MAT244 Midterm 2

Main sitting, November 24, 18:00-20:00

Please print as legibly as possible:

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Instructions:

- This test is closed book. No calculators, phones or notes are permitted.
- You have 105 minutes to complete the test.
- Do not write on the top section of the pages. This area needs to be clear for the scanning and matching to be done correctly.
- Only the front of each page will be scanned and uploaded to Gradescope for grading. THE BACK OF EVERY PAGE IS FOR ROUGH WORK ONLY AND WILL NOT BE GRADED
- **ANY WORK WRITTEN ON THE BACK OF ANY PAGE WILL NOT BE GRADED OR CONSIDERED IN ANYWAY**
- This test consists of 17 pages including the cover page. The last two pages are extra space. If you want any work on these extra pages to be considered in grading you must indicate so on the page of the relevant question that work from that question is in the extra space.
- If you require extra space beyond that included please contact an invigilator. If you include extra pages, please set your exam aside at the end and do not include it with the main pile(s).
- Unless noted otherwise justify all solutions.
- The test is out of 105 points

1. The four parts of this question are short answer questions. You should provide some justification for your answers.

(a) (5 points) Find a fundamental set of real solutions to the ODE

$$\mathcal{L}[y] = 0$$

where the constant coefficient operator $\mathcal{L}$ has characteristic polynomial

$$p(r) = 4(r+3)^3(r-1)^2(r^2+1)^3$$

$r=-3$, m3 , 1 m2 , $\sqrt{-1}$ m3

$y_1 = e^{-3t}$, te^{-3t}, t^2e^{-3t},

$y_2 = e^{t}$, te^{t}

$y_3 = e^{it}$, te^{it}, t^2e^{it}

$$\boxed{\begin{array}{lll} y_1(t)= e^{-3t} & y_2(t)= te^{-3t} & y_3(t)= t^2e^{-3t} \\ y_4(t)= e^{t} & y_5(t)= te^{t} & \\ y_6(t)= \cos t + \sin t & & \\ y_7(t)= t\cos t + t\sin t & & \\ y_8(t)= t^2\cos t + t^2\sin t & & \end{array}}$$

(b) (5 points) Find the lowest degree monic[1] polynomial $p(r) = r^n + a_{n-1}r^{n-1} + \cdots + a_0$ with real coefficients such that the differential equation

$$y^{(n)} + a_{n-1}y^{(n-1)} + \cdots + a_0 y = 0$$

has solutions t^2e^{-t} and $e^t\cos(t)$. Your answer $p(r)$ may be left as a product of linear and/or quadratic factors all with real coefficients.

→ t^2e^{-t}, need at least 2 derivatives.

$\boxed{\text{Thus lowest degree } n=2}$

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[1] Monic means that the coefficient of the highest degree term is 1. E.g., r^3+1 is monic but $2r^4+1$ is not.

(c) (5 points) Consider the third order equation

$$y^{(3)}(t) + \sin(t)y^{(2)}(t) + y(t) = \mathrm{e}^{t}.$$

$y''' + \sin t\, y'' + y = e^{t}$

Find an equivalent three-dimensional first order system of the form,

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{x} = A\mathbf{x} + \mathbf{b}.$$

The entries of A and $\mathbf{b}$ may depend on time.

Let $x_1 = y$, $x_2 = \dot{y} = \dot{x}_1$, $x_3 = \ddot{y} = \dot{x}_2$ $x_4 = \dddot{y}$

$x_3(t) + \sin(t)\, x_2(t) + x_1(t)$

$$\boxed{\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & -\sin t & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ e^{t} \end{bmatrix}}$$

$x_1 = y$

$\dot{x}_1 = x_2$

$\dot{x}_2 = x_3$

$\dot{x}_3 = x_4$

$\dot{x}_4 = -\sin(t)x_3 - x_1 + e^{t}$

(d) (5 points) What values of α will the system:

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{x} = \begin{pmatrix} 3 & 2 \\ -2 & \alpha \end{pmatrix}\mathbf{x}$$

be such that all solutions wind/wrap/spiral around the origin infinitely many times and also move off to ∞ (you only need to consider eigenvalues for this)

$(3-\lambda)(\alpha-\lambda) + 4$

$= 3\alpha - \lambda(3+\alpha) + \lambda^2 + 4 \quad = \lambda^2 - \lambda(3+\alpha) + (4+3\alpha)$

$$\lambda = \frac{3+\alpha}{2} \pm \frac{\sqrt{(3+\alpha)^2 - 4(4+3\alpha)}}{2}$$

Need $(3+\alpha)^2 - 4(4+3\alpha) < 0$ and $3+\alpha > 0$.

