



Knowledge Graph Embeddings

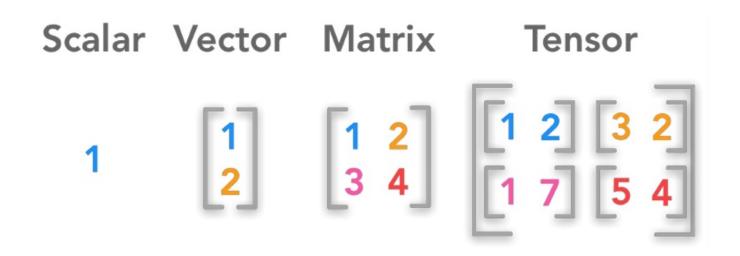
Basel Shbita

INF 558/CSCI 563: Building Knowledge Graphs Spring 2020, University of Southern California

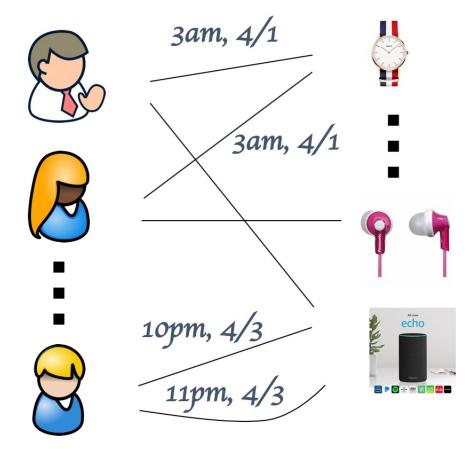
^{*} Some of the slides were provided by: Jay Pujara, Mayank Kejriwal, Luna Dong, Christos Faloutsos, Andrey Kan, Jun Ma, Subho Mukherjee, Sebastian Riedel, Antoine Bordes

Agenda

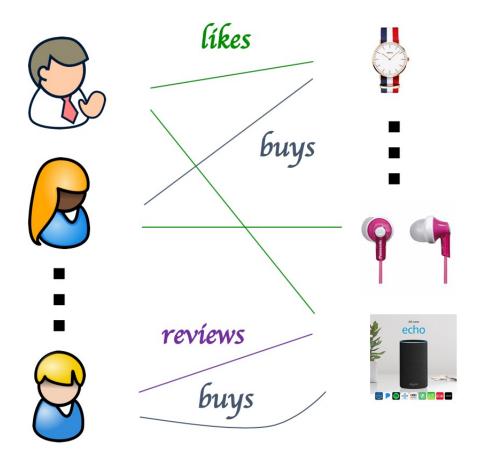
- → Motivation
 - Tensors
 - Graphs
 - Embeddings
 - Problem Definition
 - Graph Embedding
 - Tensor Embedding
 - Knowledge Graph Embedding



- e.g., Time-evolving graphs
- What is 'normal'? 'suspicious'?
 - Groups?



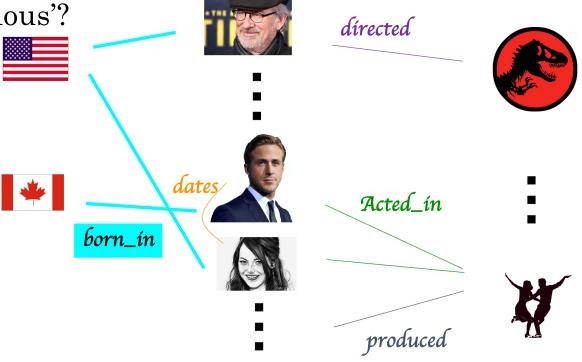
- e.g., MultiView Graph
- What is 'normal'? 'suspicious'?
 - Groups?



• e.g., Knowledge Graphs

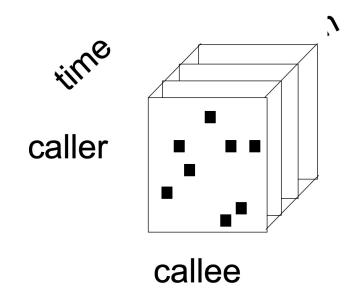
• What is 'normal'? 'suspicious'?

• Groups?



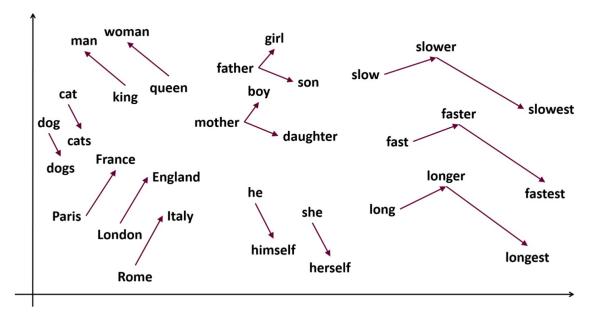
Graphs over time -> tensors!

