Authority and Delegation in Online Communities *

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Abstract

Many online platforms rely on user-generated content and need to incentivize free effort. With data from Stack Exchange, I investigate if users provide more and better quality contributions when endowed with more autonomy and authority over actions. Using a dynamic discrete choice model, I show that authority has positive marginal value that is heterogeneous across different types of users. I simulate counterfactuals with different designs. Results show that the platform would lose an important share of production and quality of content in absence of delegation. The trade-off depends on the composition of the community, as the sensitivity to the incentives is heterogeneous.

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1 Introduction

Many companies rely on voluntary contributions by internet users. User-generated content is valuable as it provides information about quality (as for product reviews, Luca 2011, Lewis and Zervas 2019), customer support at zero cost (as crowdsourced online forums[1]), or platform enhancing features, like for Spotify and Google Maps. For some platforms instead, users' content is the product itself. It is the case of social media platforms, information aggregators as Wikipedia, or question-and-answer websites like Stack Exchange. How can companies incentivize participation without remuneration? The literature has investigated the motives behind participation, identifying factors based on intrinsic utility (Roberts, Hann, and Slaughter 2006), peer effects (Zhang and Zhu 2011, Chen, Harper, Konstan, and Li 2010), or virtual rewards (Gallus and Frey 2016, Goes, Guo, and Lin 2016).[2] Theoretical literature in Personnel Economics has anyway identified another non-monetary channel to incentivize participation, that is the delegation of autonomy and decision rights (Gibbons, Matouschek, and Roberts 2013, Gambardella, Panico, and Valentini 2015). Empirical work on this channel is scarce and, to my koledgew, it has not been studied in the context of online communities.[3][4]

In this paper, I investigate if the delegation of control rights and authority induces more and better contributions online. I identify whether and to what extent users are interested in reaching more autonomy over tasks and study its role in contribution patterns.

Every online community requires moderators (Gillespie 2018), but who has authority in modifying the community content differs across platforms. Facebook does not allow users to modify content and hires professional moderators. Users are only allowed to flag what they believe not responding to Facebook's rules. On the other side, Wikipedia lets every internet user modify existing articles. Finally, Stack Exchange provides authority on moderation conditional on achieving performance thresholds. What trade-offs affect this decision?

This paper focuses on the incentive effects of the allocation of authority based on

[1]Mozilla's Firefox for instance.

[2]The main motives behind participation that the literature has identified are intrinsic utility and firm's recognition (Roberts et al. 2006, Nov 2007, Ma and Agarwal 2007, Jeppesen and Frederiksen (2006)), the community size (Zhang and Zhu 2011), reference points on others' behavior (Chen et al. 2010), within community reputation (Chen, Ho, and Kim 2010), peer recognition (Jin, Li, Zhong, and Zhai 2015, Chen, Wei, and Zhu 2017), awards (Gallus and Frey 2016), sequential targets (Goes et al. 2016), and the signaling of skills (Belenzon and Schankerman 2015, Xu, Nian, and Cabral 2020).

[3]Other relevant theoretical work includes Rajan and Zingales 1998, Blanes I Vidal and Möller 2007, Bester and Krähmer 2008. The literature has addressed other types of non-monetary incentives that are relatable. Auriol and Renault (2008) and Besley and Ghatak (2008) investigate status incentives, while the tournaments literature has studied promotion incentives (Lazear and Rosen 1981). These papers include rivalry between workers in the obtaining of status/promotions. In my work instead, delegation does not depend on other workers' actions.

[4]In this paper, I use interchangeably the terms of "delegation of decision rights", "delegation of control rights", and "autonomy over decisions". The term "authority" identifies autonomy over a decision whose outcome affects other individuals. For the definition of power and related concepts I refer to Sturm and Antonakis 2015

performance and studies the trade-off rising from conflicting incentives. It includes Facebook's and Wikipedia's strategies as limit cases, where the threshold performance is set at either infinity or zero. I address two main incentive effects. First, if users value acquiring autonomy, delegation incentivizes effort until users reach the performance threshold (*dynamic incentive*). Second, if users value to contribute when endowed with more autonomy, delegation relaxes the participation constraint, as it relatively decreases the outside option (*static incentive*). Stronger *dynamic incentive* effect would suggest increasing the performance threshold, while stronger *static incentive* to decrease it. The paper studies the platform's trade-off by quantifying both incentive effects under different counterfactual performance thresholds.

In a nutshell, the paper uses data from Stack Exchange, a family of websites where registered users ask questions and provide answers on different topics. The website is moderated by experienced users who have full autonomy over editing questions and answers and who are not paid. New users' edits need to be approved by the moderators. New users become moderators after reaching a performance threshold. In this context, I observe users' contributions before and after they receive authority on editing. After providing evidence of the *static incentive* effect via a regression discontinuity, I develop a dynamic discrete choice model to measure users' preference for authority, allowing for heterogeneity across types. The paper finds that the incentives affect differently the different types. The incentive responses depend on a heterogeneous valuation for authority and different participation costs. The final total amount of contribution is strongly affected by the composition of the community, which is then a crucial factor in designing incentives.

The data I use include the contribution history (answers, questions, edits, and comments) of all participants in the English Language Learners website of Stack Exchange.[5] The discrimination of types is data-driven and based on users' profile pages. It aims to capture the heterogeneity of the broad motives behind participation. Three types emerge: *Anonymous* users provide very little information, *Identifiable* users provide community-relevant information, and *Informative* users provide a lot of information with link to external content (as Linkedin profiles).

The analysis proceeds in two steps. First, I test for the presence of *static incentives* by looking, via a reduced-form analysis, at both the acquisition and loss of authority. To study the effect of the acquisition of authority, I use a regression discontinuity analysis where the running variable is the distance from the threshold in points, i.e. the performance measure on which the threshold depends. I compare editing with commenting activity since the latter is not affected by the threshold. I find a significant and stable increase in the number of edits just after the acquisition of autonomy, while the number of comments follows an independent pattern. The effect is mainly driven by the *Anonymous* users, while *Informative* users seem to respond in the long run. The study of the loss of autonomy exploits a variation in the platform design, which increased the performance required to obtain authority. I find that participants that anticipated the change and lost authority stopped making edits, while participation did not change in

[5] https://ell.stackexchange.com/

answering. The dynamic nature of the *dynamic incentive* effect does not allow clean identification in reduced form.

In the second step, to quantify the incentive effects and simulate counterfactuals, I use a structural model of dynamic discrete choice. In each period, users decide their contribution in terms of the number of answers, quality of answers, and the number of edits. The utility function includes a dummy variable equal to 1 if the user reaches the required performance threshold for authority. Identification of the *dynamic incentive* relies on the effort that users make when approaching the performance threshold: higher effort allows them to reach the threshold faster. Systematic higher effort approaching the threshold would identify a positive marginal utility of authority. Variation in the willingness to participate once endowed with authority identifies the *static incentive* effect. The utility function includes interactions of the dummy variable with variables capturing the intrinsic net benefit of participation, allowing for long term changes in the net cost of contribution. In addition to variables that capture the cost of participation, the utility function includes other sources of motivation potentially correlated with the threshold: the number of points and the number of privileges accumulated. I estimate the flow utility parameters using *finite dependence* (Arcidiacono and Miller 2011), a methodological tool that allows approximation of value functions without full solution of the model.

The results show a positive marginal utility of authority and a significant increase in willingness to participate in editing once endowed with authority. *Anonymous* users show the highest marginal cost of contribution. Nevertheless, they have the highest value for authority, meaning that both the *dynamic incentive* and the *static incentive* effects are particularly relevant for them. *Informative* users are very sensitive to the *dynamic incentive*, while *Identifiable* users are not. Results suggest that authority on editing does not have spillovers on answering, and only the *Anonymous* users slightly substitute answering with editing.

With estimates from the model, I simulate counterfactual contribution histories under different performance requirements to obtain authority. In particular, I consider the case with a performance threshold equal to zero (full delegation), infinity (no delegation), or two intermediate levels. Results show that in the simplified context of the simulation, *Anonymous* users do not contribute due to their high costs of participation. Since *Identifiable* users are not sensitive to the *dynamic incentive*, the choice of the threshold level should focus on *Informative* users. Their participation in answering is maximized when they have to reach a performance threshold, but a too high threshold may induce a smaller increase in participation. The optimal threshold level depends on the expected life-time of participation and on whether the reputation points that can be accumulated are capped.

This paper has two main contributions. First, I show direct evidence of non-monetary preferences, identifying in real data the intrinsic value of authority. This confirms experimental results showing that individuals value control rights and power (Fehr, Holger, and Wilkening 2013, Bartling, Fehr, and Herz 2014, Owens, Grossman, and Fackler 2014, Pikulina and Tergiman 2020).[6] The second contribution relates to the organizational implications of these non-monetary preferences. The paper shows that platform designers should take into account the incentive effects induced by the allocation of decision rights. Besides, the paper suggests that the platform would optimally target different users with different incentives. While the results of this paper are specific to the context of online communities, they may suggest implications for a broader set of environments, addressing puzzles that emerged in the promotions' literature. It can provide a plausible explanation for 1) the use of promotions rather than bonuses, even if bonuses are more flexible incentives (Baker, Jensen, and Murphy 1988, Gibbons and Waldman 1999) 2) the commitment to promote employees on the ground of observable measures not correlated to the skills required in the delegated tasks (Peter principle, Fairburn and Malcomson 2001, Benson, Li, and Shue 2019).

The paper proceeds as follows. Section 2 describes the website from which data is taken, while sections 3 presents the data and the identification of user types within the online community. I then present results from the reduced form analyses in section 4, and the structural model in section 5. Finally, sections 6 and 7 report the results and the counterfactual simulation. Section 8 conclude.

2 Stack Exchange: "self managed" platforms

I use data from Stack Exchange. Stack Exchange is a family of platforms born in 2009 that provides users the possibility to post questions and answers on a variety of topics. Each website of the group specializes on a particular topic: notably *Stack Overflow*, the largest community, hosts questions and answers about programming languages, but there are 172 other websites, each focused on a different topic, from technology to arts. These websites belong to the commercial company Stack Exchange Inc. which has, at July 2020, raised 153 million dollars in venture capital.[7] To give a sense of the welfare produced to consumers, Stack Exchange receives 418.8 million monthly visits and 805.9 million monthly page views. It contains 3.3 million questions, which received 3.6 million answers[8]. Instead of hiring experts to answer questions, Stack Exchange is crowd-based. Anyone can register and contribute to the platform: there are no registration fees, but contributions are not remunerated. Users do not need to register to browse the content. The business strategy is similar to other websites, like *Quora* or *Yahoo! Answers*, but differs from *Google Answers*, active between 2002 and 2006, where answers' providers were paid.

The objective of the platform, as described by the creators, is to provide detailed and easily-accessible solutions for specific questions.[9] For instance, duplicate questions

[6] Non-expeirmental work has identified a beneficial effect of delegation on performance, but does not investigate whether a channel is an intrinsic value for authority. Bandiera, Best, Khan, and Prat (2020) use a field experiment, while Liberti (2018) use real data from a financial institution.

[7] https://www.businesswire.com/news/home/20200728005330/en/Stack-Overflow-raises-85M-Series-funding-accelerate

[8] https://stackexchange.com/about

[9] https://www.joelonsoftware.com/2008/09/15/stack-overflow-launches/

or questions on subjective topics are *closed* (i.e. do not allow answers), and the answers to a question are ranked based on up-votes rather than publication date.

Participation in Stack Exchange is subject to an incentive system based on virtual rewards, either reputation points, or *badges*.

Badges.

Badges are comparable to medals and, to some degree, to firms' bonuses. Users obtain badges when they accomplish given performance targets, where performance depends on quantity, quality, and timing of contributions. There are bronze, silver, and golden badges, based on the degree of difficulty.

Reputation points and Privileges.

Once the user publishes a question or an answer, other community members can up-vote it or down-vote it, allocating or removing reputation points from the author. More precisely, each up-vote provides 10 points, while each down-vote removes 2 points. To vote, users need to have accumulated at least 15 reputation points. The user can receive points also by suggesting a modification to existing content: if the suggestion is accepted and gets implemented, the user gains 2 points.[10] The accumulation of points allows obtaining *privileges*. With few exceptions, privileges are rewards that give access to resources or actions. Users obtain them when reaching given threshold levels of reputation points, hierarchically. The higher the amount of points accumulated, the closer the user gets to have full administrative control of the website. Table 1 reports the list of privileges and the reputation points necessary to obtain each of them (rightmost columns).[11] Privileges may be comparable to promotions in traditional companies: more experienced *employees* receive more information and authority from the company owners, as well as more responsibilities. A difference anyway is that workers compete for a limited number of higher positions, while in Stack Exchange there is no rivalry. Users are guaranteed to get the privileges if they reach the required performance.

2.1 Hierarchy in Stack Exchange and the rationale for delegation

In Stack Exchange, community members make most of the decisions, but not all users have the same decision rights. Users can acquire decision rights and authority in two ways. One way is via elections. The platform organizes internal elections in a non-systematic frequency. Elected users have a permanent mandate and authority in moderating the platform. Users can obtain the same control rights as elected moderators by accumulating points, which allows them to reach the privileges. Privileges provide either access to actions or more authority. Via the allocation of Privileges, the platform commits to delegate control to community members that achieve given performance

[10] Figure 25 in the appendix provides the detailed rules to gain points

[11] Since these values changed during the life of the website, the table provides two values for each privilege. In section 3, I provide more details on how the change happened and when each threshold has applied.

measures. In this paper, I will focus specifically on the privilege that delegates authority in editing. Editing is the action of modifying existing content to improve or correct it. Before users reach that privilege, they are allowed to propose modifications, but their suggestions need to be approved by either the author of the modified content or by the voting of two users that already have the privilege. Once users obtain enough reputation points to get the privilege, their edits are directly implemented and do not require the approval of third parties. I consider this variation as an increase in the authority of editing.

Why the platform may want to delegate authority to community members? Compared to hiring professional moderators, to delegate has the advantage that community members work for free. It induces important savings for the platform, as it needs many moderators.[12] There are nevertheless two other important reasons. First, if users' willingness to make contributions to the platform is significantly higher when endowed with full authority, delegation relaxes the participation constraint. The intuition is equivalent to what Gibbons et al. (2013) refers to as "to pay the employee less": I define this effect as *Static Incentive*, as it is independent of the dynamics of contribution. Second, to tie delegation to performance incentivizes participation if users value to gain authority. I call this effect *Dynamic Incentive*. The incentive effects induced by the delegation are particularly relevant in the context of voluntary work. As users are volunteers, their outside option from participating is high, and the absence of formal contracts reduces the cost of leaving.

If the platform wants to leverage both the *Static Incentive* and the *Dynamic Incentive*, faces a trade-off. A positive *Static Incentive* effect would suggest delegating to every user, independently of performance, while a positive *Dynamic Incentive* effect would advise conditioning delegation on performance. Finally, if the platform condition delegation on performance, it needs to decide the level of performance required. A second trade-off emerges: a more demanding performance threshold incentivizes participants for a longer time. Nevertheless, a higher performance threshold decreases the *Static Incentive* effect.

[12] A platform like Facebook hires around 15000 moderators: Charlotte Jee, MIT Technology Review, June 2020

		Reputation Requirements	
Privilege	type	Graduated	Public Beta
access to site analytics	Milestone	25000	5000.0
trusted user	Milestone	20000	4000.0
protect questions	Moderation	15000	3500.0
access to moderator tools	Moderation	10000	2000.0
approve tag wiki edits	Moderation	5000	1500.0
cast close and reopen votes	Moderation	3000	500.0
create tag synonyms	Moderation	2500	1250.0
edit questions and answers	Moderation	2000	1000.0
established user	Milestone	1000	750.0
create gallery chat rooms	Communication	1000	
access review queues	Moderation	500	350.0
create tags	Creation	300	150.0
view close votes	Moderation	250	250.0
vote down	Moderation	125	125.0
edit community wiki	Creation	100	100.0
create chat rooms	Communication	100	
set bounties	Creation	75	75.0
comment everywhere	Communication	50	50.0
talk in chat	Communication	20	
flag posts	Moderation	15	15.0
vote up	Moderation	15	15.0
remove new user restrictions	Milestone	10	10.0
create wiki posts	Creation	10	10.0
participate in meta	Communication	5	5.0
create posts	Creation	1	1.0
vote in moderator elections		150	150.0
association bonus		200	200.0
shown in network reputation graph and flair		200	200.0
reputation leagues - top x% link in profile		201	201.0
qualify for first Yearling badge		201	201.0
run for moderator		300	300.0

Table 1: List of privileges that users can obtain when accumulating reputation points. The first column describes what the user obtains achieving each privilege, the second the category of the privilege, while the third and the forth the number of reputation points required to obtain the privileges. The *Public Beta* column applies to the platform between January 2013 and February 2016, while the *Graduation* column applies from February 2016 onwards.

3 Data

Stack Exchange is composed of many websites sharing the same structure, whose only difference is the main topic of the questions posted. In this paper, I use data from the website called *English Language Learners* (ELL), which focuses on questions and answers related to the use of English. The creation of Stack Exchange websites follows a specific procedure. First, an initial community of users makes a proposal of creation in a specific platform called *Area 51* and starts contributing.[13] When the website, within the *Area 51*, proves to have enough demand and a sustained amount of activity, the platform administrators launch it with an independent URL. The website enters the beta period, which is divided in *private beta* and *public beta*. The *private beta* allows participation only to users that have contributed to the development phase. Normally, after a week, the website moves to the *public beta* phase, characterized by no restrictions to participation. Finally, once the platform administrators assess that the website can sustain in time, it *graduates* to the final phase and receives a personalized design. Normally, the *graduation* and the new design would occur on the same date, but on the ELL website, the design occurred later.[14] The timeline of these steps for the ELL website is reported in figure 1. Once the website receives the new design, the requirements in reputation points to obtain the privileges change.[15]

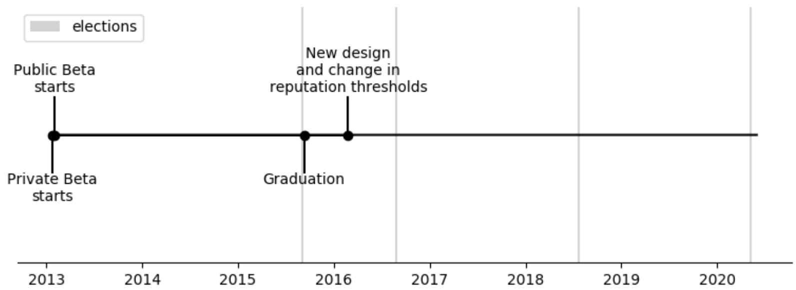

Figure 1: Timeline of the website

The data was retrieved on May 31^{st}, 2020, and contains both information displayed in the user profile pages, as well as content and modification history of posts (questions and answers). While the company makes most of the data available for free, I web-scraped the daily histories of reputation points obtained by users. At the time of download of the data, the website counted 92,853 registered users, 121,633 published answers, and 77,357 questions. I constructed a panel of users' participation in the website. I include users that have published at least one answer or edit while exclude who has not gained a

[13] https://area51.stackexchange.com/

[14] The shift was due to a backlog of the designer team.

