PHY224 - Kater Pendulum

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1 Introduction

The Kater Pendulum is an asymmetrical double ended pendulum with an adjustable mass in the centre. In principle, one could position the mass in such a way so that the period of the swing is identical on either end. Measuring the period swing and the pendulum length accurately gives a precise measurement for the acceleration due to gravity. In the 1800's, these measurements allowed precise globe navigation and temperature compensated, torsion-springed clocks.

1.1 Theory

For small angles or equivalently, low amplitude oscillations, we have that $\sin\theta \approx \theta$. In such a case, the period of a swinging pendulum is given by

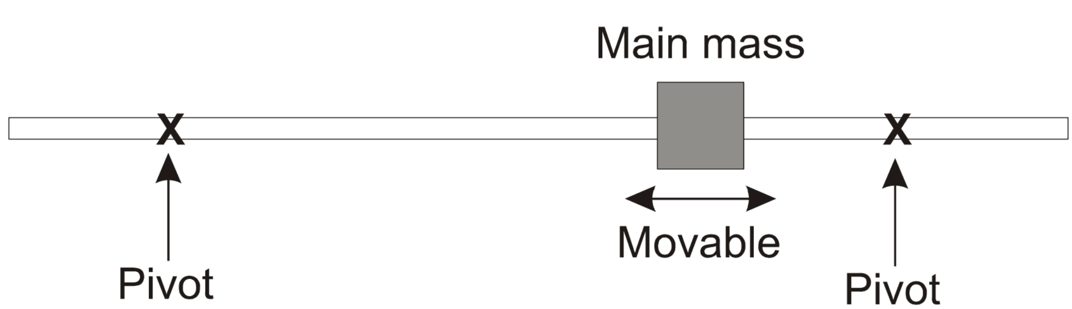
$$T = 2\pi\sqrt{\frac{L}{g}}. \tag{1}$$

This yields

$$g = L\left(\frac{2\pi}{T}\right)^2. \tag{2}$$

Realistically, a swinging pendulums' effective length L_e is difficult to define, in which $L_e = \frac{k^2}{r_m}$, where k is the radius of gyration of the pendulum and r_m is the distance from the center of mass to the pendulum fulcrum. These individual parameters are also quite difficult to determine due to the moment of inertia of the pendulum and its changing mass position or length.

Kater's innovation was to swing an asymmetrical pendulum from either pivot point on opposite ends and find the position of the moveable mass such that the period on either pivot is the same. When this is fulfilled, the effective length of the pendulum is just the distance between either pivot.



2 Materials and Methods

2.1 Materials

- UofT Physics Kater Pendulum (1 main mass, 1 coarse adjustment mass, 1 fine adjustment mass)
- Voltage gate, ruler
- Interval timer
- Cathetometer

2.2 Methods

1. The pendulum was set in grooves and let loose until it stopped swinging entirely.
2. The voltage gate was placed directly underneath the pendulum tip. The pendulum was let to swing to adjust the orientation of the voltage gate:

3. The interval timer was turned on and the pendulum was let swing from a 5cm distance from the center of the voltage gate by using the ruler placed on the voltage gate device. Each time the pendulum was release, it was not pushed and minimal movement was made while it was swinging.
4. The pendulum was let swing. The interval timer recorded 22 passes and the time was measured, which allowed the period to be determined in latter steps.
5. The pendulum was then inverted and the time was again measured. Afterwards, the coarse adjustment mass was moved by 1cm at a time, as indicated by the ruler on the face of the pendulum arm.

6. Steps 3 and 4 were repeated multiple times until an equivalent time point was determined, where the coarse mass position on the upright pivot measured a time equivalent to the time measured on the inverted pivot, without moving the mass.

7. From this point, the fine adjustment mass was moved and again, the pendulum was released from an amplitude of 5cm from equilibrium. The time for 22 passes was recorded, and another 2 measurements were made for certainty purposes.

8. The pendulum was then inverted and more time measurements were made by adjusting the position of the fine mass. These steps were repeated multiple times for a series of upright and inverted measurements of fine mass position.

9. Using the Cathetometer, the pendulum was hung and the distance between both fulcrums was measured. This distance is the effective length of the pendulum

10. Using Python, the data was imported and separated into the separate trials of the upright time measurements and the inverted time measurements. These times were converted into periods by
$$T = 2 \cdot \frac{time}{22}.$$
It was plotted.

11. Using **scipy.optimize.curve-fit**, a line of best fit was created to go through each data set, forming an 'X'. Furthermore, using **scipy.optimize.brentq**, a program was designed to locate the roots of a function. This was used to determine the point of intersection between the upright data curve fit and the inverted data curve fit.

