

# Ontologies and Reasoning

Pedro Szekely

University of Southern California

**computers are  
dimwits**

name	rank
Bob	Captain
Sue	Sargent
Mary	Admiral
Joe	Sargent

select ?x { ?x rank "Sargent" }

select ?x { ?x a Person }

select ?x { ?x a MilitaryPerson }

# inferencing

# inferencing

think RDF triples

# inferencing

I tell the computer some triples  
it infers more triples

name	rank
Bob	Captain
Sue	Sargent
Mary	Admiral
Joe	Sargent

bob	a foaf:Person; foaf:name "Bob"; ex:rank "Captain" .
sue	a foaf:Person; foaf:name "Sue"; ex:rank "Sargent" .

select ?x { ?x rank "Sargent" }

select ?x { ?x a Person }

select ?x { ?x a MilitaryPerson }

name	rank
Bob	Captain
Sue	Sargent
Mary	Admiral
Joe	Sargent

bob	a foaf:Person; foaf:name "Bob"; ex:rank "Captain" .
sue	a foaf:Person; foaf:name "Sue"; ex:rank "Sargent" .

```
select ?x { ?x a MilitaryPerson }
```

**what inference do I need?**

name	rank		bob	a foaf:Person; foaf:name "Bob"; ex:rank "Captain" .
Bob	Captain			
Sue	Sargent		sue	a foaf:Person; foaf:name "Sue"; ex:rank "Sargent" .
Mary	Admiral			
Joe	Sargent			

select ?x { ?x a MilitaryPerson }

ONE SMALL **RDFSHEMA** STATEMENT

ex:rank rdfs:domain ex:MilitaryPerson .

# RDF Schema Vocabulary

xmlns:rdfs="http://www.w3.org/2000/01/rdf-schema#"

## Classes

rdfs:Resource  
rdfs:Class  
rdfs:Literal  
rdfs:Datatype  
rdf:XMLLiteral  
rdf:Property

## Properties

rdfs:range  
rdfs:domain  
rdf:type  
rdfs:subClassOf  
rdfs:subPropertyOf  
rdfs:label  
rdfs:comment

## Utility Properties

rdfs:seeAlso  
rdfs:isDefinedBy  
rdf:value

# Limitations of RDFS

RDF Schema is a simple modelling language:

- Cannot define: Boolean combinations of classes, disjointness, cardinality restrictions, ..
- Cannot define properties of properties: unique, inverse, transitive, ...
- No mechanism of specifying necessary and sufficient conditions for class membership.
  - Ex: If John has a vehicle which is 7ft high, has wide wheels and loading space is 3 m<sup>3</sup>, then we should be able to reason that John has a SUV,  
if the necessary and sufficient conditions for a vehicle being an SUV are height > 6ft & wide wheels & loading space > 2 m<sup>3</sup>

Therefore we need an ontology layer on top of RDF and RDF Schema → OWL

# OWL vs RDFS

Ideally, OWL would extend RDF Schema to be consistent with the layered architecture of the Semantic Web

However, directly combining RDF Schema with classical logic leads to problems:

- RDF allows resources to be instances and classes (elements and sets) at the same time, which leads to paradoxes
- RDF/RDFS was formalized using non-standard semantics:
  - RDF Semantics: <http://www.w3.org/TR/rdf-mt/>

**inferencing**

RDFSchema

**OWL**

# Basic Idea

I state a few OWL axioms

I load lots of triples

the system infers lots of new triples

# **OWL Building Blocks**

Classes

Properties

Individuals

# Unique Name Assumption

bob a foaf:Person;  
foaf:name "Bob";  
ex:rank "Captain" .

sue a foaf:Person;  
foaf:name "Sue";  
ex:rank "Sargent" .

Unique Name Assumption = **TRUE**:

bob and sue refer to **different** individuals in the world

# Unique Name Assumption

bob a foaf:Person;  
foaf:name "Bob";  
ex:rank "Captain" .

sue a foaf:Person;  
foaf:name "Sue";  
ex:rank "Sargent" .

Unique Name Assumption = **TRUE**:

bob and sue refer to **different** individuals in the world

Unique Name Assumption = **FALSE**:

bob and sue **may** refer to the **same** individual in the world

# **OWL**

Unique Name Assumption = **FALSE**

# **Open World, Closed World**

I tell the computer some triples

# **Open World, Closed World**

I tell the computer some triples

Closed World:

# Open World, Closed World

I tell the computer some triples

Closed World:

if I don't tell it something, assume it's **false**

# Open World, Closed World

I tell the computer some triples

Closed World:

if I don't tell it something, assume it's **false**

Open World:

# Open World, Closed World

I tell the computer some triples

Closed World:

if I don't tell it something, assume it's **false**

Open World:

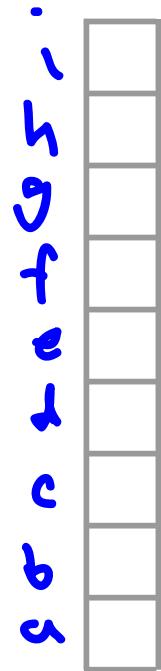
if I don't tell it something, **don't** assume **anything**  
I might tell it later that it is **true**

