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IDSS

# Applied Data Science Program

## DECISION TREES

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# Introduction

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# Outline

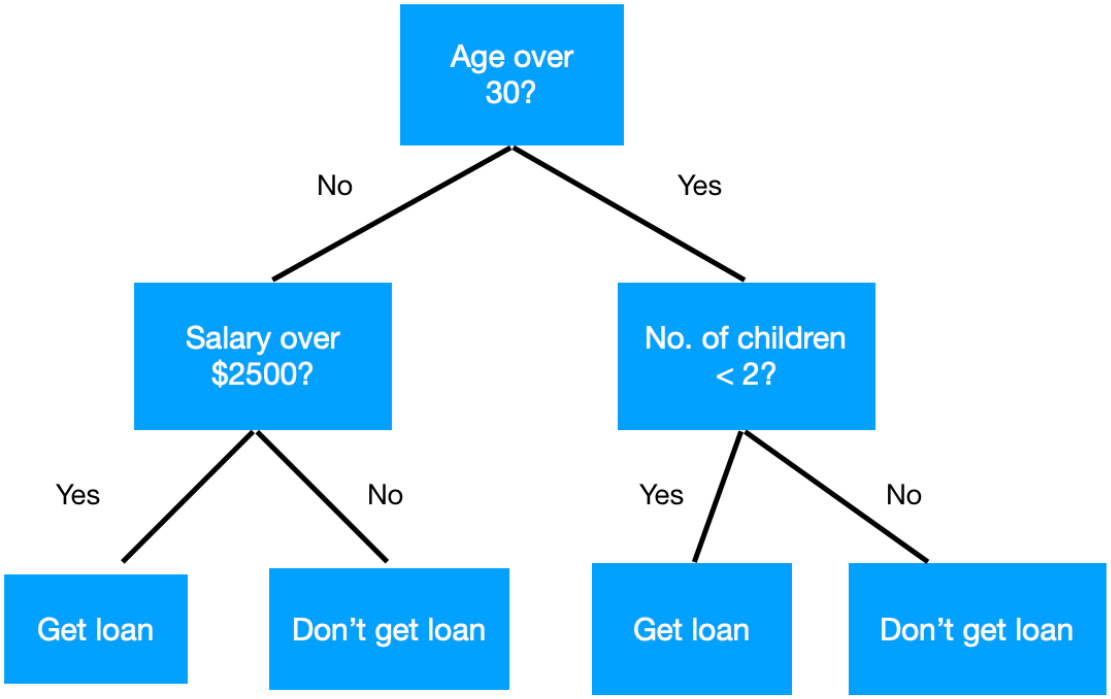
- Learning may require more than linear regression
- Data may not be static (IID)
- Next three lectures:
  - Decision Trees
  - Random Forest
  - Time Series

# Decision Trees Outline

- Part I: Definitions, interpretations
- Part II: Learning a Decision Tree
- Part III: Entropy-based Greedy algorithm

# Part I: Introduction

# Example: Who Gets a Loan?



# A Decision Tree is a Flow Chart Classifier

- Each internal node represents a "test" on an attribute (feature)
- Each branch represents the outcome of the test
- Each leaf node represents a class label (decision taken after computing all attributes)
- The paths from root to leaf represent classification rules

# Advantages

- Human-Algorithm Interaction
  - Simple to understand and interpret.
  - Mirrors human decision making more closely
  - Uses an open-box  model (as opposed to a black box)—Contrast to NNT

- Versatile
  - Able to handle both numerical and categorical data
  - Powerful: Can model arbitrary functions
  - Requires little data preparation
  - Performs well with large datasets.
  - Robust against boosting

# Advantages

- Built-in feature selection
  - Naturally de-emphasizes irrelevant features
  - Develops a a hierarchy in terms of relevance

- Testable: Possible to validate a model using statistical tests

# Limitations

- Trees can be non-robust:
  - A small change in the training data can result in a large change in the tree and consequently the final predictions.

- The problem of learning an optimal decision tree is known to be NP-Complete
  - Practical decision-tree learning algorithms are based on heuristics (greedy algorithm)
  - Such algorithms cannot guarantee to return the globally optimal decision tree.

- Overfitting: Decision-tree learners can create over-complex trees that do not generalize well from the training data.
  - Motivates pruning and Random Forests

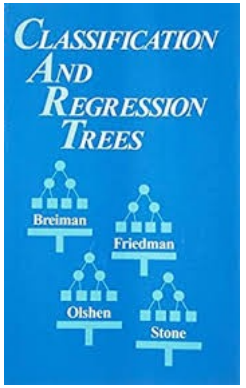
# Implementation

- Many data mining software packages provide implementations of decision tree algorithms.
  - Salford Systems
  - IBM SPSS Modeler
  - RapidMiner
  - SAS Enterprise Miner
  - Matlab
  - R
  - MS SQL Server
  - Scikit learn (a free and open-source machine learning library for the Python programming language)

# History and Reference

- Quinlan, J. R. 1986. Induction of Decision Trees. Mach. Learn. 1, 1 (Mar. 1986), 81-106

- Breiman, Friedman, Olshen, and Stone: Classification and Regression Trees.

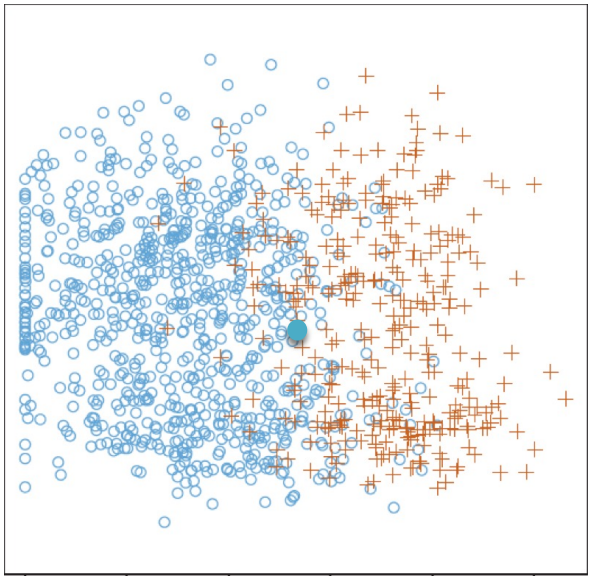
# Classification: Main Idea

| | |
|---|---|
| $\mathbf{X}_1$ | $Y_1$ |
| $\vdots$ | $\vdots$ |
| $\mathbf{X}_n$ | $Y_n$ |
| $\mathbf{X}$ | $Y$? |

- Classify a data record into one of multiple categories, based on examples

$Y = 0, 1$ (binary)

$Y = 1, \ldots, m$ ($m$-ary)



$\mathbf{X}$: symptoms, test results — $Y$: cancer?