[15] Table 1 reports the amount of reputation points required to obtain each privilege. The *Public Beta* column reports the requirements before the design, while the *Graduated* column reports the requirements after the design.

positive number of reputation points. Users are assumed to exit the platform after three months of inaction in answering or editing. The data is right-censored at the download date.

In the panel there are 9797 users, who participated on average 713 days (with a range between 1 and 2685 days). They published a total of 114,926 answers, on average 11.7 each, but with a very skewed distribution, ranging from 1 to 4173. The edits I consider are edits to answers, either modifications of answer's content or rollbacks, i.e. the recovery of a previous version. The users of the sample made 8168 edits, of which 1409 were suggested and the rest were directly implemented. Each user on average made 0.8 edits, with a range between 0 and 1174. Figure 2 shows the cumulative production of users in both absolute terms and in shares. Users are ranked by the intensity of answering. It shows that activity in answers and edits is very concentrated, while questions are more homogeneously distributed.

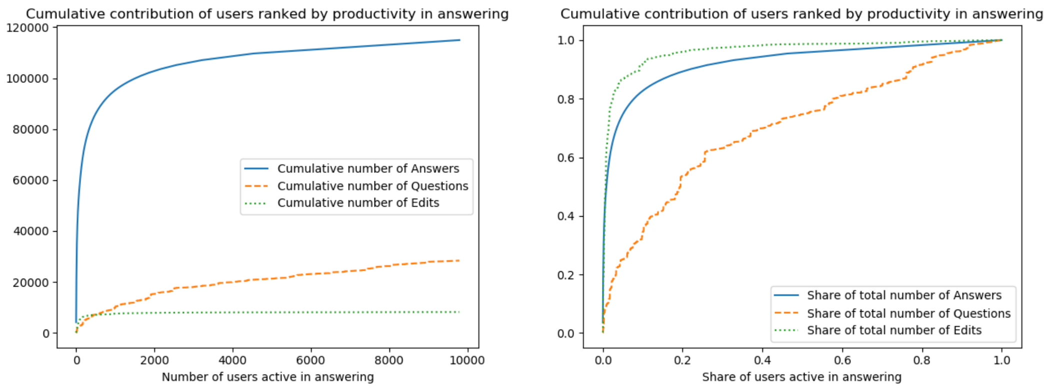

Figure 2: Production of answers, questions, and edits between the active users. x-axis reports the number of users (left) or the share of users (right) ranked by amount of answers published, while the y-axis reports the absolute number of answers, questions, and edits (left) and the respective shares (right).

In the sample, users reached on average 487 reputation points, with a range going from 0 to 175,955, the 75^th percentile being 208 (the zero is due to a particular case that got included in the sample). Figure 3 reports the number of users, at each point in time, who have reached the threshold to obtain more control over editing. Note that when the threshold value changed, this made some users lose the privilege.

I construct a proxy for answers' quality using textual measures, including the number of words, links, and pictures. Details on this process can be found in the appendix A.1. On average, users made answers with quality equal to 0.16, and the quality range spans between 0.004 and 14.107.

Finally, I construct a variable to measure the number of daily open questions in the topics of experience of the users. A question is open if it does not have an accepted answer. In a nutshell, the variable is constructed by 1) clustering tags around topics, identified exploiting the co-occurrence of tags in questions, 2) allocating open questions to topics, 3) recovering user's expertise on each topic based on her contributions, and 4) weighting user's available questions by her expertise. Appendix A.2 provides more details on the construction of this variable. On average, users have about 8,000 available questions on a given day, ranging from 22 to 21744 (note that at the time of data retrieval, out of the total 77,357 questions, 38,015 do not have an accepted answer). Table 2 reports some descriptive statistics.

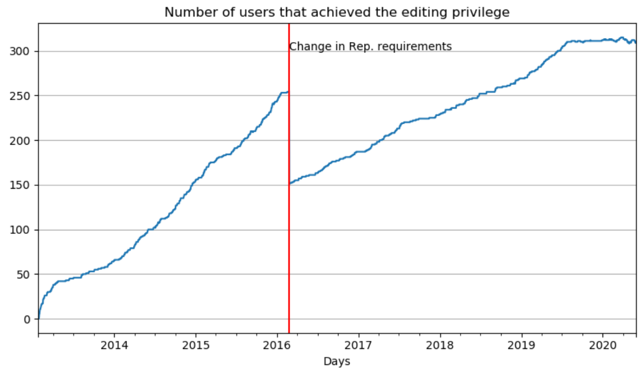

Figure 3: Number of users that have obtained control over the editing task. In February 2016, an increase in the requirement of points to obtain the privilege induced the loss of the privilege for some users.

	Periods active	Questions	Answers	Edits	Avg Quality	Avg Availability of Q.
count	9797.00	9797.00	9797.00	9797.00	9797.00	9797.00
mean	713.16	2.90	11.73	0.83	14.18	7985.31
std	722.51	26.88	94.31	15.81	1.62	5168.64
min	0.00	0.00	1.00	0.00	6.01	22.73
25%	92.00	0.00	1.00	0.00	13.43	3770.96
50%	414.00	0.00	1.00	0.00	13.92	7532.39
75%	1183.00	0.00	4.00	0.00	14.47	11277.32
max	2685.00	906.00	4173.00	1174.00	44.96	21744.97

Table 2: Descriptive statistics across active users. Columns report, in order, the number of periods spent in the platforms (it does not control for right censoring), the number of answers, questions, and edits made, the average answer's quality, and the mean number of questions available, on average, in a given day.

3.1 User characteristics

I construct a database to proxy for users' motivation. Users can choose what information to upload on their user pages and the type of information provided can be informative on the underlying motives of participation. The dataset contains the information of whether users are providing the location, a personal website, a Linkedin profile, and a full name. In addition to these dummies, I include measures from the biographical note that users can include, in particular the number of words and the number of links appearing. Table 3 and 4 report summary statistics on these variables for the whole sample of registered users and the sample of users in the panel respectively. It emerges that most users do not have a lot of information, but there is some heterogeneity.

	Share of users
has full name	34.17 %
has website	16.67 %
has location	31.93 %
has Linkedin	1.54 %
has bio note	25.82 %
has links in bio	4.57 %
Sample size	92,853

	net AboutMe	AboutMe	links AboutMe
mean	21.92	31.12	2.84
std	33.27	47.66	3.93
min	1.00	1.00	1.00
25%	5.00	6.00	1.00
50%	10.00	14.00	1.00
75%	25.00	36.00	3.00
max	535.00	542.00	55.00
Sample size	23,979	23,979	4,239

Table 3: Statistics of user characteristics. **(Left)** Share of users that have the given characteristic. **(Right)** Distribution of, respectively, number of words in the biographical note (net of *stopwords*), number of words in the biographical note (all), number of links in the biographical note. Right table statistics are conditional on observing a positive value of each measure. Sample include all users registered in the website at May 31^st, 2020

	Share of users
has full name	26.06%
has website	24.37%
has location	40.98%
has Linkedin	1.18%
has bio	39.33%
has links	6.71%
Sample size	9,797

	net AboutMe	AboutME	links AboutMe
mean	25.4	37.74	2.94
std	36.8	55.89	4.67
min	1.0	1.0	1.0
25%	5.0	7.0	1.0
50%	13.0	18.0	1.0
75%	30.0	45.0	3.0
max	340.0	510.0	55.0
Sample size	3,853	3,853	657

Table 4: Statistics of user characteristics, conditional on the user being part of the panel of active users. **(Left)** Share of users that have the given characteristic. **(Right)** Distribution of, respectively, number of words in the biographical note (net of *stopwords*), number of words in the biographical note (all), number of links in the biographical note. Right table statistics are conditional on observing a positive value of each measure. Sample include all users registered in the website at May 31^st, 2020

3.2 User Types

If the motives behind participation are strongly heterogeneous, then the incentive effects may differ across users. In such a context, a platform incentive design that targets the average consumer may not be maximizing participation. To address this concern, I use the user characteristics to identify types, so that the platform could 1) ex-ante assess the composition of the participants, 2) combine different incentive strategies to target different types of users.

The identification of types relies on the assumption that the type of information provided is informative on the broad motives of participation.[16]. I use the data summarized in table 3. It includes dummy variables taking value equal to 1 if the given type of information is provided, the variables with the number of words/links in the biographical description, which I bin in three categories each, and the year of registration.

The approach implemented is to use a K-Means clustering algorithm to cluster observation in a data-driven (or unsupervised) way. The challenges to address are twofold: first, the K-Means algorithm does not apply to categorical variables. Second, the algorithm requires as input the number of clusters to be identified, which ex-ante is not known. To address these challenges I adopt a Multiple Correspondence Analysis (MCA, similar to PCA but for categorical variables). The procedure transforms the data exploiting cross-frequencies tables of all variables, and outputs new variables (*components*) that aggregate information in a hierarchical way from the first to the last *component*. The new data, while capturing the same information of the original data, provides continuous variables, addressing the first issue. I then decide how many clusters should be selected iteratively. Pick an arbitrary number of clusters k, run the K-Means clustering, and plot the individual observations clustered in the k groups on the first two *components* plane. Repeat the procedure with $k' \neq k$ cluster and evaluate if, in the plot, the new distributions of groups better separate the observations in clusters. If observations do not separate in groups in the plot, then the evaluation must rely on some arbitrariness. This process leads to the identification of three types. More details on the procedure and on why I adopted this specific approach rather than alternatives are given in appendix A.3.

Tables 5 and 6 provide summary statistics of individual characteristics by type, for the whole sample of registered users and the sample of active users respectively. It is possible to notice that the main discriminant of types is the number of pieces of information provided. The largest group, including more than half of the registered users, displays little if any information and I define them as *Anonymous*. A second group includes information about themselves (location, biography) but not much information on their life outside the community. This is in contrast to users in the last group, who generally provide a personal website and sometimes a Linkedin profile. I refer to these two groups with the terms *Identifiable* and *Informative* respectively. It is relevant to notice that some variables are not informative about the types. To have a full name is quite homogeneous across types, and the year of registration is also orthogonal information.[17]

[16] The idea that users self-select in types by some of their choices in the platform is also adopted by Belenzon and Schankerman (2015), where they infer types from the choice of contributing to more or less open open-source software

[17] The year of registration is used in the analysis but not reported in the table.

user type	Num. users	Share of users who have ... full name	website	location	linkedin	bio	links
Anonymous	65134	35.31%	1.65%	9.04%	0.00%	1.73%	0.00%
Identifiable	23260	29.25%	47.18%	85.09%	0.00%	79.82%	2.88%
Informative	4459	43.13%	76.74%	88.99%	32.05%	96.08%	80.06%

user type	stat.	net AboutMe	AboutMe	links AboutMe
Anonymous	25%	9.00	15.00	
	50%	15.00	22.00	
	75%	25.00	37.00	
	count	1129.00	1129.00	
	max	208.00	397.00	
	mean	21.80	33.38	
	min	1.00	1.00	
	std	25.03	40.68	
Identifiable	25%	4.00	5.00	1.00
	50%	8.00	11.00	1.00
	75%	17.00	25.00	1.00
	count	18566.00	18566.00	669.00
	max	361.00	505.00	3.00
	mean	15.48	22.51	1.11
	min	1.00	1.00	1.00
	std	23.59	35.79	0.35
Informative	25%	19.00	24.00	1.00
	50%	33.00	44.00	2.00
	75%	61.00	83.00	4.00
	count	4284.00	4284.00	3570.00
	max	535.00	542.00	55.00
	mean	49.90	67.83	3.16
	min	1.00	1.00	1.00
	std	51.56	71.11	4.20

Table 5: User characteristics by user type. The first table reports the share of user, for each type, to have the given information displayed. The second table reports the distribution of the number of words (without and with stop-words) and of the number of link contained in the biographical note (if any).

3.3 Type's behavior

While types are not directly based on behavior, participation patterns are very different across them.[18] In this section, I present descriptive statistics of differences in behavior between types. I use the sample of active users from the panel, corresponding to the sample of table 6. In the sample, 5414 of type *Anonymous*, 3705 users are of type *Identifiable*, and 678 of type *Informative*.

[18] Some literature addresses unobserved heterogeneity by inferring types from observed actions (Arcidiacono and Miller 2011). In this paper, I do not adopt that approach for two reasons. First, I want the platform-designer to be able to assess the composition of the community ex-ante and in a simple manner. Second, since I observe an unbalanced panel of participation with censoring at the download date, inferring motives from observed actions may be biased by the selection of action I observe.

Badges

Do types differ in the collection of badges? Badges are virtual medals rewarding the accomplishment of a performance target. The accumulation of badges may suggest sensitivity to short term incentives. More challenging targets, i.e. silver and golden badges, point even more in this direction, as it is impossible to achieve them without tailoring behavior to the target. Figure 4 shows the average number of badges obtained by users in each group, where the vertical black bars are the standard errors of the means. It suggests that users more informative in their user pages are also the ones reacting more to short term incentives, type *Informative* obtaining more badges than type *Identifiable*, and type *Identifiable* than type *Anonymous*.

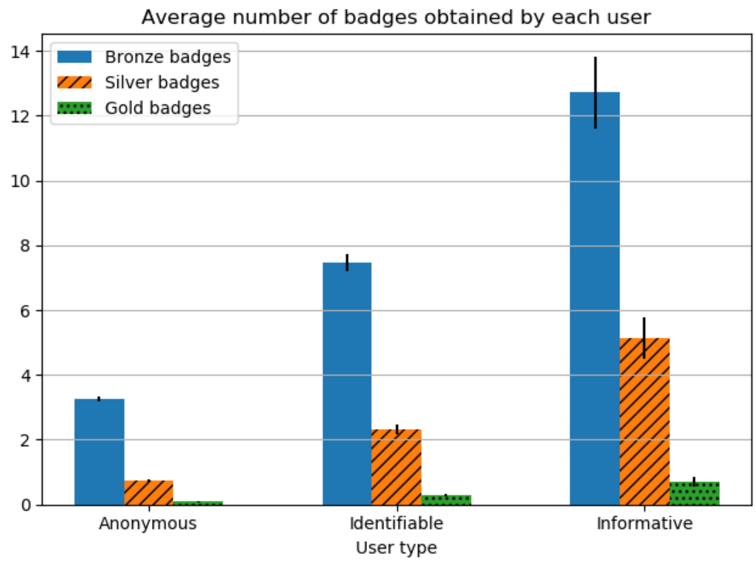

Figure 4: Average number of badges obtained by the active users of each type. Sample of users included in the panel. Vertical bars are standard error of the mean.

Time to reach the editing threshold

The types show heterogeneity also on the probability, at each point in time, of having reached the delegation threshold.
I estimate the survival function, where the failing event is the achievement of the threshold number of points. Since in the data I use the value of the threshold changed, I estimate two survival functions, one for the users that registered before the change, and one for users that registered when the threshold was already in its final value. Figure 5 shows the plot of the survival functions for each type. Users of the *Informative* obtain control the fastest.

Share of production

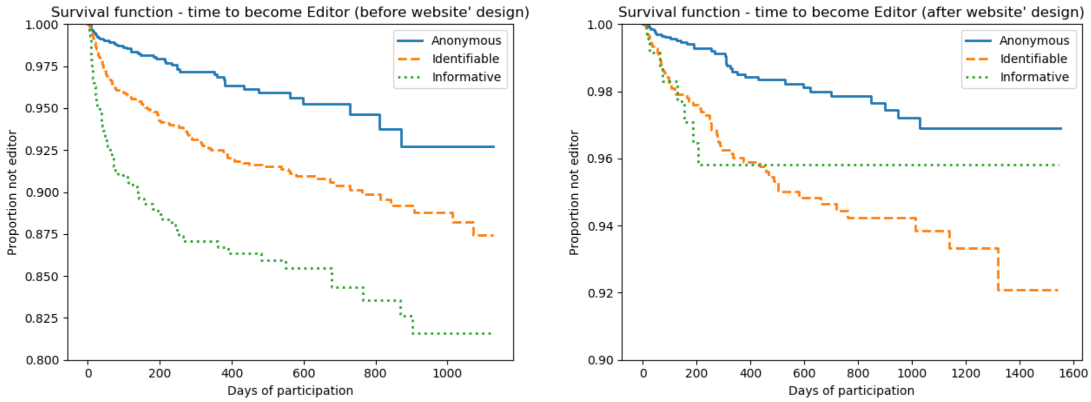

Figure 5: Survival function estimated on users who registered before the threshold change (left) and after the change (right). For the left graph, the data includes time series cut at the date when the reputation threshold changed.

Table 7 reports instead the total and average production by type. The marginal contribution of *Informative* users is the highest, followed by the one of type *Identifiable*. The most relevant observation is on direct edits, where the 678 users of type *Informative* made nearly 60% of the total. Similar patterns can be identified in figure 6, where it is plotted the share of total content produced each month by each type. It is possible to notice that *Informative* users reduced their contribution in answering while remaining the main editors of the website across time.