# **OWL**

Open world or closed world = **OPEN**

# **Classes**

all individuals



T

owl:Thing

i	X
h	X
g	X
f	X
e	X
d	X
c	X
b	X
a	X

$\top$   
 $\text{owl:Thing}$

i	X
h	X
g	X
f	X
e	X
t	X
c	X
b	X
a	X

$\perp$   
 $\text{owl:Nothing}$


$\top$

owl:Thing

i	X
h	X
g	X
f	X
e	X
t	X
c	X
b	X
a	X

$\perp$

owl:Nothing


Person

X
X
X

$\top$

owl:Thing

i	X
h	X
g	X
f	X
e	X
t	X
c	X
b	X
a	X

$\perp$

owl:Nothing


Person

X
X
X

Woman

X
X
X

T

owl:Thing

i	X
h	X
g	X
f	X
e	X
t	X
c	X
b	X
a	X

Person

X
X

Woman

X
X

Man

X

T

owl:Thing

i	X
h	X
g	X
f	X
e	X
t	X
c	X
b	X
a	X

Person

X
X

Woman

X
X

Man

X
X
X
X

Man  $\subseteq$  Person

subclass

T

owl:Thing

i	X
h	X
g	X
f	X
e	X
t	X
c	X
b	X
a	X

Person

X
X

Woman

X
X

Man

X

Man  $\subseteq$  Person

subclass

T

owl:Thing

i	X
h	X
g	X
f	X
e	X
t	X
c	X
b	X
a	X

Person

X
X

Woman

X
X

Man

X

Person  $\equiv$  Man  $\sqcup$  Woman

equivalent      union

T

owl:Thing

i	X
h	X
g	X
f	X
e	X
t	X
c	X
b	X
a	X

Person

X
X
+
X
X

Woman

X
X

Man

X
X
X

Person  $\equiv$  Man  $\sqcup$  Woman

equivalent union

T

owl:Thing

i	X
h	X
g	X
f	X
e	X
t	X
c	X
b	X
a	X

Person

X
X
X
X
X

Woman

X
X
O
X

Man

X
X
X
X

Person  $\equiv$  Man  $\sqcup$  Woman

{g}  $\sqcap$  Woman  $\sqsubseteq$  ⊥

$\top$

owl:Thing

i	X
h	X
g	X
f	X
e	X
t	X
c	X
b	X
a	X

Person

X
X

Woman

X
X

Man

X

Man  $\sqsubseteq$  Person

Man  $\sqcap$  Woman  $\sqsubseteq \perp$

$\top$

owl:Thing

i	X
h	X
g	X
f	X
e	X
t	X
c	X
b	X
a	X

Person

X
X

Woman

X
X

Man

X
X
X
X

Contradiction

$\text{Man} \sqsubseteq \text{Person}$

$\text{Man} \sqcap \text{Woman} \sqsubseteq \perp$

# **Properties**

son of

i									
h									
g									
f									
e									
d									
c		x							
b			x						
a				x					
b	a	b	c	d	e	f	g	h	i

b son of c

b son of f

e son of c

sonOf

i											
h											
g											
f											
e											
d											
c	x										
b		x									
a			x								
	a	b	c	d	e	f	g	h	i		

childOf

i											
h											
g											
f											
e											
d											
c											
b											
a											
	a	b	c	d	e	f	g	h	i		

sonOf  $\subseteq$  childOf

sonOf

i									
h									
g									
f									
e									
d									
c	x								
b		x							
a			x						
	a	b	c	d	e	f	g	h	i

childOf

i									
h									
g									
f									
e									
d									
c		x							
b			x						
a				x					
	a	b	c	d	e	f	g	h	i

sonOf  $\subseteq$  childOf

sonOf

i	j	h	g	f	e	d	c	b	a

T

X
X
X
X
X
X
X
X
X
X

	X			X					
a	b	c	d	e	f	g	h	i	

$\exists \text{sonOf}.\text{T}$

sonOf

i									
h									
g									
f									
e									
d									
c									
b									
a									
	a	b	c	d	e	f	g	h	i

T

X
X
X
X
X
X
X
X
X

	X			X					
	a	b	c	d	e	f	g	h	i

$\exists \text{sonOf}. T \subseteq \text{Person}$

sonOf

i									
h									
g									
f									
e									
d									
c									
b									
a									
	a	b	c	d	e	f	g	h	i

T

X
X
X
X
X
X
X
X
X

	X			X					
	a	b	c	d	e	f	g	h	i

$\exists \text{sonOf}. T \subseteq \text{Person}$   
 $\text{domain}(\text{sonOf}) = \text{Person}$

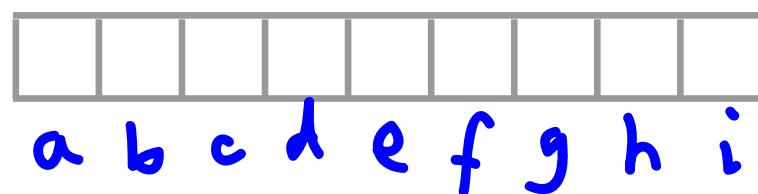
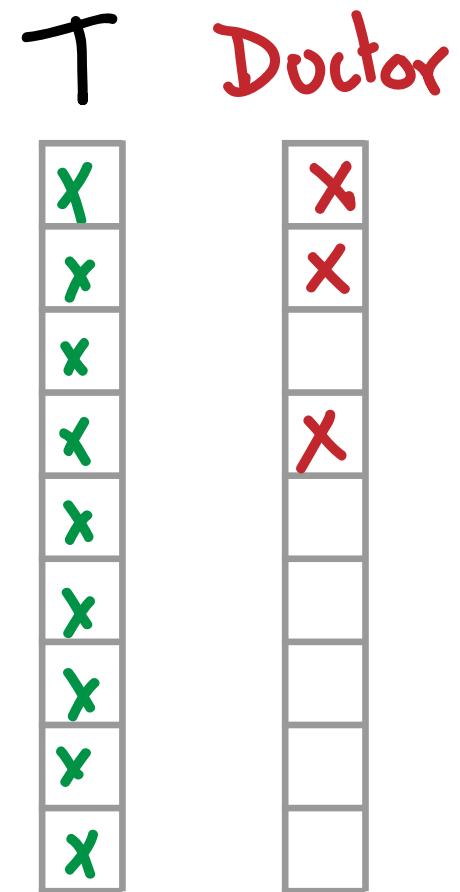
P rdfls:domain C

$\exists P. \top \sqsubseteq C$

parent

	i	h	g	f	e	d	c	b	a

i h g f e d c b a



$\exists$  parent . Doctor

parent

T Doctor

A horizontal row of ten boxes for handwriting practice. The first box contains a green 'x'. Below the boxes, the lowercase letters 'a' through 'i' are written in blue cursive script.

