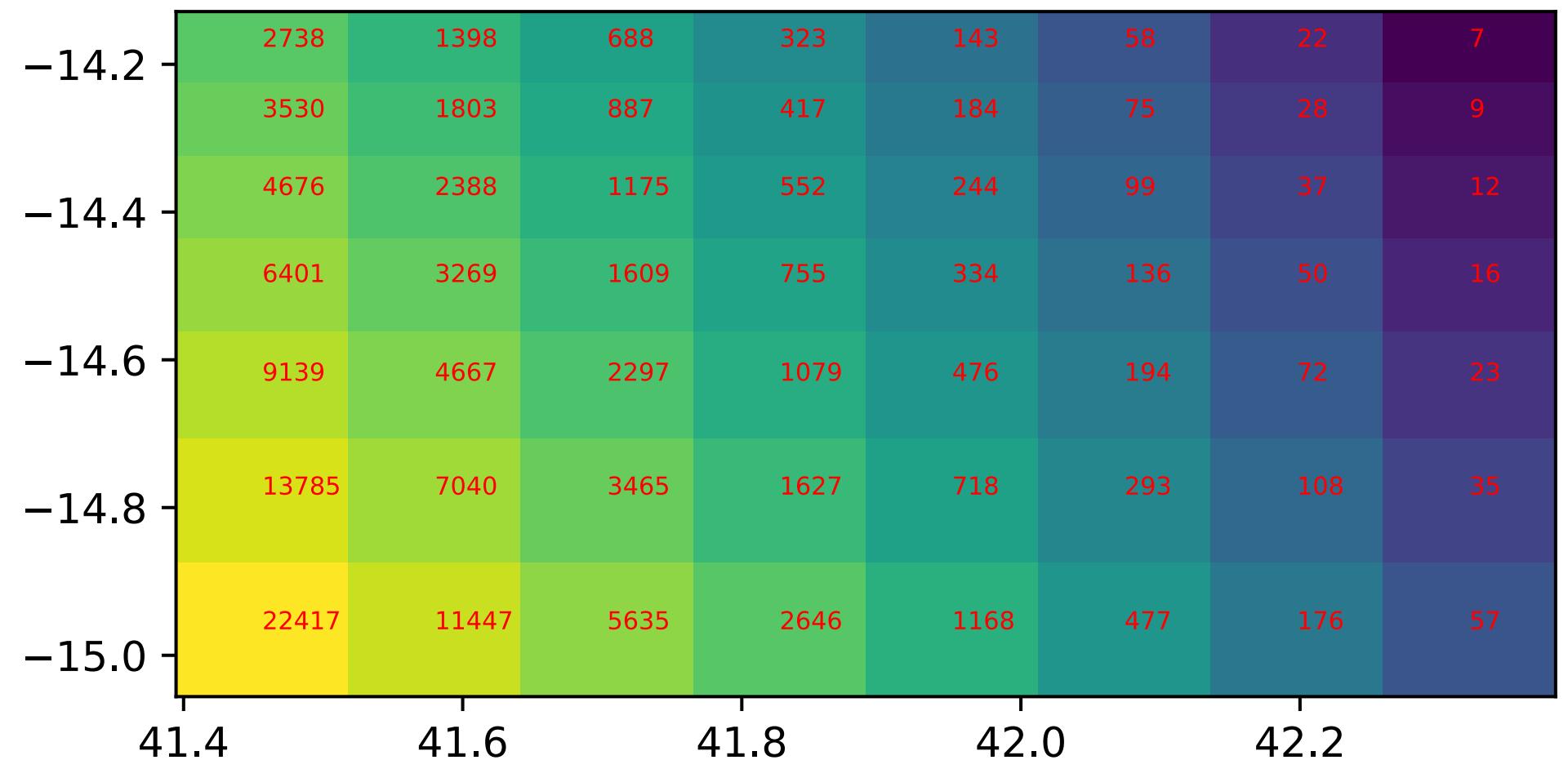


# kalepy sampling

“outlier”  
sampling

for bins with  $N > N_0$  [ $O(10 - 100)$ ], use centroids, not samples

distribution



samples