Participation to elections

Participation and win of an election may as well reveal information on the motives of participation. The participation in an election may signal in fact that the user has a specific commitment towards the community. To candidate in an election you need to have at least 300 reputation points, while to vote for candidates the requirement is of 150 points[19]. Figure 7 reports the number of candidates by type, and the number of winners. It is possible to notice that candidates are generally of type *Identifiable* or *Informative*, while elected users are mostly *Informative*.

3.4 Summary on types

Overall, the profile of each type emerges quite clearly from the descriptive evidence. The online community is in large part populated by *Anonymous* users, who are not particularly active in production. Low production implies a longer average time to achieve the delegation threshold. Nevertheless, the size of this group is such that it still contributes to nearly 30% of the total production of answers. On the contrary, *Informative*

[19]For more details, see https://stackoverflow.blog/2010/12/02/stack-exchange-moderator-elections-begin/

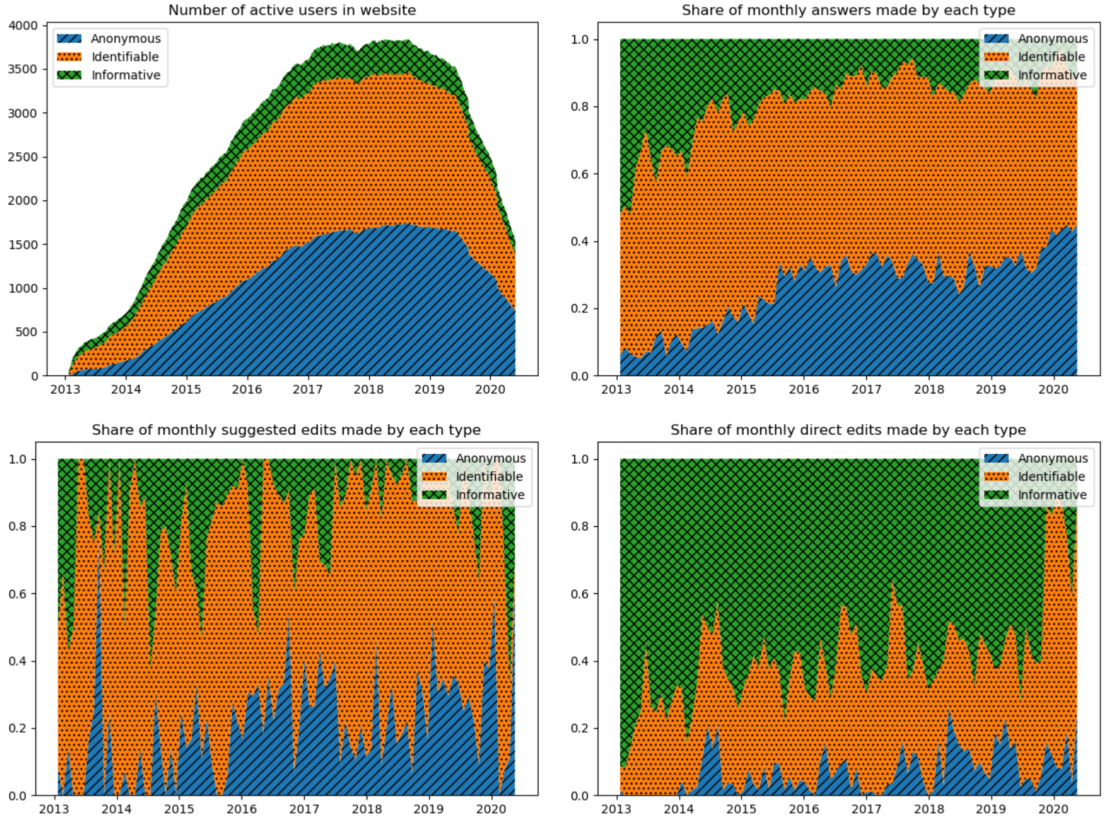

Figure 6: Time distribution of the number of active participants (top left) and of the share of content published by each type. Proceeding clockwise, graphs report the share of answers, the share of suggested edits, and the share of direct edits.

users are few members but the most active. They provide a lot of information on their profile, suggesting important extrinsic motives. They produce the most and provide the majority of the editing activity. Their very high activity may also justify the higher likelihood of winning elections. Finally, *Identifiable* users are in between: they provide some information about themselves, but no links or Linkedin profile, suggesting that they do not aim to signal outside of the platform. They contribute significantly, but are still achieving the delegation threshold in more time than *Informative* users.

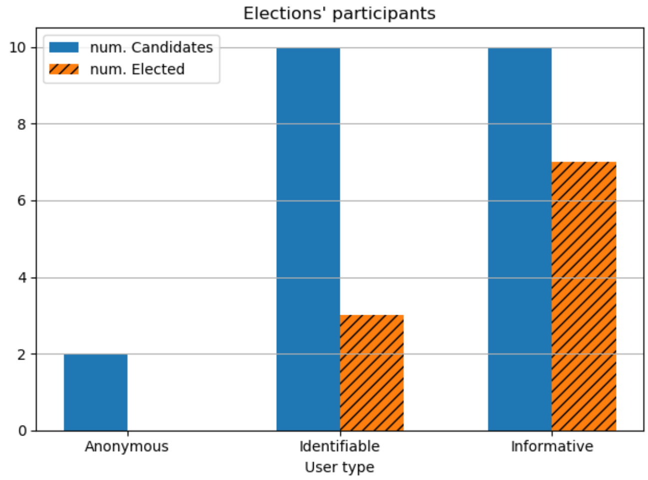

Figure 7: Number of candidates and number of winners of the elections, by type.

user type	Num. users	Share of users who have ... full name	website	location	linkedin	bio	links
Anonymous	5414	24.9%	2.4%	8.53%	0.0%	3.77%	0.0%
Identifiable	3705	25.32%	47.72%	80.08%	0.0%	80.54%	2.05%
Informative	678	39.38%	72.27%	86.43%	17.11%	98.08%	85.69%

user type	stat.	net AboutMe	AboutMe	links AboutMe
Anonymous	25%	10.00	16.00	
	50%	16.00	24.00	
	75%	28.00	43.00	
	count	204.00	204.00	
	max	208.00	397.00	
	mean	25.47	40.69	
	min	1.00	1.00	
	std	29.84	50.51	
Identifiable	25%	5.00	6.00	1.00
	50%	9.00	13.00	1.00
	75%	21.00	31.00	1.00
	count	2984.00	2984.00	76.00
	max	338.00	505.00	2.00
	mean	17.82	26.95	1.07
	min	1.00	1.00	1.00
	std	25.18	39.99	0.25
Informative	25%	23.00	32.00	1.00
	50%	40.00	56.00	2.00
	75%	71.00	104.00	3.00
	count	665.00	665.00	581.00
	max	340.00	510.00	55.00
	mean	59.39	85.23	3.18
	min	1.00	1.00	1.00
	std	57.62	85.84	4.92

Table 6: User characteristics by user type, for sample of active users in the panel. The first table reports the share of user, for each type, to have the given information displayed. The second table reports the distribution of the number of words (without and with stop-words) and of the number of link contained in the biographical note (if any).

Type	Num Users	num answers Total	Avg. per user	num suggested edits Total	Avg. per user	num direct edits Total	Avg. per user
Anonymous	5414	32511.0	6.00	309.0	0.06	465.0	0.09
Identifiable	3705	63500.0	17.14	836.0	0.23	2272.0	0.61
Informative	678	18915.0	27.90	264.0	0.39	4022.0	5.93

Table 7: Total and average users' production by type.

4 Reduced Form Analysis

In this section, I provide reduced-form evidence of a positive *static incentive* effect.[20] In other words, I test the hypothesis that users are more willing to contribute when endowed with authority and find that users are significantly more willing to contribute when they obtain more authority over an action. This result is confirmed by the literature on power, where "[...] Generally, research has shown that power increases an action orientation and, thus, leads directly to the taking of action for those who possess it [...] " (Sturm and Antonakis 2015). Only the action concerned by variation in authority sees a significant change. A comparable action does not increase or decrease significantly, suggesting the absence of both complementarity and substitutability effects.

The analysis proceeds by presenting initially the effect of removing the *static incentive*, by looking at participation when users lose authority, and then the effect of introducing the *static incentive*, observing behavior when users gain authority.

4.1 Loss of Authority

Due to a change in reputation requirements, users could lose the editing privilege. In practice, before February 2016 authority on editing was allocated when the user reached 1000 points, while after the requirements were 2000 points. Every user that on that date had a number of points in between 1000 and 2000 lost authority on editing.[21]

Users partly anticipated the change in reputation requirement. At a previous date, the *graduation* date, the platform stepped into its last phase of development, and users knew since then that the reputation requirements were going to change. To account for the anticipation, I select users participating in the website at the time of the *graduation* date, and exclude those registering later. I then select users at risk to be affected by the change in reputation requirements, i.e. users who had less than 2000 points at the *graduation* date. These users knew that, if they had 1000 points or may have reached 1000 points, but not 2000, they would have lost the editing privilege.

Figures 8 and 9 show the share of users, out of the considered sample, making a positive amount of edits and answers respectively. Different colors/patterns separate participants based on the number of points accumulated in the given week. The orange/striped sections identify the share of users that, if the change in reputation requirement would happen in that given week, would lose the authority.

It is possible to see that users who lose the privilege stop making edits, as the orange/striped sections disappear after the change in reputation requirement. In other words, the share of users who did not reach the new threshold before the change, but had already reached the previous threshold, do not participate in the next four months

[20]The *dynamic incentive* effect cannot be identified in reduced form with standard tools like difference-in-difference, regression discontinuity, etc. because of its dynamic nature: reduced form models cannot account for forward-looking behavior, and the identifying assumptions do not hold. Goes et al. (2016) claim to identify the dynamic incentive effect in reduced form relying on functional form assumptions and modifying the data to account for forward-looking behavior.

[21]Please refer to section 3 for more details.

after the change.[22] The share of users who have reached the new threshold or have never reached the previous threshold remains positive after the change (except for the first week after the change). The same pattern is not observed in answering behavior. The users who lost the privilege maintain a similar participation level after the change in reputation requirement, independently of the reputation point bracket to which they belong.

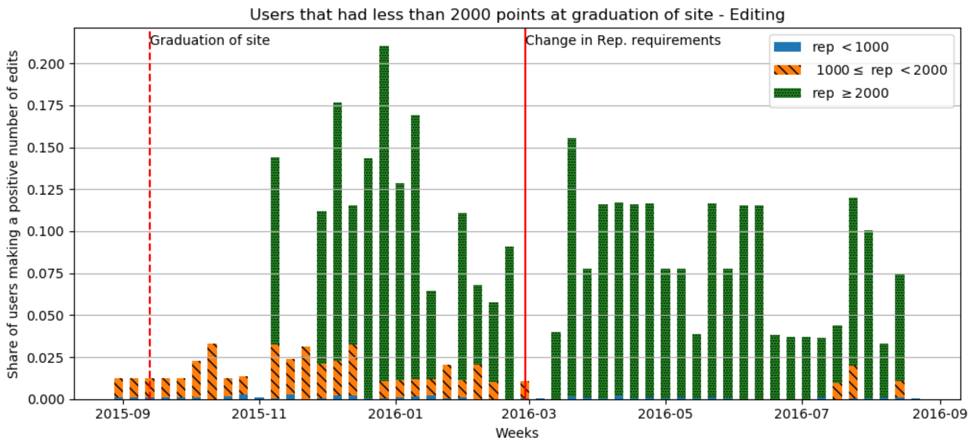

Figure 8: Number of users making a positive number of edits, out of the ones having less than 2000 points at the graduation week.

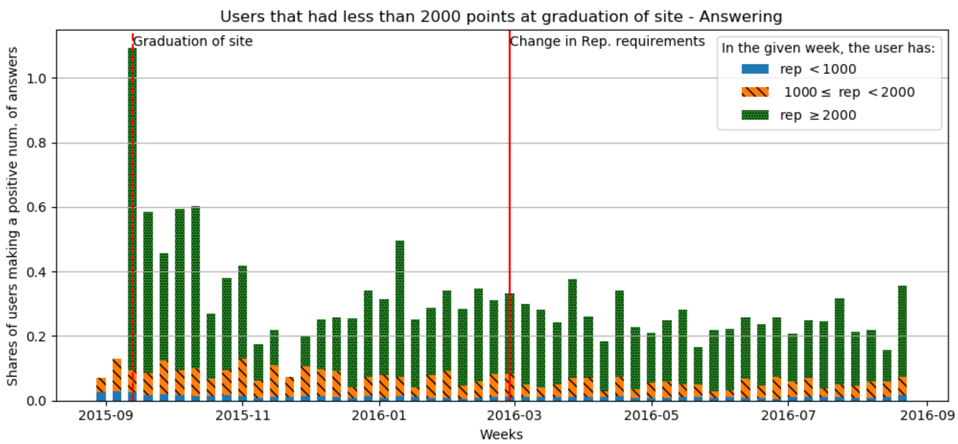

Figure 9: Number of users making a positive number of answers, out of the ones having less than 2000 points at the graduation week.

[22] This is not driven by the fact that everyone either reached the 2000 points or never reached the 1000 points. Figure 26 in the appendix shows that every week had a positive number of users with an amount of point between 1000 and 2000, even conditioning on the selected sample.

4.2 Gain of Authority

To test the hypothesis I also look at contribution behavior when users obtain authority. I exploit the discontinuity created by the allocation of authority and implement a regression discontinuity design.[23] The outcome variable is either the number of edits published or the number of comments published. Editing and commenting are comparable actions because 1) both are not main drivers of the accumulation of points 2) they share the purpose of improving or helping improve existing content. They differ anyway on the treatment status. Once users achieve the threshold number of points, they acquire authority in editing. At that same threshold instead, authority on commenting is not affected (users have already full authority on comments).

I estimate the following specification, where the running variable is the number of reputation points[24]

$$Y_{it} = \alpha_i + \gamma_t + \beta_{r_{it}-\bar{R}} + a_{m_{it}} + b_{c_{it}} + \varepsilon_{it} \tag{1}$$

where Y is either the number of edits or the number of comments made, based on which is the outcome of interest. α_i identifies the user fixed effect, γ_t the week fixed effect, r_{it} the number of reputation points that the user i has in period t (binned in 50-points intervals), and $\bar{R}$ the number of reputation points required to obtain authority on editing. Note that this value depends on the calendar date since it is set at 1000 points before February 2016 and at 2000 points after. The parameters of interest are the $\{\beta_{r-\bar{R}}\}\forall_r$, which identify the fixed effects of being $r - \bar{R}$ points distant from the threshold $\bar{R}$. Note an important remark: while these fixed effects are not period-specific, yet unit of observation is still weeks. This means that the fixed effects capture a **weekly average** number of edits (or comments) at a given reputation point interval.[25] Finally, I include a dummy equal to 1 if the user is an elected moderator in time t, and a dummy equal to 1 if the user is a candidate in a moderators' election in time t (a_m and b_c) respectively. ε_{it} is an error term.

In figure 10 I report the estimates for $\{\beta_{r-\bar{R}} | r - \bar{R} \in [-6, 6]\}$. The outcome variable is standardized to enable comparisons between editing and commenting. On the x-axis then, the value at 0 identifies the fixed effect estimate of having a number of points included in $[\bar{R}, \bar{R} + 50)$, at 1 the fixed effect of having between $\bar{R} + 50$ and $\bar{R} + 100$ points, and so on.

[23] Sometimes called event study with two-way fixed effects.

[24] I do not use time as running variable for two main reasons. The first is that users are aware of the allocation rule, so they may adjust their behavior to receive the privilege sooner or later. The treatment date is then endogenous. The second reason is more technical: Sun and Abraham (2020) show that in an OLS regression with individual fixed effects, time fixed effects, and relative time fixed effects (i.e. fixed effects for the nth period before or after the treatment), trends before and after are not identified. If the treatment would be completely unexpected, then the researcher would just be interested in the effect at the treatment time. Anyway in my context there is anticipation, and it is then relevant to identify the trends.

[25] If the observational units were the reputation points intervals themselves, the total amount of editing would depend on the time "spent" on each interval, and, as a consequence, on the rate of answering and of accumulation of points.

Identification of the effect relies on the assumption that, when users have a number of points in the neighborhood of the threshold, the only variable affecting behavior is the acquisition of the privilege. It is possible to see that the number of edits made increases significantly when users have a number of reputation points just above the threshold. The pattern of the number of comments made does not seem, instead, to depend on the threshold. This comparison suggests that authority increases the willingness to participate only for the action for which the users receive more authority, and there are no clear spillovers over the effort made in similar actions. In section A.4 of the appendix, I provide some robustness checks, that look at effort levels around the precedent and the following privilege.

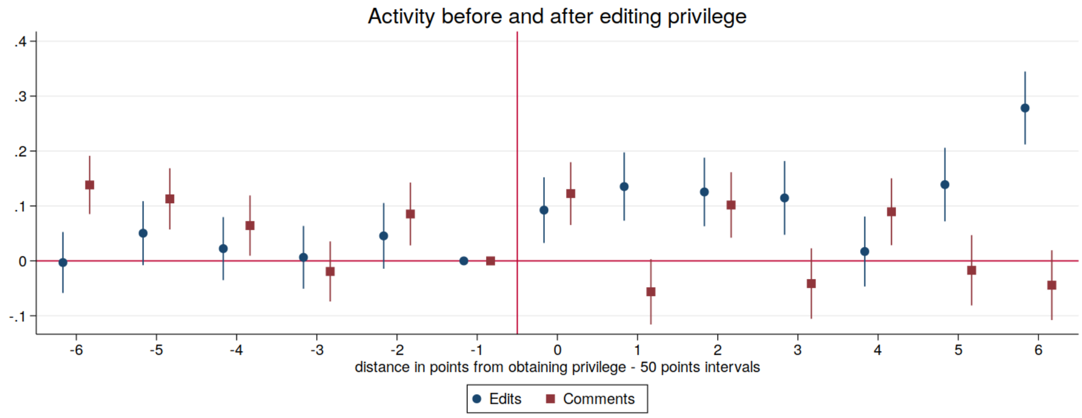

Figure 10: Estimates for the fixed effects of being in the nth reputation point interval above or below the threshold, that is the parameters $\{\hat{\beta}_{r-\bar{R}}|r - \bar{R} \in [-6, 6]\}$ in the regression specification 1. Vertical bars are confidence intervals. Outcome variables are the standardized number of edits (circles) and number of comments (squares).

