The Binomial and Bernoulli Distributions

The Binomial Distribution

A Bernoulli trial (or a Binomial trial) is a random experiment with exactly 2 possible outcomes, “success” or “failure”, in which the probability of success is the same every time the experiment is conducted.

Example 1:

Let’s say we perform an experiment of tossing a coin. In this scenario, the outcome of the experiment can be either heads or tails. Here, we have 2 possible outcomes and we can consider any 1 out of 2 outcomes as a success (depending on the experiment).

If we consider heads as success, the probability of getting heads (probability of success) is always the same, i.e., 0.5. Similarly, if we consider tails as success, the probability of getting tails (probability of success) is always the same, i.e., 0.5. Hence, this experiment of tossing a coin can be considered a Bernoulli trial.

Example 2:

Let’s assume that the probability of a woman having breast cancer all over the world is 0.2, hence, the probability of not getting breast cancer is 1 - 0.2 = 0.8.

Let’s say we perform an experiment of randomly selecting a female candidate from anywhere in the world, to check whether she has breast cancer or not. Here, we have 2 possible outcomes and we can consider any 1 out of 2 outcomes a success. The probability is also the same every time. Hence, this experiment can also be considered a Bernoulli trial.

The binomial distribution represents the probability for **'x' successes** of an experiment in **'n' trials**, given a **success probability 'p'** for each trial of the experiment.

The formula for the binomial distribution for a random variable X is given as,

$$P(x: n, p) = \frac{n!}{(n-x)!x!} p^{x} q^{n-x}$$

Where:

p = Probability of Success

q = 1 - p = Probability of Failure

n = Number of Trials

x = Number of successes desired

The Notation for a binomial distribution is

X ~ B (n, p)

which is read as ‘X follows a binomial distribution with n trials and probability of success in each trial is equal to p’.

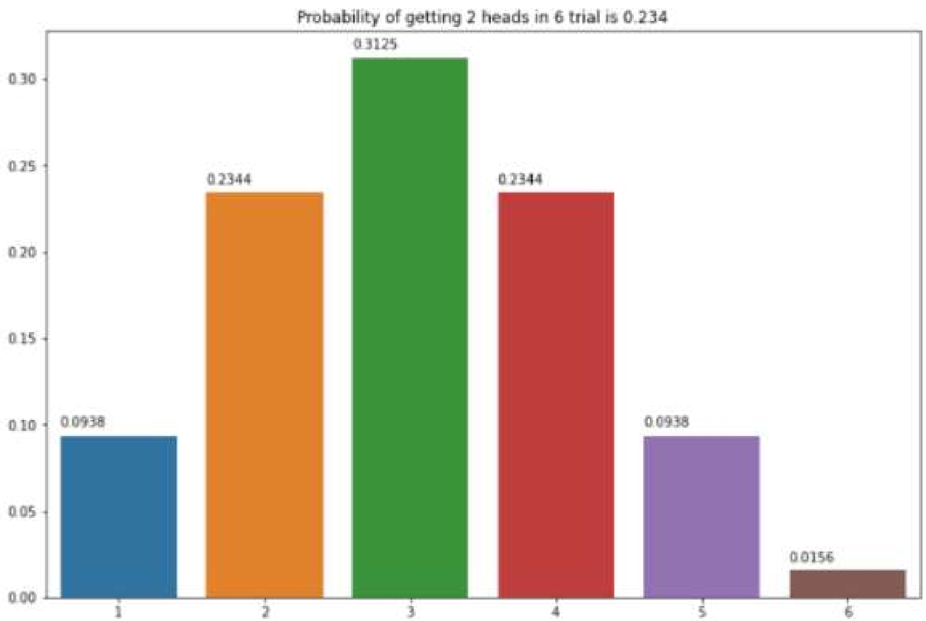
There are some of the assumptions that need to be satisfied for a random variable to follow a binomial distribution. They are as follows:

1. There are a fixed number of trials. (represented by the variable n)
2. Each trial has 2 outcomes (called "success" and "failure" for convenience)
3. The trials are all independent of one another.
4. The probability of success, i.e., p is the same for all trials.

Let's consider an example. We can use the above formula to find the probability of 2 heads in 6 coin tosses

Here the experiment of tossing a coin is **repeated** 6 times.