$9 + \alpha^2 + \cancel{6\alpha} - 16 - 12\alpha = -7 + \alpha^2 - 6\alpha < 0.$

$= (\alpha - 7)(\alpha + 1) < 0$

$\alpha < 7$

$\boxed{\alpha < -1}$ but $\alpha > -3$

$\boxed{\text{Thus } -3 < \alpha < -1.}$

2. Draw a direction field for the following descriptions of a 2 × 2 system of ODEs. Your answers do not need to be exact, but they need to convey the relevant characteristics of the direction field.

   (a) (3 points) $\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{x} = \begin{pmatrix} 3 & 5 \\ 0 & -2 \end{pmatrix} \mathbf{x}$

$(3-\lambda)(-2-\lambda) = 0$

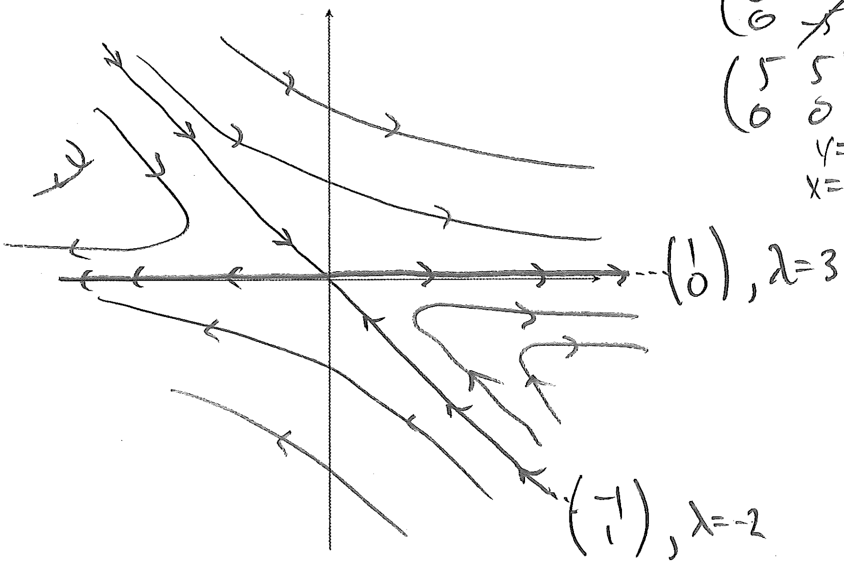
$\lambda = 3, -2$

$\begin{pmatrix} 0 & 5 \\ 0 & \cancel{-5} \end{pmatrix}$ $\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \to \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

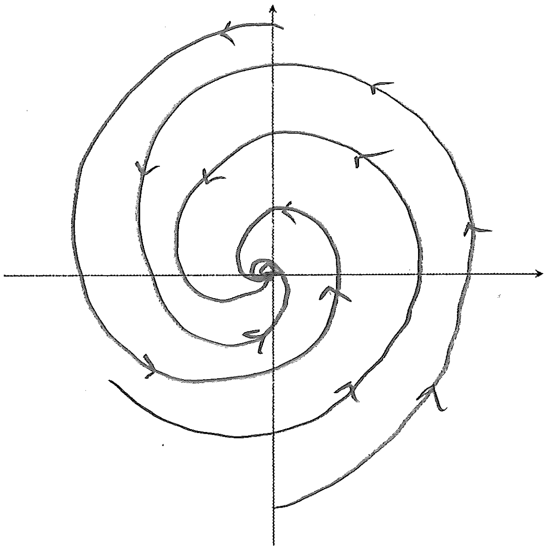
$\begin{pmatrix} 5 & 5 \\ 0 & 0 \end{pmatrix} \to \begin{pmatrix} -1 \\ 1 \end{pmatrix}$

