PS1

My score
100% (16/16)

Q1

0 / 0

Problem Set Policy: Read the Problem Set Policy on this course Syllabus (page 5). Did you read and understand the policy? A " Yes" should be answered truthfully before continuing with this problem set.

PHY254 PS1

Q1/ yes.

[illegible]
1006940802

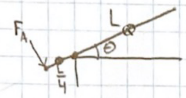
Q2

4 / 4

Statics. Morin, Exercise 2.33. [Hints: It can be assumed that the normal force of the corner on the stick points perpendicular to the stick.]

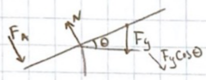
PHY254 - PS1

Q2) Morin 2.33



- Force of gravity acts at centre of mass.

Balancing Torques:



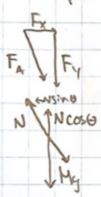
Require that

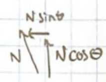
$\tau_A = \tau_g$ $\qquad \tau = F R \sin\theta$

$$N \cdot \frac{L}{4} = Mg \cdot \frac{L}{2} \cdot \cos\theta$$

$$\Rightarrow N = 2Mg\cos\theta$$

Decompose Forces:





$$F_x = N\sin\theta \quad = 2Mg\cos\theta\sin\theta$$

$$F_y = Mg - N\cos\theta \quad = Mg - 2Mg\cos^2\theta$$

Net Force

$$F_A = \sqrt{F_x^2 + F_y^2}$$

$$= \left(4M^2g^2\cos^2\theta\sin^2\theta + M^2g^2 + 4M^2g^2\cos^4\theta - 4M^2g^2\cos^2\theta\right)^{1/2}$$

$$= \left(\cancel{4M^2g^2\cos^2\theta\,(\underbrace{\sin^2\theta + \cos^2\theta}_{1})} + M^2g^2 - \cancel{4M^2g^2\cos^2\theta}\right)^{1/2}$$

$$= \left(M^2g^2\right)^{1/2}$$

$$\boxed{F_A = Mg}$$

Horizontal Force: $F_y = 0 \Rightarrow Mg(1 - 2\cos^2\theta) = 0$

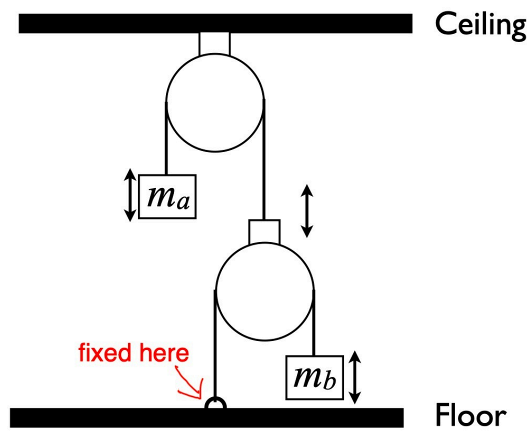
$$\Rightarrow \cos^2\theta = \frac{1}{2}$$

$$\Rightarrow \boxed{\theta = \frac{\pi}{4}, \text{ or } 45^\circ}$$

Q3

4 / 4

3. **Newton's second law.** In the figure below, masses m_a and m_b are connected to a system consisting of ideal strings and pulleys (massless, frictionless, etc.) as shown. Find the accelerations of the masses and the tensions in the strings. When is the system static?



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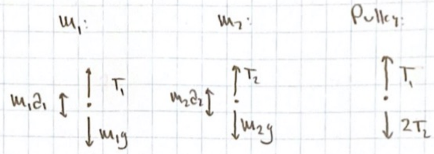
Q3



To balance forces.
$T_1 = 2T_2$.

How do you know this?

- For every distance x_0 m_1 moves, m_2 moves $-2x_0$. Then $\Delta x_2 = -2\Delta x_1$. Implies acceleration constraint: $\boxed{a_2 = -2a_1}$

Equations:

(1) $2T - m_1 g = m_1 a_1$

(2) $T - m_2 g = m_2 a_2$

$\Rightarrow T = m_2(a_2 + g)$

- Sub into (1):

$$2(m_2(g + a_2)) - m_1 g = m_1 a_1$$
$$= -\frac{1}{2} m_1 a_2 \qquad \text{Apply acceleration constraint}$$
$$2m_2 g + 2m_2 a_2 = m_1 g - \frac{1}{2} m_1 a_2$$
$$a_2\left(2m_2 + \frac{1}{2} m_1\right) = m_1 g - 2m_2 g = g(m_1 - 2m_2)$$
$$\Rightarrow \boxed{a_2 = 2 \cdot \frac{g(m_1 - 2m_2)}{(m_1 + 4m_2)}} \Rightarrow a_1 = -\frac{g(m_1 - 2m_2)}{(m_1 + 4m_2)}$$
$$\cancel{\boxed{a_2 = \frac{g(2m_2 - m_1)}{(m_1 + 4m_2)}}}$$