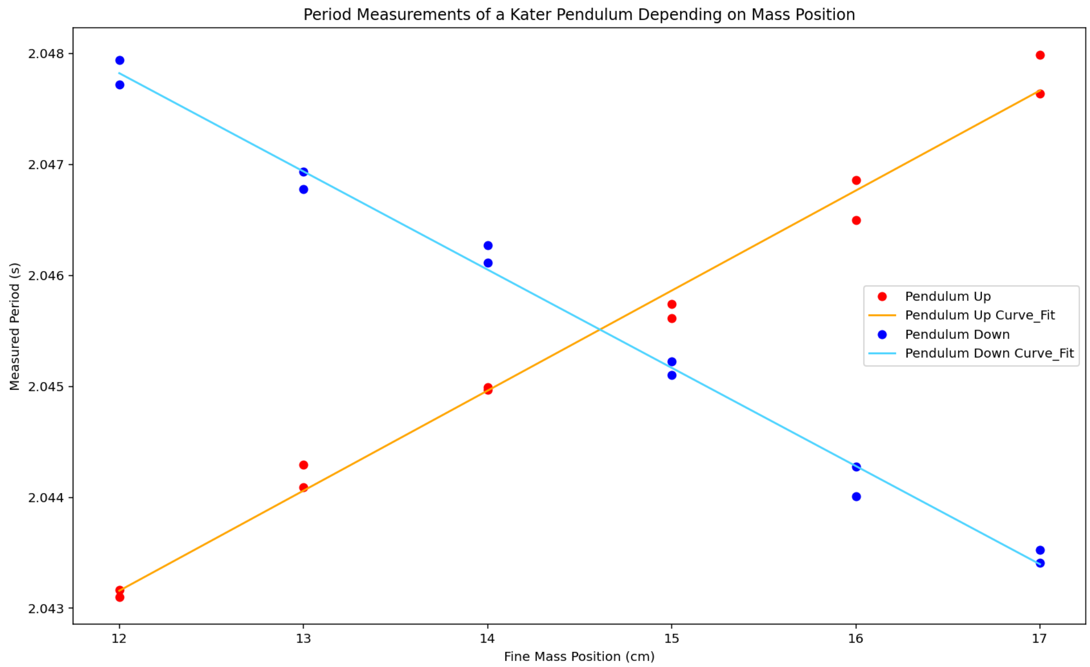
12. Lastly, the intersection period was using in determining the measurement of gravity by using (1), where L was the effective pendulum length measured before. Uncertainties were recorded and propagated. Percentage error was determined.

3 Results

The following table is a sample of measurements made throughout the trials of the Kater Pendulum:

Trial	Coarse Mass (cm) ($\pm$0.001cm)	Fine Mass (cm) ($\pm$0.001cm)	Passes	Measured Time (s) ($\pm$0.05 s)	Direction (0=up, 1=down)
1	37	17	22	15.740528	0
2	37	17	22	15.724722	1
3	37	16	22	15.737527	0
4	37	16	22	15.727609	1
5	37	15	22	15.733232	0
6	37	15	22	15.730778	1
7	37	14	22	15.730255	0
8	37	14	22	15.735274	1

The following figure is the plot of periods of each upright and inverted measurements, with their respective curve fits. An intersection is observed:



Furthermore, the data analyzed from Python was printed:

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The measured value of g is 9.80705 m/s^2.
The uncertainty in the measurement of g is ± 0.00021 m/s^2.
The percentage error in measuring g is 0.00412 %.
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4 Comparison of Results, Analysis, Discussion

Result Comparison

As determined from the Python analysis, the percentage error of the measured value of g compared to the actual value of earth gravity is quite low, and thus we can conclude that the measurement obtained in this experiment is consistent with other results.

Analysis, Discussion

It was observed throughout the process of the experiment that consistent measurements were quite difficult to obtain. While finalizing the data, many trials were discarded due to incredibly poor time measurement for 22 pendulum passes through the voltage gate. This is why it was so difficult to determine the uncertainty of the measured time. We believe this is due to the quality of measurement of the voltage gate or the interval timer, since both devices are quite old and since every trial was executed the same way.

Throughout the first 10 trials, it was noticed that greater distance the pendulum was released from caused a greater period. Furthermore, it was frequently observed that any movement during the duration of the pendulum swing may cause deviations in measured time due to air resistance existing.

One interesting hypothesis is the the affect of height above sea level on Kater's pendulum, since the magnitude of gravity decreases as altitude increases. We believe the pendulum should emit an equivalent value of g at that height, however this hypothesis shows that height above sea level may be contributing to period measurement.

Uncertainty

As mentioned before, the uncertainty in this experiment was quite difficult to determine. While it came to the mass position uncertainty, this was relatively straightforward because the position of the masses depended on ones ability to read a ruler. Thus, the uncertainty in mass position was relatively low just because it was considered reading uncertainty.

The uncertainty due to the timer was quite different. Throughout trials, there were some occurrences in which an decrease in fine mass or coarse mass position led to a greater period and vice versa, although the period should be shorter since adjusting the mass up the pendulum arm is expected to decrease the period.

The greatest deviations during trials were observed to be $\pm 0.05s$ of the measured time from the interval timer, and thus this is what we took our uncertainty to be.

Strengths and Limitations of Experimental Design

Overall, the experiment was relatively straightforward and each trial was completed consistently and accurately with the others. The main strength of the experimental design was the consistency with how the pendulum was swung every trial due to the grooves in the wall mount and how the pendulum was designed: thin and heavy, to avoid as much air effects. Another strength to the pendulum design was the measurement of the effective length, which was just fulcrum to fulcrum, which saved a lot of dirty math.

The one limitation to the experimental design with the biggest impact on measurement was some of the digital equipment used. It is theorized that the large deviations in recorded times for each trials and the outlier times when compared with the rest of the data originated from a poor interval timing per each swing of the pendulum. We believe this was accounted for in the uncertainty choice for the measured time, which again was $\pm 0.05s$.

5 Conclusion

In conclusion, the results we had obtained are as expected and are as desired and are consistent with the actual measurement of gravity. Therefore it is verifiable that Kater's innovation is effective in measuring gravity, which proves why this pendulum was so useful in the 1800's for finding the value of g.

Suggested Further Research:

- Bessel's Improved Kater Pendulum

6 References

1. https://www.isobudgets.com/propagation-of-uncertainty-rules/
2. https://en.wikipedia.org/wiki/Gravimetry
3. https://www.physics.brocku.ca/Labs/SampleLabs/Bessel/1.1349544.pdf