Heterogeneity

Do these effects differ across the different types of users? Figure 11 (graph above) reports the estimates for the editing activity for the different types of users separately. Note that outcome variables are standardized within each user type. Estimates seems to suggest that the effect is strongest for *Anonymous* users, who increase the number of edits right after the threshold. *Informative* users increase contribution significantly when they have more than 150 points on top of the threshold requirement. It would suggest a long term effect of authority. The identifying assumptions are anyway stronger moving away from the threshold, and the causal interpretation is less reliable. Even accounting for heterogeneity, the number of comments still do not seem to depend on the threshold. Figure 11 (graph below) reports the estimates by type when the outcome variable is the number of comments.

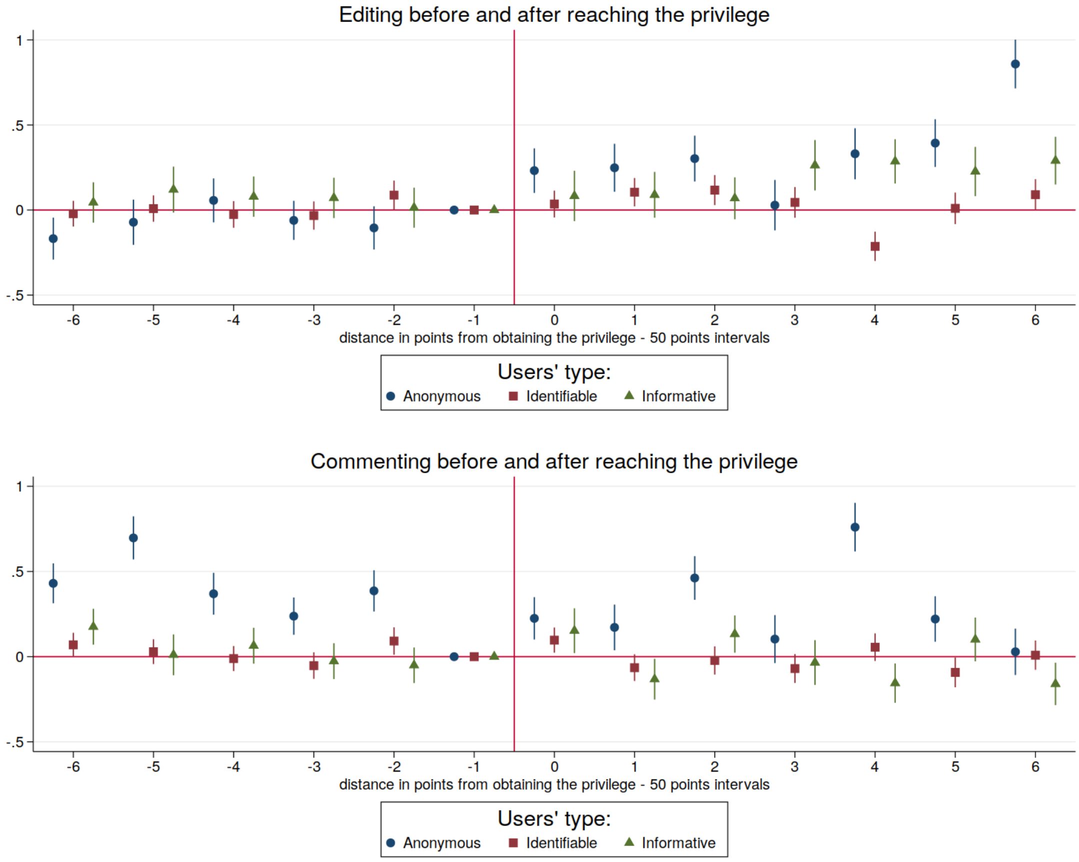

Figure 11: Estimates for the fixed effects of being in the nth reputation point interval above or below the threshold, that is the parameters $\{\hat{\beta}_{r-\bar{R}} | r - \bar{R} \in [-6, 6]\}$ in the regression specification 1. The outcome variable is the standardized (by type) number of edits (graph above) or the standardized (by type) number of comments (graph below). Vertical bars are confidence intervals. Shapes differentiate the types *Anonymous* (circles), *Identifiable* (squares), and *Informative* (triangles).

5 Dynamic Discrete Choice Model

The reduced-form evidence tests for the presence of *static incentive* effects. Nevertheless, it has multiple limitations. First of all, it does not allow us to compare the incentive effect of allocating authority relative to other types of motives. Second, it does not test and does not quantify the *dynamic incentive* effect. Finally, it does not allow to simulate counterfactual behavior.

To overcome these limitations, I develop a dynamic discrete choice model that studies inter-temporal choices and accounts for forward-looking behavior. Dynamic discrete choice models estimate preference parameters based on the concept of *revealed preferences*, that is the assumption that choices are the outcome of (random) utility maximization and, as such, provide information on users' preferences. In the context of participation in online communities, users choose their effort in contributing to the platform. Their choice depends on the cost of effort, net of choice's intrinsic utilities, and expected future benefits. Benefits could be, for instance, a certain number of reputation points or the achievement of authority.[26]

5.1 Model setup

At each period, the user decides whether to participate in the online community and, if she does, she decides effort levels in two tasks, answering and editing. Effort is defined as a combination of quantity and quality of answers, and quantity of edits. An action choice in period t is then a vector:

$$\boldsymbol{\alpha_t} = \begin{bmatrix} A_t \\ Q_t \\ E_t \end{bmatrix} \ni \mathcal{A}$$

where A identifies the quantity of answers, Q the average quality of answers, and E the quantity of edits. $\mathcal{A}$ represents the choice set, including all possible combinations of effort levels in the two tasks.[27] Choices affect the utility in two ways: first, the user pays the cost of effort, net of all the benefits that the actions provide in the given period. Second, choices affect the transition and future realizations of states.

The net cost of effort is specific to the action made. The net cost of answering, for a user i in period t, is defined as:

$$C_{it}^{A} \equiv Q_{it} + A_{it}^{scarsity_{it}}$$

where $A^{scarcity}$ is the number of answers made raised to a measure of scarcity of questions to answers. The variable *scarcity* captures the inverse of the availability of questions to

[26] Note that the application of dynamic discrete choice models to this context is conceptually similar to works that study dynamic investment decisions with discrete choice models. A typical application is to human capital investment decisions. Examples of this literature are Arcidiacono, Aucejo, Maurel, and Ransom (2016), De Groote (2019).

[27] Effort levels are discretized to have 21 possible combinations of quantity of answers, quality of answers, and quantity of edits. More details are provided in appendix A.6.

answer. This variable is user-specific as it accounts for the topics the user can address. More details about the construction of this variable are available in appendix A.2. The *scarcity* variable takes values in $[1, \infty]$ so that when there are many questions to answer, the cost tend to be linear, while with fewer questions available the cost becomes more and more convex in the quantity of answers.

The net cost of editing is instead the number of edits that the user decided to make:

$$C^E_{it} \equiv E_{it}$$

The motives of participation directly included in the utility function are the ones directly related to the accumulation of reputation points: the number of accumulated reputation points themselves (R), a dummy equal to 1 if the individual has control over editing and 0 otherwise ($Authority$), and a variable with the cumulative number of privileges obtained by the user ($cumT$ for "cumulative thresholds"), so to control for the possibility that users do not value authority per se, but rather virtual vertical rewards (either perceiving them as virtual promotions or as sequential targets in a game). All other motives driving participation are assumed to not be correlated with the accumulation of points. They are captured by a choice-specific preference shock (ε). Note that unobserved motives may anyway affect the cost of participation. The marginal cost of answering and editing is then net of all benefits deriving from participation. These include the intrinsic value of participation ("having fun" in contributing per-se), altruism, and reciprocity.

The per period flow utility of user i is then defined as:

$$U_{it} = \beta_0 R_{it} + \beta_1 C^A_{it} + \beta_2 C^E_{it} + \beta_3 cumT_{it} + Authority_{it} \left(\beta_4 + \beta_5 C^A_{it} + \beta_6 C^E_{it}\right) + \varepsilon_{it}. \quad (2)$$

The parameters β_4, β_5, and β_6 together capture user's marginal utility from the acquisition of authority and, as a consequence, how the user responds to the dynamic incentive effect. The *static incentive* effect is instead captured by the coefficients β_5, and β_6. They capture, respectively, a change in the willingness to make answers and edits, after the user achieves authority in editing. β_5 corresponds to a spillover effect on answering, while $beta_6$ a direct effect of participation on the task endowed with authority. The latter coefficient captures the same incentive effect that was observed in the reduced form analysis, with the difference that $beta_6$ is an average for the change in participation for all reputation levels greater than the threshold.

The user chooses an optimal sequence of choices to maximize the total sum of the discounted utility from all her periods of participation. Let $\boldsymbol{\alpha^*} \equiv \{\boldsymbol{\alpha_t}\}_{t<T}$ be the sequence of optimal choices, where T is her last period of participation in the website. Then she chooses:

$$\boldsymbol{\alpha^*} = \arg\max_{\boldsymbol{\alpha}} \mathbb{E}\left[\sum_{t=1}^{T} \delta^{t-1} U_{it}(\boldsymbol{\alpha_t})\right]$$

Timing of a period

As represented in figure 12, the timing is the following. 1) The agent observes the values of the states realized at the end of the previous period. This includes the total number of reputation points she has obtained, the number of points she expects to receive from the past effort, how many privileges she has collected, whether she has already obtained control over editing or not, the availability of questions to answer, and her experience in terms of time spent in the website and number of contributions. She then forms beliefs over the value of the states that may realize in the next periods, conditional to the possible choices she could make. 2) She makes an effort decision over two tasks, maximizing her conditional value function. 3) The flow payoff realizes, and 4) at the end of the period the new value of the states realizes.

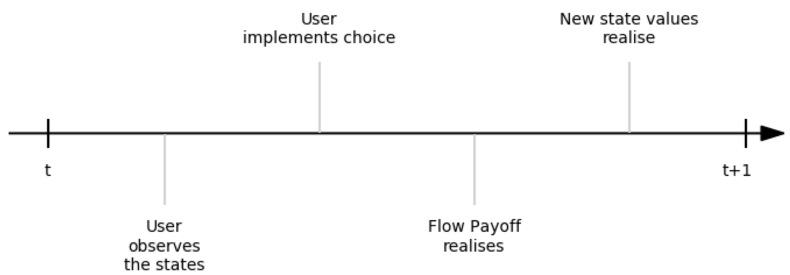

Figure 12: Timing of a period

5.2 Beliefs

Users form beliefs and expectations over the evolution of the state space, given the choices they make.

Evolution of reputation points

The points that the user expects to receive in the future depend on current and past actions, in particular on the choice of quantity and quality of answers, as well as of edits in case they are suggested. Points depend as well on edits that the user receive from other community members.[28]

Consider for simplicity the beliefs that the user form in the first period of participation t_0. She considers to choose a triplet $\{A, Q, E\}$ of, respectively, number of answers,

[28]For a detailed explanation of the rules to obtain points, please refer to figure 25 in the appendix

average quality of answers, and number of edits. The number of received edits on an answer at publication day is modeled as a Poisson process where the mean depends on the answer's quality and the user's experience. Similarly, also the number of up-votes and down-votes arriving on the answer at the creation date are modeled as Poisson processes. Let j identify a given answer that the user published at a publication day t_0, which is also the first day of participation of the user. Then, the following random variables are, respectively, the number of modification that in period t_0 the answer j receives, the number of up-votes and the number of down-votes, also received by j in t_0:

$$\begin{aligned} \text{Received Edits}_{j,t_0} &\sim \mathscr{P}(\lambda_{E,j,t_0}), \\ \text{Up-votes}_{j,t_0} &\sim \mathscr{P}(\lambda_{U,j,t_0}), \\ \text{Down-votes}_{j,t_0} &\sim \mathscr{P}(\lambda_{D,j,t_0}). \end{aligned}$$

The expected values of these random variables, are given by:

$$\lambda_{E,j,t_0} = \exp\left(\beta_0 + \beta_1 Q_{t_0} + \boldsymbol{EXP}_{t_0}\boldsymbol{\beta_2}\right) \tag{3}$$

$$\lambda_{U,j,t_0} = \exp\left(\gamma_0 + \gamma_1 Q_{t_0} + \gamma_2 \lambda_{E,j,t_0} + \boldsymbol{EXP}_{t_0}\boldsymbol{\gamma_3}\right) \tag{4}$$

$$\lambda_{D,j,t_0} = \exp\left(\delta_0 + \delta_1 Q_{t_0} + \delta_2 \lambda_{E,j,t_0} + \boldsymbol{EXP}_{t_0}\boldsymbol{\gamma_3}\right) \tag{5}$$

where $\boldsymbol{EXP}$ is a vector of variables capturing user's experience. Specifically it includes the number of days for which the user has been participating in the website, and the cumulative number of answers that she has published.

If the user, in its first period of participation, published A_{t_0} answers, then he will expect to receive by the end of the period:

$$\begin{aligned} \Lambda_{U,t_0} &= A_{t_0} \times \lambda_{U,j,t_0} \\ \Lambda_{D,t_0} &= A_{t_0} \times \lambda_{D,j,t_0} \end{aligned}$$

Which are, respectively, the total expected amount of up-votes and down-votes.

Finally, the number of approved suggested edits is modeled as a binomial distribution:

$$\text{ApprovedEdits}_{t_0} \sim \mathscr{B}(E_{t_0}, \pi)$$

The expected number of points that the user expects to receive at the end of period t_0 is given by:

$$\mathbb{E}[\rho_{t_0}|\boldsymbol{\alpha}_{t_0}] = 10 \times \Lambda_{U,t_0} - 2 \times \Lambda_{D,t_0} + 2 \times \pi \times E_{t_0}.$$

The answers produced in period t_0 may as well induce arrival of up-votes and down-votes in the next periods. This is modeled deterministically. Let Δt be the number of days passed from the publication day, such that if $t = t_0 + 1$, then $\Delta t = 1$. Then:

$$\begin{aligned} \lambda_{U,j,t_0+\Delta t} &= \lambda_{U,j,t_0} \times \exp\left(\frac{-\Delta t}{\tau_U}\right) \\ \lambda_{D,j,t_0+\Delta t} &= \lambda_{D,j,t_0} \times \exp\left(\frac{-\Delta t}{\tau_D}\right) \end{aligned}$$

$\lambda_{U,j,t_0+\Delta t}$ being the expected number of up-votes that the answer j, published in t_0, receives in period $t_0 + \Delta t$, and similarly for down-votes. τ_U and τ_D are parameters.

Given these assumptions, effort induces the arrival of a number points, which is decreasing in time. If the user chooses positive effort in several periods, these processes aggregate. The user expects to receive in the future an amount of points resulting from all present and past efforts. In general, the expected number of up-votes and down-votes arriving at the end of a given period y are:

$$\Lambda_{U,t} = \Lambda_{U,t-1} \times \exp\left(\frac{-1}{\tau_U}\right) + A_t \times \lambda_{U,j,t}(Q_t)$$
$$\Lambda_{D,t} = \Lambda_{D,t-1} \times \exp\left(\frac{-1}{\tau_D}\right) + A_t \times \lambda_{D,j,t}(Q_t)$$

and the expected number of points arriving at the end of the period is:

$$\mathbb{E}[\rho_t | \{\boldsymbol{\alpha}_{\tilde{t}}\}_{\tilde{t} \leq t}] = 10 \times \Lambda_{U,t} - 2 \times \Lambda_{D,t} + 2 \times \pi \times E_t.$$

To conclude, let R_t be the cumulative number of points that the user observes to have at the beginning of period t. Then the user expect to have, at the end of the period:

$$\mathbb{E}[R_{t+1} | R_t, \{\boldsymbol{\alpha}_{\tilde{t}}\}_{\tilde{t} \leq t}] = R_t + \mathbb{E}[\rho_t | \{\boldsymbol{\alpha}_{\tilde{t}}\}_{\tilde{t} \leq t}]$$

Evolution of the Experience variables

The variables for users' experience evolve in a deterministic way. The number of days of participation in the platform increases by one unit each period, while the cumulative number of answers published increases based on the choice of quantity of answers published.

Evolution of the Scarcity variable

The availability (or scarcity) of questions to answer evolves in an exogenous way, based on the general trend in the platform. Let *avail* be the variable capturing the number of available questions in the platform. Then:

$$avail_{it} = avail_{it-1} + \nu_1$$

where ν_1 is identified in reduced form from the linear regression:

$$avail_t = \nu_0 + \nu_1 t + \epsilon_t$$

As shown in the graphs in the appendix A.2, the availability of questions to answer increases monotonically over time. Users then expect a steady increase, given by an estimated parameter. Note that the rate of increase in availability is not topic-specific.

5.3 Identification

The identification of preference parameters relies on the concept of *revealed preferences*. Since choices affect the value of the states, observed choices are informative on what the user cares about. In Stack Exchange, choices affect users' utility in two ways. First, they have a direct impact on the present utility. Direct effects on utility include the cost of effort and the intrinsic value of contributing. They are captured in the utility specification by C^A and C^E. Marginal utilities of these direct effects are identified as in a standard static conditional logit model. Everything else equal, a higher cost of effort would imply that the user takes the action less frequently. Beliefs do not play a role as the costs and intrinsic values do not affect future utilities. Second, choices induce returns in future periods that may affect future utilities, as the arrival of points and the achievement of authority. These returns do not have any impact on the utility that the user receives in the same period that she makes her contribution choice. In this case, the choice affects only the discounted future payoff. The identification of the marginal utilities of these returns is strictly based on variation across value functions. If users value obtaining these returns, they will be more willing to choose an effort level that allows to obtain them. That choice would then be observed with a higher frequency. It is relevant to note that the parameters $beta_0$, $beta_3$, and $beta_4$ are not identified in a static model.

