Physics 411: Midterm III

Tuesday April 5, 2016 before class (i.e. 10:10 AM)

Emanuel Gull

Note: questions get progressively more difficult. You do not need to solve the questions in order. Read through all questions before you start. Collaboration is *not* allowed. Do not hesitate to ask questions on Piazza if clarification is needed.

Random Number Generator

1. Implement a linear congruential generator for which $a = 16807$, $c = 0$, and $m = 2^{31} - 1$. Generate a list of 607 numbers, starting from the initial value 66779, and print the last 10 numbers.
2. Implement a lagged fibonacci pseudo random number generator using the pair of numbers $(273, 607)$ and the same m. Initialize it using the sequence you generated with your linear congruential generator. Convert your numbers to floating point numbers between zero and one, and print the first 10 pseudo-random numbers.
3. Test your results by computing the mean, the variance, and the error of the mean of your lagged fibonacci numbers using the first 100000 pseudo-random numbers.

For full credit: hand in an implementation of your LCG and your lagged Fibonacci code, a table with 10 numbers $(\#597 - \#606)$ *obtained by the LCG, a table with the first 10 random numbers obtained by the lagged Fibonacci code, and the mean, variance, and error of the mean obtained from* 100000 *samples.*

Monte Carlo Integration

1. Write a code to compute the integral of a function using 'hit-or-miss' Monte Carlo. Use it to compute the integral of $f(x) = \sin(x)$ between 0 and π. Compute the value of the integral and estimate its error for $N = 100000$ samples.
2. Write an importance sampling Monte Carlo integration to determine the value of the integral $F(x) = \int_0^\infty \exp(-\frac{x^2}{2})$. Use a uniform random number generator to generate exponentially distributed random numbers as weight functions, and compute the value of the integral and its error for 100000 samples.

For full credit: hand in the computer programs that estimate the value of the integral and its error, along with your estimates for integral and error in the two cases.

The Finite Element Method in one dimension

Consider the one-dimensional Poisson problem

$$\partial_x^2 \phi(x) = -4\pi\rho(x) \tag{1}$$

in the interval $[0, 1]$, with a charge distribution $\rho(x) = \sin(4\pi x)$ and the boundary conditions $\phi(0) = \phi(1) = 0$.

1. Provide a plot of the initial charge distribution.
2. Using a discretization of $\Delta x = \frac{1}{64}$, introduce finite element 'hat' basis functions $u_i(x)$ and compute the matrix A_{ij} and b_j.
3. Solve the matrix problem to obtain the coefficients of the solution and plot the solution.
4. Adapt your code to respect the boundary condition $\phi(0) = 0$ and $\phi(1) = 1$ using one of the methods discussed in class. Solve the problem and plot the solution.