- Problem #1:
 - Given who calls whom, and when
 - Find patterns / anomalies



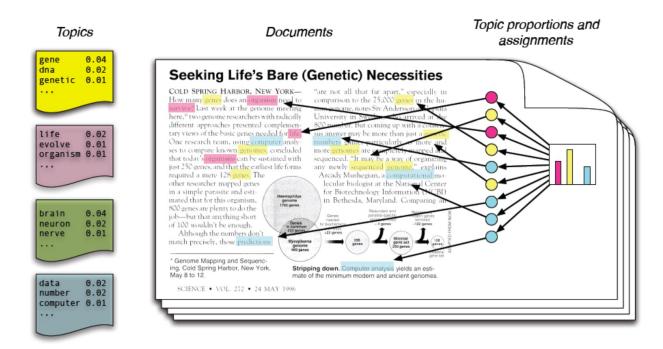
Embedding

- Mapping of discrete variable to a vector of continuous numbers
- Low-dimensional
- Very popular for documents, graphs, words...



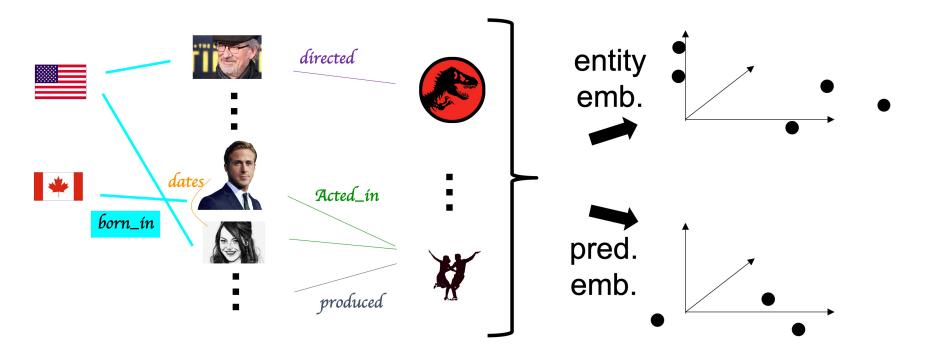
Embedding

• Embeddings are not a 'new' invention... topic models are an early example still widely used



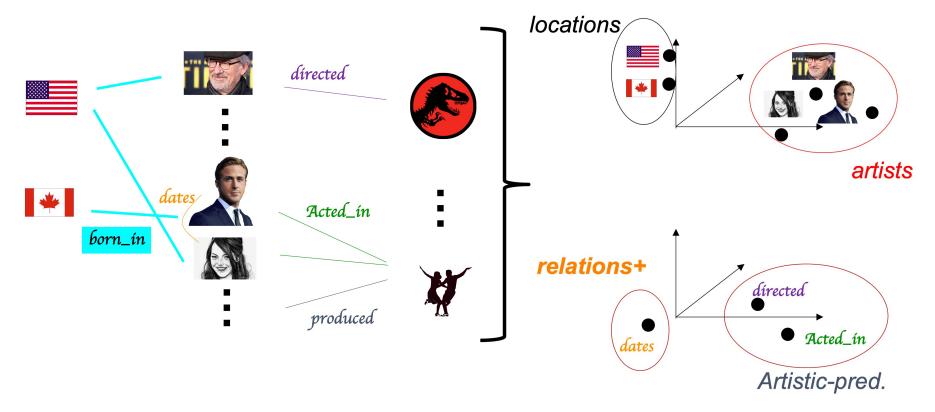
Problem Definition

• Given entities & predicates, find mappings



Problem Definition

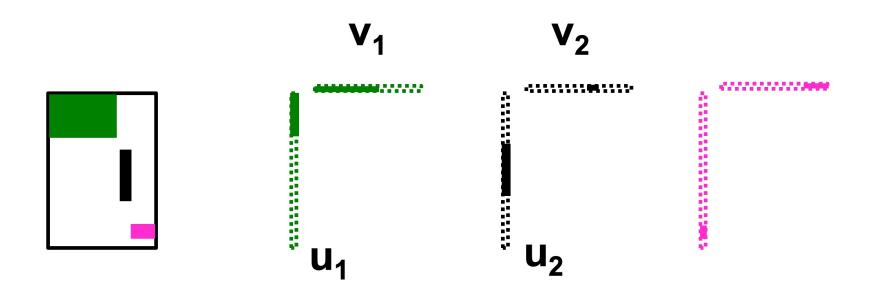
• Given entities & predicates, find mappings



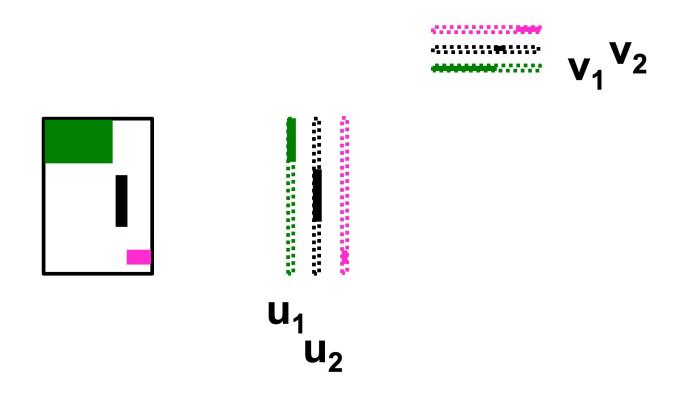
Agenda

- Motivation
- → Graph Embedding
 - SVD
 - Deep Graph
 - Tensor Embedding
 - Knowledge Graph Embedding