The computation of the value functions under the different possible choices relies on a technique called *finite dependence* (Arcidiacono and Miller (2011)).[29] This approach reduces substantially the computational burden since it allows the estimation without a full solution of the model. The computation of the value functions requires the evaluation of the future expected utility for only a few periods ahead. This approach has anyway a drawback. Whenever returns are not smooth but are step functions, which is the case of *cumT* and *Authority*, their marginal utility is identified only when the user can obtain the given return in the few periods ahead used for the computation of the value function. In practice, only the choices of users who are close to reaching the threshold(s) identify the marginal utility of *cumT* and *Authority*. For these users, certain levels of effort will allow them to obtain a privilege in the next periods, but others not. The observation of a significant increase in effort when users have a number of points just below the threshold identifies a positive utility for the acquisition of the privilege.

5.4 Estimation

The estimation proceeds on several steps. First, I set the discount factor at 0.95. Second, I estimate in either reduced-form or nonparametrically all parameters not appearing in the utility function. This includes the probability that a suggested edit is approved, the rate of arrival of new questions, the parameters to predict return for given levels of effort and experience, and the decay rate for the returns of effort. Third, I estimate the preference parameters following Arcidiacono and Miller (2011). The estimation relies

[29] The work by Arcidiacono and Miller (2011) roots on a large econometric literature. Seminal papers are the works by Rust (1987), Hotz and Miller (1993), and Magnac and Thesmar (2002).

on the conditional logit assumption, assuming that the idiosyncratic preference shocks follow an extreme value Type 1 distribution. The derivation of the log-likelihood function is presented in appendix A.5. The algorithm used preserves computational feasibility even without binning the state variables, i.e. without reducing the dimensionality of the state space.

6 Results

6.1 First stage estimate of reduced-form parameters

When users decide what action to take, they form beliefs on the arrival of points next period. For a given amount of answer and given quality, they first predict the number of edits that they would receive in the publication date, and then the number of up-votes and down-votes that their content can receive next period.

The expected number of edits made on an answer, excluding edits made by the author of the answer, is modeled as in equation 3 and table 8 reports the different specification estimates (the specification used in the structural model is the number 2). It is possible to notice that experience is correlated negatively with the arrival of edits. It implies that experience captures some skills that the quality variable is not able to measure: even controlling for quality, experienced users produce content that needs to be corrected less.

	(1)	(2)	(3)	(4)	(5)
Received Edits	Poisson	Poisson	Poisson	Poisson	OLS
Answer Quality	-0.0135*	-0.00178	-0.00414	-0.00178	-0.000224
	(-2.29)	(-0.30)	(-0.70)	(-0.20)	(-0.64)
Experience: num Answers		-0.000382***	-0.000389***	-0.000382**	-0.00000871***
		(-9.59)	(-9.79)	(-2.71)	(-4.94)
Experience: days in platform		-0.000609***	-0.000584***	-0.000609***	-0.0000208***
		(-15.85)	(-14.59)	(-8.07)	(-9.30)
_cons	-2.946***	-2.788***	-2.504***	-2.788***	0.0598***
	(-33.72)	(-32.12)	(-25.31)	(-20.59)	(10.40)
N	118552	118552	118552	118552	118552
Year FE	NO	NO	YES	NO	NO
std. err. clustered at author	NO	NO	NO	YES	YES

t statistics in parentheses
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 8: Estimates for beliefs over arrival of edits on publication day, given answer's publication quality and experience.

Once users have expectations of the number of edits they will receive, they predict how many up-votes and down-votes their content will receive in expectations. Parameter estimates of equations 4 and 5 are reported in table 9 and 10 respectively. As expected, higher quality correlates with more up-votes and fewer down-votes. Received edits have a positive coefficient in both cases. One explanation is that edits improve quality, inducing more up-votes, but, at the same time, users may want to penalize content that was of bad quality. Finally, more experienced users can expect more up-votes and fewer down-votes.

	(1)	(2)	(3)	(4)	(5)
Num Up-Votes	Poisson	Poisson	Poisson	Poisson	OLS
Answer Quality	0.0528***	0.0513***	0.0463***	0.0463***	0.0887***
	(69.61)	(67.06)	(59.88)	(9.58)	(8.84)
Received Edits	0.457***	0.485***	0.472***	0.472***	0.935***
	(55.61)	(58.79)	(57.26)	(20.99)	(15.46)
Experience: num Answers		0.0000465***	0.0000332***	0.0000332	0.0000563
		(11.79)	(8.29)	(0.61)	(0.58)
Experience: days in platform		0.000112***	0.000271***	0.000271***	0.000372***
		(24.04)	(52.35)	(7.17)	(6.34)
_cons	-0.414***	-0.469***	-0.101***	-0.101	0.532**
	(-35.47)	(-39.59)	(-6.87)	(-1.10)	(2.97)
N	118552	118552	118552	118552	118552
Year FE	NO	NO	YES	YES	YES
st. err. clustered at author	NO	NO	NO	YES	YES

t statistics in parentheses
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 9: Expected number of up-votes arriving on publication day

When users choose effort, they form beliefs about the number of up-votes and down-votes not only of the next period but also of the periods following the next. If effort directly affects the number of up-votes and down-votes of the next period, after the next they decrease deterministically following an exponential function, as shown in figures 13 and 14.[30] The model was estimated via a non-linear fit. Table 11 reports the estimates. In the figures, the red dots are the data values for λ_U and λ_D, and on the x-axis is reported Δt. The figures also compare the chosen model (exponential) with alternatives and present the decay both at daily and weekly level.

Finally, the last parameter estimated in the first step is the rate of increase in the availability of questions along time. Since the community is increasing, and some questions remain unanswered, then users expect a steady increase in availability. Estimates are reported in table 12.

6.2 Flow payoff parameters

Tables 14 and 15 present the flow payoff parameters obtained from the dynamic discrete choice model. The former presents parameter estimates when the utility function does

[30]The functional form is the standard function to model the amplitude of the cycles of a pendulum. Users' effort corresponds to the strength that starts the oscillation.

Num Down-votes	(1) Poisson	(2) Poisson	(3) Poisson	(4) OLS
Answer Quality	-0.0517***	-0.0488***	-0.0488***	-0.00434***
	(-14.98)	(-13.27)	(-7.31)	(-7.53)
Received Edits	0.732***	0.665***	0.665***	0.107***
	(27.00)	(24.28)	(18.98)	(12.44)
Experience: num. Answers		-0.000236***	-0.000236*	-0.0000156**
		(-10.85)	(-2.44)	(-3.10)
Experience: days in platform		-0.000239***	-0.000239***	-0.0000202***
		(-10.79)	(-4.32)	(-4.78)
_cons	-1.664***	-1.528***	-1.528***	0.169***
	(-32.97)	(-28.44)	(-14.74)	(17.97)
N	118552	118552	118552	118552
std. err. clustered at author	NO	NO	NO	YES

t statistics in parentheses
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 10: Expected number of down-votes arriving on the publication day

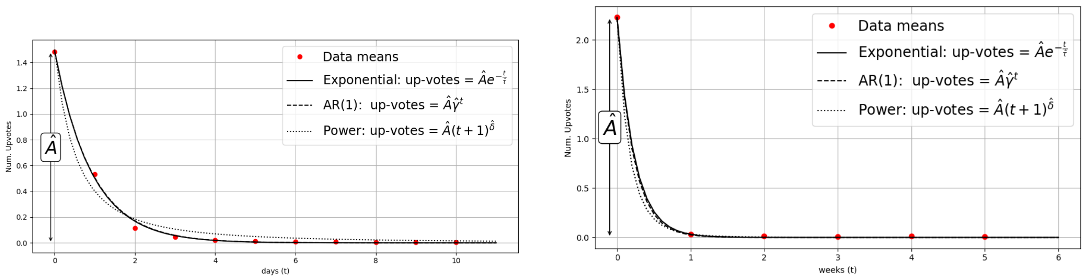

Figure 13: Return in up-votes from published content

not include the interaction terms i.e. does not allow that authority to affect the willingness to participate in answering and/or editing. This specification aims to omit the sensitivity to the *static incentive*, to focus on the *dynamic incentive*. Under this specification, the coefficient of *Authority* captures the marginal utility from the acquisition of authority. A higher parameter implies a higher willingness to reach the threshold and, therefore, a stronger *dynamic incentive* effect. It is possible to notice that *Anonymous* and *Informative* users value to obtain authority, while the threshold does not motivate *Identifiable* users. Scaling the parameters by the marginal values of points provides a more concrete quantification of the value of authority. Table 13 reports the value of authority in terms of points for each user type separately. It also includes the average number of posts (including questions and answers) that a user of the given type had to do to achieve that number of points.

Estimates suggest that only *Anonymous* and *Informative* users are sensitive to the

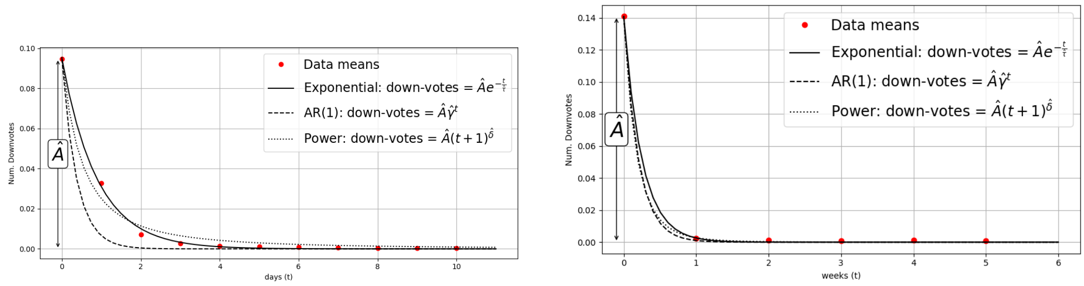

Figure 14: Return in down-votes from published content

period length	τ_U (up-votes)	τ_D (down-votes)
day	0.91	0.89
week	0.23	0.25

Table 11: Estimates of the parameters for the rate of decay on the arrival of up-votes and down-votes on past answers.

dynamic incentive.

Table 15 reports estimates for the flow utility parameters when the specification includes interaction terms of the variable authority with the net cost of participation. The coefficients of *CA x Authority* and *CE x Authority* capture the sensitivity to the *static incentive*. The former identifies possible changes in the willingness to answer questions and the latter capture possible changes in the willingness to make edits. It is possible to notice that *Anonymous* users are less willing to contribute to answering after they reach the threshold. There are two possible interpretations: either they lose interest in participating because their main motive was the achievement of authority, or the substitute answering with editing. The effect is anyway small: *Anonymous* are 1% less willing to make answers. The other types of users are not affected in answering by the achievement of the threshold. On the contrary, all users are significantly more willing to make edits. *Anonymous* users are 8% more willing to make edits, *Identifiable* users 4%, and *Informative* users 5%. Besides this positive *static incentive* effect on editing, the cost of participation remains high for all users.

period length	Number of additional available question next period
day	13.68
week	95.76

Table 12: Estimates of increase in availability of answers each period

User type	coef. authority	value in points	value in actions (avg)
Anonymous	1.5394	252 points	33 posts
Identifiable	0.1702	30 points	4 posts
Informative	1.4503	329 points	28 posts

Table 13: Marginal value of acquiring authority, by type. Proceeding from left to right, the table reports the parameter estimates for the marginal utility of authority, it's counterpart value in terms of points (parameter scaled by the coefficeint of R), and the average number of posts that a user of the given type needed to make to achieve those points. Posts include answers and questions.

Variables	(no Heterogeneity)	(*Anonymous*)	(*Identifiable*)	(*Informative*)
R	0.0074***	0.0061***	0.0056***	0.0044***
	(0.0001)	(0.0005)	(0.0002)	(0.0003)
CA	0.0004*	-0.3669***	0.00003	0.0007**
	(0.0002)	(0.0192)	(0.0004)	(0.0002)
CE	-0.6133***	-3.3660***	-4.4860***	-2.0967***
	(0.1661)	(0.6046)	(0.3161)	(0.2319)
cumT	-0.8409***	-0.4032***	-0.7842***	-0.8019***
	(0.0205)	(0.0310)	(0.0276)	(0.0548)
Authority	1.2052***	1.5394***	0.1702	1.4503**
	(0.1207)	(0.3577)	(0.2536)	(0.5118)
N. users	9,783	3,700	5,407	676
Sample size	991,657	471,837	407,098	112,722

$* \ p < 0.05$, $** \ p < 0.01$, $*** \ p < 0.001$

Table 14: Estimates for the flow payoff parameters considering the whole sample, or estimating separately for each type of user. Standard errors in parenthesis

Variables	(no Heterogeneity)	(*Anonymous*)	(*Identifiable*)	(*Informative*)
R	0.0069***	0.0064***	0.0057***	0.0045***
	(0.0001)	(0.0005)	(0.0002)	(0.0004)
CA	-0.0001	-0.3563***	.00005	0.0007***
	(0.0008)	(0.0196)	(.0006)	(0.0002)
CE	-10.3311***	-7.9549***	-6.1724***	-5.7740***
	(0.4979)	(0.8927)	(0.4051)	(0.4757)
cumT	-0.7745***	-0.4177***	-0.7855***	-0.7681***
	(0.0206)	(0.0322)	(.028)	(0.0563)
Authority	1.3162***	1.5223***	0.1713	1.4709***
	(0.1203)	(0.3577)	(0.2535)	(0.5118)
CA x Authority	0.0609***	-0.0048***	-0.0018	-0.0008
	(0.0036)	(0.0016)	(0.0011)	(0.0014)
CE x Authority	12.2064***	0.6338***	0.2507***	0.2703***
	(0.5247)	(0.0593)	(0.0308)	(0.0274)
N. users	9,783	3,700	5,407	676
Sample size	991,657	471,837	407,098	112,722

$* \ p < 0.05$, $** \ p < 0.01$, $*** \ p < 0.001$

Table 15: Estimates for the flow payoff parameters for the whole sample or by type of user. The specification includes interaction terms of the net costs of actions with the control dummy. Standard errors in parenthesis

7 Counterfactual Analysis: Incentive effect of delegation on contributions

The estimated flow payoff parameters allow predicting behavior under different delegation designs. Via counterfactual analysis, I provide evidence of the trade-off faced by the platform when designing the allocation of authority. In particular, I simulate counterfactual contribution in answering when the performance threshold is set to 1) zero, so that everyone is endowed with authority since their registration in the platform, 2) infinity, such that no one will ever get authority, and 3) two intermediate levels, where authority is allocated conditional on the user reaching a pre-established positive but finite performance level. Note that all these scenarios are realistic. Wikipedia is a leading example of the case where agents have full authority. In Wikipedia, every internet user is allowed to both contribute by writing new articles or modify existing content. On the other side, most online retailers do not allow users to modify reviews provided by other contributors. In this case, there is no delegation. Users can sometimes rate existing reviews or flag inappropriate ones but have no right to modify them. Stack Exchange represents instead an example of the intermediate cases, where the allocation of authority depends on the achievement of a performance threshold.

To simulate contribution levels, I cannot rely on the results of Arcidiacono and Miller (2011). Some additional restrictions are then necessary to grant computational feasibility. The approach used is to solve backward the maximization problem, assuming that users participate in the website for a fixed amount of time.

The simulation proceeds in three steps. First, I compute the choice-specific transition probabilities. These are matrices mapping each possible combination of state values to future combinations of state values and provide the probability distribution of future state values given a choice made. The state variables that I consider are the number of accumulated reputation points, the expected up-votes and down-votes arriving from past decisions, the availability of questions to answer, and the variables capturing experience: the number of answers already made and the number of days of participation in the platform. Details on the restrictions on the dimensionality of the state variables are in appendix A.8.1.

Second, I compute the value function backward, starting from the last period. I assume users participate for 100 periods and then exit the platform definitively. Finally, in the third step, I forward-simulate decisions at each period.

Each simulation is characterized by a different performance threshold.[31] The considered threshold levels are 0, 500, 1000, and 99999. Since I set the maximum amount of points that users can achieve to 1500, in the last scenario none of the users obtain authority.[32] Figure 15 reports the simulated contributions in answering under the different delegation thresholds. The estimates used for the simulations are the ones of the utility function that includes the interaction terms. It reports the average number of answers made by users of each type. On average, users reach the threshold at the vertical solid

[31] In a given simulation, the performance threshold is fixed i.e. does not change across time.

[32] Please refer to appendix A.8.1 for more details on the accumulation of points in the simulations.

line and reach 1500 points at the vertical dotted line. Since users cannot accumulate more points than 1500, after that line that source of motivation is no more valid.

It is possible to see that the incentive effects are inducing very heterogeneous responses across types. *Anonymous* users participate very little, even though they should be the most sensitive to the incentives. The low production is caused by their high cost of participation, which, in the simplified context of the simulation, is not compensated by the incentive effects. The *Identifiable* users are instead not sensitive to the dynamic incentive effect. Their participation is not much affected by changes in the performance threshold. Finally, *Informative* users are instead very reactive to the incentive design. Their participation increases faster when approaching the threshold while it is slacker in case of full or no delegation.

To understand how the different incentive designs translate in final production in the platform, I sample users of each type following the proportion appearing in the real data. This corresponds to 55% of *Anonymous* users, 38% of *Identifiable* users, and 7% of *Informative* users. As shown in table 16, the platform reaches the highest level of production with the performance threshold set at 500 points. Most of the increase is accountable to the *Informative* users. Since they are a small share of the participants, the final change in production is limited. Figure 16 shows how the production of answers would occur during the lifetime of the platform.

Performance required	Answers	Change	*Anonymous*	*Identifiable*	*Informative*
0 Points	12562.0		92	10967	1503
500 Points	13374.0	+6.46%	+13.04%	+1.6%	+41.52%
1000 Points	13300.0	+5.87%	+13.04%	+2.43%	+30.54%
NO Delegation	12886.0	+2.58%	+13.04%	+2.01%	+6.12%

Table 16: Total number of answers produced in the platform under the different delegation designs. Columns report the number of answers produced and the relative change compared to the full delegation design, overall and by type.