$\mathbf{X}$: an email message — $Y$: spam?

$\mathbf{X}$: image of a digit $Y$ — $Y$: which digit is it? ($m = 10$)

$\mathbf{X}$: image of an animal $Y$ — $Y$: cat, dog, cow,...

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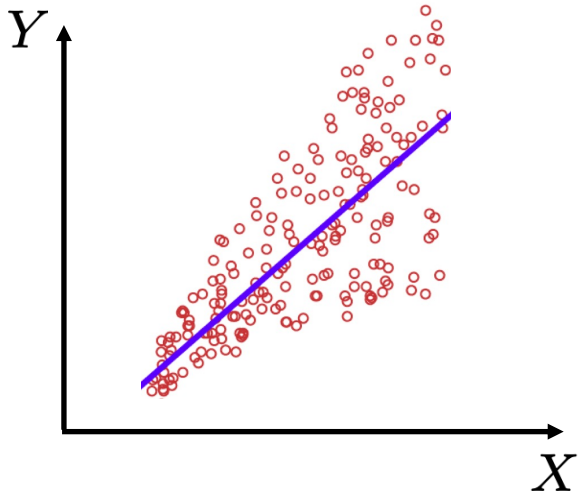
# Predictor vs Classifier

- In regression:

  predictor $\hat{Y} = g(\mathbf{X})$

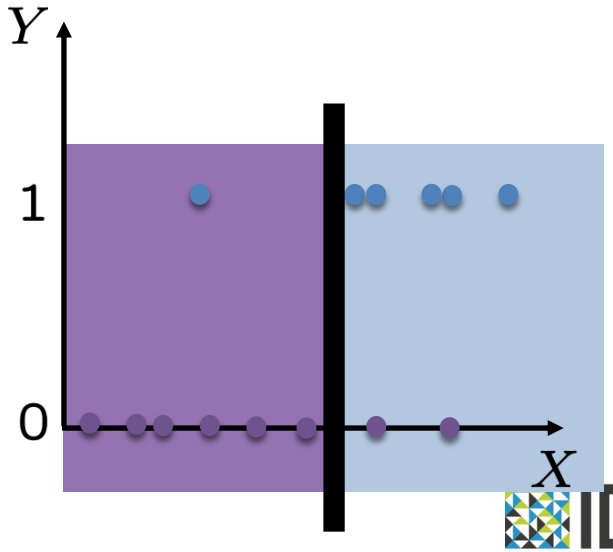
  metric $\mathbb{E}\left[(\hat{Y} - Y)^2\right]$



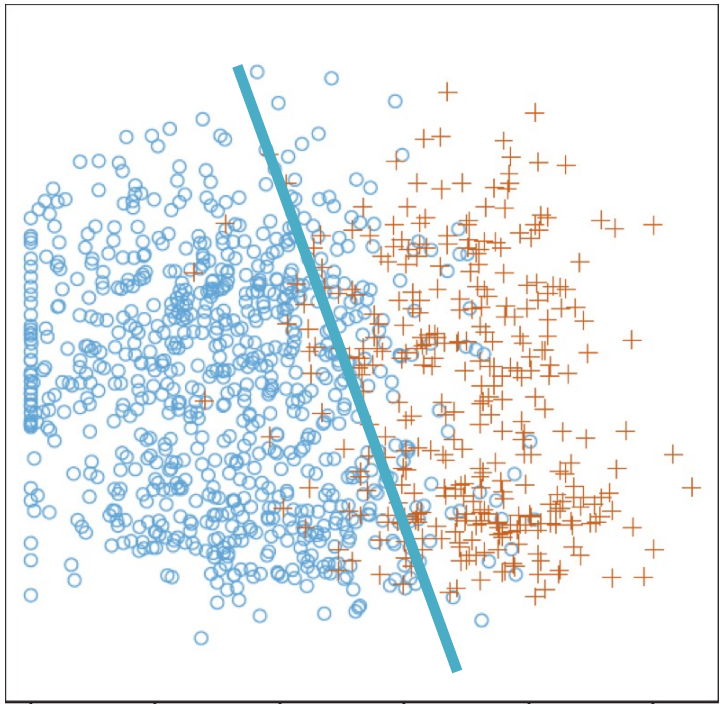

- In classification:

  classifier $\hat{Y} = g(\mathbf{X}) \in \{1, \ldots, m\}$

  metric $\mathbb{P}(\text{mistake})$

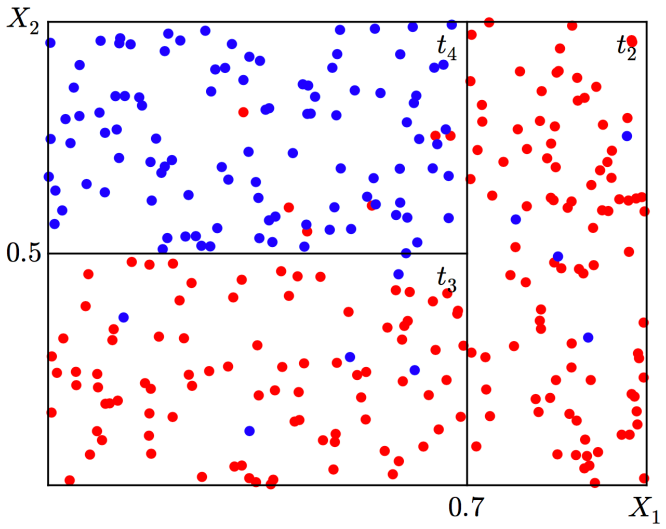


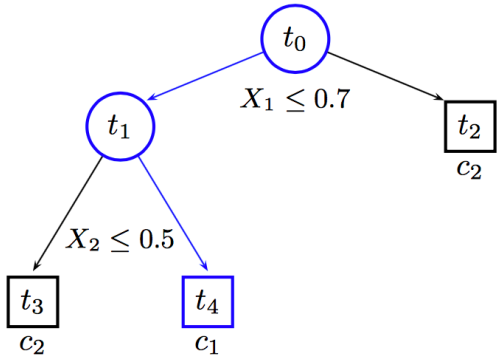
# Type of Classifier: Linear



- Linear classifiers: compare $\boldsymbol{\theta}^T \mathbf{X}$ to a threshold
  - learn "good" vector $\boldsymbol{\theta}$ and threshold

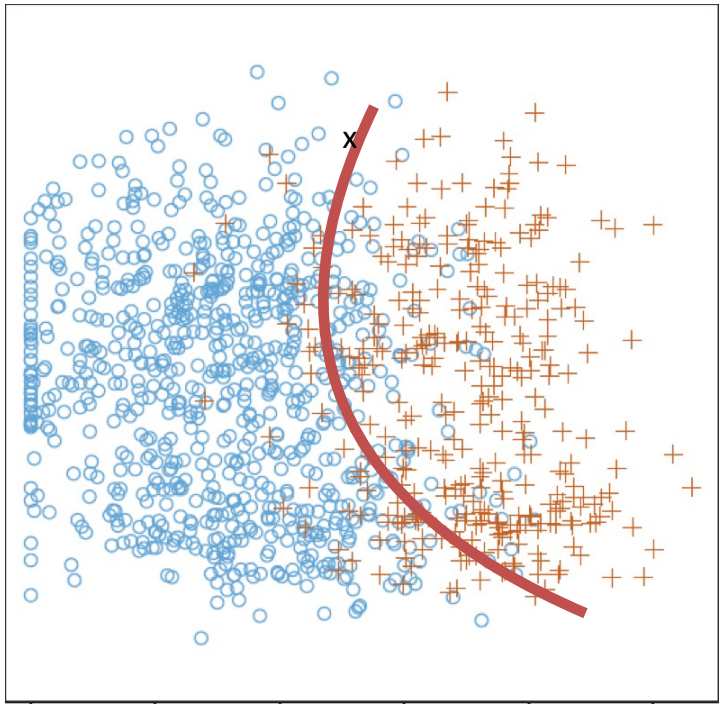
# Type of Classifier: A Decision Tree




Boolean Functions of Features

# Type of Classifier: Nonlinear

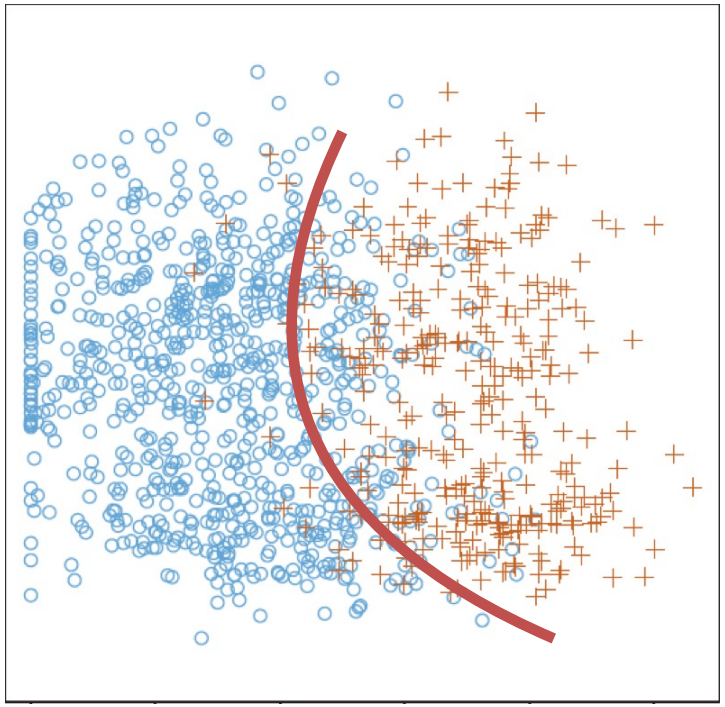



- Nonlinear classifiers: compare $h(\mathbf{X})$ to a threshold
  - learn "good" function $h$ and threshold

- Nonlinear classifier $\theta_1 X_1 + \theta_2 X_2 + \theta_{12} X_1 X_2$ is actually linear if we redefine $\mathbf{X} = (X_1, X_2, X_1 X_2)$

- Feature-based linear classifier: compare $\boldsymbol{\theta}^T \phi(\mathbf{X})$ to a threshold

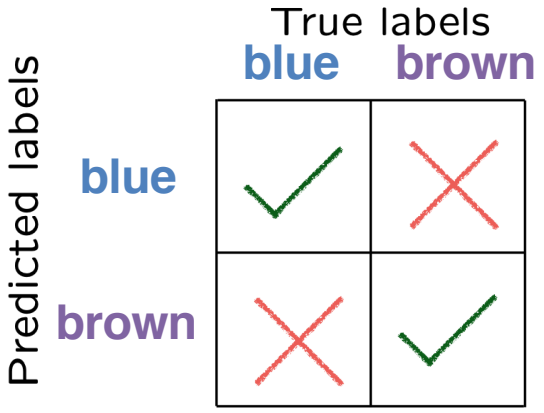
# Error Types and Confusion Matrix



- Binary classification, two error types:

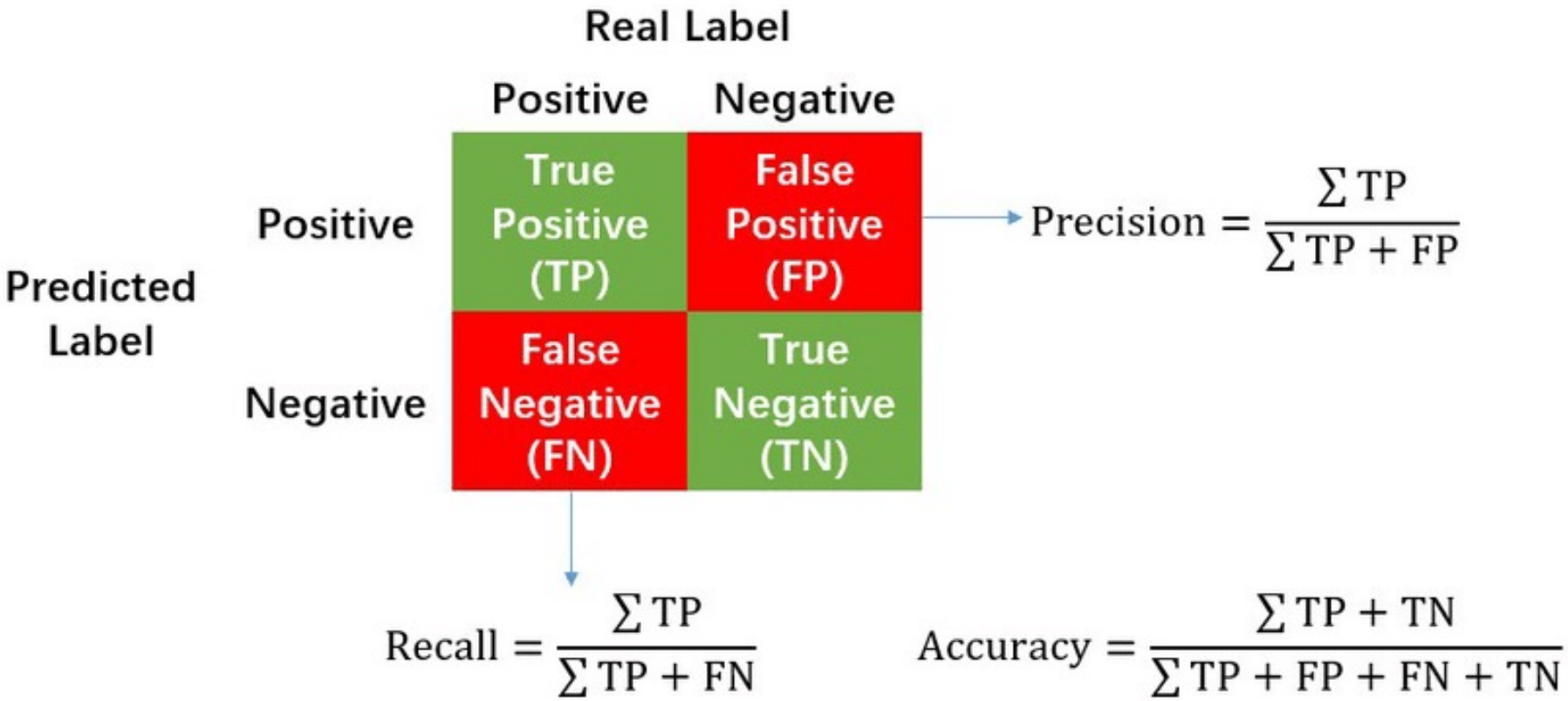
  truth is "blue", decide $\hat{Y}$ = "brown"

  truth is "brown", decide $\hat{Y}$ = "blue"




- Interested in having few errors on new data records
- Aim at few errors on training set
- Tradeoff between the two error types
- Similar confusion matrix, for $m$-ary classification

# Confusion Matrix



# Numerical vs. Categorical Data

- So far, you focused on continuous variables
- Decision trees are simpler when variables are categorical:
  - Variables take on discrete values
  - Refer to this as *finite outcomes*
  - Binary: equivalent to $\{0, 1\}$
- Will revisit continuous variables at the end

# *Questions and Break*

# *Part II: Learning a DT*

# Illustrative Example: Waiting at a restaurant

| Example | Input Attributes | | | | | | | | | | Goal |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | *Alt* | *Bar* | *Fri* | *Hun* | *Pat* | *Price* | *Rain* | *Res* | *Type* | *Est* | *WillWait* |
| $\mathbf{x}_1$ | *Yes* | *No* | *No* | *Yes* | *Some* | *$$$* | *No* | *Yes* | *French* | *0–10* | $y_1 =$ *Yes* |
| $\mathbf{x}_2$ | *Yes* | *No* | *No* | *Yes* | *Full* | *$* | *No* | *No* | *Thai* | *30–60* | $y_2 =$ *No* |
| $\mathbf{x}_3$ | *No* | *Yes* | *No* | *No* | *Some* | *$* | *No* | *No* | *Burger* | *0–10* | $y_3 =$ *Yes* |
| $\mathbf{x}_4$ | *Yes* | *No* | *Yes* | *Yes* | *Full* | *$* | *Yes* | *No* | *Thai* | *10–30* | $y_4 =$ *Yes* |
| $\mathbf{x}_5$ | *Yes* | *No* | *Yes* | *No* | *Full* | *$$$* | *No* | *Yes* | *French* | *>60* | $y_5 =$ *No* |
| $\mathbf{x}_6$ | *No* | *Yes* | *No* | *Yes* | *Some* | *$$* | *Yes* | *Yes* | *Italian* | *0–10* | $y_6 =$ *Yes* |
| $\mathbf{x}_7$ | *No* | *Yes* | *No* | *No* | *None* | *$* | *Yes* | *No* | *Burger* | *0–10* | $y_7 =$ *No* |
| $\mathbf{x}_8$ | *No* | *No* | *No* | *Yes* | *Some* | *$$* | *Yes* | *Yes* | *Thai* | *0–10* | $y_8 =$ *Yes* |
| $\mathbf{x}_9$ | *No* | *Yes* | *Yes* | *No* | *Full* | *$* | *Yes* | *No* | *Burger* | *>60* | $y_9 =$ *No* |
| $\mathbf{x}_{10}$ | *Yes* | *Yes* | *Yes* | *Yes* | *Full* | *$$$* | *No* | *Yes* | *Italian* | *10–30* | $y_{10} =$ *No* |
| $\mathbf{x}_{11}$ | *No* | *No* | *No* | *No* | *None* | *$* | *No* | *No* | *Thai* | *0–10* | $y_{11} =$ *No* |
| $\mathbf{x}_{12}$ | *Yes* | *Yes* | *Yes* | *Yes* | *Full* | *$* | *No* | *No* | *Burger* | *30–60* | $y_{12} =$ *Yes* |