Familiar embedding: SVD



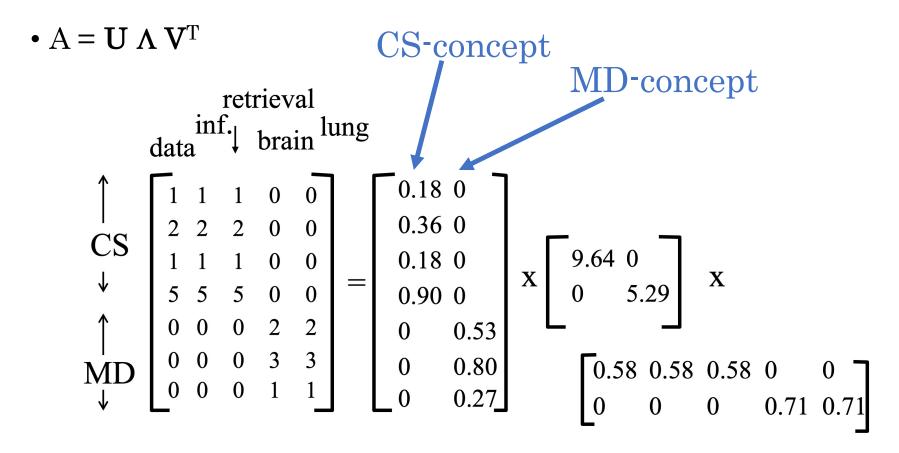
Familiar embedding: SVD



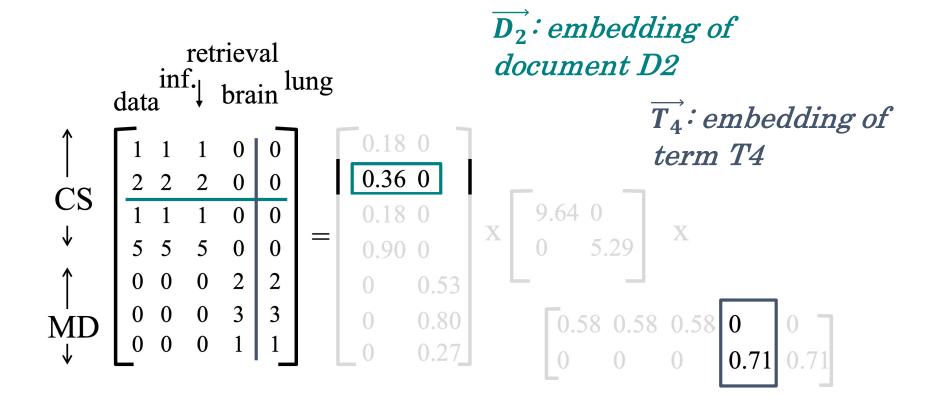
SVD as embedding

• $A = U \wedge V^T$

SVD as embedding



SVD as embedding



Deep Graph Embeddings

- Skip-gram

- DeepWalk
- Node2Vec
- Metapath2Vec
- LINE
- UltimateWalk
- AutoEncoder
- Struc2Vec
- GraphSAGE
- GCN
- •

Skip-gram

- Borrowed from work on language model
- Sample a set of paths with random walk from node v_i
 - $min log \sum_{v_j \in N(v_i)} P(v_j | v_i)$

•
$$P(v_j|v_i) = \frac{\exp(v_i v_j)}{\sum_{v_k \in |V|} \exp(v_i v_k)}$$

- Solved with
 - Hierarchical Softmax (DeepWalk)
 - Negative Sampling (Node2Vec)

Deep Graph Embeddings

- DeepWalk
- Node2Vec
- Metapath2Vec heterogenous graph
- LINE 1st order + 2nd order proximity
- UltimateWalk closed form, unifies DeepWalk and Node2Vec
- AutoEncoder reconstruct W, similar to SVD
- Struc2Vec focuses on structural similarity
- GraphSAGE "inductive", sample and aggregate
- GCN interesting! borrowed the idea from CNN

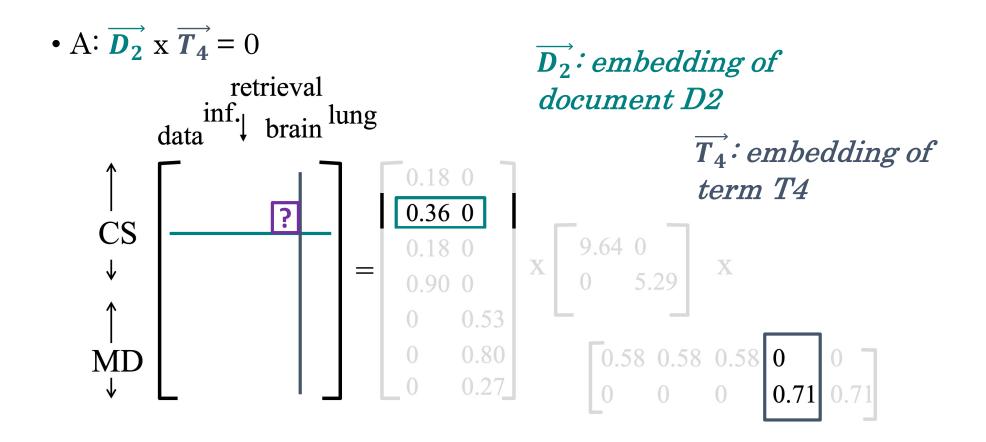
• ...

Embedding can help with...