$y=5$

$x=-5$

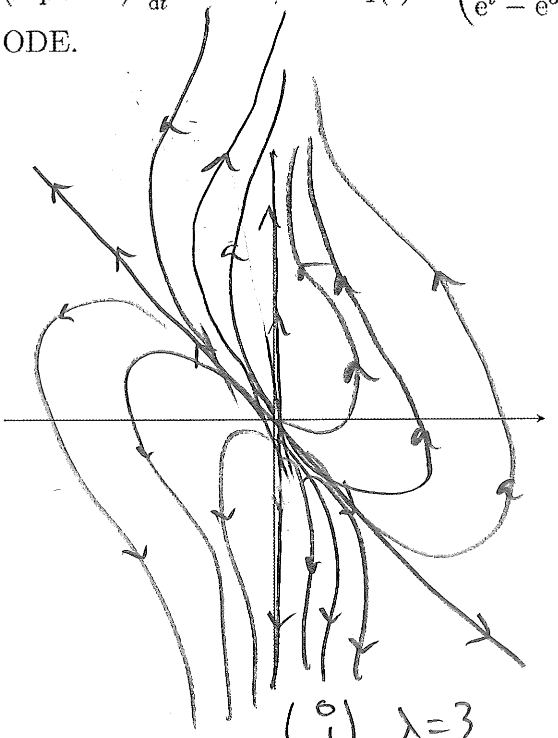


   (b) (3 points) $\frac{\mathrm{d}}{dt}\mathbf{x} = A\mathbf{x}$, and the 2 × 2 matrix A has eigenvalues $-2 \pm 2i$, and $\underbrace{A\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}}_{\text{counter-clockwise}}$



(c) (3 points) $\frac{d}{dt}\mathbf{x} = A\mathbf{x}$, and $\mathbf{x}_1(t) = \begin{pmatrix} -e^t \\ e^t - e^{3t} \end{pmatrix}$ and $\mathbf{x}_2(t) = \begin{pmatrix} 0 \\ 2e^{3t} \end{pmatrix}$ are both solutions of the ODE.

$$:= C_1 \begin{pmatrix} -1 \\ 1 \end{pmatrix} e^{t} + C_2 \begin{pmatrix} 0 \\ 1 \end{pmatrix} e^{3t}$$



$\begin{pmatrix} 0 \\ 1 \end{pmatrix}, \lambda = 3$

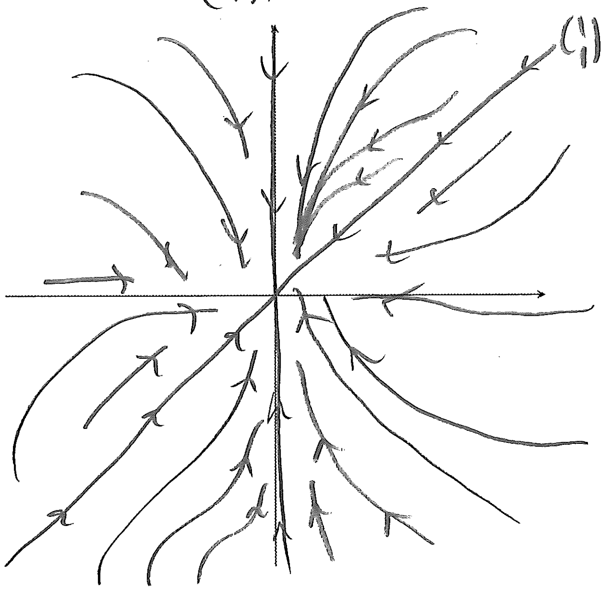
$\begin{pmatrix} -1 \\ 1 \end{pmatrix}, \lambda = 1$

(d) (3 points) $\frac{d}{dt}\mathbf{x} = A\mathbf{x}$, and $\mathbf{x}_1(t) = \begin{pmatrix} e^{-t} \\ e^{-t} - e^{-2t} \end{pmatrix}$ and $\mathbf{x}_2(t) = \begin{pmatrix} 0 \\ 4e^{-2t} \end{pmatrix}$ are both solutions of the ODE

$\begin{pmatrix} 0 \\ 1 \end{pmatrix}, \lambda = -2$

$\begin{pmatrix} 1 \\ 1 \end{pmatrix}, \lambda = -1$

$$C_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{-t} + C_2 \begin{pmatrix} 0 \\ 1 \end{pmatrix} e^{-2t}$$



Something like this?

e^{-2t} dominates as $t \to \infty$.

Solutions converge to (0,0).