Attributes:

1. Alternate: whether there is a suitable alternative restaurant nearby.
2. Bar: whether the restaurant has a comfortable bar area to wait in.
3. Fri/Sat: true on Fridays and Saturdays.
4. Hungry: whether we are hungry.
5. Patrons: how many people are in the restaurant (values are None, Some, and Full).
6. Price: the restaurant's price range ($, $$, $$$).
7. Raining: whether it is raining outside.
8. Reservation: whether we made a reservation.
9. Type: the kind of restaurant (French, Italian, Thai or Burger).
10. WaitEstimate: the wait estimated by the host (0-10 minutes, 10-30, 30-60, >60).

[from: Russell & Norvig]

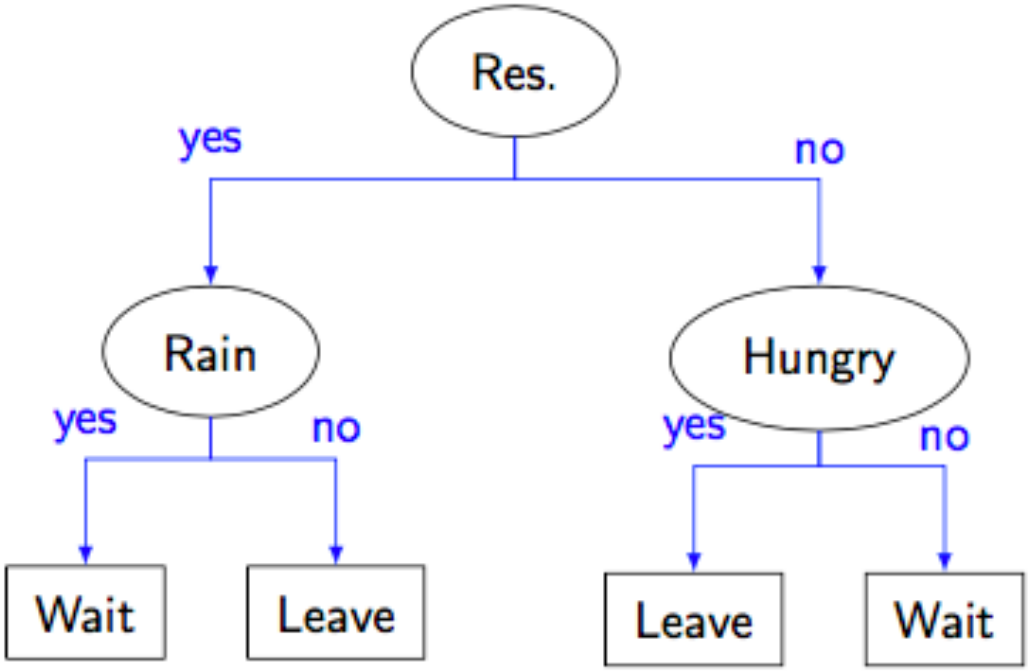
# Notation

**Feature Space- A vector of categorical data**: $\mathcal{X}$.

**Outcome Class (categorical)**: $\mathcal{Y}$

**A Decision Rule**: $f : \mathcal{X} \rightarrow \mathcal{Y}$

# One possible Tree




Depth = 2

*Misclassification = ?*

# Misclassification Error

**Data Set:** $\{(x_i, y_i) \in \mathcal{X} \times \mathcal{Y}, i = 1, \ldots N\}$

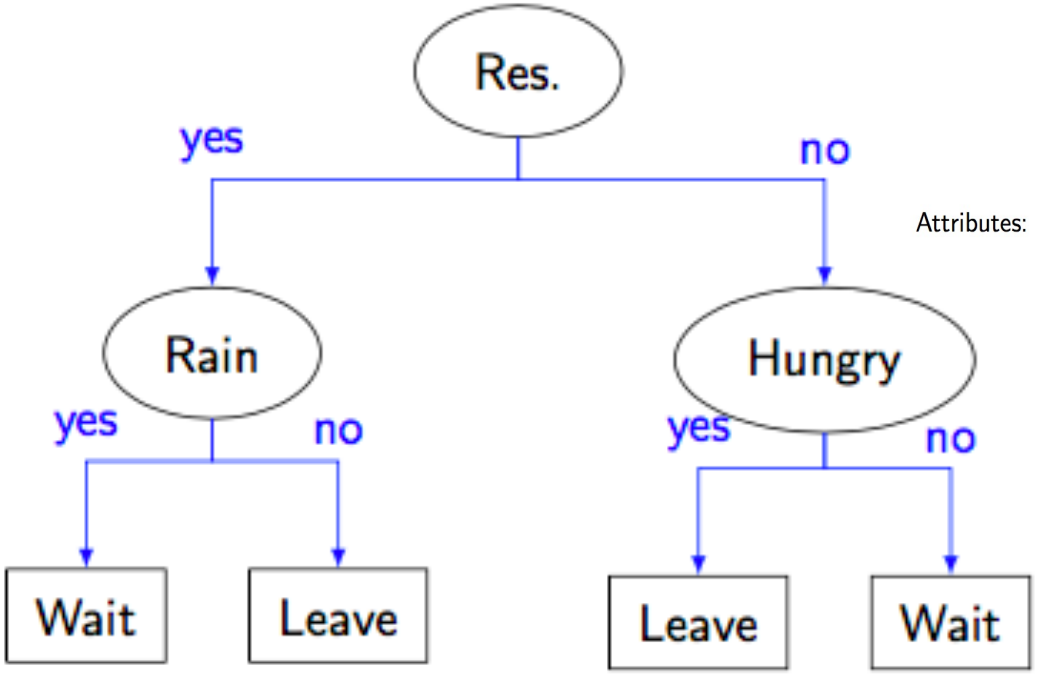
**Empirical Error of a decision Rule** $f$ :

