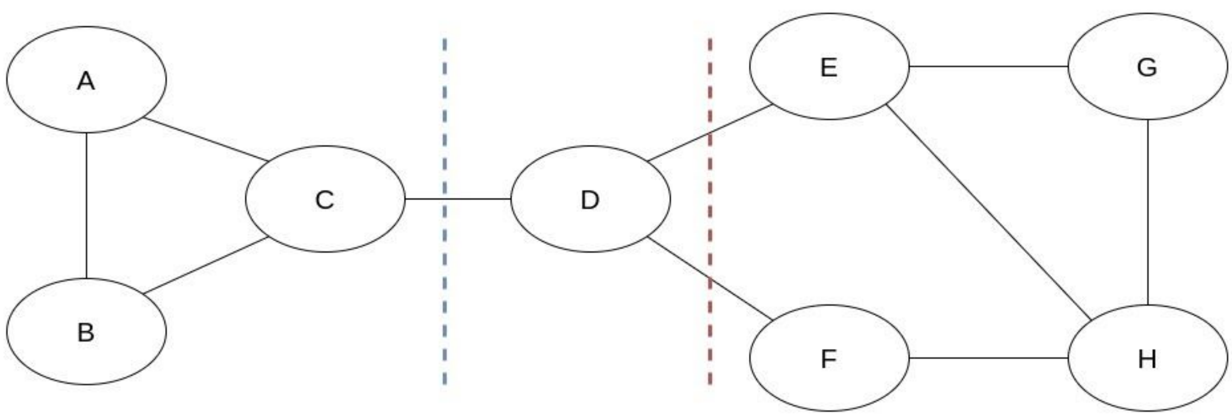
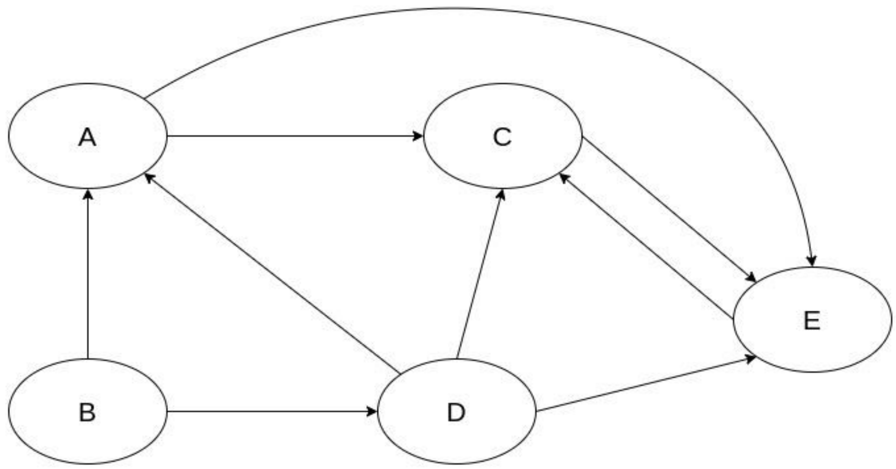
Tuesday Quiz

1. [1 point] What is the value of the smallest eigenvalue for every Laplacian Matrix
   a) -2
   b) -1
   c) 0
   d) 1
   e) 2
   Ans: c
2. [2 points] Explain briefly the three stages of the spectral partitioning algorithm.
   Ans: Pre-processing: Build Laplacian matrix L of the graph
   Decomposition: Find eigenvalues λ and eigenvectors x of the matrix L. Map vertices to corresponding components of λ2
   Grouping: Sort components of reduced 1-dimensional vector. Identify clusters by splitting the sorted vector in two
3. [1 point] How the second smallest eigen-value and its corresponding eigen-vector will help with partitioning graphs in Spectral clustering techniques.
   For the eigen-vector corresponding to the 2nd smallest eigenvalue, we have sum(x[i]) = 0.
   Therefore, by default, we have some numbers < 0 and some numbers >= 0. These 2 sets of numbers form the two subgraphs that we want to partition our graph into.
4. [2+1 points] Consider the following graph. Here, the blue and red dotted lines are 2 possible cuts. For each of these cuts, calculate the value of the normalized cut. Finally, comment which of the 2 cuts is better in terms of breaking the graph into 2 communities.



   Normalized cut for blue line: 11/28 [1 point]
   Normalized cut for red line: 8/12 [1 point]
   Hence the blue line is a better cut, as it has a smaller normalized cut value. [1 point]

5. [1+2 points] Trawling: with a support threshold = 2, find one **K(s,t=2)** bipartite sub-graph from the graph below (2 points). You need to first convert the graph to a market basket model (i.e., write down baskets and their contents) (1 point). The bipartite subgraph K(s,t) should be represented as follows: s -> {m, n} ; t -> {p, q}



Market basket model: [1 point]
a -> {c, e}
b -> {a, d}
c -> {e}
d -> {a. c, e}
e -> {c}

The K(s,2) bipartite subgraphs is:
S -> {a, d} [1 point]
T -> {c, e} [1 point]