3. (10 points) Find a particular solution to the ODE

$$\mathcal{L}[y](t) = \sin(3t)$$

where the constant coefficient operator $\mathcal{L}$ has characteristic polynomial

$$p(r) = (r^2 + 9)(r^2 - 1) = r^4 + 8r^2 - 9.$$

$\to (r^2 - 9)(r^2 + 1)$

$r = \pm 3, \quad r = \pm i.$

$y^{(4)} + 8y^{(2)} - 9y = \sin(3t).$

$c_1 e^{3t} + c_2 e^{-3t} + c_3 \cos t + c_4 \sin t.$

$8 \cdot 9 = 72$

$A\sin(3t)$. $\cos(t)$ gets cancelled.

$\downarrow y''$

$y'''' \downarrow -9A\sin(3t)$

$81 A\sin(3t).$

$81 A\cancel{\sin(3t)} + 72 A\cancel{\sin(3t)} + A\cancel{\sin(3t)} = \cancel{\sin(3t)}$

$9A + A = 1 \quad , A = \frac{1}{10}.$

Thus $\boxed{y_p(t) = \frac{1}{10}\sin(3t).}$

4. (8 points) Given the third order ODE:

$$y''' - y'' - 6y' = e^{3t}$$

Use variation of parameters to give the solution to this ODE with $y(0) = y'(0) = y''(0) = 0$. You must use variation of parameters or you will not receive marks for this question.

$r^{\cancel{3}} - r^{\cancel{2}} - 6\cancel{r} = 0.$

$r^2 - r - 6 = 0. \quad = (r-3)(r+2).$ $\qquad e^{3t} \quad e^{-2t}$

$\rightarrow y'' - y' - 6y = \frac{1}{3}e^{3t}, \qquad g(t) = \frac{1}{3}e^{3t},$

$W = \det\begin{pmatrix} e^{3t} & e^{-2t} \\ 3e^{3t} & -2e^{-2t} \end{pmatrix} \rightarrow -2e^{t} - 3e^{t} = -5e^{t}$

$-y_1 \int_0^{t} \frac{g(s)\, y_2(s)}{-5e^{s}} ds = -e^{3t} \int_0^{t} \frac{\frac{1}{3}\cancel{e^{3s}}\, e^{-2s}}{-5\cancel{e^{s}}} ds = \frac{1}{15} t e^{3t}$

$y_2 \int_0^{t} \frac{\frac{1}{3} e^{3t} e^{2t}}{-5e^{t}} ds = -\frac{1}{15} e^{-2t} \int e^{5t} dt = \frac{1}{15(5)} e^{3t}$

$-\frac{1}{15}e^{3t}t - \frac{1}{5}e^{3t} - \frac{3}{5}e^{3t}$

$y = -\frac{1}{15}te^{3t} \rightarrow y' = -\frac{1}{15}e^{3t}(t+3) \rightarrow y'' = -\frac{1}{15}e^{3t}(t+3) - \frac{3}{5}e^{3t}$

$-\frac{1}{15}te^{3t} + \left(-\frac{1}{5}e^{3t}\right) \quad y''' = -\frac{1}{11}e^{2t}(t+3) - \frac{12}{5}e^{3t}.$

$W = -5e^{t}. \quad y_1 = e^{3t}. \quad y_2 = e^{-2t}. \quad g(t) = \frac{1}{3}e^{3t}.$

$u_1(t) = -y_2 \int_0^{t} \frac{g(s)\, W_1(s)}{W(s)} ds = e^{-2t}\int_0^{t} \frac{\frac{1}{3}e^{3s} \cdot e^{3s}}{-5e^{s}} ds \rightarrow e^{-2t}\frac{-1}{15}\int e^{5s} ds \rightarrow e^{-2t}\frac{-1}{15\cdot 5}e^{5t}$

$= \frac{1}{75}e^{3s}.$

This is a guess.

lol this test is ridiculous.