$\exists$  parent . Doctor

parent

i									
h									
g									
f									
e									
d									
c									
b									
a									
i	b	c	d	e	f	g	h	i	

Doctor

X
X
X
X

a	b	c	d	e	f	g	h	i

H parent . Doctor

parent									Doctor
i	h	g	f	e	d	c	b	a	
j									

	X			X					
a	b	c	d	e	f	g	h	i	

H parent . Doctor

# parent

i	h	g	f	e	d	c	b	a	

# Doctor

# Parent Doctor

									parent	Doctor
i	h	g	f	e	d	c	b	a		
j										
x										
	x									
		x								
			x							
				x						
					x					
						x				
							x			
								x		
									x	
										x

x	x	x	x	x	x	x	x	x
a	b	c	d	e	f	g	h	i

Λ parent . Doctor

parent

	a	b	c	d	e	f	g	h	i
i									X
h									
g									
f									
e									
d									
c									
b									
a									

Doctor

X	X								

X	X	X	X	X	X	X	X	X	
a	b	c	d	e	f	g	h	i	

$$T \leq \forall \text{parent}.\text{Doctor}$$

parent

	a	b	c	d	e	f	g	h	i
i									X
h									
g									
f									
e									
d									
c									
b									
a									

Doctor

X	X								
X									

X	X	X	X	X	X	X	X	X	
a	b	c	d	e	f	g	h	i	

$T \subseteq \forall \text{parent}.\text{Doctor}$

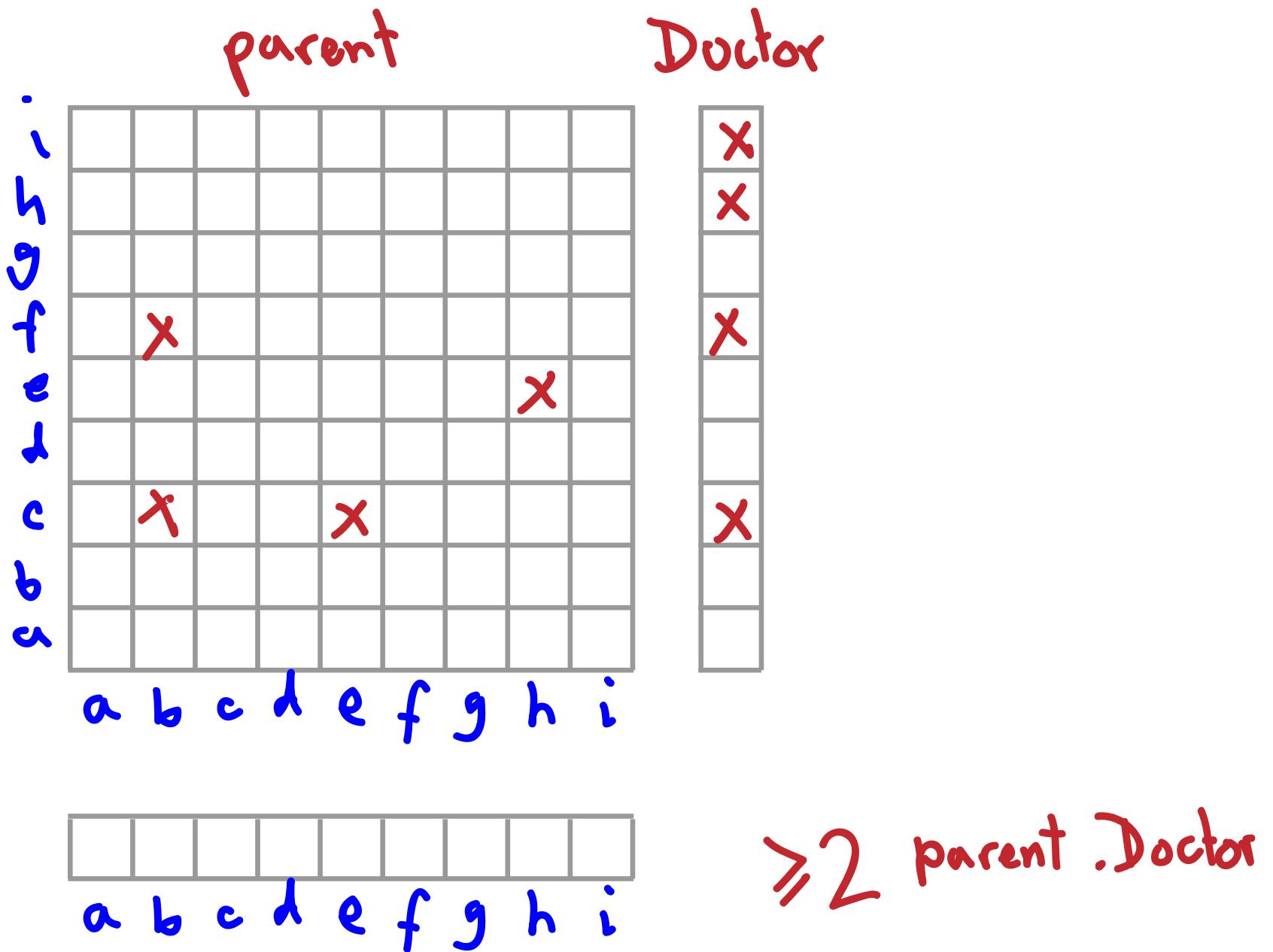
$\text{range}(\text{parent}) = \text{Doctor}$