- Reconstruction / Fact checking
 - Triples completion
- Classification
 - Triples classification
- 'Featurizing'
 - (Link prediction)
 - (Recommendation)



Example: Reconstruction of (2,4)



Agenda

- Motivation
- Graph Embedding
- → Tensor Embedding
 - Pairs and Relations as Matrix
 - Tensor Formulation of KG
 - Knowledge Graph Embedding

"Distant" Supervision

John was born in Liverpool, to Julia and Alfred Lennon.

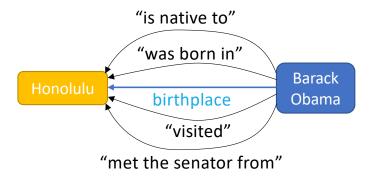


No direct supervision gives us this information.

Supervised: Too expensive to label sentences

Rule-based: Too much variety in language

Both only work for a small set of relations, i.e. 10s, not 100s



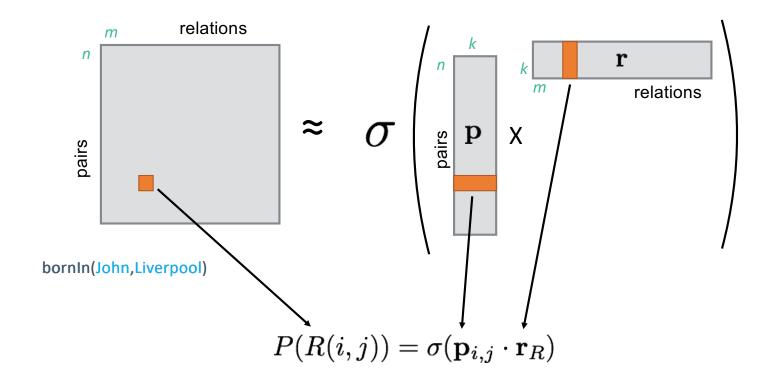
Relation Extraction as a Matrix

John was born in Liverpool, to Julia and Alfred Lennon.

	Was born in Was born in Was born to	ρ_{U_Q}	birthplacers	Spousedth,
John Lennon, Liverpool	1		?	
John Lennon, Julia Lennon	1			
John Lennon, Alfred Lennon	1			
Julia Lennon, Alfred Lennon		1		?
Barack Obama, Hawaii	1		1	
Barack Obama, Michelle Obama		1		1
			I	

intity Pairs

Matrix Factorization



Training: Stochastic Updates

relations $\frac{g}{g} = \frac{R'(x,y)}{R(i,j)} \approx \frac{g}{g} + \frac{r}{g} + \frac{r}{g}$

- Pick an observed cell, R(i, j):
 - Update \mathbf{p}_{ij} & \mathbf{r}_R such that R(i,j) is higher
- Pick any random cell, assume it is negative:
 - Update \mathbf{p}_{xy} & $\mathbf{r}_{R'}$ such that R'(x,y) is lower

Relation Embeddings

 $\mathbf{r}_{ ext{is-native-of}} \ \mathbf{r}_{ ext{bornIn}} \ \mathbf{p}_{ ext{Barack,USA}}$

 $\boldsymbol{r}_{\mathrm{livedIn}}$

 $\mathbf{p}_{ ext{Barack,Michelle}} \ \mathbf{p}_{ ext{George,Laura}}^{\mathbf{r}_{ ext{spouse}}}$

Embeddings ~ Logical Relations

Relation Embeddings, r

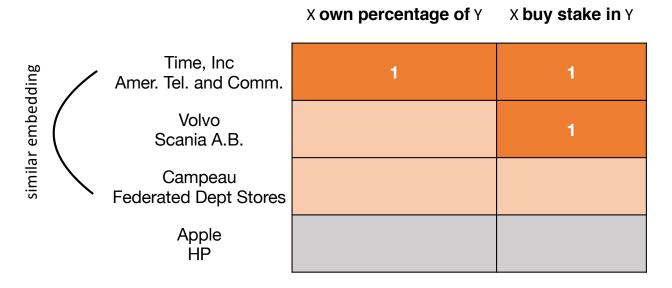
- Similar embedding for 2 relations denote they are paraphrases
 - isMarriedTo(X,Y), spouseOf(X,Y)
- One embedding can be contained by another
 - $r(topEmployeeOf) \subset r(employeeOf)$
 - $topEmployeeOf(X,Y) \rightarrow employeeOf(X,Y)$
- Can capture logical patterns, without needing to specify them!

Entity Pair Embeddings, p

- Similar entity pairs denote similar relations between them
- Entity pairs may describe multiple "relations"
 - independent foundedBy and employeeOf relations

Similar Embeddings





Successfully predicts "Volvo owns percentage of Scania A.B." from "Volvo bought a stake in Scania A.B."