$$\boxed{y_p(t) = -\frac{1}{18 \cdot 3}te^{3t} = -\frac{1}{54}te^{3t}}$$

5. (10 points) Find the solution of

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{x} = \begin{pmatrix} -2 & 0 & 1 \\ 0 & 3 & 0 \\ 0 & 4 & -1 \end{pmatrix}\mathbf{x}, \quad \mathbf{x}(0) = \begin{pmatrix} -2 \\ 10 \\ -5 \end{pmatrix}$$

$$(-2-\lambda)(3-\lambda)(-1-\lambda) + 0 + 0 - 0 - 0 - 0$$

$A - \lambda I$

$A + 2I$

$$\lambda = -2, 3, -1$$

$$\lambda=-2 \quad \begin{pmatrix} 0 & 0 & 1 \\ 0 & 5 & 0 \\ 0 & 4 & \cancel{1} \end{pmatrix} \to \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \to v = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$$

$$\lambda=-1 \quad \begin{pmatrix} -1 & 0 & 0 \\ 0 & \cancel{4} & 0 \\ 0 & \cancel{4} & 0 \end{pmatrix} \to \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \to v = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$

$$\lambda = 3 \quad \begin{pmatrix} -5 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 1 & -1 \end{pmatrix} \to \begin{pmatrix} 1 \\ 5 \\ 5 \end{pmatrix}$$

$z=5$

$x = -\frac{5}{5}$

$y=5$

$$x(t) = c_1\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}e^{-2t} + c_2\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}e^{-t} + c_3\begin{pmatrix} 1 \\ 5 \\ 5 \end{pmatrix}e^{3t}$$

$$\begin{pmatrix} -2 \\ 10 \\ -5 \end{pmatrix} = \begin{matrix} & + c_2 & + c_3 \\ c_1 & & + 5c_3 \\ c_1 & + c_2 & + 5c_3 \end{matrix} \to \left(\begin{array}{ccc|c} 0 & 1 & 1 & -2 \\ 1 & 0 & 5 & 10 \\ \overset{0}{\cancel{1}} & 1 & \overset{0}{\cancel{5}} & \overset{-15}{\cancel{-5}} \end{array}\right) \to \left(\begin{array}{ccc|c} 0 & \overset{0}{\cancel{1}} & 1 & \overset{13}{\cancel{2}} \\ 1 & 0 & 5 & 10 \\ 0 & 1 & 0 & -15 \end{array}\right)$$

$-2-(-15)$
$= 13$

$c_3 = 13$

$c_2 = -15$

$c_1 = 10 - 5(13) = -65$

$$\boxed{x(t) = -65\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}e^{-2t} + 15\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}e^{-t} + 13\begin{pmatrix} 1 \\ 5 \\ 5 \end{pmatrix}e^{3t}}$$

6. Consider the two-dimensional first order system,

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{x}(t) = A\mathbf{x}(t), \qquad A = \begin{pmatrix} 1 & 4 \\ 2 & -1 \end{pmatrix}$$

(a) (5 points) Find the eigenvectors and eigenvalues of A.

$(1-\lambda)(-1-\lambda) - 8$

$0 = -1 \cancel{-\lambda} \cancel{+\lambda} + \lambda^2 - 8$

$\lambda^2 = 9$, $\boxed{\lambda = \pm 3.}$

$$\begin{pmatrix} 1-3 & 4 \\ 2 & -1-3 \end{pmatrix} \rightarrow \begin{pmatrix} \overset{1}{\cancel{-2}} & \overset{-2}{\cancel{4}} \\ \cancel{2} & \cancel{-4} \end{pmatrix} \rightarrow v_1 = \begin{pmatrix} 2 \\ 1 \end{pmatrix}, \lambda = 3$$

$$\begin{pmatrix} 1+3 & 4 \\ 2 & -1+3 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix} \rightarrow v_2 = \begin{pmatrix} -1 \\ 1 \end{pmatrix}, \lambda = -3$$

(b) (5 points) Find the general real solution, as well as the solution satisfying $\mathbf{x}(0) = \begin{pmatrix} 3 \\ 0 \end{pmatrix}$

$$\boxed{x(t) = c_1 \begin{pmatrix} 2 \\ 1 \end{pmatrix} e^{7t} + c_2 \begin{pmatrix} -1 \\ 1 \end{pmatrix} e^{-3t}}$$ Gen Sol$^{\underline{n}}$

$$x(0) = \begin{pmatrix} 3 \\ 0 \end{pmatrix} = \begin{matrix} 2c_1 \\ c_1 \end{matrix} + \begin{matrix} -c_2 \\ c_2 \end{matrix}$$