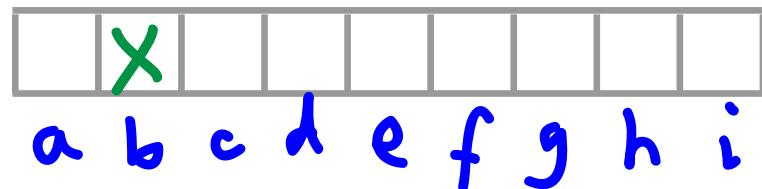
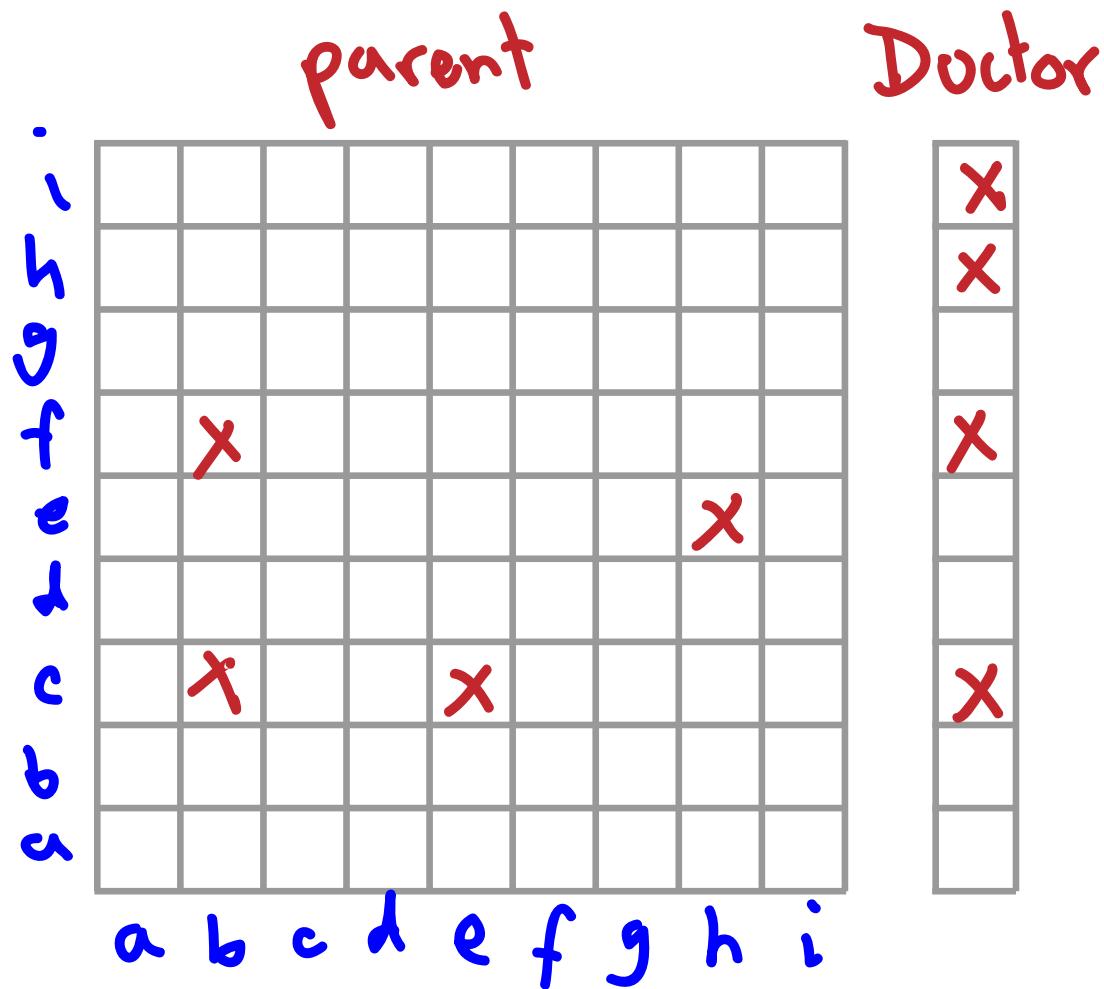
P rdfs:domain C

$\exists P.T \subseteq C$

P rdfs:range C

$T \subseteq \forall P.C$





$\geq 2$  parent . Doctor

parent

Doctor

X X X

a b c d e f g h i

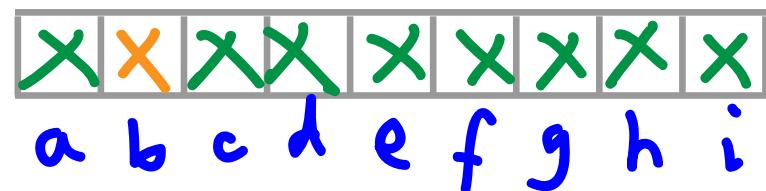
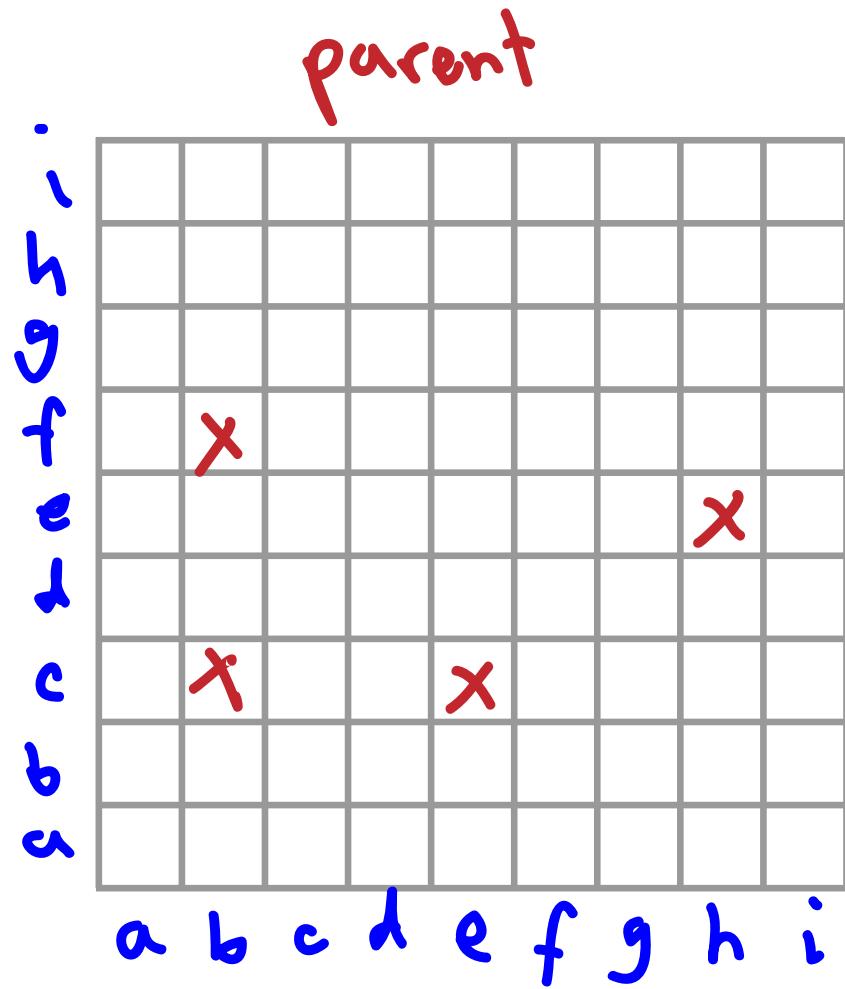
$\geq 2$  parent .Doctor

