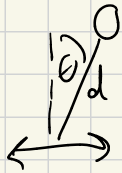
Lagrangian:



$$x = x_0 \sin(\omega t) \qquad \dot{x} = \omega x_0 \cos(\omega t)$$

$$V = d\cos(\theta)mg \qquad T = \tfrac{1}{2}m(d\dot{\theta} + \dot{x})^2$$

$$T = \tfrac{1}{2}md^2\dot{\theta}^2 + \tfrac{1}{2}m\dot{x}^2 + md\dot{\theta}\dot{x}\cos(\theta)$$

$$= \tfrac{1}{2}m\left(d^2\dot{\theta}^2 + \omega^2 x_0^2\cos^2(\omega t) + 2d\dot{\theta}\omega x_0\cos(\omega t)\cos(\theta)\right)$$

$$\Rightarrow \boxed{L = \tfrac{1}{2}m\left(d^2\dot{\theta}^2 + \omega^2 x_0^2\cos^2(\omega t) + 2d\dot{\theta}\omega x_0\cos(\omega t)\cos(\theta)\right) - d\cos(\theta)mg}$$

Removing dimensions:

$$\tau/\omega = t$$

$$\Rightarrow \mathcal{L} = \tfrac{1}{2}m\left(d^2\dot{\theta}^2 + \omega^2 x_0^2\cos^2(\tau) + 2d\dot{\theta}\omega x_0\cos(\tau)\cos\theta\right) - dmg\cos\theta$$

Euler lagrange:

$$\frac{\partial \mathcal{L}}{\partial \theta} = md\dot{\theta}\omega x_0\cos\tau \cdot -\sin\theta + dmg\sin\theta$$

$$\frac{\partial \mathcal{L}}{\partial \dot{\theta}} = md^2\dot{\theta} + md\omega x_0\cos\tau\cos\theta$$

$$\Rightarrow \frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{\theta}}\right) = md^2\ddot{\theta} - md\omega^2 x_0\cos\theta\sin\tau - md\omega x_0\cos\tau\sin\theta \cdot \dot{\theta}$$

$$\Rightarrow md\dot{\theta}\omega x_0\cos\tau \cdot -\sin\theta + dmg\sin\theta = md^2\ddot{\theta} - md\omega^2 x_0\cos\theta\sin\tau - md\omega x_0\cos\tau\sin\theta \cdot \dot{\theta}$$

$$\cancel{m}\cancel{d}\dot{\theta}\omega x_0\cos\tau \cdot \sin\theta + \cancel{d}\cancel{m}g\sin\theta = \cancel{m}d^{\cancel{2}}\ddot{\theta} - \cancel{m}\cancel{d}\omega^2 x_0\cos\theta\sin\tau - \cancel{m}\cancel{d}\omega x_0\cos\tau\sin\theta\cdot\dot{\theta}$$

$$\sin\theta\left(g-\dot{\theta}\omega x_0\cos\tau\right) = d\ddot{\theta} - \omega^2 x_0\cos\theta\sin\tau - \omega x_0\cos\tau\sin\theta$$

$$\underline{\sin\theta\left(g-\dot{\theta}\omega x_0\cos\tau\right) = d\ddot{\theta} - \omega x_0\left(\omega\cos\theta\sin\tau + \cos\tau\sin\theta\right)}$$

$$H = \Theta\dot{\theta} - L \qquad \Theta = \frac{\partial L}{\partial\dot{\theta}} = md^2\dot{\theta} + md\omega x_0\cos\tau\cos\theta$$

$$L = \tfrac{1}{2}m\left(d^2\dot{\theta}^2 + \omega^2 x_0^2\cos^2(\tau) + 2d\dot{\theta}\omega x_0\cos(\tau)\cos\theta\right) - dmg\cos\theta$$

$$md^2\dot{\theta}^2 \qquad \cancel{md\dot{\theta}\omega x_0\cos\tau\cos\theta}$$

$$-\left(\tfrac{1}{2}md^2\dot{\theta}^2 + \tfrac{1}{2}m\omega^2 x_0^2\cos^2\tau + \cancel{md\dot{\theta}\omega x_0\cos\tau\cos\theta} - dmg\cos\theta\right)$$

$$\underline{H = \tfrac{1}{2}md^2\dot{\theta}^2 - \tfrac{1}{2}m\omega^2 x_0^2\cos^2\tau + dmg\cos\theta}$$

Code and graph of $\dot{\theta}$ and θ every $\tau = 2\pi n \quad n \in \mathbb{Z}$:

$$\sin\theta(g - \theta^0 w x_0\cos t) = d^0 e^0 - \omega x_0(w\cos\theta\sin\pi + \cos t\sin\theta)\frac{1}{n} + \cos\theta$$