- Success = 'heads'
- n = 6 trials
- p = 0.5
- x = number of heads in 6 tosses which are 2 here
- X follows a binomial distribution with n = 6 and p = 0.5, i.e., X ~ B (6, 0.5)



This is computed using the formula below.

$$P(x: n, p) = \frac{n!}{(n-x)!x!} p^{x} q^{n-x}$$

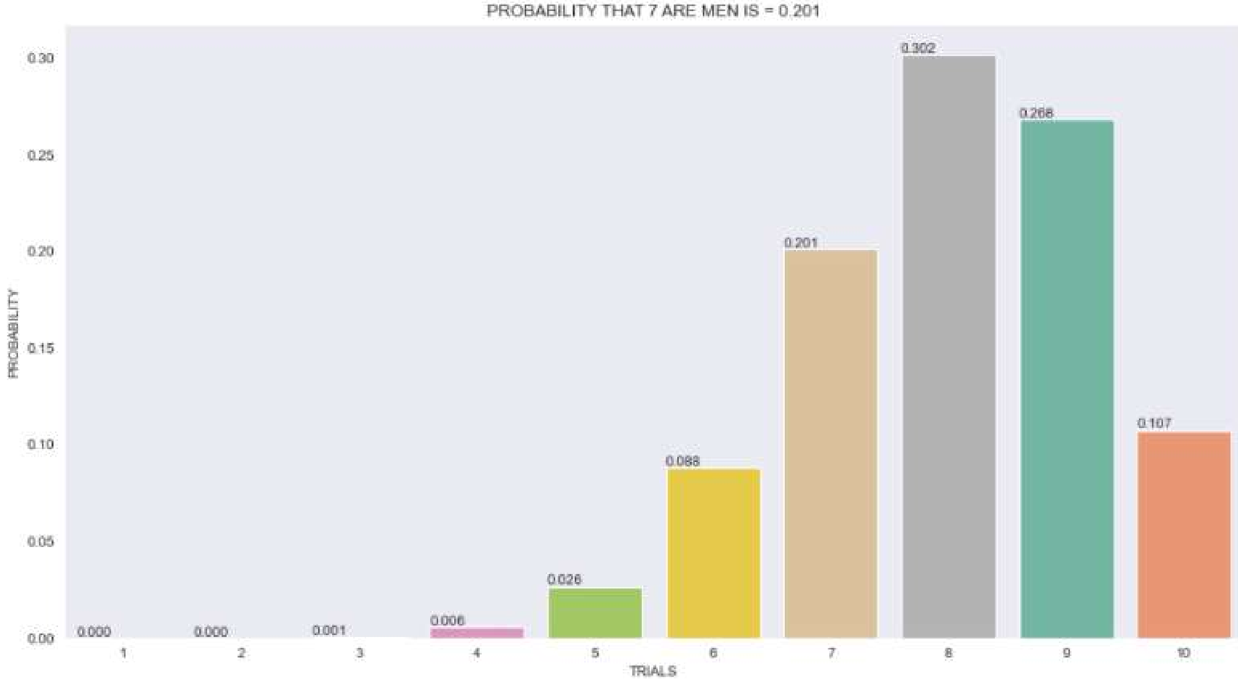
$$P(x = 2: n = 6, p = 0.5) = \frac{6!}{(6-2)!2!} 0.5^{2}(1 - 0.5)^{6-2} = 0.234$$

So, the probability of getting 2 heads out of 6 coin tosses (trials) is 0.234. Hence, it means that there is a ~23% chance that we will get exactly 2 heads if we repeatedly toss a coin 6 times.

Let's consider another example. Suppose 80% of people who purchase shirts are men, then what would be the probability that exactly 7 are men if 10 shirt owners are randomly selected?

Here the experiment of selecting random shirt owners is repeated 10 times,

- Success = "person is male"
- n = 10 trials
- p = 0.8
- x = 7 (Number of male shirt owners desired)
- X has a binomial distribution with n=10 & p=0.8, i.e., X ~ B(10,0.8)



This is computed using the formula below,

$$P(x: n, p) = \frac{n!}{(n-x)!x!} p^x q^{n-x}$$

$$P(x = 7: n = 10, p = 0.8) = \frac{10!}{(10-7)!7!} 0.8^7 (1 - 0.8)^{10-7} = 0.201$$

So, the probability of getting 7 male shirt owners out of 10 randomly picked shirt owners(trials) is 0.201. Hence, it means that there is a ~20% chance that we will get exactly 7 male shirt owners if we randomly select 10 shirt owners.

The Bernoulli Distribution

The Bernoulli Distribution is a special case of the Binomial Distribution where the number of trials is equal to 1. Hence it represents the probability of an event occurring in 1 single trial, for example, the probability of getting heads when a coin is tossed only 1 time is 0.5.