Implications

X historian at $Y \rightarrow X$ professor at Y

(Freeman, Harvard)
→ (Boyle, Ohio State)

Kevin Boyle
Ohio State
R. Freeman
Harvard

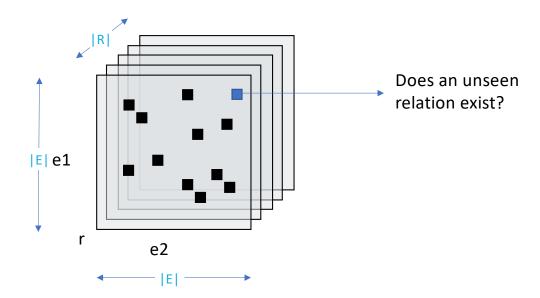
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Learns asymmetric entailment:

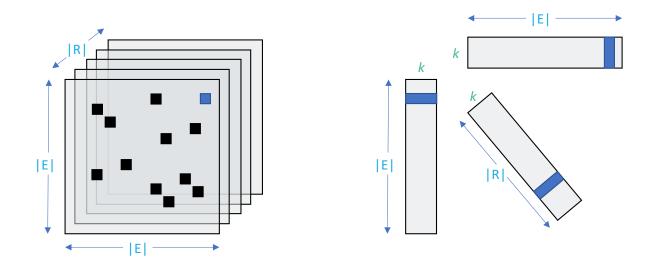
PER historian at UNIV \rightarrow PER professor at UNIV But,

PER professor at UNIV $\not\rightarrow$ PER historian at UNIV

Tensor Formulation of KG

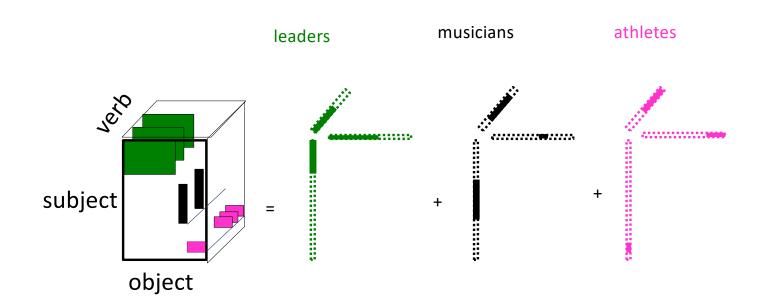


Factorize that Tensor

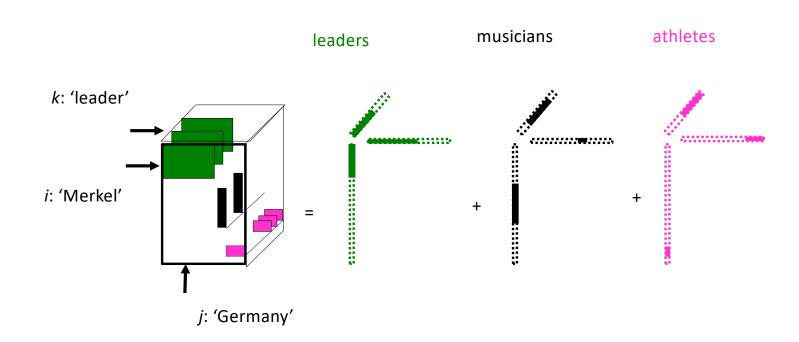


$$S(r(a,b)) = f(\mathbf{v}_r, \mathbf{v}_a, \mathbf{v}_b)$$

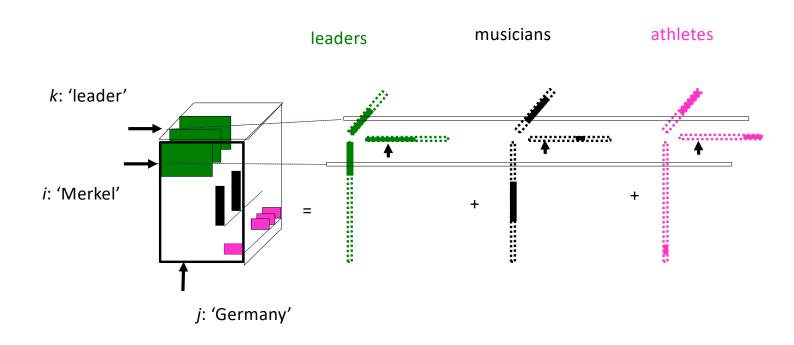
PARAFAC: as embedding



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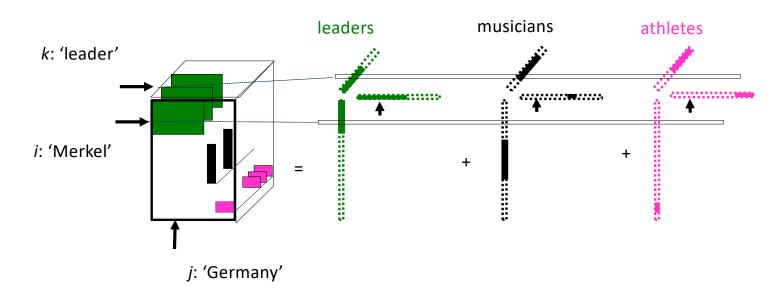


PARAFAC: as embedding



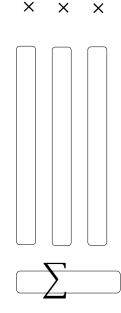
PARAFAC: as embedding

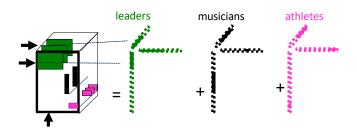
- 'Merkel': i-th subject vector: (1,0,0)
- 'Germany': j-th object vector: (1,0,0)
- 'is_leader': k-th verb vector: (1,0,0)



