PHY489 Problem Set 2

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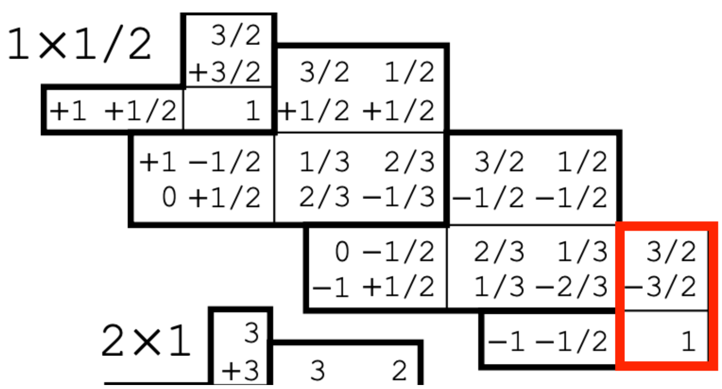
Problem 1

Consider the decay $\Delta^- \to \pi^- n$. This decay proceeds via the strong interaction, and therefore the isospin (quark content) of the particles must be conserved throughout the process. The Δ^- is a baryon, and hence has total isospin $\frac{3}{2}$. It, however, is comprised of three down quarks ddd which each have isospin $-\frac{1}{2}$, so the only ket representing the Δ^- baryon, adding the z-components of isospin (such that their sum has magnitude $3/2$), is $\left|\frac{3}{2}\ -\frac{3}{2}\right\rangle$. As given (in lecture and the textbook), the isospin kets of the π^- meson (isospin 1 meson) and the neutron (isospin $\frac{1}{2}$ baryon) are $|1\ -1\rangle$, $\left|\frac{1}{2}\ -\frac{1}{2}\right\rangle$ respectively. We may therefore proceed to show that isospin is conserved in the interaction by adding the isospin on the right hand side of the decay equation:

$$|1\ -1\rangle \left|\frac{1}{2}\ -\frac{1}{2}\right\rangle \tag{1.1}$$

which, according to the $1 \times \frac{1}{2}$ Clebsch Gordon table, $m = -1 - \frac{1}{2} = -\frac{3}{2}$, so $J = \frac{3}{2}$ and the resulting ket with probability 1 is $\left|\frac{3}{2}\ -\frac{3}{2}\right\rangle$ which is equivalent to the isospin of the Δ^-.



Therefore isospin is conserved in this strong decay, and the magnitude and z-component of isospin is $\frac{3}{2}$ and $-\frac{3}{2}$, respectively.

Problem 2

Consider the decay of the ρ^0 meson $\rho^0 \to \pi^+\pi^-$. We desire to show that total angular momentum (spin and orbital) and parity J^p are conserved throughout this process.

First consider the parity and angular momentum of the ρ^0. In the rest frame of the ρ^0, there is no orbital angular momentum (there's nothing for it to orbit), while it being a spin-1 meson, has spin 1 (we can always choose our axis such that spin-1 aligns with the z-axis of the particle in its rest frame) so it's spin ket is $|0\,0\rangle\,|1\,1\rangle$. Consulting the Clebsch-Gordon table, it is easy to see that any addition of spin to a spin-0 state remains the same with probability 1 (they just yield Kronecker delta's: $\delta_{j,j_1}\delta_{m,m_1}$): $|0\,0\rangle\,|1\,1\rangle = |1\,1\rangle$. For mesons, then, the parity is given by $(-1)^{\ell+1}$ (ℓ is orbital angular momentum), which implies that $J^P_{\rho^0} = 1^-$.

For the pions, however, the only possible way for angular momentum to be conserved is if both pions orbit each other in the first orbital, ie. $\ell = 1$. Let's choose the post-decay rest frame to be that of the π^-, so the π^+ acquires the orbital momentum ket $|1\,1\rangle$. Furthermore, since both pions have spin-0, their spin kets are both $|0\,0\rangle$. Thus, for the π^-: $|0\,0\rangle\,|0\,0\rangle = |0\,0\rangle$ (this is trivial), while for the π^+: $|1\,1\rangle\,|0\,0\rangle = |1\,1\rangle$ (as before) and therefore the totaly angular momentum of the pion final state is $|1\,1\rangle$, equivalent to that of the ρ^0. Therefore angular momentum is conserved, but we must now check the parity conservation of the pions.