$$c_1 = -c_2$$

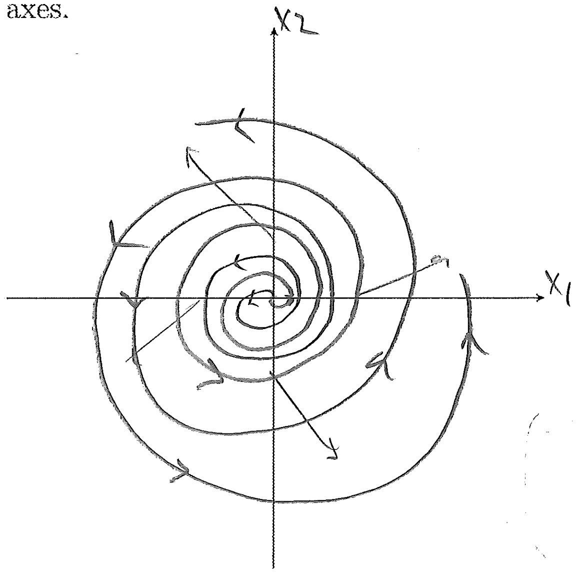
$$3 = 2c_1 + c_1 \rightarrow 1 = c_1 \,,\; c_2 = -1$$

$$\boxed{x(t) = \begin{pmatrix} 2 \\ 1 \end{pmatrix} e^{3t} - \begin{pmatrix} -1 \\ 1 \end{pmatrix} e^{-3t}}$$ IVP Sol$^{\underline{n}}$.

7. Consider the two dimensional first order system,

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{x}(t) = A\mathbf{x}(t), \qquad A = \begin{pmatrix} 3 & -5 \\ 1 & 5 \end{pmatrix}$$

(a) (12 points) Find the general real solution, and sketch the phase portrait on the provided axes.



$= (3-\lambda)(5-\lambda) + 5$

$= 15 + 5 - 8\lambda + \lambda^2$

$= \lambda^2 - 8\lambda + 20$

$\lambda = \frac{8}{2} \pm \frac{\sqrt{64 - 4(1)(20)}}{2}$

$= 4 \pm \frac{\sqrt{64-80}}{2} = \boxed{4 \pm 2i}$

$80 - 64$

$= 16$

$= \frac{i4}{2} = 2i$

$\begin{pmatrix} 3-(4+2i) & -5 \\ 1 & 5-(4+2i) \end{pmatrix} \rightarrow \begin{pmatrix} -1+2i & -5 \\ 1 & 1+2i \end{pmatrix} \cdot (1-2i) \rightarrow \begin{pmatrix} -1+2i & -5 \\ \cancel{1-2i} & \cancel{5} \end{pmatrix}$

$(1-2i) \quad (\cancel{1+4})$

$\rightarrow \begin{pmatrix} -1+2i \\ 5 \end{pmatrix}$

$\lambda = 4 \pm 2i, \quad v = \begin{pmatrix} -1 \\ 2 \end{pmatrix} \pm i \begin{pmatrix} 2 \\ 0 \end{pmatrix}$

$a\cos t \quad -b\sin t$

$a\sin t \quad b\cos t$

$$\boxed{x(t) = c_1 e^{4t}\left[\begin{pmatrix} -1 \\ 2 \end{pmatrix}\cos(2t) - \begin{pmatrix} 2 \\ 0 \end{pmatrix}\sin(2t)\right] + c_2 e^{4t}\left[\begin{pmatrix} -1 \\ 2 \end{pmatrix}\sin(2t) + \begin{pmatrix} 2 \\ 0 \end{pmatrix}\cos(2t)\right]}$$

(b) (3 points) Consider a non-zero solution of the ODE you just solved and let $N(t)$ be the number of times the solution crosses the x_2 axis in the interval $[0, t]$. Find,

$$\lim_{t\to\infty} \frac{N(t)}{t}$$

$N(t)$ depends on $\cos(2t)$, $\sin(2t)$.

Sps. $x(t) = e^{4t}\begin{pmatrix} 7 \\ 2 \end{pmatrix}\cos(2t) + e^{4t}\begin{pmatrix} 2 \\ 0 \end{pmatrix}\sin(2t)$.

Because the solution oscillates, $\lim_{t\to\infty} N(t) = \infty$.

every $t = k\cdot\frac{\pi}{2}$, the solution will pass through x_1 axis.

every $t = k\pi$, the solution will pass through x_2 axis.

