Physics 411: Midterm IV

Tuesday April 19, 2016 before class (i.e. 10:10 AM)

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Note: questions get progressively more difficult. You do not need to solve all questions for a top grade, and you do not need to solve the questions in order. Read through all questions before you start. Collaboration is *not* allowed. Do not hesitate to ask questions on Piazza if clarification is needed.

The Ising model in two dimensions

The Hamiltonian of the Ising model is given by

$$H = -J \sum_{\langle i,j \rangle} \sigma_i \sigma_j \tag{1}$$

Simulate the model in the ferromagnetic regime (choose $J = 1$). The model is exactly solvable and has a critical temperature of

$$T_c = \frac{2}{\log(1+\sqrt{2})} \tag{2}$$

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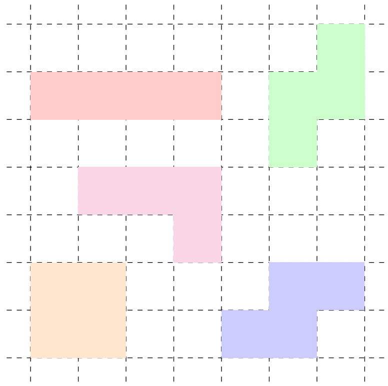
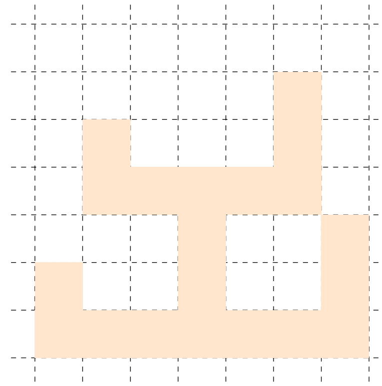
- implement (or finish implementing) a Metropolis simulation for the 2d Ising model.
- show that for $T \gg T_c$ the (absolute value of the) magnetization per site $\langle |m| \rangle$ and the square of the magnetization per site $\langle m^2 \rangle$ is near zero.
- show that for $T \ll T_c$ the (absolute value of the) magnetization per site and the square of the magnetization per site is near one, and the energy per site is near the ground state energy.
- Simulate for some time. Then make a plot of the spin configuration at high T, showing a disordered state, and at low T, showing an ordered state.
- Obtain curves of $m(T)$, the magnetization per site as a function of T, and of $e(T)$, the energy per site as a function of T. Plot this for lattices of size 6×6, 8×8, and 10×10.
- The Binder cumulant is given as

  $$U_4 = 1 - \frac{\langle m^4 \rangle}{3 \langle m^2 \rangle^2}. \tag{3}$$

  At the critical temperature the Binder cumulant $U_4^L(T)$ shows a crossing when plotted as a function of T for different lattice sizes L. It is often used to identify phase transition points of continuous phase transitions. Make a plot of the binder cumulant for the lattices of size 6×6, 8×8, and 10×10 in the vicinity of the crossing point. What is your numerical estimate for T_c?

Cluster Labeling Algorithm





1. Write a function implementing the Hoshen Kopelman algorithm described in class. Test the Hoshen Kopelman algorithm at the example configurations given above and verify that your cluster labeling works.

2. Generate a plot with 100×100 sites. Occupy them randomly with probability 0.58. Label your clusters using the Hoshen Kopelman algorithm, and provide a color plot showing your labels. Hint: use `numpy`'s `pcolor` function to generate pictures.

The Percolation Problem

1. Write a function that, given a matrix with cluster labels, checks if there is a percolating cluster (defined as a cluster that goes from the top to the bottom of your grid).

2. Write a Monte Carlo simulation that, given a probability p of occupying a site, computes the probability P of having a percolating cluster. In order to do this:

   (a) generate a random configuration with site occupation probability p.

   (b) label its clusters using the Hoshen Kopelman algorithm.

   (c) compute if there is a percolating cluster

   (d) repeat this procedure at least 1000 times.

   (e) compute $P(p)$ as the number of times you obtained a percolating clusters divided by the total number of attempts.

3. Make a graph showing $P(p)$ for three different lattice sizes. Can you provide the approximate critical probability p_c of the transition?