parent

i									
h									
g									
f									
e	x								
d									
c	x			x					
b									
a									
	a	b	c	d	e	f	g	h	i

x	xx	x	x	x	x	x	x	
a	b	c	d	e	f	g	h	i

$\leq 1$  parent.T


 $T \subseteq \leq^{\leq 1} \text{parent}.T$

parent

	i	h	g	f	e	d	c	b	a
→									

a b c d e f g h i

X	X	X	X	X	X	X	X	X	X
a	b	c	d	e	f	g	h	i	

$f \approx c$

$f \text{ owl:sameAs } c$

$T \sqsubseteq \leq 1 \text{ parent } . T$

talksTo

i	X								X
h	X								
g	X								
f	X								
e	X								
d	X								
c	X	X							
b	X	X	X						
a	X	X							
	a	b	c	d	e	f	g	h	i

	a	b	c	d	e	f	g	h	i

$\exists$  talksTo. Self

talksTo

i	X								X
h	X								
g	X								
f	X								
e	X								
d	X								
c	X		X						
b	X		X	X					
a	X	X							
	a	b	c	d	e	f	g	h	i

X	X		X						X
a	b	c	d	e	f	g	h	i	

$\exists$  talksTo. Self

# CLASSES

# PROPERTIES

# AXIOMS

T

$\exists P.C$

$P_1 \circ P_2$

$P_1 \subseteq P_2$

$\perp$

$\exists P.\text{Self}$

$P^-$

$P_1 \equiv P_2$

$C_1 \sqcap C_2$

$\forall P.C$

$C_1 \sqcup C_2$

$\forall P.\text{Self}$

$\neg C_1$

$C_1 \subseteq C_2$

$\{a\}$

$\geqslant P.C$   
 $\leqslant P.C$

$a:C$   
 $\langle a, b \rangle : R$   
 $a_1 \equiv a_2$   
 $a_1 \neq a_2$

# REASONING

⊓ rule:

A-box: our triples

⊔ rule:

Example

⊑ rule:

$A = \{ a:c_1, b:c_2 \}$

an A-box with 2 triples

⊐ rule:

# REASONING

$\sqcap$  rule:  $a : (c_1 \sqcap c_2) \in A$

what else goes in the A-box?

$\sqcup$  rule:

$\exists$  rule:

$\forall$  rule:

# REASONING

$\sqcap$  rule: if 1.  $a : (c_1 \sqcap c_2) \in A$  and  
2.  $\{a : c_1, a : c_2\} \notin A$

$\sqcup$  rule: then ...

$\exists$  rule:

$\forall$  rule:

# REASONING

$\sqcap$  rule: if 1.  $a : (c_1 \sqcap c_2) \in A$  and  
2.  $\{a : c_1, a : c_2\} \notin A$

$\sqcup$  rule: then set  $A_1 = A \cup \{a : c_1, a : c_2\}$

$\exists$  rule:  $A = \{a : \text{Engineer} \sqcap \text{Artist}\}$

$$A_1 =$$

$\forall$  rule:

# REASONING

$\sqcap$  rule: if 1.  $a : (C_1 \sqcap C_2) \in A$  and  
2.  $\{a : C_1, a : C_2\} \notin A$

$\sqcup$  rule: then set  $A_1 = A \cup \{a : C_1, a : C_2\}$

$\exists$  rule:

Example

$$A = \{a : \text{Engineer} \sqcap \text{Artist}\}$$

$\forall$  rule:

$$A_1 = \{a : \text{Engineer}, a : \text{Artist},\}  
a : \text{Engineer} \sqcap \text{Artist}$$

# REASONING

$\sqcap$  rule: if 1.  $a:(c_1 \sqcap c_2) \in A$  and  
2.  $\{a:c_1, a:c_2\} \notin A$   
then set  $A_1 = A \cup \{a:c_1, a:c_2\}$

$\sqcup$  rule: if 1.  $a:(c_1 \sqcup c_2) \in A$  and  
2.  $\{a:c_1, a:c_2\} \cap A = \emptyset$

$\exists$  rule: then set

$\forall$  rule:

# REASONING

$\sqcap$  rule: if 1.  $a:(c_1 \sqcap c_2) \in A$  and  
2.  $\{a:c_1, a:c_2\} \notin A$   
then set  $A_1 = A \cup \{a:c_1, a:c_2\}$

$\sqcup$  rule: if 1.  $a:(c_1 \sqcup c_2) \in A$  and  
2.  $\{a:c_1, a:c_2\} \cap A = \emptyset$

$\exists$  rule: then set  
 $A_1 = A \cup \{a:c_1\}$  and we branch  
 $A_2 = A \cup \{a:c_2\}$

$\forall$  rule:

# REASONING

$\sqcap$  rule: if 1.  $a:(c_1 \sqcap c_2) \in A$  and  
2.  $\{a:c_1, a:c_2\} \notin A$   
then set  $A_1 = A \cup \{a:c_1, a:c_2\}$

$\sqcup$  rule: if 1.  $a:(c_1 \sqcup c_2) \in A$  and  
2.  $\{a:c_1, a:c_2\} \cap A = \emptyset$   
then set  
 $A_1 = A \cup \{a:c_1\}$  and we branch  
 $A_2 = A \cup \{a:c_2\}$

## Example

$A = \{a: \text{Engineer} \sqcup \text{Artist}\}$  then  
 $A_1 = \{a: \text{Engineer} \sqcup \text{Artist}, a: \text{Engineer}\}$   
 $A_2 = \{a: \text{Engineer} \sqcup \text{Artist}, a: \text{Artist}\}$

# REASONING

$\sqcap$  rule: if 1.  $a:(c_1 \sqcap c_2) \in A$  and  
2.  $\{a:c_1, a:c_2\} \notin A$   
then set  $A_1 = A \cup \{a:c_1, a:c_2\}$