For the π^+, $\ell = 1$ and for the π^- $\ell = 0$. Thus for mesons, the total parity is the multiplicative product of the consituents, hence $(-1)^{1+1}(-1)^{0+1} = (1)(-1) = (-1)$, so the final state parity is -1 and therefore parity is also consvered: $J^P_{\pi^+\pi^-} = 1^- = J^P_{\rho^0}$, which is what I wanted to show.

Problem 3

Consider the fixed-target deuterium-pion interaction decay of the bound state (no orbital angular momentum) $\pi^- d \to nn$. It is given that $J^P = 1^+$ for deuteriumm, the π^- spin ket is $|0\,0\rangle$, and the orbital angular momentum for the pair of neutrons is $\ell = 1$. This process proceeds via the strong interaction, and therefore angular momentum and parity must be conserved.

We first begin by determining the parity of the neutron state. Note that $\ell = 1$ for one of the neutrons (if we choose the rest frame to be one of the neutrons post-decay) and the parity of baryons is given by $(-1)^\ell$ (ℓ = orbital angular momentum). Since the total parity is the product of consituents, the parity of the neutron pair is $(-1)^1(-1)^0 = -1$.

Since the parity of the right-hand-side of the interaction is -1 and $J_d^P = 1^+$, it must be that the π^- has negative parity for it to be conserved: $(-1)(1) = -1$.

Furthermore, this may be verified by knowing that $\ell = 0$ in the $d\pi^-$ bound state and that the parity of mesons is given by $(-1)^{\ell+1}$, thus for the pion, $(-1)^{0+1} = -1$.

Problem 4

Suppose that quarks and antiquarks have spin-0. The possible meson states are formed via two quarks $q_1 q_2$. Since there is no spin on the quarks, the only total angular momentum arises from the contributions of orbital angular momentum: given by the spherical harmonics $Y_l^m(\theta, \varphi)$. Since the harmonics are independent of the radius, a parity operation on them takes them into the new function

$$\begin{aligned} PY_l^l(\theta, \varphi) &= Y_l^l(\theta - \pi, \varphi + \pi) \\ &\sim e^{i\ell\varphi} e^{i\ell\pi} \sin^\ell(\theta) \\ &= (-1)^l Y_l^l(\theta, \varphi). \end{aligned} \tag{4.1}$$

However, the parity of a orbital state is governed by the angular component, and since angular momentum operators are pseudovectors, they do not change sign under a parity operation. Therefore the Y_l^l and the Y_l^m have the same parity, so $PY_l^m(\theta, \varphi) = (-1)^l Y_l^m(\theta, \varphi)$.

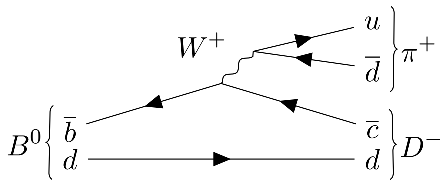
In terms of the mesons (whose quarks are spin-0), the parity of the spherical harmonics determine the parity of the meson state according the orbital angular momentum value: $P = (-1)^\ell$ hence $J^P = \ell^{(-1)^\ell}$ which, in series, is

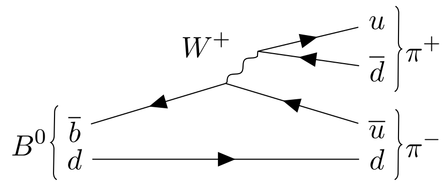
$$J^P = 0^+, 1^-, 2^+, 3^-, \ldots. \tag{4.2}$$

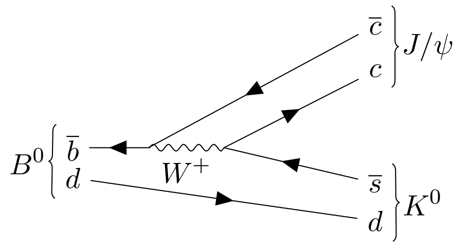
which is what I wanted to show. Since meson states are never observed with such a J^P, quarks must not have spin-0.