$$R(f) = \frac{1}{N} \sum_i^N \mathbf{I}(f(x_i) \neq y_i)$$

$$\mathbf{I}(x) = 1 \; if \; x \neq 0, otherwise \; it \; is \; 0$$

# Misclassification Error

| Example | Input Attributes | | | | | | | | | | Goal |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | *Alt* | *Bar* | *Fri* | *Hun* | *Pat* | *Price* | *Rain* | *Res* | *Type* | *Est* | *WillWait* |
| $\mathbf{x}_1$ | *Yes* | *No* | *No* | *Yes* | *Some* | *$$$* | *No* | *Yes* | *French* | *0–10* | $y_1 = Yes$ |
| $\mathbf{x}_2$ | *Yes* | *No* | *No* | *Yes* | *Full* | *$* | *No* | *No* | *Thai* | *30–60* | $y_2 = No$ |
| $\mathbf{x}_3$ | *No* | *Yes* | *No* | *No* | *Some* | *$* | *No* | *No* | *Burger* | *0–10* | $y_3 = Yes$ |
| $\mathbf{x}_4$ | *Yes* | *No* | *Yes* | *Yes* | *Full* | *$* | *Yes* | *No* | *Thai* | *10–30* | $y_4 = Yes$ |
| $\mathbf{x}_5$ | *Yes* | *No* | *Yes* | *No* | *Full* | *$$$* | *No* | *Yes* | *French* | *>60* | $y_5 = No$ |
| $\mathbf{x}_6$ | *No* | *Yes* | *No* | *Yes* | *Some* | *$$* | *Yes* | *Yes* | *Italian* | *0–10* | $y_6 = Yes$ |
| $\mathbf{x}_7$ | *No* | *Yes* | *No* | *No* | *None* | *$* | *Yes* | *No* | *Burger* | *0–10* | $y_7 = No$ |
| $\mathbf{x}_8$ | *No* | *No* | *No* | *Yes* | *Some* | *$$* | *Yes* | *Yes* | *Thai* | *0–10* | $y_8 = Yes$ |
| $\mathbf{x}_9$ | *No* | *Yes* | *Yes* | *No* | *Full* | *$* | *Yes* | *No* | *Burger* | *>60* | $y_9 = No$ |
| $\mathbf{x}_{10}$ | *Yes* | *Yes* | *Yes* | *Yes* | *Full* | *$$$* | *No* | *Yes* | *Italian* | *10–30* | $y_{10} = No$ |
| $\mathbf{x}_{11}$ | *No* | *No* | *No* | *No* | *None* | *$* | *No* | *No* | *Thai* | *0–10* | $y_{11} = No$ |
| $\mathbf{x}_{12}$ | *Yes* | *Yes* | *Yes* | *Yes* | *Full* | *$* | *No* | *No* | *Burger* | *30–60* | $y_{12} = Yes$ |




Attributes:

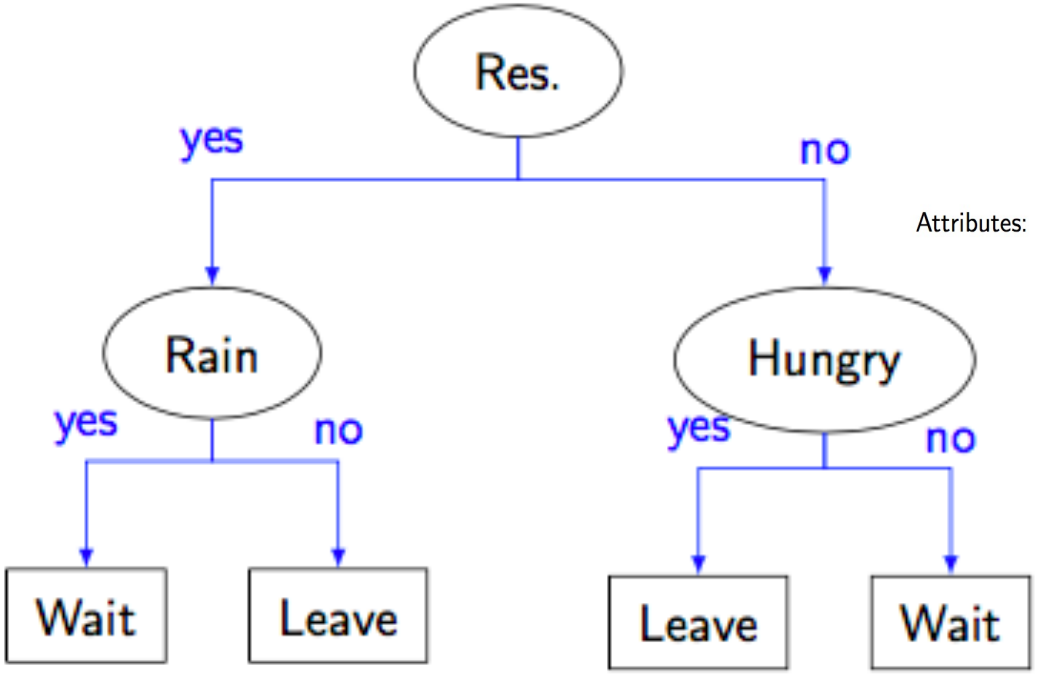
| | |
|---|---|
| 1. | Alternate: whether there is a suitable alternative restaurant nearby. |
| 2. | Bar: whether the restaurant has a comfortable bar area to wait in. |
| 3. | Fri/Sat: true on Fridays and Saturdays. |
| 4. | Hungry: whether we are hungry. |
| 5. | Patrons: how many people are in the restaurant (values are None, Some, and Full). |
| 6. | Price: the restaurant's price range ($, $$, $$$). |
| 7. | Raining: whether it is raining outside. |
| 8. | Reservation: whether we made a reservation. |
| 9. | Type: the kind of restaurant (French, Italian, Thai or Burger). |
| 10. | WaitEstimate: the wait estimated by the host (0-10 minutes, 10-30, 30-60, >60). |

