Physics 411: Homework VII

Tuesday March 29, 2016 before class (i.e. 10:10 AM)

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Lorentzian Distribution

1. Explain how to generate random numbers with a Lorentzian distribution

$$L(x) = \frac{1}{\pi}\frac{1}{1+x^2} \tag{1}$$

2. Generate a (fairly large) sample of numbers with this distribution (dont print them out!) and calculate what fraction lie in the range $|x| < 1$. Compare your numerical result with the exact answer.

Monte Carlo integration with error bars

1. evaluate the integral

$$\int_1^2 \log x dx \tag{2}$$

   using $N = 2^4$ to 2^{20} samples, each time multiplying the number of samples by 2, using Monte Carlo integration.

2. Estimate an error bar from this integration. Make a table of values and your Monte Carlo error estimates. Plot the value of the integral with its error bar versus $\frac{1}{N}$ to show how it converges.

Bound states in a potential well (version 2)

Find the bound states solutions of the 1D Schrödinger equation with $E < 0$ using the Runge Kutta algorithm and a root solver. Note that a finite number of bound state solutions with discrete energy eigenvalues exists. The potential well should be zero for $x < 0$ and $x > 1$. For $0 < x < 1$, the potential is given by

$$V(x) = c(x^2 - x), \tag{3}$$

where c is a constant. What are the eigen energies as a function of c, and how many bound states are there as a function of c? Count the number of nodes (zeros) of your wave function to make sure that you find all eigen energies.

Proceed as follows:

1. Implement the Runge Kutta algorithm.
2. Implement a root solver.
3. Find the bound state of the potential by searching for the position b for which $V(b) = 1$, and proceed as in class.
4. Are there any excited states for this c? If so, find them, or show that there are none. The first excited state will have one node, the second two, etc...
5. Plot your ground state and excited state energies as a function of c. Show that as c is increased, more bound states are possible.

For full credit hand in an implementation of the Runge Kutta algorithm, a root solver routine, the ground state energy for $c = 1$ and your excited state information, and a plot showing all possible eigen energies as a function of c.