Physics 411: Homework VI

Tuesday March 8, 2016 before class (i.e. 10:10 AM)

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Harmonic Oscillator

- Show (analytically) that the Leapfrog and the Verlet method (both of which are described in your textbook) are time-reversal invariant, while forward and backward Euler are not.
- Write ODE integrator routines for the
  1. Forward Euler algorithm
  2. Backward Euler algorithm
  3. Verlet method
  4. Leapfrog method

  and integrate the equations of motion of a harmonic oscillator over 10 oscillations. Plot the resulting positions and velocities as a function of time for two step sizes. Plot the energy as a function of step size.

For full credit hand in a clear derivation (make sure we can follow every step), your implementation of the four algorithms, and the plots.

Bound states in a potential well

Find the bound states solutions of the 1D Schrödinger equation with $E < 0$ using the Runge Kutta algorithm and a root solver. Note that a finite number of bound state solutions with discrete energy eigenvalues exists. The potential well should be zero for $x < 0$ and $x > 1$. For $0 < x < 1$, the potential is given by

$$V(x) = c(x^2 - x), \tag{1}$$

where c is a constant. What are the eigen energies as a function of c, and how many bound states are there as a function of c? Count the number of nodes (zeros) of your wave function to make sure that you find all eigen energies.

Proceed as follows:

1. Implement the Runge Kutta algorithm.
2. Implement a root solver.
3. Choose a $c = 1$ and find the ground state energy. Note that the ground state wave function does not have any nodes. Plot the ground state wave function in the interval $0 < x < 1$. (The way function will decay exponentially outside this interval).
4. Are there any excited states for this c? If so, find them, or show that there are none. The first excited state will have one node, the second two, etc...
5. Plot your ground state and excited state energies as a function of c. Show that as c is increased, more bound states are possible.

For full credit hand in an implementation of the Runge Kutta algorithm, a root solver routine, the ground state energy for $c = 1$ and your excited state information, and a plot showing all possible eigen energies as a function of c.