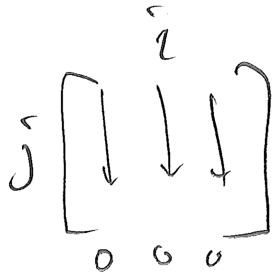
Find RoC of $N(t)$, $N'(t)$. every π radians soln should pass through, so

$$\boxed{\lim_{t\to\infty} \frac{N(t)}{t} = \frac{\pi}{2}}$$

?

since every $t = (2k+1)\frac{\pi}{2}$, $\cos(2t) = 0$.

8. Let Q be an $n \times n$ symmetric matrix with real entries such that,



$$\sum_{j=1}^{n} Q_{ij} = 0$$

for every i and $Q_{ij} \geq 0$ for $i \neq j$. Let $\mathbf{u}(t)$ be a solution of the nth order system,

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{u}(t) = Q\mathbf{u}(t)$$

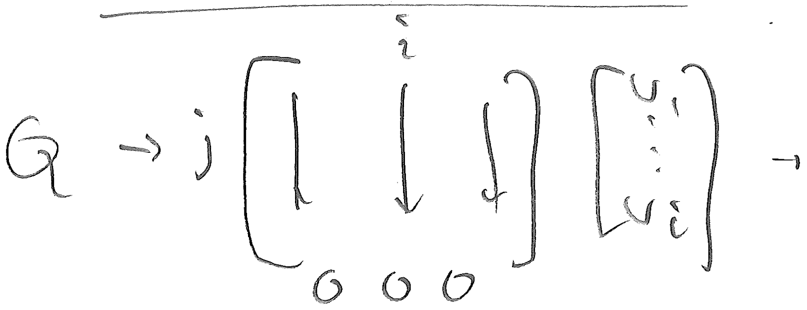
(a) (5 points) Show that the average

$$f(t) := \sum_{i=1}^{n} u_i(t)$$

is constant. Here, $u_i(t)$ denotes the ith coordinate of $\mathbf{u}(t)$.

The coordinates of $u_i(t)$ are scalar ~~multiples~~ of Q's colums, which add to zero no matter which column.

Thus, white summing each column, the average is constant and is 0.



(b) (5 points) Consider the function,

$$g(t) := \sum_{i=1}^{n} (u_i(t))^2$$

and show that

$$\frac{\mathrm{d}}{\mathrm{d}t} g(t) = -\sum_{i=1}^{n}\sum_{j=1}^{n} Q_{ij}(u_i(t) - u_j(t))^2.$$

Use this to show that $g(t)$ is decreasing. If every entry of Q is non-zero what are the equilibrium solutions of the system $\mathbf{u}' = Q\mathbf{u}$?

$$\frac{d}{dt} g(t) = \frac{d}{dt} \sum_{j=1}^{n} (u_i(t))^2$$

$$= \sum_{i=1}^{n} \cdot 2 \cdot \frac{d}{dt} u_i(t) \rightarrow 2 \cancel{\sum_{i=1}^{n}} \frac{d}{dt}\left[u(t)\right]$$

Nice.

$$\int_{-\infty}^{\infty} \frac{\sin x}{x} dx \rightarrow I(\kappa) = \int e^{-\kappa x} \frac{\sin x}{x} dx \quad , I'(\kappa) = \int$$

9. (10 points) Find a first order three-dimensional system of the form

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{x}(t) = A\mathbf{x}(t)$$

where A is 3×3 matrix such that:

- There exist non-zero, non-constant solutions $\mathbf{x}(t)$ such that $\|\mathbf{x}(t)\| = 1$ for all times t.
- There exist non-zero solutions that converge to 0 as $t \to \infty$.

Explain why your answer is correct.

$$\begin{pmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$

$\lambda = -2, \quad v_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad \|v_1\| = 1$

$\lambda = -1, \quad v_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \quad \|v_2\| = 1$

$\lambda = -3, \quad v_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \quad \|v_3\| = 1$

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}.$$

$$X(t) = C_1 \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} e^{-t} + C_2 \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} e^{-2t} + C_3 \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} e^{-3t}.$$

For all t, $\|X(t)\| = 1$. (?)

my answer is correct because neither of e^{-t}, e^{-2t}, e^{-3t} are 0 for all t, and $\lim_{t \to \infty} [e^{-t}, e^{-2t}, e^{-3t}] = 0$ so they converge to 0.

v, the norm of $\|x\|$ is 1 for any of x_1, x_2, and x_3.

Extra work for Question ____

(Please write "EXTRA WORK AT END OF EXAM" on original question page)

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