$\sqcup$  rule: if 1.  $a:(c_1 \sqcup c_2) \in A$  and  
2.  $\{a:c_1, a:c_2\} \cap A = \emptyset$   
then set  
 $A_1 = A \cup \{a:c_1\}$  and we branch  
 $A_2 = A \cup \{a:c_2\}$

$\exists$  rule: if 1.  $a:(\exists P.c) \in A$  and  
2. there is no  $b$  such that

$\{\langle a,b \rangle : P, b:c\} \subseteq A$   
then set

# REASONING

$\sqcap$  rule: if 1.  $a:(c_1 \sqcap c_2) \in A$  and then set  $A_1 = A \cup \{a:c_1, a:c_2\}$   
2.  $\{a:c_1, a:c_2\} \notin A$

$\square$  rule: if 1.  $a:(c_1 \sqcup c_2) \in A$  and then set  $A_1 = A \cup \{a:c_1\}$  and  
2.  $\{a:c_1, a:c_2\} \cap A = \emptyset$   $A_2 = A \cup \{a:c_2\}$

$\exists$  rule: if 1.  $a:(\exists P.c) \in A$  and  
2. there is no  $b$  such that

$$\{\langle a,b \rangle : P, b:c\} \subseteq A$$

then set

$$A_1 = A \cup \{\langle a,d \rangle : P, d:c\}$$

where  $d$  is new in  $A$

$\forall$  rule:

# REASONING

$\sqcap$  rule: if 1.  $a:(c_1 \sqcap c_2) \in A$  and then set  $A_1 = A \cup \{a:c_1, a:c_2\}$   
2.  $\{a:c_1, a:c_2\} \notin A$

$\sqcup$  rule: if 1.  $a:(c_1 \sqcup c_2) \in A$  and then set  
2.  $\{a:c_1, a:c_2\} \cap A = \emptyset$   $A_1 = A \cup \{a:c_1\}$  and  
 $A_2 = A \cup \{a:c_2\}$

$\exists$  rule: if 1.  $a:(\exists P.c) \in A$  and then set  
2. there is no  $b$  such that  $A_1 = A \cup \{\langle a,d \rangle : P, d:c\}$   
 $\{\langle a,b \rangle : P, b:c\} \subseteq A$  where  $d$  is new in  $A$

## Example

$$A = \{ a:\exists \text{parent}. \text{Doctor}, b:\text{Doctor} \}$$

$$A_1 = \{ a:\exists \text{parent}. \text{Doctor}, b:\text{Doctor}, \langle a,d \rangle : \text{parent}, d:\text{Doctor} \}$$

$\forall$  rule:

# REASONING

$\sqcap$  rule: if 1.  $a:(c_1 \sqcap c_2) \in A$  and  
2.  $\{a:c_1, a:c_2\} \notin A$  then set  $A_1 = A \cup \{a:c_1, a:c_2\}$

$\sqcup$  rule: if 1.  $a:(c_1 \sqcup c_2) \in A$  and  
2.  $\{a:c_1, a:c_2\} \cap A = \emptyset$  then set  
 $A_1 = A \cup \{a:c_1\}$  and  
 $A_2 = A \cup \{a:c_2\}$

$\exists$  rule: if 1.  $a:(\exists P.c) \in A$  and  
2. there is no  $b$  such that  
 $\{\langle a,b \rangle : P, b:c\} \subseteq A$  then set  
 $A_1 = A \cup \{\langle a,d \rangle : P, d:c\}$   
where  $d$  is new in  $A$

$\forall$  rule: if 1.  $\{a:\forall P.c, \langle a,b \rangle : P\} \subseteq A$  and  
2.  $b:c \notin A$  then  $A_1 = A \cup \{b:c\}$

Example  $A = \{a: \forall \text{hasChild}. \text{Doctor}, \langle a,b \rangle : \text{has Child}\}$

$A_1 = \{a: \forall \text{hasChild}. \text{Doctor}, \langle a,b \rangle : \text{has Child}\}$   
 $b: \text{Doctor}$

# REASONING

⊓ rule: if 1.  $a:(C_1 \cap C_2) \in A$  and      then set  $A_1 = A \cup \{a:C_1, a:C_2\}$   
          2.  $\{a:C_1, a:C_2\} \notin A$

⊔ rule: if 1.  $a:(C_1 \cup C_2) \in A$  and      then set  
          2.  $\{a:C_1, a:C_2\} \cap A = \emptyset$        $A_1 = A \cup \{a:C_1\}$  and  
   $A_2 = A \cup \{a:C_2\}$

⊑ rule: if 1.  $a:(\exists P.c) \in A$  and      then set  
          2. there is no  $b$  such that       $A_1 = A \cup \{\langle a,d \rangle : P, d:c\}$   
  where  $d$  is new in  $A$   
 $\{ \langle a,b \rangle : P, b:c \} \subseteq A$

⊐ rule: if 1.  $\{a:\forall P.c, \langle a,b \rangle : P\} \subseteq A$  and      then  $A_1 = A \cup \{b:c\}$   
          2.  $b:c \notin A$

# CLASSES

# PROPERTIES

# AXIOMS

T

$\exists P.C$

$P_1 \circ P_2$

$P_1 \leq P_2$

1

$\exists P.\text{Self}$

$P^-$

$P_1 \equiv P_2$

$\text{disjoint}(P_1, P_2)$

$C_1 \sqcap C_2$

$\forall P.C$

$C_1 \leq C_2$

$C_1 \sqcup C_2$

$\forall P.\text{Self}$

$C_1 \equiv C_2$

$\neg C_1$

{a}

$\geq P.C$   
 $\leq P.C$

$a:C$   
 $\langle a, b \rangle : P$   
 $a_1 \equiv a_2$   
 $a_1 \neq a_2$

$\exists P.C$

	hasChild									Doctor
i										
h										
g										
f										
e										
d										
c										
b										
a	x									
	a	b	c	d	e	f	g	h	i	

X									
a	b	c	d	e	f	g	h	i	

$\exists \text{hasChild}.\text{Doctor}$

set of triples  $\{x_i\}$

such that

$\langle x_i, y_i \rangle : P \wedge y_i : C$

individuals who

have a child

who is a Doctor

$\exists$  P. C

$\exists$  hasChild . Doctor

individuals who  
have a child who is a Doctor

$\exists$  hasChild . Doctor  $\sqcap$  Person

persons who  
have a child who is a Doctor

$\exists$  hasChild . Doctor  $\sqcap$  Person  $\leq_T$  meaning?