Problem 5

Consider the three decays of the B^0 meson: $B^0 \to D^-\pi^+$, $B^0 \to \pi^-\pi^+$, and $B^0 \to (J/\psi)K^0$. The corresponding Feynman diagrams are







Now, the decay rate is proportional to the square of the Feynman amplitude, which therein is proportional to the product of the CKM matrix coupling probabilities between the mass and quark bases. That is, $\Gamma \propto |\mathcal{M}|^2 \propto |\prod_i V_i|^2$ where i in the product represents the CKM matrix element associated with the i-th vertex of the diagrammatic perturbation.

For the first decay, the W boson couples via V_{cb} and V_{ud}. Thus $\Gamma_1 \propto |V_{cb}V_{ud}|^2$. Similarly, the second decay corresponds to $\Gamma_2 \propto |V_{ub}V_{ud}|^2$, and the third decay $\Gamma_3 \propto |V_{cb}V_{cs}|^2$. Explicitly, we have that (from PDG)

$$
\begin{aligned}
V_{ud} &= 0.97420 \pm 0.00021 \\
V_{cb} &= 0.0422 \pm 0.0008 \\
V_{ub} &= 0.00394 \pm 0.00036 \\
V_{cs} &= 0.997 \pm 0.017.
\end{aligned}
$$

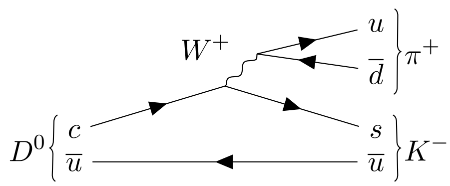
Calculating each of the products, we find that $\Gamma_1 \sim 1.69 \times 10^{-3}, \Gamma_2 \sim 1.47 \times 10^{-5}$, and $\Gamma_3 \sim 1.77 \times 10^{-3}$. This implies that

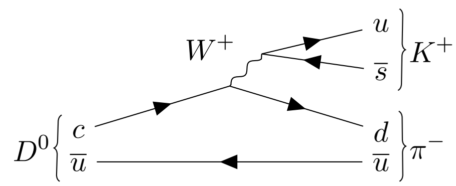
$$\Gamma(B^0 \to (J/\psi)K^0) > \Gamma(B^0 \to D^-\pi^+) > \Gamma(B^0 \to \pi^+\pi^-)$$

are the decay rates in decreasing order.

Problem 6

Lastly, consider the decays of the D^0 meson $D^0 \to K^-\pi^+$ and $D^0 \to K^+\pi^-$. From the Feynman diagrams and the CKM matrix elements, we are able to determine which decay occurs more frequently.





Similarly to Problem 5, the square of the Feynman amplitude is directly proportional to the probability of the event occuring, thus utilizing the CKM matrix elements

$$
\begin{aligned}
V_{ud} &= 0.97420 \pm 0.00021 \\
V_{cd} &= 0.023 \pm 0.01 \\
V_{us} &= 0.22 \pm 0.0001 \\
V_{cs} &= 0.997 \pm 0.017
\end{aligned}
$$

the square of the probabilities (square of the products of the elements) yield the most likely outcomes for such a decay:

$$
\begin{aligned}
|\langle f|S-1|i\rangle|^2 &\propto |V_{ud}V_{cs}|^2 \approx 0.943 \\
|\langle f|S-1|i\rangle|^2 &\propto |V_{cd}V_{us}|^2 \approx 2.56 \times 10^{-5}
\end{aligned}
$$

($S-1$ indicates that leading order scattering must occur) and therefore the π^+K^- decay is significantly more likely to occur than the π^-K^+ decay, which is what I wanted to determine.

Problem 7

Not a graduate student.