[from: Russell & Norvig]

*Misclassification = ?*

# Misclassification Error

| Example | Input Attributes | | | | | | | | | | Goal |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | *Alt* | *Bar* | *Fri* | *Hun* | *Pat* | *Price* | *Rain* | *Res* | *Type* | *Est* | *WillWait* |
| $\mathbf{x}_1$ | *Yes* | *No* | *No* | *Yes* | *Some* | *$$$* | *No* | *Yes* | *French* | *0–10* | $y_1 = Yes$ |
| $\mathbf{x}_2$ | *Yes* | *No* | *No* | *Yes* | *Full* | *$* | *No* | *No* | *Thai* | *30–60* | $y_2 = No$ |
| $\mathbf{x}_3$ | *No* | *Yes* | *No* | *No* | *Some* | *$* | *No* | *No* | *Burger* | *0–10* | $y_3 = Yes$ |
| $\mathbf{x}_4$ | *Yes* | *No* | *Yes* | *Yes* | *Full* | *$* | *Yes* | *No* | *Thai* | *10–30* | $y_4 = Yes$ |
| $\mathbf{x}_5$ | *Yes* | *No* | *Yes* | *No* | *Full* | *$$$* | *No* | *Yes* | *French* | *>60* | $y_5 = No$ |
| $\mathbf{x}_6$ | *No* | *Yes* | *No* | *Yes* | *Some* | *$$* | *Yes* | *Yes* | *Italian* | *0–10* | $y_6 = Yes$ |
| $\mathbf{x}_7$ | *No* | *Yes* | *No* | *No* | *None* | *$* | *Yes* | *No* | *Burger* | *0–10* | $y_7 = No$ |
| $\mathbf{x}_8$ | *No* | *No* | *No* | *Yes* | *Some* | *$$* | *Yes* | *Yes* | *Thai* | *0–10* | $y_8 = Yes$ |
| $\mathbf{x}_9$ | *No* | *Yes* | *Yes* | *No* | *Full* | *$* | *Yes* | *No* | *Burger* | *>60* | $y_9 = No$ |
| $\mathbf{x}_{10}$ | *Yes* | *Yes* | *Yes* | *Yes* | *Full* | *$$$* | *No* | *Yes* | *Italian* | *10–30* | $y_{10} = No$ |
| $\mathbf{x}_{11}$ | *No* | *No* | *No* | *No* | *None* | *$* | *No* | *No* | *Thai* | *0–10* | $y_{11} = No$ |
| $\mathbf{x}_{12}$ | *Yes* | *Yes* | *Yes* | *Yes* | *Full* | *$* | *No* | *No* | *Burger* | *30–60* | $y_{12} = Yes$ |




Attributes:

| | |
|---|---|
| 1. | Alternate: whether there is a suitable alternative restaurant nearby. |
| 2. | Bar: whether the restaurant has a comfortable bar area to wait in. |
| 3. | Fri/Sat: true on Fridays and Saturdays. |
| 4. | Hungry: whether we are hungry. |
| 5. | Patrons: how many people are in the restaurant (values are None, Some, and Full). |
| 6. | Price: the restaurant's price range ($, $$, $$$). |
| 7. | Raining: whether it is raining outside. |
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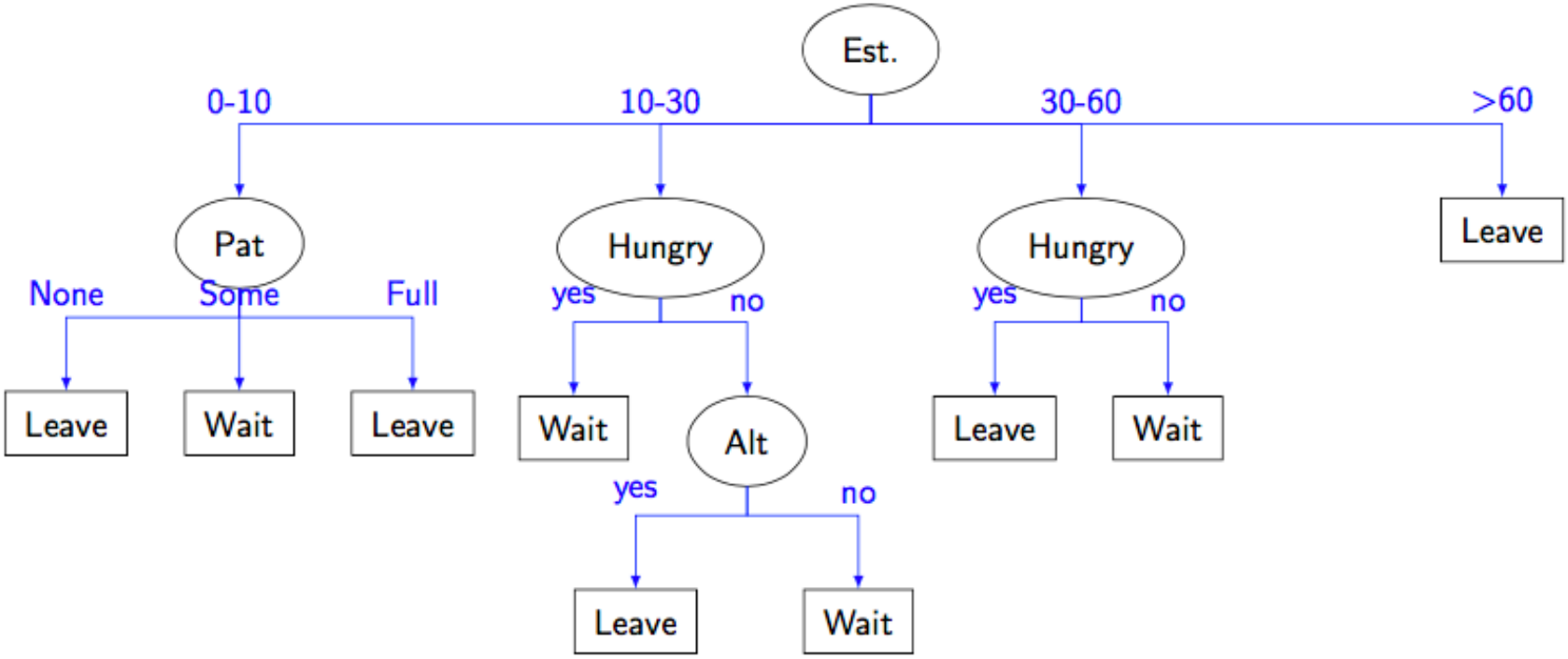
| 9. | Type: the kind of restaurant (French, Italian, Thai or Burger). |
| 10. | WaitEstimate: the wait estimated by the host (0-10 minutes, 10-30, 30-60, >60). |