These results show that the platform could exploit the *dynamic incentive* effect to increase the number of answers provided. Nevertheless, it faces a trade-off: since users are more willing to edit when endowed with authority, to postpone delegation induces a lower production of edits. Table 17 reports the contribution in editing under the different delegation designs.[33] It is relevant to notice that any design different from the full delegation scenario produces fewer edits. The full delegation design is the setting that maximizes the *static incentive* effect.

[33]Notice that the sample used in the simulation is much smaller than the actual number of participants. In reality, the number of edits would be more substantial.

Performance required	Edits	Change	*Anonymous*	*Identifiable*	*Informative*
0 Points	16.0		2	11	3
500 Points	7.0	-56.25%	-50.0%	-54.55%	-66.67%
1000 Points	8.0	-50.0%	-50.0%	-54.55%	-33.33%
NO Delegation	7.0	-56.25%	-50.0%	-45.45%	-100.0%

Table 17: Total number of edits produced in the platform under the different delegation designs. Columns report the number of edits produced and the relative change compared to the full delegation design, overall and by type.

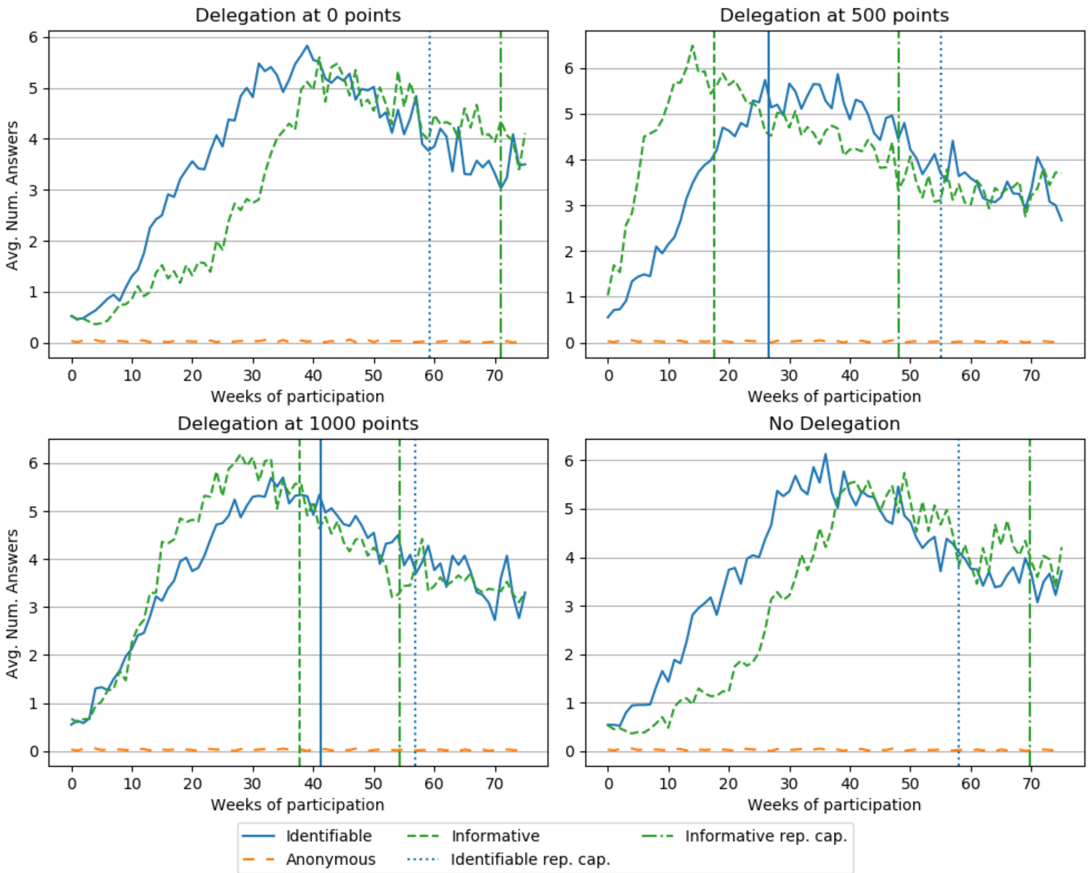

Figure 15: Average number of answers that users of each type make under different delegation designs. X-axis reports weeks of participation in the platform. Vertical lines of the same pattern as the series identify the average period in which users achieve the threshold. Vertical lines with dots (named as re. cap. in the legend) identify the average period in which users reach the cap of 1500 reputation points. After those lines, users cannot accumulate more points. Anonymous users never reach this limit. After 100 periods (weeks) the users exit the platform.

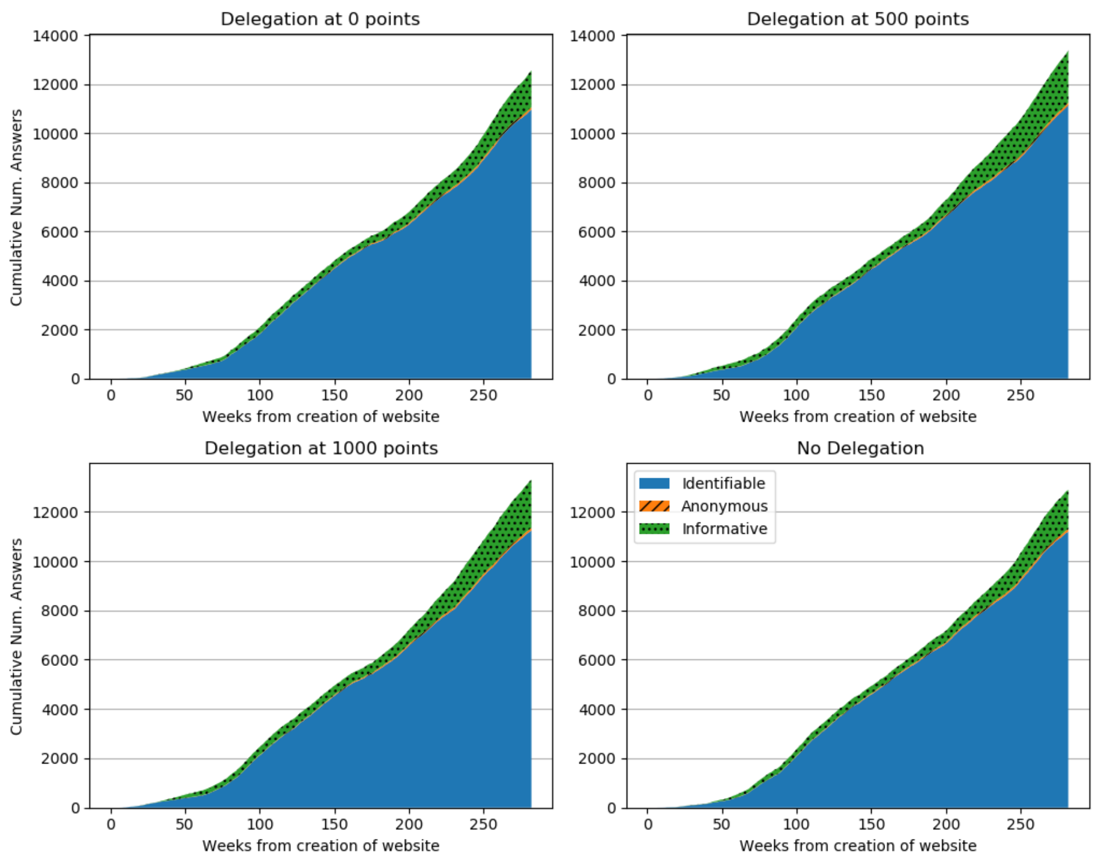

Figure 16: Cumulative number of answers made during the life of the platform, by type and delegation design.

8 Conclusion

In this paper, I investigate whether users participating in online communities value the allocation of control rights and authority. I then study the implication for the platform design, investigating the incentive role of delegation.

I find evidence that users value authority in the community. First, the willingness to contribute to a given task depends on the level of autonomy and authority the user has on the task. The paper indeed finds that users do significantly more edits if their edits are directly implemented and do not require third party approval. To my knowledge, this is novel evidence in real data and contributes to the growing literature that studies the role of autonomy and authority for incentives and the optimal delegation structure (Liberti 2018, Bandiera et al. 2020). Interestingly, the allocation of authority on a task does not seem to affect contributions in other tasks. The paper finds evidence that the production of both comments and answers is not affected by the allocation of authority. These results contribute to the literature on multitasking (Holmstrom and Milgrom 1991), suggesting that incentives may not backfire in these contexts. Differently from the others, they slightly substitute answers with edits when they have more authority.

Second, the paper finds heterogeneity in the value of acquiring authority. *Anonymous* and *Informative* users are motivated by the acquisition of authority and increase their contribution reaching the threshold. On the contrary, *Identifiable* users seem to be motivated by other factors.

The results on the preference for authority have important implications for platform design. For what concerns the moderation task, the platform can incentivize participation via the *static incentive*. If the objective is to maximize contributions in the moderation task, the platform would need to provide authority to all participants from the registration date. The *dynamic incentive* has no impact in participation in editing and, as a consequence, there is no good reason to delegate authority based on performance. This is because suggested edits provide very few points and cannot be the main tool to reach the performance threshold. The scenario of full delegation would be comparable to the design adopted by Wikipedia.

Nevertheless, to delay delegation and commit to allocating authority based on performance incentivizes answering. The effect is mainly driven by *Informative* users, who increase by 40% their participation when incentivized via the *dynamic incentive*. The answering task is not much affected by the *static incentive* instead. If the platform aims to maximize as much as possible the number of answers produced in the platform, it should delay delagation and commit to provide authority based on performance in answering. The optimal performance threshold depends on users' cost of answering, and the average time users plan to stay in the platform.

The optimal organizational design depends on 1) the type of action the platform needs to incentivize and 2) the composition of the community. On the first dimension, I show that participation in the different tasks (answering and editing) depends on different incentives. On the second dimension, I show that the sensitivity to the incentives is heterogeneous. If the community is not populated by *Informative* users, the platform's trade-off simplifies: the *dynamic incentive* becomes irrelevant and full delegation emerges as leading strategy. Otherwise, the platform would be better off in targeting different types with different incentives. It would allocate authority based on performance for *Informative users*, and full authority to the other types. This paper provides a way for the platform to identify user types ex-ante, before observing their actions. It then allows the platform to assess the composition of the community and adopt the design that suits best.

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Appendix A Details and Robustness

A.1 Construction of quality variable

The variable quality captures the variation of points received by an answer at its publication day explained by text characteristics.
Let $\boldsymbol{X}_j$ be a vector of text characteristics of an answer j, right after publication, so before any modification occurs. Let $\bar{t}_j$ be the publication date of answer j. I estimated the following linear model:

$$points_{j,\bar{t}_j} = \beta_0 + \boldsymbol{X}_j\boldsymbol{\beta}_1 + \boldsymbol{X}_j^2\boldsymbol{\beta}_2 + \varepsilon_j$$

with $points_{j,\bar{t}_j}$ being the points obtained by answer j's author at the publication day. The quality of answer j is then defined as the predicted number of points from the above model.

The vector of text characteristics includes:

- number of words,
- precision, defined as the number of words excluding the stop-words, over total number of words,
- number of links,
- number of images.

Table 18 reports the estimates for the linear regression models used to predict the variable quality. The specification adopted corresponds to column (5).

A.2 Construction of scarcity variable

The construction of the variable scarcity follows several steps.

- Construct the variable *availability*, given by the cumulative number of questions appearing in the platform, that, each day, don't have yet an answer selected as best answer. This is equivalent for every users. The cumulative number of questions and the number of questions without an accepted answer are plotted in figure 17.
- Recover topics from question tags[34]. To do this, I first construct a graph of tags, where a link between two tags exists if the two tags appear at least once in the same question. The intensity of the links are given by the number of times that the two tags have appeared in a same question. I then identify topics using the Page rank algorithm[35], i.e. a topic will be those tags that are connected to the

[34] Questioners can add tags when posting a question
[35] This is the Google search algorithm of the early times of the search engine

Dep. var: **points**	(1)	(2)	(3)	(4)	(5)
Length	0.00440***	0.00997***	0.00921***	0.00927***	0.00859***
	(5.65)	(7.48)	(6.80)	(6.85)	(6.34)
Precision	9.219***	9.495***	32.20***	32.02***	30.91***
	(8.81)	(9.06)	(4.38)	(4.35)	(4.20)
Num. figures	3.504***	3.554***	3.555***	6.915***	6.474***
	(8.98)	(9.11)	(9.11)	(10.42)	(9.73)
Num. links	1.818***	1.807***	1.806***	1.784***	2.235***
	(21.86)	(21.73)	(21.72)	(21.44)	(23.50)
Length^2		-0.00000991***	-0.00000926***	-0.00000932***	-0.00000861***
		(-5.15)	(-4.78)	(-4.81)	(-4.44)
Precision^2			-22.54**	-22.39**	-21.81**
			(-3.12)	(-3.10)	(-3.02)
Num. figures ^{2}				-1.231***	-1.172***
				(-6.26)	(-5.95)
Num. Links					-0.0393***
					(-9.78)
_cons	8.978***	8.437***	2.909	2.944	3.292
	(17.16)	(15.81)	(1.57)	(1.59)	(1.78)
N	118552	118552	118552	118552	118552

t statistics in parentheses
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 18: Regressions to predict the quality variables. Model finally used is in column (5).

most other tags. I identify 6 topics, since the Page ranking value drops suddenly after those first 6 tags. The topics are: 'grammar', 'word-usage', 'meaning', 'sentence-construction', 'meaning-in-context', 'word-choice'. I then partition the graph around these 6 tags, using a Voronoi diagram. After this process I then have 6 topics, and a mapping from every tag to each of these topics. The word-clouds of tags related to each topic are plotted in figure 18.

- I allocate topics at the questions still to answer, recovered at the first bullet point: using the tags assigned to those questions, I obtain the share of each topic in each of the questions.
- I do a similar process for each user, on all questions he/she has answered, and recover the share of each topic in which he/she is expert about
- for each user i, I weight the available questions at each period t by his/her expertise, call this variable $avail_{it}$. Figure 19 shows the distribution of time of this variable, in average across users' lifetime in the website.

The variable scarcity is then defined as:

$$scarcity_{it} \equiv \frac{maxavail}{\log(avail_{it})}$$

where *maxavail* is the maximum value that $\log(avail_{it})$ takes in the data, across all i, t.

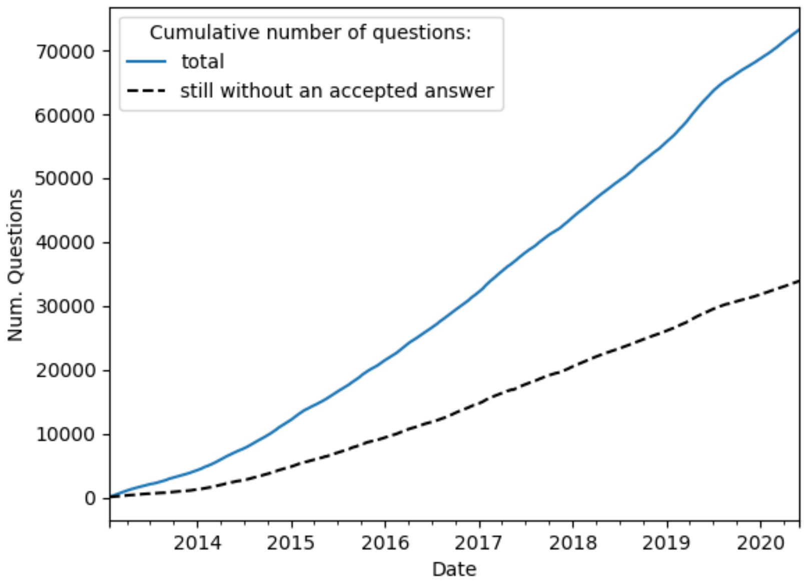

Figure 17: Cumulative number of questions in the platform, both total number and net of questions that have already selected an answer as *best answer*.

A.3 Construction of Types

The procedure I used to identify types follows few steps with the combination of quantitative assessment and interpretative assessment. First I aggregate information to reduce the dimensions of the individual characteristics. Then I employ an algorithm to identify clusters within the reduced space.

Information aggregation. The most simple approach to reduce dimensionality would be to aggregate the variables via, for example, sum. Since the individual characteristics include dummy variables taking value 1 if the user decided to display some given information, as well as the length of the biographical description, summing over them gives a measure of the amount of information displayed. This approach turned to not be a good solution, as behavior is not linearly correlated with the amount of information. The aggregated variable is then not informative on the different types of users.
A common alternative is to perform the Principal Component Analysis (PCA). This approach transforms the data by creating orthogonal vectors, each containing the largest possible variance of the original variables. The first vector will be the most representative of the original variance, the second will be the most representative of the residual variance, and so on. PCA anyway relies on quantitative continuous variables, as it relies

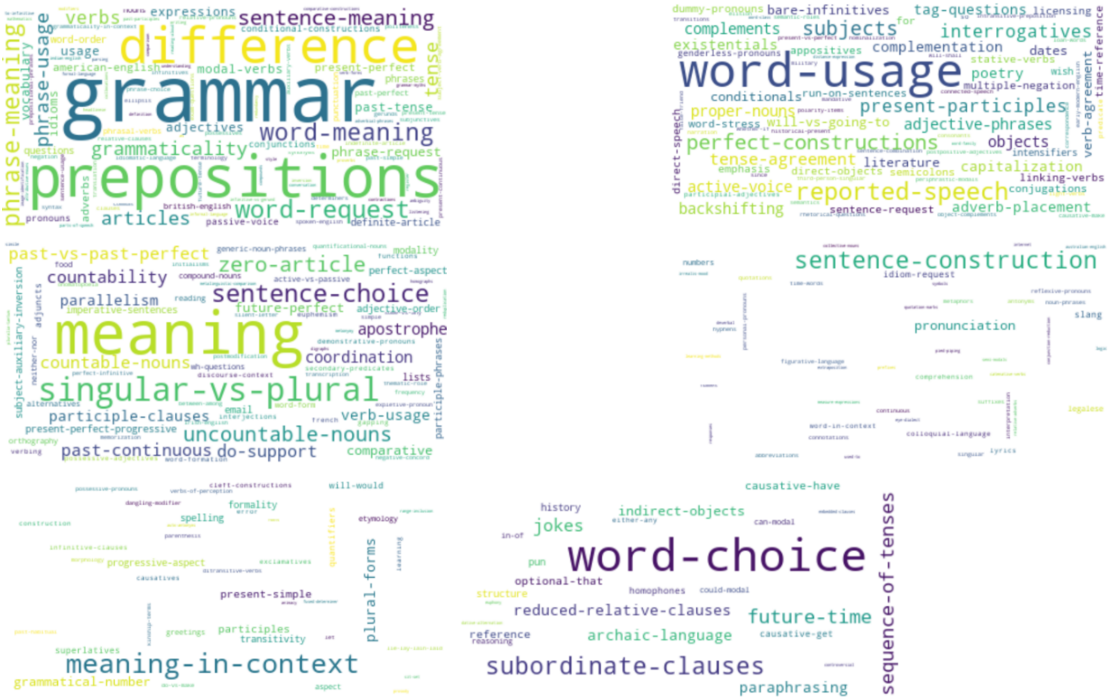

Figure 18: Word-clouds for each topic identified

on the computation of the variance, and it is not suitable to dummy variables.

