Physics 411: Homework VIII

Tuesday April 12, 2016 before class (i.e. 10:10 AM)

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Site Percolation on the 2d Square Lattice

In class, you have defined the RG flow equation for site percolation in the 2d square lattice as

$$R(p) = p^4 + 3p^3(1-p) + 2p^2(1-p)^2 \tag{1}$$

when coarse graining a 2×2 square lattice.

1. derive the analogous RG transformation equation for a 3×3 square lattice.
2. find the fixed points of the equation (numerically). What is the critical percolation threshold p_c?
3. evaluate the critical exponent of the correlation length according to

$$\nu = \frac{\log b}{\log \frac{dR}{dp}|_{p_b^*}}. \tag{2}$$

Monte Carlo Integration

Consider the following 10-dimensional integral

$$I = \int_0^1 dx_1 \int_0^1 dx_2 \int_0^1 dx_3 \cdots \int_0^1 dx_{10} (x_1 + x_2 + \cdots + x_{10})^2 \tag{3}$$

1. Show that the exact answer is $\frac{155}{6}$.
2. Estimate the answer numerically using a Monte Carlo method. Obtain the error bar on your estimate and compare with the exact answer.

Monte Carlo Integration II

Consider a random walker who starts at $x = 0$ and walks along a line along the x-axis. At each time step, $t = 1, 2, 3, \cdots$, the walker moves one step to the right or one step to the left with equal probability. By averaging over a sufficiently large number of walks show numerically that

$$\langle x(t) \rangle \sim 0 \tag{4}$$

$$\langle x(t)^2 \rangle \sim t \tag{5}$$

where the average $\langle \cdots \rangle$ is over your sample of walks. plot (or produce a neat table of) $\langle x(t) \rangle$ and $\langle x(t)^2 \rangle$ against t for a range of t.