$\exists$  hasChild . Doctor  $\sqcap$  Person  $\exists^T \}$  different?

$\forall$  hasChild . Doctor  $\sqcap$  Person

# EXAMPLE

Doctor  $\in$  Person

Parent  $\equiv$  Person  $\cap \exists \text{hasChild}.\text{Person}$

HappyParent  $\equiv$  Parent  $\cap \forall \text{hasChild}.(\text{Doctor} \sqcup \exists \text{hasChild}.\text{Doctor})$

translate to English

# EXAMPLE

Doctor  $\in$  Person

Parent  $\equiv$  Person  $\cap$   $\exists$  hasChild. Person

HappyParent  $\equiv$  Parent  $\cap$   $\forall$  hasChild. (Doctor  $\sqcup$   $\exists$  hasChild. Doctor)

A = John: HappyParent,  
 $\langle$  John, Mary  $\rangle$ : hasChild      is Mary: Doctor ?

is Mary:Doctor?

Doctor  $\subseteq$  Person

Parent  $\equiv$  Person  $\cap$   $\exists$  hasChild. Person

HappyParent  $\equiv$  Parent  $\cap$   $\forall$  hasChild. (Doctor  $\sqcup$   $\exists$  hasChild. Doctor)

A = John: HappyParent,  
 $\langle$  John, Mary  $\rangle$ : hasChild

Mary:  $\neg$  Doctor add the negation, and apply the rules

is Mary : Doctor ?

Doctor ⊑ Person

Parent ≡ Person  $\sqcap \exists \text{hasChild}.\text{Person}$

HappyParent ≡ Parent  $\sqcap \forall \text{hasChild}.(\text{Doctor} \sqcup \exists \text{hasChild}.\text{Doctor})$

A = John : HappyParent,  
 $\langle \text{John}, \text{Mary} \rangle : \text{hasChild}$

Mary :  $\neg \text{Doctor}$

John : Parent

John :  $\forall \text{hasChild}.(\text{Doctor} \sqcup \exists \text{hasChild}.\text{Doctor})$

John : Person

John :  $\exists \text{hasChild}.\text{Person}$

# is Mary:Doctor?

Doctor ⊑ Person

Parent ≡ Person  $\sqcap \exists \text{hasChild}.\text{Person}$

HappyParent ≡ Parent  $\sqcap \forall \text{hasChild}.(\text{Doctor} \sqcup \exists \text{hasChild}.\text{Doctor})$

---

John: HappyParent,

$\langle \text{John}, \text{Mary} \rangle : \text{hasChild}$

Mary:  $\neg \text{Doctor}$

John: Parent

John :  $\forall \text{hasChild}.(\text{Doctor} \sqcup \exists \text{hasChild}.\text{Doctor})$

John: Person

John :  $\exists \text{hasChild}.\text{Person}$

Mary: Doctor

Mary:  $\exists \text{hasChild}.\text{Doctor}$



# is Mary:Doctor?

Doctor ⊑ Person

Parent ≡ Person  $\sqcap \exists \text{hasChild}.\text{Person}$

HappyParent ≡ Parent  $\sqcap \forall \text{hasChild}.(\text{Doctor} \sqcup \exists \text{hasChild}.\text{Doctor})$

---

John: HappyParent,

$\langle \text{John}, \text{Mary} \rangle$ : hasChild

Mary:  $\neg \text{Doctor}$  inconsistent

John: Parent

John:  $\forall \text{hasChild}.(\text{Doctor} \sqcup \exists \text{hasChild}.\text{Doctor})$

John: Person

John:  $\exists \text{hasChild}.\text{Person}$

Mary: Doctor

Mary:  $\exists \text{hasChild}.\text{Doctor}$

is Mary:Doctor?

Doctor ⊑ Person

Parent ≡ Person  $\sqcap \exists \text{hasChild}.\text{Person}$

HappyParent ≡ Parent  $\sqcap \forall \text{hasChild}.(\text{Doctor} \sqcup \exists \text{hasChild}.\text{Doctor})$

---

John: HappyParent,

$\langle \text{John}, \text{Mary} \rangle$ : hasChild

Mary:  $\neg$  Doctor

John: Parent

John :  $\forall \text{hasChild}.(\text{Doctor} \sqcup \exists \text{hasChild}.\text{Doctor})$

---

John: Person

John :  $\exists \text{hasChild}.\text{Person}$

Mary:  $\exists \text{hasChild}.\text{Doctor}$

$\langle \text{John}, a \rangle$ : hasChild

a: Doctor  $\sqcup \exists \text{hasChild}.\text{Doctor}$

# is Mary:Doctor?

Doctor ⊑ Person

Parent ≡ Person  $\sqcap \exists \text{hasChild}.\text{Person}$

HappyParent ≡ Parent  $\sqcap \forall \text{hasChild}.(\text{Doctor} \sqcup \exists \text{hasChild}.\text{Doctor})$

---

John: HappyParent,

$\langle \text{John}, \text{Mary} \rangle$ : hasChild

Mary:  $\neg$  Doctor

John: Parent

John:  $\forall \text{hasChild}.(\text{Doctor} \sqcup \exists \text{hasChild}.\text{Doctor})$

John: Person

John:  $\exists \text{hasChild}.\text{Person}$

Mary:  $\exists \text{hasChild}.\text{Doctor}$

---

$\langle \text{John}, a \rangle$ : hasChild

a: Doctor  $\sqcup \exists \text{hasChild}.\text{Doctor}$

$\langle \text{Mary}, b \rangle$ : hasChild

b: Doctor

More inferences

a: Person

b: Person

# is Mary:Doctor?

Doctor ⊑ Person

Parent ≡ Person  $\cap \exists \text{hasChild}.\text{Person}$

HappyParent ≡ Parent  $\cap \forall \text{hasChild}.(\text{Doctor} \sqcup \exists \text{hasChild}.\text{Doctor})$