Tension

$$T = m_2(a_2 + g) = m_2 \cdot 2\frac{g(m_1 - 2m_2)}{(m_1 + 4m_2)} + \frac{m_2 g(m_1 + 4m_2)}{(m_1 + 4m_2)}$$
$$= \frac{2g m_1 m_2 \cancel{- 4g m_2^2} + g m_2 m_1 \cancel{+ 4g m_2^2}}{m_1 + 4m_2}$$
$$\boxed{T = \frac{3g m_1 m_2}{m_1 + 4m_2}}$$

System is static when $a = 0$ $\Rightarrow$ $\boxed{\text{Static System when } m_1 = 2m_2}$

Q4

4 / 4

4. **Integrating Newton's second law.**

A particle of mass m moves on the axis, starting at $t_0 = 0$ at $x = x_0$, with velocity v_0. It is acted on by a retarding force $F = -mbe^{cv}$, where b and c are positive constants. Find analytic solutions for the position $x(t)$, velocity $v(t)$, and acceleration $a(t)$. [*Hint:* $\int \ln(1+x)dx = (1+x)\ln(1+x) - (1+x)$.]

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Q4/ Given x_0, $t_0 = 0$, v_0. $F = -mbe^{cv}$, $b, c \in \mathbb{R}^+$.

Steps: · Solve ODE to get $v(t)$. (1)
· Integrate to get $x(t)$. (2)
· Differentiate to get $a(t)$. (3)

(1)
$$F(v) = m\frac{dv}{dt} = -mbe^{cv}$$

$$dv = -be^{cv}\,dt \Rightarrow -\frac{dv}{be^{-cv}} = dt \Rightarrow -\frac{1}{b}e^{cv}\,dv = dt$$

$$-\frac{1}{b}\int_{v_0}^{v} e^{cv'}dv' = \int_{t_0}^{t} dt'$$

$$-\frac{1}{b}\left[\frac{1}{c}e^{cv'}\right]_{v_0}^{v} = -\frac{1}{bc}\left[e^{cv} - e^{cv_0}\right] = t - t_0$$

$$\Longrightarrow \frac{e^{cv_0}}{bc} - \frac{e^{cv}}{bc} = t - t_0 \Rightarrow e^{cv_0} - e^{cv} = bc(t - t_0)$$

$$\Longrightarrow e^{cv} = e^{cv_0} - bc(t - t_0) = e^{cv_0} + bc(t_0 - t)$$

$$\log\left[e^{cv}\right] = cv = \log\left[e^{cv_0} + bc(t_0 - t)\right]$$

$$\Longrightarrow \boxed{v(t) = \frac{1}{c}\log\left[bc(t_0 - t) + e^{cv_0}\right]}$$

(2)
$$\frac{dx}{dt} = v(t) \Longrightarrow dx = v(t)\,dt. \quad \text{Then}$$

$$\int_{x_0}^{x} dx' = \int_{t_0}^{t} \frac{1}{c}\log\left(bc(t_0 - t) + e^{cv_0}\right)dt$$

$$u = bc(t_0 - t) + e^{cv_0}$$
$$du = -bc\,dt \Rightarrow dt = -\frac{du}{bc}$$

$$x - x_0 = \frac{1}{c}\int_{u(t_0)}^{u(t)} \log(u')\cdot\left(-\frac{du'}{bc}\right)$$

$$= -\frac{1}{bc^2}\left[u'\log u' - u'\right]_{u(t_0)}^{u(t)}$$

$$= -\frac{1}{bc^2}\left[\left(bc(t_0 - t) + e^{cv_0}\right)\log\left(bc(t_0 - t) + e^{cv_0}\right) - \left(bc(t_0 - t) + e^{cv_0}\right)\right]_{t_0}^{t}$$

$$= -\frac{1}{bc^2}\Big[(bc(t_0-t)+e^{cv_0})\log(bc(t_0-t)+e^{cv_0}) - (bc(t_0-t)+e^{cv_0}) -$$

$$(\cancel{bc(t_0-t_0)}^{0}+e^{cv_0})\log(\cancel{bc(t_0-t_0)}^{0}+e^{cv_0}) + (\cancel{bc(t_0-t_0)}^{0}+e^{cv_0})\Big]$$

$$= -\frac{1}{bc^2}\Big[(bc(t_0-t)+e^{cv_0})\log(bc(t_0-t)+e^{cv_0}) - (bc(t_0-t)+e^{cv_0}) - e^{cv_0}\cdot cv_0 + e^{cv_0}$$

$$= -\frac{1}{bc^2}\Big[(bc(t_0-t)+e^{cv_0})\log(bc(t_0-t)+e^{cv_0}) - bc(t_0-t) \cancel{- e^{cv_0} + e^{cv_0}}^{0} - e^{cv_0}cv_0$$

$$= -\frac{1}{c}\left[\left(t_0-t+\frac{e^{cv_0}}{bc}\right)\log(bc(t_0-t)+e^{cv_0}) + (t-t_0) - \frac{v_0e^{cv_0}}{b}\right]$$