In this work I adopt the Multiple Correspondence Analysis (MCA, Greenacre and Blasius 2006), a sort of PCA counterpart for categorical variables, which is a generalization of the Correspondence Analysis (CA). This method relies on the cross tabulation of each pair of variables, with the single categories being the rows and columns, and the joint frequency the measure in the cells.

As the PCA, the MCA algorithm outputs dimensions (or factors) that aggregate the information of the original variables. Individual users can then be plotted in the reduced bi-dimensional space formed by each pair of dimensions. In the discussion that follows I will focus on the plane formed by the first and second dimensions. Note that, since this algorithm is applied to categorical variables, I bin in three groups the variables representing the length of the biographical note and the variable with the number of links appearing in the biographical note.

Figure 20 shows the variable representation in the first two dimensions space. First it is possible to notice, on the axes, that the first dimension contains about 17% of the information of the individual characteristics, while the second dimension about 8%. The location of the variables on the plain tells the extent to which that dimension include information from the given variables. It is possible to see that the length of the biographical note is the most important source of information for both dimensions, while the inclusion of location and website in the user page is only captured by the first di-

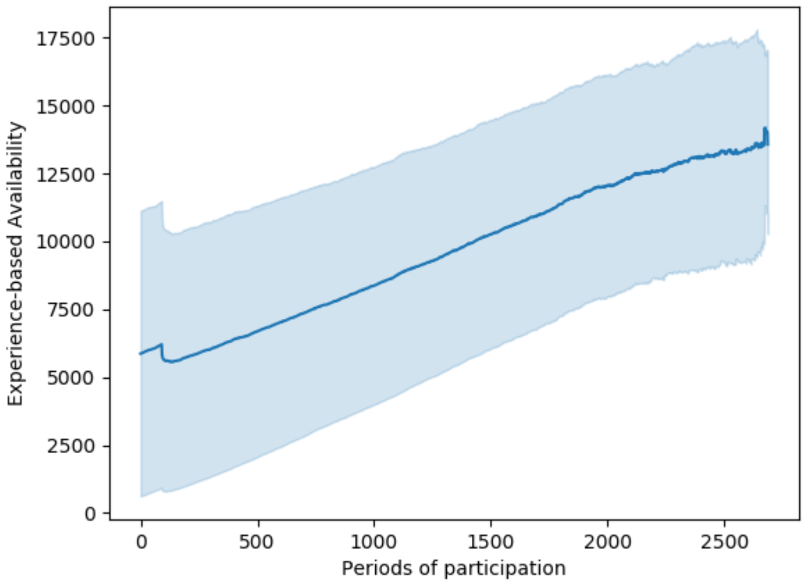

Figure 19: Average expertise-weighted availability of questions per period of participation on the website, across users. Shadow areas identify the standard deviation.

mension.
Figure 21 instead represents on the same dimensions the individuals, i.e. the sample of users. This graph may help to understand if individuals cluster in groups, based on the information of the first two factors. It is possible to observe that clear clusters are not emerging. Nonetheless, points are not displayed in an uniform cloud with respect to the axis. While some are grouping around the (0,0) point, meaning that they have characteristics close to the average of the sample, others appear on the positive side of the first dimension. Users appearing in the upper right quadrant are more likely to have a Linkedin profile, a website, and the location, compared to the average user, as well as longer biography with more links. Users in the bottom right quadrant are also more likely to have a website and the location, they tend to have a biography, but a short one.

Identification of groups. A typical clustering algorithm is the so called K-Means clustering. This algorithm requires the number k of groups that want to be identified, it picks k centroids (i.e. means of partitions of the observations) and updates the centroids so to minimize the within-cluster variance. This algorithm is also meant to work with continuous quantitative variables, so is not suitable to be directly applied on the original individual characteristics. I then apply the K-Means clustering procedure to the first 5 dimensions recovered after the application of the MCA procedure. These are continuous variable and still represent the information of the original data.

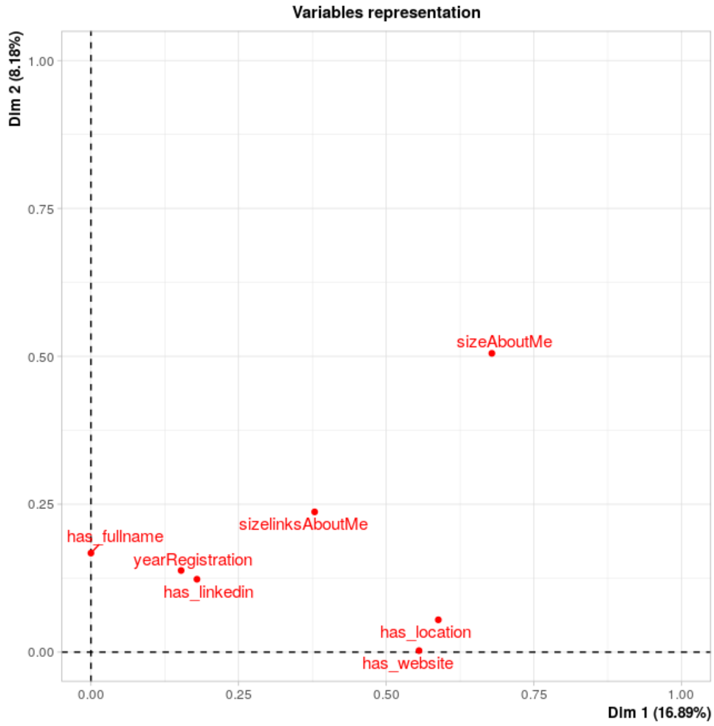

Figure 20: Variable representation on the first two dimensions plane, where the dimensions are obtained via the MCA of the individual characteristics.

By choosing three clusters (i.e. $k = 3$), the resulting individual representation is shown in figure 22, with individuals colored based on the allocated cluster.

A.4 Reduced form - robustness checks

A possible concern on the reduced form analysis is that the effect observed is not specific of the privilege allocating control on editing. In other words, we could observe a significant increase in the editing activity after each achievement of privileges. To check for this possibility, I estimate the exact same specification of section 4.2 around different thresholds.
In particular I consider the two privileges achieved just before and just after the allocation of authority. Figure 23 reports the estimates of the reputation-point intervals fixed effects, around the privilege "Established User". This privilege does not allocate any resource, and it is just a recognition. It is obtained with 750 points during the beta phase of the site, and with 1000 points during the final phase. It is possible to notice that right around the threshold it is not observed a significant increase in editing. Moving further from the threshold shows instead an increase, but pre-treatment effects seems to suggest the presence of a trend, rather than a causal effect of the treatment.
Finally, figure 24 shows again estimates for the same specification, but this time around

Figure 21: Representations of users on the first two dimensions plane, where the dimensions are obtained via the MCA of the individual characteristics.

the allocation of the "Creat Tag Synonyms" privilege. This privilege allows users to corrects tags. It is achieved either with 1250 points in the beta phase, or with 2500 points otherwise. Looking at the effects just in the neighborhood of the treatment, it is not really possible to identify a clear pattern.

A.5 Derivation of Likelihood function

Let $D \in \{1, 0\}$ be a binary variable that takes value equal to 1 when the user is given full ex-ante control over Edits. In addition, denote $\boldsymbol{d_t}$ a vector of dummy variables, $d_{\alpha t}$, for each possible choice $\boldsymbol{\alpha} \in \mathcal{A}$, such that $d_{\alpha t}$ is equal to 1 if in period t is selected choice $\boldsymbol{\alpha}$, and zero otherwise.
Choosing an action $\boldsymbol{\alpha^*}$ in period t, the one period flow utility of user i is then given by:

$$U_{it}\left(d_{\boldsymbol{\alpha^* t}} = 1\right) = \boldsymbol{\beta}_0' \boldsymbol{x_{it}}\left(d_{\boldsymbol{\alpha^* t}} = 1\right) + \mathbf{1}\{D_t = 1\}\boldsymbol{\beta}_1' \boldsymbol{x_{it}}\left(d_{\boldsymbol{\alpha^* t}} = 1\right) + \varepsilon_{i\alpha^* t}$$

Where the vector $\boldsymbol{x_t}$ is described in section 5.1.

The term $\varepsilon_{i\alpha^* t}$ is instead a choice specific utility term not measurable by the econometrician.

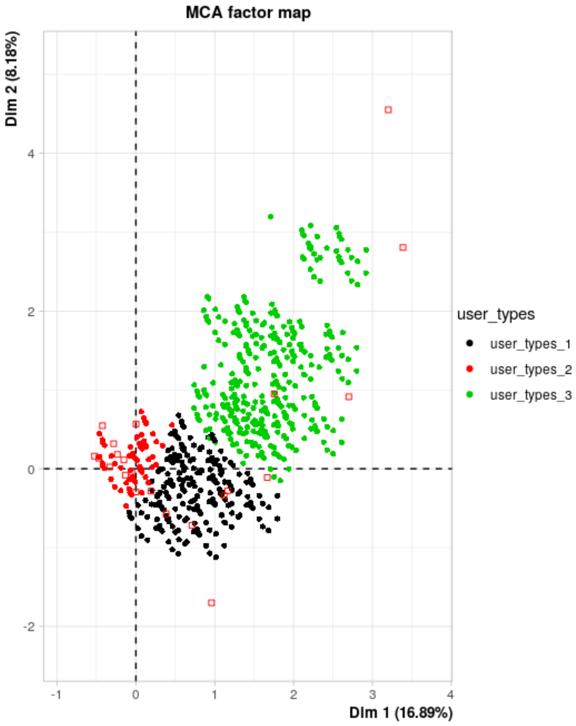

Figure 22: Representations of users on the first two dimensions plane, where the dimensions are obtained via the MCA of the individual characteristics. Colors refer to cluster groups identified with k-means clustering on the MCA dimensions.

Individual problem

Define as $\mathcal{Z}$ the set of all possible states z, i.e. all possible combinations of state variwhichables, at t. This does not consider only the variables that enter the utility function (i.e. $\boldsymbol{x_t}$), but also variables that may affect users' beliefs on the probability distribution over future states.
A user selects a sequence of optimal decisions $\boldsymbol{d}^* \equiv \{\boldsymbol{d}_t^*\}_{t\leq T}$ that satisfies[36]:

$$\boldsymbol{d}^* = \arg\max_{\boldsymbol{d}} \mathbb{E}\left[\sum_{t=1}^{T}\sum_{\boldsymbol{\alpha}\in\mathcal{A}} \delta^{t-1} d_{\alpha,t} U_{\alpha t}(z_t)\right] = \mathbb{E}\left[\sum_{t=1}^{T}\sum_{\boldsymbol{\alpha}\in\mathcal{A}} \delta^{t-1} d_{\alpha,t}\left(u_{\alpha t}(z_t) + \varepsilon_{\alpha t}\right)\right],$$

where δ is a discount factor and, at each period t, the expectation is taken with respect to z_τ and ε_τ, for $\tau \geq t+1$.
In words, the agent, at each period, will choose whether to contribute in the platform

[36]To make notation more readable, for any function f that depends on the agent's choice, I will use the following:

$$f_{\alpha t}() \equiv f_t(d_{\alpha t} = 1)$$

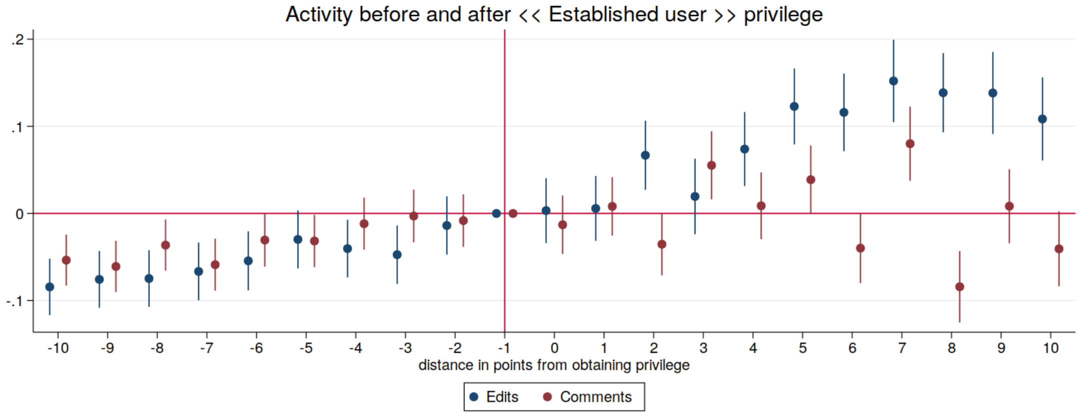

Figure 23: Estimates for reduced form effect around the *Establish User* privilege

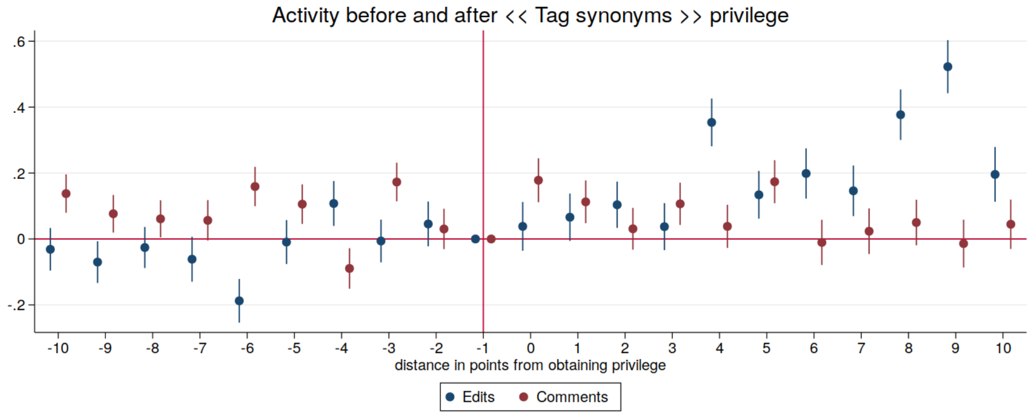

Figure 24: Estimates for reduced form effect around the *create tag synonyms* privilege

and eventually what type of contribution to make, between producing content (answers), performing moderation task (edits), or both.

Identification and estimation

For the characterization of the problem I follow Arcidiacono and Miller (2011).
Define the ex-ante value function at period t as the discounted sum of the expected future payoff under optimal behavior, and before the shock ε_t is realized[37]. In other words, it is the continuation value of being in state z_t, before ε_t is realized and the decision at t

[37] The reason why it is considered the ex-ante value function is because the shock is not observed by the researcher. Note nevertheless that at the time of the decision in period t, the shock is observed by the agent, who'll take it into account in her choice.

is taken. By applying Bellman's principle, it is then given by:

$$V_t(z_t) = \mathbb{E}\left[\sum_{\boldsymbol{\alpha}\in\mathcal{A}} d^*_{\alpha,t}\left(u_{\alpha t}(z_t) + \varepsilon_{\alpha t} + \delta \sum_{z_{t+1}\in\mathcal{Z}} V_{t+1}(z_{t+1}) f_{\alpha t}(z_{t+1}|z_t)\right)\right]$$

where the expectation is taken with respect to $\varepsilon_{\alpha t}$, and $f_{\alpha t}(z_{t+1}|z_t)$ is the probability that the vector of states will take a certain value in the next period, given the choice made. This transition probability does not depend on all the history of past choices due to the assumptions made in the previous section.

Define then the conditional value function $\nu_{\alpha t}(z_t)$ as the value function $V_t(z_t)$ for a given choice $\boldsymbol{\alpha}$ and net of the preference shock ε_t:

$$\nu_{\alpha t}(z_t) = u_{\alpha t}(z_t) + \delta \sum_{z_{t+1}\in\mathcal{Z}} V_{t+1}(z_{t+1}) f_{\alpha t}(z_{t+1}|z_t).$$

Finally, define the conditional choice probabilities $\boldsymbol{p}_t(z_t)$ as the vector that gives the probabilities of choosing option $\boldsymbol{\alpha} \in \mathcal{A}$ given state z_t, taking expectations on the preference shock, so to explain different choices in the data given the same states:

$$p_{\alpha t}(z_t) = \int d^*_{\alpha t} g(\varepsilon_t) d\varepsilon_t,$$

with $g(\varepsilon_t)$ being the density of ε_t which is assumed to have continuous support.