Reconstruction

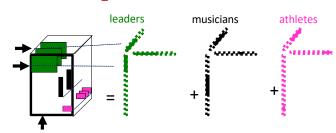
- 'Merkel': i-th subject vector: $\vec{s} = (1, 0, 0)$
- 'Germany': j-th object vector: $\vec{o} = (1, 0, 0)$
- 'is_leader': k-th verb vector: \vec{v} =(1, 0, 0)
- A: $x_{i,j,k} = \sum_{h=1}^{3} s_{i,h} o_{j,h} v_{k,h}$

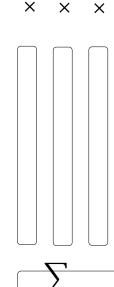




Reconstruction

- 'Merkel': i-th subject vector: $\vec{s} = (1, 0, 0)$
- 'Germany': j-th object vector: $\vec{o} = (1, 0, 0)$
- 'is_leader': k-th verb vector: \vec{v} =(1, 0, 0)
- A: $x_{i,j,k} = \sum_{h=1}^{3} s_{i,h} o_{j,h} v_{k,h}$
- Intuitively:
 - s,v,o: should have common 'concepts'





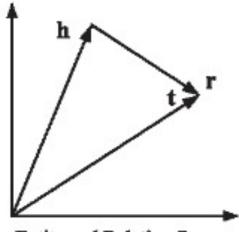
Agenda

- Motivation
- Graph Embedding
- Tensor Embedding
- → Knowledge Graph Embedding
 - Triple Scoring
 - Addition
 - Multiplication
 - Loss
 - Applications

Knowledge Graph Embedding

- Triple scoring: what is the relationship among sub (h), pred (r), and obj (t)?
 - Addition: h + r = ?= t
 - Multiplication: $h \circ r = ?= t$
- Loss: what shall we optimize?
 - Closed-world assumption
 - Open-world assumption

- Addition: h + r = ?= t
 - TransE
 - $score(h,r,t) = ||h+r-t||_{1/2}$



Entity and Relation Space

TransE

```
'Merkel': \vec{h}=(1, 0, 0)

'Germany': \vec{t}=(1, 1, 0)

'is_leader': \vec{r}=(0, 1, 0)

score (h, r, t) = -|| \vec{h} + \vec{r} - \vec{t}||<sub>1/2</sub> = 0

'Merkel': \vec{h}=(1, 0, 0)

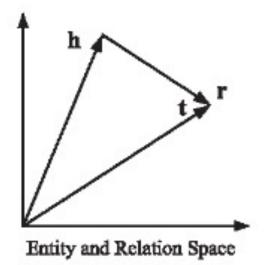
'Beatles': \vec{t'}=(0, 0, 1)

'plays_bass': \vec{r'}=(0, 0, 1)

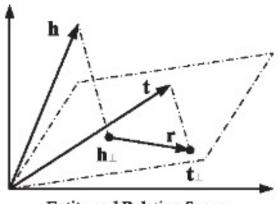
score (h, r, t) = -|| \vec{h} + \vec{r'} - \vec{t'}||<sub>1/2</sub> = -1
```

- Addition: h + r = ?= t
 - TransE
 - $score(h,r,t) = ||h+r-t||_{1/2}$
 - What if multiple objects apply??



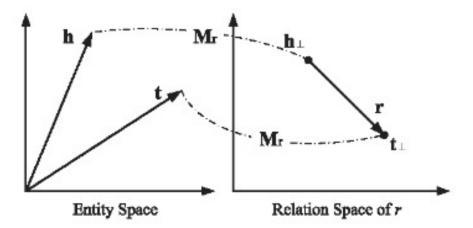


- Addition: h + r = ?= t
 - TransE
 - $score(h,r,t) = ||h+r-t||_{1/2}$
 - TransH
 - project to relation-specific hyperplanes

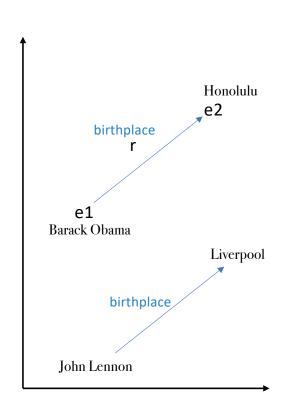


Entity and Relation Space

- Addition: h + r = ?= t
 - TransE
 - $score(h,r,t) = ||h+r-t||_{1/2}$
 - TransH
 - project to relation-specific hyperplanes
 - TransR
 - translate to relation-specific space



- Addition: h + r = ?= t
 - TransE
 - $score(h,r,t) = ||h+r-t||_{1/2}$
 - TransH
 - project to relation-specific hyperplanes
 - TransR
 - translate to relation-specific space
 - Many simplifications of TransH and TransR
 - STransE is reported to be the best in Dat Quoc Nguyen. An overview of embedding models of entities and relationships for knowledge base completion



TransE

$$S(r(a,b)) = -\|\mathbf{e}_a + \mathbf{R}_r - \mathbf{e}_b\|_2^2$$

TransH

$$S(r(a,b)) = -\|\mathbf{e}_a^{\perp} + \mathbf{R}_r - \mathbf{e}_b^{\perp}\|_2^2$$
$$\mathbf{e}_a^{\perp} = \mathbf{e}_a - \mathbf{w}_r^T \mathbf{e}_a \mathbf{w}_r$$