Then

$$\boxed{x(t) = x_0 - \frac{1}{c}\left(t_0-t+\frac{e^{cv_0}}{bc}\right)\log(bc(t_0-t)+e^{cv_0}) + \frac{t_0-t}{c} + \frac{v_0e^{cv_0}}{bc}}$$

(3) $$\frac{dv}{dt} = a(t) = \frac{d}{dt}\left[\frac{1}{c}\log\left[bc(t_0-t)+e^{cv_0}\right]\right]$$

$$= \frac{1}{c(bc(t_0-t)+e^{cv_0})}\cdot(-bc)$$

$$\Longrightarrow \boxed{a(t) = -\frac{b}{bc(t_0-t)+e^{cv_0}}}$$

When does the particle stop? When $v=0$ ...

$$0 = \frac{1}{c}\log\left[bc(t_0-t)+e^{cv_0}\right]$$

$\log(x)$ is zero only when $x=1$:

$$bc(t_0-t)+e^{cv_0} = 1$$

$$bct_0 - bct + e^{cv_0} = 1 \Rightarrow bct = bct_0 + e^{cv_0} - 1$$

$$\Longrightarrow \boxed{t = t_0 + \frac{e^{cv_0}-1}{bc}}$$

This is when the particle stops moving.

Q5

4 / 4

5. Motion in polar coordinates/circular motion.

Morin, Exercise 3.59.

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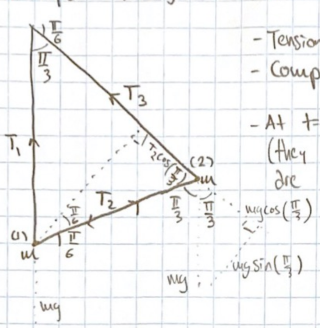
Q5/ Morin 3.59

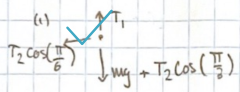
- Analyze Triangle + FBD's



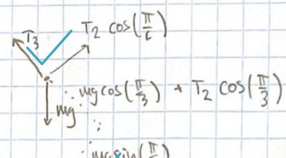
- Tension on T_1, T_3
- Compression in T_2

- At $t=0$, radial accelerations must be zero (they cannot move radially because the rods are stiff)

(1)



(2)



Radial: $T_1 - T_2\cos(\frac{\pi}{3}) = mg$ $\qquad$ $T_3 - T_2\cos(\frac{\pi}{3}) = mg\cos(\frac{\pi}{3})$

Tangential: $T_2\cos(\frac{\pi}{6}) = ma$ $\qquad$ $mg\sin(\frac{\pi}{3}) - T_2\cos(\frac{\pi}{6}) = ma$

$\downarrow$

4 eqn's, 4 unknowns

$T_1 - \frac{1}{2}T_2 = mg \quad (1)$ $\qquad$ $T_3 - \frac{1}{2}T_2 = \frac{1}{2}mg \quad (3)$

$\frac{\sqrt{3}}{2} T_2 = ma$ (2) $\qquad \frac{\sqrt{3}}{2} mg - \frac{\sqrt{3}}{2} T_2 = ma$ (4)

- (2) into (4)

$\frac{\sqrt{3}}{2} mg - \frac{\sqrt{3}}{2} T_2 = \frac{\sqrt{3}}{2} T_2 \Rightarrow mg = 2T_2 \qquad \boxed{T_2 = \frac{1}{2} mg}$ - Compression

- (1) $T_1 - \frac{1}{2}\left(\frac{1}{2} mg\right) = mg \Rightarrow \boxed{T_1 = \frac{5}{4} mg}$
- (3) $T_3 - \frac{1}{2}\left(\frac{1}{2} mg\right) = \frac{1}{2} mg \Rightarrow \boxed{T_3 = \frac{3}{4} mg}$

} Tension

- (2) $\frac{\sqrt{3}}{2} T_2 = \frac{\sqrt{3}}{2} \cdot \frac{1}{2} mg = ma \Rightarrow \boxed{a = \frac{\sqrt{3}}{4} mg}$