Building on Hotz and Miller (1993), Arcidiacono and Miller (2011) show that, under certain conditions, it exists a function ω for each $\boldsymbol{k} \in \mathcal{A}$ such that:

$$\omega_k(\boldsymbol{p}_t(z_t)) = V_t(z_t) - \nu_{kt}(z_t).$$

It follows that:

$$\nu_{\alpha t}(z_t) = u_{\alpha t}(z_t) + \delta \sum_{z_{t+1}\in\mathcal{Z}} \left(\nu_{kt+1}(z_{t+1}) + \omega_k(\boldsymbol{p}_{t+1}(z_{t+1}))\right) f_{\alpha t}(z_{t+1}|z_t),$$

which can be rewritten as:

$$\nu_{\alpha t}(z_t) = u_{\alpha t}(z_t) + \sum_{\tau=t+1}^{T} \sum_{\boldsymbol{k}\in\mathcal{A}} \sum_{z_\tau\in\mathcal{Z}} \delta^{\tau-t}(u_{k\tau}(z_\tau) + \omega_k(\boldsymbol{p}_\tau(z_\tau))) d^*_{k\tau}(z_\tau, d_{\alpha t}=1) \kappa^*_{\tau-1}(z_\tau|z_t, d_{\alpha t}=1), \tag{6}$$

where the function $\kappa^*_\tau(z_{\tau+1}|z_t, d_{\alpha t}=1)$ represents the cumulative probability of being in state $z_{\tau+1}$ in period $\tau+1$ conditional on having been in state z_t and having chosen α in period t, i.e.

$$\kappa^*_\tau(z_{\tau+1}|z_t, d_{\alpha t}=1) \equiv \begin{cases} f_{\alpha t}(z_{t+1}|z_t) & \text{for } \tau = t \\ \sum_{z_\tau\in\mathcal{Z}} \sum_{\boldsymbol{k}\in\mathcal{A}} d^*_{k\tau} f_{k\tau}(z_{\tau+1}|z_\tau) \kappa^*_{\tau-1}(z_\tau|z_t, d_{\alpha t}=1) & \text{for } \tau = t+1, ..., T. \end{cases}$$

To write the conditional value function as in 6 is functional to implement the *Finite Dependence* property, generalized by Arcidiacono and Miller (2011). This property allows to rewrite the problem such that the agent considers only a subset of the future periods to make her decision.

The intuition behind the property goes as follows.
First of all the identification of the structural parameters will be based on the comparison of conditional value functions, since the likelihood of observing at t a choice α rather than α' given a specific state z_t corresponds to the probability that $\nu_{\alpha t}(z_t) - \nu_{\alpha' t}(z_t) > \varepsilon_{\alpha t} - \varepsilon_{\alpha' t}$.
Consider now two alternative choices, α and α'. If, by choosing either of the two, it is possible to follow sequences of decisions such that the probability distribution of the state variables is exactly equivalent, then, when substituting equation 6 into the difference $\nu_{\alpha t}(z_t) - \nu_{\alpha' t}(z_t)$, all future periods after the sequence of choices will cancel out.

Assumption over the distribution of the stochastic term.
Consider again two alternative choices, α and α'. Since we are interested in measuring the probability that $\nu_{\alpha t}(z_t) - \nu_{\alpha' t}(z_t) > \varepsilon_{\alpha t} - \varepsilon_{\alpha' t}$, we need to make assumptions on the distribution of the stochastic term $\varepsilon_{\alpha t t}$. I will assume a Type I extreme value distribution. This allows to express the choice probabilities as:

$$p_{\tilde{\alpha} t}(z_t) = \frac{\exp\left(\nu_{\tilde{\alpha} t}(z_t)\right)}{\sum_{\boldsymbol{\alpha} \in \mathcal{A}} \exp\left(\nu_{\alpha t}(z_t)\right)} = \frac{1}{\sum_{\boldsymbol{\alpha} \in \mathcal{A}} \exp\left(\nu_{\alpha t}(z_t) - \nu_{\tilde{\alpha} t}(z_t)\right)}$$

and the ex-ante value function as:

$$V_t(z_t) = \ln\left(\sum_{\boldsymbol{\alpha} \in \mathcal{A}} \exp\left(\nu_{\alpha t}(z_t)\right)\right) + \gamma = -\ln\left(p_{\tilde{\alpha} t}(z_t)\right) + \nu_{\tilde{\alpha} t}(z_t) + \gamma$$

where γ is the Euler's constant and $\tilde{\boldsymbol{\alpha}}$ is an arbitrary reference choice from $\mathcal{A}$. It follows that:

$$\omega_{\tilde{\alpha}}(\boldsymbol{p}_t(z_t)) = -\ln\left(p_{\tilde{\alpha} t}(z_t)\right) + \gamma.$$

Given a reference choice $\tilde{\boldsymbol{\alpha}}$ then it is possible to write the difference of conditional value functions as:

$$\begin{aligned}\nu_{\alpha t}(z_t) - \nu_{\tilde{\alpha} t}(z_t) = & u_{\alpha t}(z_t) - u_{\tilde{\alpha} t}(z_t) + \\ & \sum_{\tau = t+1}^{t+\Delta_t} \sum_{\boldsymbol{k} \in \mathcal{A}} \sum_{z_\tau \in \mathcal{Z}} \delta^{\tau - t}\left(u_{k\tau}(z_\tau) - \ln(p_{k\tau}(z_\tau))\right)\left[d^*_{k\tau}(z_\tau, d_{\alpha t} = 1)\kappa_{\tau-1}(z_\tau | z_t, d_{\alpha t} = 1) + \right. \\ & \left. - d^*_{k\tau}(z_\tau, d_{\tilde{\alpha} t} = 1)\kappa_{\tau-1}(z_\tau | z_t, d_{\tilde{\alpha} t} = 1)\right]\end{aligned}$$

where Δ_t is the number of periods after which the agent faces the same probability distribution over the states, independently of having initially chosen $\boldsymbol{\alpha}$ or $\tilde{\boldsymbol{\alpha}}$.

The Log-likelihood function of the data is given by:

$$L\left(\boldsymbol{\beta}_0, \boldsymbol{\beta}_1, \gamma\right) = \sum_{i=1}^{N}\sum_{t=1}^{T}\sum_{\alpha\in\mathcal{A}} \log\left(\frac{\exp(\nu_{\alpha it}(z_{it}))}{\sum_{k\in\mathcal{A}}\exp(\nu_{kit}(z_{it}))}\right) \times d_{\alpha it}$$
$$= \sum_{i=1}^{N}\sum_{t=1}^{T}\sum_{\alpha\in\mathcal{A}} \log\left(\frac{\exp(\nu_{\alpha it}(z_{it}) - \nu_{\tilde{\alpha}it}(z_{it}))}{\sum_{k\in\mathcal{A}}\exp(\nu_{kit}(z_{it}) - \nu_{\tilde{\alpha}it}(z_{it}))}\right) \times d_{\alpha it}$$

A.6 Details on Estimation of Structural model

A.6.1 Choice set

Because of computational time, the choice set must be constrained to a finite and limited number of options.[38] In my specification, users are allowed to make 21 possible choices of effort. They may not participate at all, make effort only in answering, only in editing, or in both. Answering effort is a combination of quantity and quality of answers, with two possible levels for quantity, and three possible levels of quality. Quantity of edits can take two possible levels. All options in the choice set are listed in the table 19.
The value of the possible levels are obtained by looking at the distribution of actions taken in the data by individuals at each week of participation. For what concerns the quantity of answers, I split the distribution at the 70^{th} quantile, corresponding to three answers, so to categorize effort between low (1 to 3 answers) and high (4 or more). I then select, as possible option for the user, the median values of these two categories, so either 1 or 7 answers. A similar process is made for quality and edits. The distribution of quality is split in three categories at the 33^{th} and 66^{th} quantiles. The median values are 13.33, 14.12, and 15.97. Finally, the distribution of number of edits is split at the 75^{th} quantile, leading to two categories: low effort, which includes 1 or 2 edits, and high effort, including 3 or more edits. The distribution of values within each category is plotted in figure 27 in the appendix. The choice of the quantile levels is arbitrary.

A.7 Conditional Choice Probabilities

Conditional choice probabilities are computed before estimation via a static logit[39] Before estimation, the data is scaled so that each variable would be in the range $(0, 1)$. The scaling algorithm subtracts the minimum and divide by the difference between the maximum and the minimum. The multinomial logit model implemented is the following:

$$\alpha^*_{it} = \beta_0 R_{it-1} + \beta_1 \Lambda_{U,it-1} + \beta_2 \Lambda_{D,it-1} + \beta_3 avail_{it} + \beta_4 AnswerNum_{it} + \beta_5 Seniority_{it} + \\ + \beta_6 t + \beta_7 date_{it} + cumT_{it}$$

[38] A more natural assumption would be that users make discrete choices of tasks, and continuous choices for effort levels. As of today, the econometric literature is not providing a way to do so. A first solution to this problem is provided in the recent work by Bruneel-Zupanc (2020).

[39] Logistic regression in Scikit-learn (Pedregosa, Varoquaux, Gramfort, Michel, Thirion, Grisel, Blondel, Prettenhofer, Weiss, Dubourg, Vanderplas, Passos, Cournapeau, Brucher, Perrot, and Duchesnay 2011) with *saga* solver.

A	Q	E
0.0	0.00	0.0
0.0	0.00	1.0
0.0	0.00	4.0
1.0	13.33	0.0
1.0	13.33	1.0
1.0	13.33	4.0
1.0	14.12	0.0
1.0	14.12	1.0
1.0	14.12	4.0
1.0	15.97	0.0
1.0	15.97	1.0
1.0	15.97	4.0
7.0	13.33	0.0
7.0	13.33	1.0
7.0	13.33	4.0
7.0	14.12	0.0
7.0	14.12	1.0
7.0	14.12	4.0
7.0	15.97	0.0
7.0	15.97	1.0
7.0	15.97	4.0

Table 19: Possible effort levels that users are allowed to choose in estimation. Columns report, from left to right, the possible choice of effort in the number of answers, the average quality of answers, and the number of edits

where α^*_{it} is the choice made by user i in period of participation t, R is the number of reputation points, Λ_U and Λ_D are the expected number of up-votes and down-votes arriving from past effort, $avail$ is the number of available questions to answer, $AnswerNum$ is the number of answers already published up to period t, $Seniority$ the number of days passed since the registration day, $date$ is the calendar week, and $cumT$ the number of privileges obtained by the user. All parameters are choice specific.

A.8 Details on Simulation of Counterfactuals

A.8.1 Restrictions on the state values

Reputation points. It is assumed that users can accumulate at most 1500 reputation points. To adjust for this limit, which is not present in the real design, I scale the returns in points from up-votes / down-votes. Every up-votes provides 5 reputation points to the author, while every down-votes removes 1 point. The approval of suggested edits provide 1 point.

Expected number of points arriving from past actions. The variables Λ_U and Λ_D, which are normally continuous, are discretized. Λ_U can take value from zero to 0.2, with steps of 0.01, while Λ_D can take value from zero to 0.03, with steps of 0.01. The boundaries of these sets are generally never hit, and do not impose important restrictions. On the contrary, the discretization reduces the sensitivity of the model.

Availability of questions. I randomly allocate to users a registration date. Based on the dates of participation, I allocate the number of available questions to each user, as it appears to be in the real platform. To reduce dimensionality, I bin the variable so that the number of available question can be one of 5 unique values. Note that the number of available questions could still change across the time of a user's participation.

Experience variables. The number oaf answers already made and the days of participation are set to zero and are not allowed to increase. In other words, in the simulations I do not allow for learning while participating.

Appendix B Other figures

You can earn a maximum of 200 reputation per day from any combination of the activities below. Bounty awards, accepted answers, and association bonuses are not subject to the daily reputation limit.

You gain reputation when:

- question is voted up: +5
- answer is voted up: +10
- answer is marked “accepted”: +15 (+2 to acceptor)
- suggested edit is accepted: +2 (up to +1000 total per user)
- bounty awarded to your answer: + full bounty amount
- one of your answers is awarded a bounty automatically: + half of the bounty amount (see more details about how bounties work)
- site association bonus: +100 on each site (awarded a maximum of one time per site)
- example you contributed to is voted up: +5
- proposed change is approved: +2
- first time an answer that cites documentation you contributed to is upvoted: +5

If you are an experienced Stack Exchange network user with 200 or more reputation on at least one site, you will receive a starting +100 reputation bonus to get you past basic new user restrictions. This will happen automatically on all current Stack Exchange sites where you have an account, and on any other Stack Exchange sites at the time you log in.

You lose reputation when:

- your question is voted down: −2
- your answer is voted down: −2
- you vote down an answer: −1
- you place a bounty on a question: − full bounty amount
- one of your posts receives 6 spam or offensive flags: −100

All users start with one reputation point, and reputation can never drop below 1. Accepting your own answer does not increase your reputation. Deleted posts do not affect reputation, for voters, authors or anyone else involved, in most cases. If a user reverses a vote, the corresponding reputation loss or gain will be reversed as well. Vote reversal as a result of voting fraud will also return lost or gained reputation.

At the high end of this reputation spectrum there is little difference between users with high reputation and ♦ moderators. That is intentional. We don’t run this site. The community does.

Figure 25: Rules to obtain or loose reputation in Stackexchange(`https://stackoverflow.com/help/whats-reputation`)

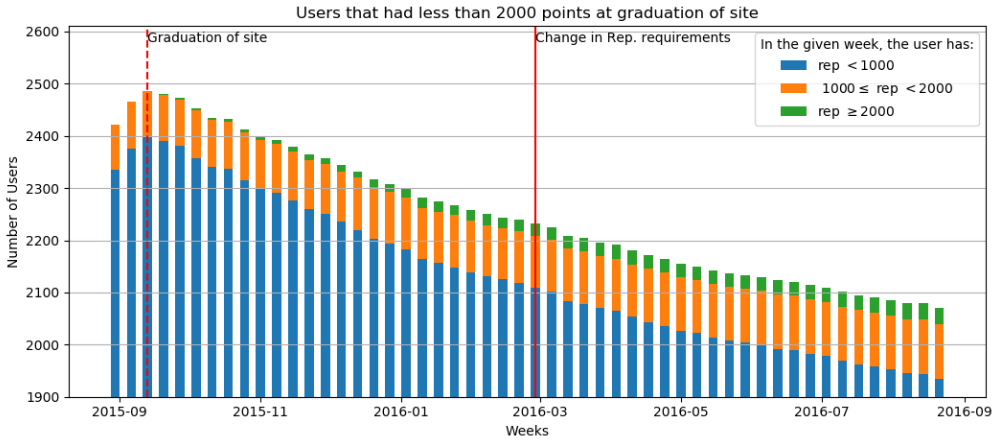

Figure 26: Number of users that have accumulated different amount of reputation points, conditional on having less than 2000 points at the graduation week. The decreasing value is due to exiting of the platform. It is possible to see that some users are reaching the level of 2000 points and they will not loose the privilege at the design date, some never reached the privilege, and others, the orange ones, loose it.

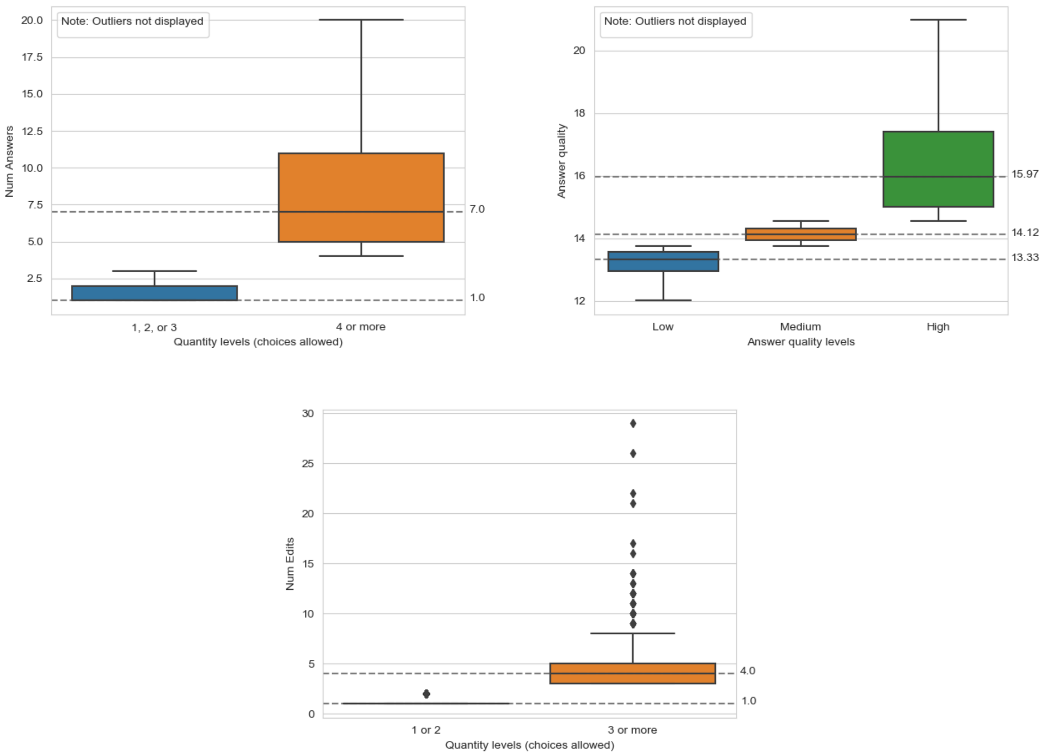

Figure 27: Categories of possible actions that users in the estimation are allowed to take, with the distribution of actual actions in each category. Values on the right vertical axis are the median value of each category, which make the set of options that users are allowed to choose.

Appendix C Credits for the software used

Pedregosa et al. (2011), Seabold and Perktold (2010), Hagberg, Schult, and Swart (2008), McKinney (2010), Lê, Josse, and Husson (2008), Virtanen, Gommers, Oliphant, Haberland, Reddy, Cournapeau, Burovski, Peterson, Weckesser, Bright, van der Walt, Brett, Wilson, Jarrod Millman, Mayorov, Nelson, Jones, Kern, Larson, Carey, Polat, Feng, Moore, Vand erPlas, Laxalde, Perktold, Cimrman, Henriksen, Quintero, Harris, Archibald, Ribeiro, Pedregosa, van Mulbregt, and Contributors (2020), Hunter (2007)

Other software used:
StataCorp. 2017. Stata Statistical Software: Release 15. College Station, TX: StataCorp LLC.