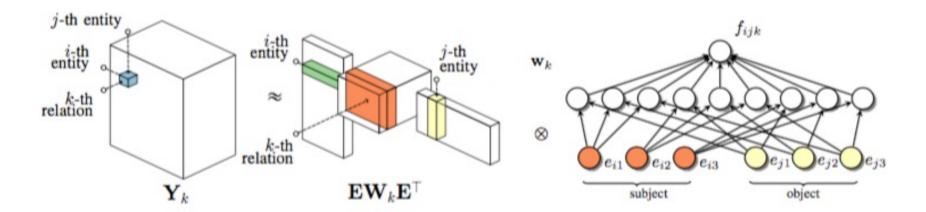
TransR

$$S(r(a,b)) = -\|\mathbf{e}_a\mathbf{M}_r + \mathbf{R}_r - \mathbf{e}_b\mathbf{M}_r\|_2^2$$

Triple Scoring - Multiplication

- Multiplication: $h \circ r = ?= t$
 - RESCAL: $score(h,r,t) = \mathbf{h}^{\top} \mathbf{W}_r \mathbf{t}$

Too many parameters?!



Triple Scoring - Multiplication

- Multiplication: $h \circ r = ?= t$
 - RESCAL: $score(h,r,t) = \mathbf{h}^{\top} \mathbf{W}_r \mathbf{t}$
 - DistMult: $score(h,r,t) = \mathbf{h}^{\top} diag(\mathbf{r})\mathbf{t}$ Simplify RESCAL by using a diagonal matrix

RESCAL

'Merkel': h=(1,0)

'Germany': $t = (0, 1)^T$ 'is_leader': $W_r = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ score (h, r, t) = $h^T W_r t$ $= \sum (h \otimes t) \odot W_r = 1$

DistMult

'Merkel': $h=(1, 0)^T$ 'Germany': $t = (1, 0)^T$

'is_leader': $W_r = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ score (h, r, t) = $h^{\top}W_rt$ $= \sum (h \odot t) \odot diag(W_r) = 1$

Triple Scoring - Multiplication

- Multiplication: $h \circ r = ?= t$
 - RESCAL: $score(h,r,t) = \mathbf{h}^{\top} \mathbf{W}_r \mathbf{t}$
 - DistMult: $score(h,r,t) = \mathbf{h}^{\top} diag(\mathbf{r})\mathbf{t}$ Simplify RESCAL by using a diagonal matrix

Cannot deal with asymmetric relations!!

Triple Scoring - Multiplication

- Multiplication: $h \circ r = ?= t$
 - RESCAL: $score(h,r,t) = \mathbf{h}^{\top} \mathbf{W}_{r} \mathbf{t}$
 - DistMult: $score(h,r,t) = \mathbf{h}^{\top} diag(\mathbf{r})\mathbf{t}$ Simplify RESCAL by using a diagonal matrix
 - ComplEx: $score(h,r,t) = Re(\mathbf{h}^{\top} diag(\mathbf{r})\mathbf{t})$ Extend DistMult by introducing complex value embedding, so can handle asymmetric relations

ComplEx

•
$$h = R(h) + iI(h)$$
, $t = R(t) + iI(t)$, $r = R(r) + iI(r)$

•
$$h \odot \overline{t} = (R(h) + iI(h)) \odot (R(t) + iI(t))$$

= $R(h) \odot R(t) + I(h) \odot I(t)$
+ $i(I(h) \odot R(t) - R(h) \odot I(t))$

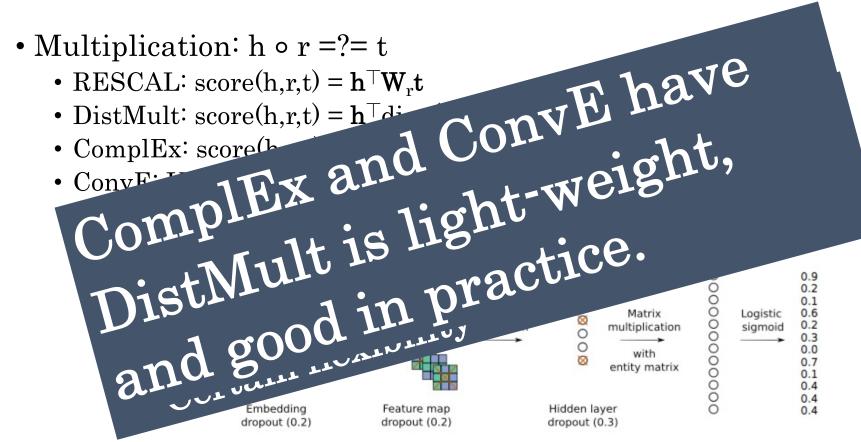
•
$$Re\{(h \odot \overline{t}) \odot r\} = R(h) \odot R(t) \odot R(r)$$

+ $I(h) \odot I(t) \odot R(r)$
+ $R(h) \odot I(t) \odot I(r)$
- $I(h) \odot R(t) \odot I(r)$

ComplEx

```
• score(h,r,t) = \sum Re\{(h \odot \overline{t}) \odot r\}
= \sum R(h) \odot R(t) \odot R(r) \Longrightarrow DistMult
+ \sum I(h) \odot I(t) \odot R(r)
+ \sum R(h) \odot I(t) \odot I(r)
- \sum I(h) \odot R(t) \odot I(r)
• \neq score(t,r,h)
\Longrightarrow Asymmetry
```