---

John: HappyParent,

$\langle \text{John}, \text{Mary} \rangle: \text{hasChild}$

Mary:  $\neg \text{Doctor}$

John: Parent

John:  $\forall \text{hasChild}.(\text{Doctor} \sqcup \exists \text{hasChild}.\text{Doctor})$

John: Person

John:  $\exists \text{hasChild}.\text{Person}$

Mary:  $\exists \text{hasChild}.\text{Doctor}$

$\langle \text{John}, a \rangle: \text{hasChild}$

a: Doctor  $\sqcup \exists \text{hasChild}.\text{Doctor}$

We are stuck, and we  
didn't reach a contradiction!

we cannot  
infer Mary: Doctor

$\langle \text{Mary}, b \rangle: \text{hasChild}$

b: Doctor

a: Person

b: Person

# is Mary:Doctor?

Doctor ⊑ Person

Parent ≡ Person  $\sqcap \exists \text{hasChild}.\text{Person}$

HappyParent ≡ Parent  $\sqcap \forall \text{hasChild}.(\text{Doctor} \sqcup \exists \text{hasChild}.\text{Doctor})$

---

John: HappyParent,  
 $\langle \text{John}, \text{Mary} \rangle: \text{hasChild}$

suppose we add one more axiom  
Mary:  $\forall \text{hasChild. } \perp$

---

Mary:  $\neg \text{Doctor}$

John: Parent

John:  $\forall \text{hasChild}.(\text{Doctor} \sqcup \exists \text{hasChild}.\text{Doctor})$

John: Person

John:  $\exists \text{hasChild}.\text{Person}$

Mary:  $\exists \text{hasChild}.\text{Doctor}$

$\langle \text{John}, a \rangle: \text{hasChild}$

a: Doctor  $\sqcup \exists \text{hasChild}.\text{Doctor}$

$\langle \text{Mary}, b \rangle: \text{hasChild}$   
b: Doctor

a: Person

b: Person

# is Mary:Doctor?

Doctor  $\equiv$  Person

Parent  $\equiv$  Person  $\cap \exists \text{hasChild}.\text{Person}$

HappyParent  $\equiv$  Parent  $\cap \forall \text{hasChild}.(\text{Doctor} \sqcup \exists \text{hasChild}.\text{Doctor})$

---

John: HappyParent,  
 $\langle \text{John}, \text{Mary} \rangle: \text{hasChild}$

suppose we add one more axiom

Mary:  $\forall \text{hasChild. } \perp$

Mary:  $\neg \text{Doctor}$

John: Parent

John:  $\forall \text{hasChild}.(\text{Doctor} \sqcup \exists \text{hasChild}.\text{Doctor})$

John: Person

John:  $\exists \text{hasChild}.\text{Person}$

Mary:  $\exists \text{hasChild}.\text{Doctor}$

$\langle \text{John}, a \rangle: \text{hasChild}$

a: Doctor  $\sqcup \exists \text{hasChild}.\text{Doctor}$

$\langle \text{Mary}, b \rangle: \text{hasChild}$

b: Doctor

b:  $\perp$  YEAH! contradiction!

a: Person

b: Person



# PROFILES

SROIQ exponential

OWL-EL polynomial

OWL-RL polynomial

OWL-QL aka DL-Lite Logspace,  
same as SQL

# SROIQ

$T$

$\exists P.C$

~~$P_1 \circ P_2$~~

$P_1 \sqsubseteq P_2$

$\perp$

$\exists P.\text{Self}$

$P_1 \circ P_2 \sqsubseteq P_3$

$P_1 \equiv P_2$

$\text{disjoint}(P_1, P_2)$

$C_1 \sqcap C_2$

$\forall P.C$

$P^-$

$C_1 \sqsubseteq C_2$

$C_1 \sqcup C_2$

$\forall P.\text{Self}$

$C_1 \equiv C_2$

$\neg C_1$

$\{a\}$

$\geq P.C$   
 $\leq P.C$

$a:C$   
 $\langle a, b \rangle : P$   
 $a_1 \equiv a_2$   
 $a_1 \neq a_2$

decidable, but exponential

# OWL-EL

T  
⊥

$\exists P.C$

$\exists P.\text{Self}$

~~$P_1 \circ P_2$~~   
 $P_1 \circ P_2 \sqsubseteq P_3$

restricted

$P_1 \sqsubseteq P_2$

$P_1 \equiv P_2$

$\text{disjoint}(P_1, P_2)$

$C_1 \sqcap C_2$

~~$C_1 \sqcup C_2$~~

~~$\neg C_1$~~

$\forall P.C$

$\forall P.\text{Self}$

$P^-$

$C_1 \sqsubseteq C_2$

$C_1 \equiv C_2$

{a}

~~$\gg P.C$~~   
 ~~$\ll P.C$~~

polynomial

$a:C$   
 $\langle a, b \rangle : P$   
 $a_1 \equiv a_2$   
 $a_1 \neq a_2$

# OWL-QL

$T$

$\exists P. C$

$\exists P. \text{Self}$

$P_1 \sqsubset P_2$

$P_1 \sqsubseteq P_2$

$P_1 \equiv P_2$

$\text{disjoint}(P_1, P_2)$

$C_1 \sqcap C_2$

$C_1 \sqcup C_2$

$\neg C_1$

$\forall P. C$

$\forall P. \text{Self}$

$P^-$

$C_1 \sqsubseteq C_2$

$C_1 \equiv C_2$

$\{a\}$

$\neq P. C$

$\leq P. C$

$a : C$

$\langle a, b \rangle : P$

$a_1 \neq a_2$

$a_1 \neq a_2$

Fastest useful profile

# OWL-QL

$\top \times$

$C_1 \cap C_2$

$P^-$

$\text{domain}(P, C)$

$\text{range}(P, C)$

$P_1 \sqsubseteq P_2$

$P_1 \equiv P_2$

$C_1 \sqsubseteq C_2$

$C_1 \equiv C_2$

$a : C$

$\langle a, b \rangle : P$

$a_1 \neq a_2$

$\text{disjoint}(C_1, C_2)$

$\text{disjoint}(P_1, P_2)$

$\text{symmetric}(P)$

# **OWL Profile Validator**

<http://mowl-power.cs.man.ac.uk:8080/validator/>