Triple Scoring - Multiplication



Loss

• Closed world assumption: square loss

$$L = \sum_{h,t \in E, r \in R} (y_{h,r,t} - f(h,r,t))^2$$

Loss

• Closed world assumption: square loss

$$L = \sum_{h,t \in E, r \in R} (y_{h,r,t} - f(h))$$

$$L = \sum_{h,t \in E, r \in R} (y_{h,r,t} - f(h))$$
• Open world as:
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• Open world as:
$$L = \sum_{h,t \in E, r \in R} (y_{h,t} - f(h))$$

KGE Applications

• Learn embeddings from IMDb data and identify WikiData errors, using DistMult

Subject	Relation	Target	Reason	
The Moises Padilla Story	writtenBy	César Ámigo Aguilar	Linkage error	
Bajrangi Bhaijaan	writtenBy	Yo Yo Honey Singh	Wrong relationship	
Piste noire	writtenBy	Jalil Naciri	Wrong relationship	
Enter the Ninja	musicComposedBy	Michael Lewis	Linkage error	
The Secret Life of Words	musicComposed By		Cannot confirm	

Comparing Real KGs with Benchmarks

- Examine statistics of real KGs and derived benchmarks
- Two metrics for capturing data distribution and sparsity:
 - entity & relation entropy (EE/RE) measure diversity of facts
 - entity & relation density (ED/RD) concentration of facts

	KG	Triples	Entities	Rels	EE	RE	ED	RD	Prec
	Freebase	1B	124M	15K	14	3.2	16	68K	1
Real	NELL1000	92M	4.8M	435	21	4.9	19	210K	0.45
	WordNet	380K	116K	27	21	2.3	7	21K	1
h.	FB15K	592K	15K	1.3K	16	5.1	79	440	1
Bench.	NELL165	1M	820K	221	25	1.5	3	4.7K	0.35
B	WN18	151K	40K	18	19	2.1	7	8.4K	1

Comparing Real KGs with Benchmarks

	KG	Triples	Entities	Rels	1010	\mathbf{RE}	ED	RD	Prec
	Freebase	1B	124M	15K	14	3.2	16	68K	1
Real	NELL1000	92M	4.8M	435	21	4.9	19	210K	0.45
	WordNet	380K	116K	27	21	2.3	7	21K	1
h.	FB15K	592K	15K	1.3K	16	5.1	79	440	1
Bench.	NELL165	1M	820K	221	25	1.5	3	4.7K	0.35
B	WN18	151K	40K	18	19	2.1	7	8.4K	1

Observations:

- Freebase is largest KG with highest RD, but lowest EE
- NELL1000 is diverse (high EE/RE), highest RD, low precision
- WN/WN18 are much smaller, low rels, low RE, low ED
- FB15K has very high ED, very low RD, more diverse than FB
- NELL165 has lowest ED, highest EE, lowest RE, low precision

Do embeddings work for extracted KGs?

• Approach:

• Evaluate on the NELL knowledge graph, containing millions of candidates extracted from WWW text

• Observations:

- Baseline (threshold input) wins against embeddings
- Best results from graphical model (PSL-KGI) using rules & uncertainty
- More complex embedding methods have the worst performance

• Conclusion:

Embeddings have poor performance on sparse & noisy KGs extracted from text

Method	AUC	F1
TransH	0.701	0.783
HolE	0.710	0.783
TransE	0.726	0.783
STransE	0.784	0.783
Baseline	0.873	0.828
PSL-KGI	0.891	0.848

Do embeddings require complete KGs?

Approach:

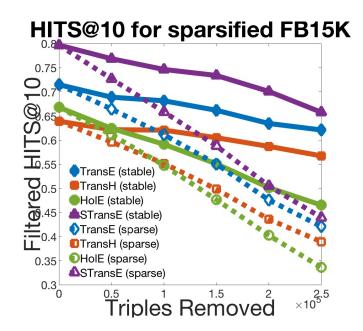
• Remove training data, either in clusters to maintain relation density (stable) or randomly (sparse)

Observations:

- All methods perform much worse with sparse KGs relative to stable baseline
- At 50% removal, stable can outperform sparse by 60%
- STransE most sensitive, HolE least sensitive to sparsity

• Conclusion:

performance quickly degrades with sparsity



Do embeddings require reliable KGs?

Approach:

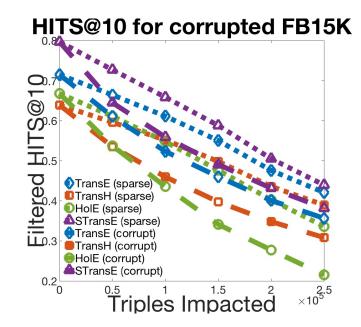
• Randomly "corrupt" training data by altering subject, predicate, or object

Observations:

- corrupt training data is worse than sparse data
- Deficit between sparse and corrupt remains stable
- HolE most sensitive, STransE least sensitive to corruption

• Conclusion:

 Unreliable data harms training more than missing data



When is noisy data worth using?

• Approach:

• Start with sparse training set and add new training data with differing noise levels

• Observations:

- All methods receive boost from initial noisy data
- Enough low noise data can allow recovery
- Even very noisy data doesn't degrade performance much

• Conclusion:

• Extending sparse training data with noisy inputs can help performance

Trading off sparse and noisy training data