[from: Russell & Norvig]

$(x_6\ x_8)$ $(x_1\ x_5\ x_{10})$ $(x_2\ x_4\ x_{12})$ $(x_3\ x_7\ x_9\ x_{11})$

*Misclassification = 5/12*

# Detailed Tree




*Misclassification: ?*

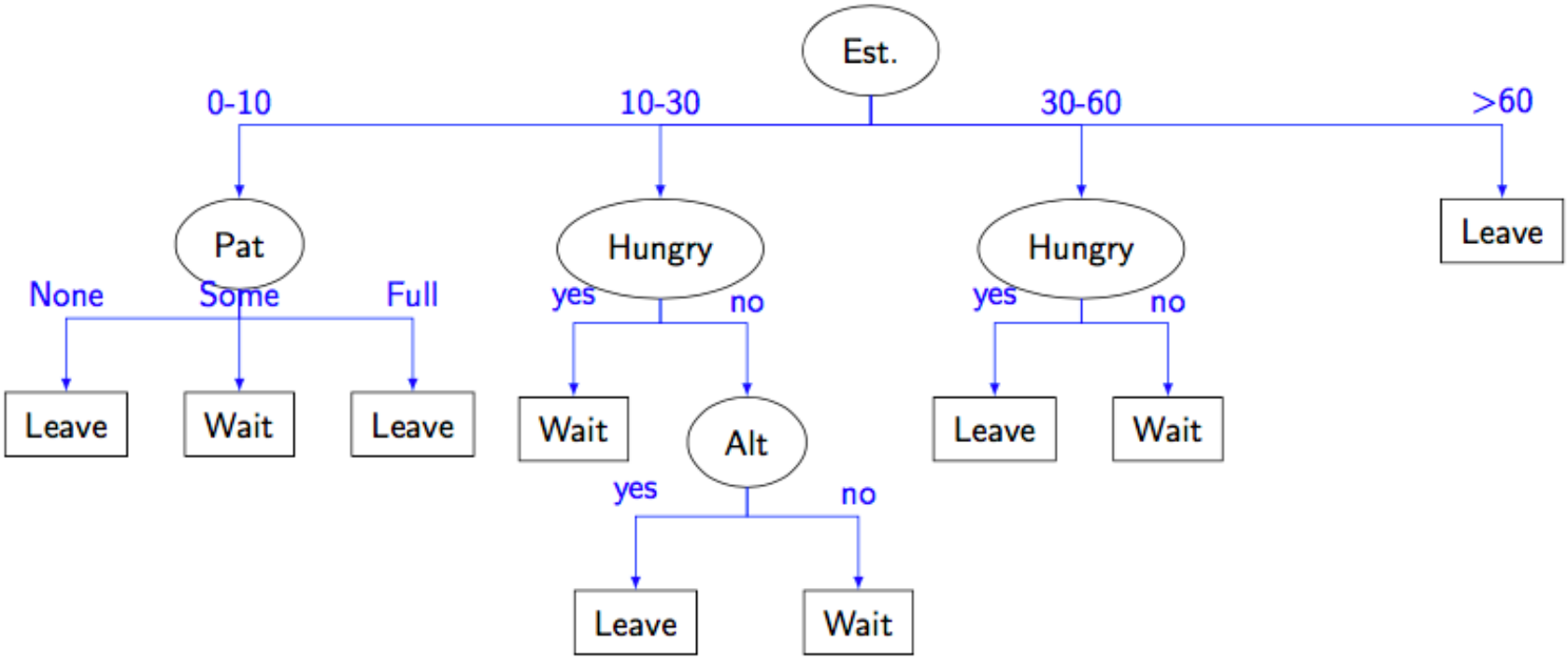
# Detailed Tree

| Example | Input Attributes | | | | | | | | | | Goal |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | *Alt* | *Bar* | *Fri* | *Hun* | *Pat* | *Price* | *Rain* | *Res* | *Type* | *Est* | *WillWait* |
| $\mathbf{x}_1$ | *Yes* | *No* | *No* | *Yes* | *Some* | *$$$* | *No* | *Yes* | *French* | *0–10* | $y_1 = Yes$ |
| $\mathbf{x}_2$ | *Yes* | *No* | *No* | *Yes* | *Full* | *$* | *No* | *No* | *Thai* | *30–60* | $y_2 = No$ |
| $\mathbf{x}_3$ | *No* | *Yes* | *No* | *No* | *Some* | *$* | *No* | *No* | *Burger* | *0–10* | $y_3 = Yes$ |
| $\mathbf{x}_4$ | *Yes* | *No* | *Yes* | *Yes* | *Full* | *$* | *Yes* | *No* | *Thai* | *10–30* | $y_4 = Yes$ |
| $\mathbf{x}_5$ | *Yes* | *No* | *Yes* | *No* | *Full* | *$$$* | *No* | *Yes* | *French* | *>60* | $y_5 = No$ |
| $\mathbf{x}_6$ | *No* | *Yes* | *No* | *Yes* | *Some* | *$$* | *Yes* | *Yes* | *Italian* | *0–10* | $y_6 = Yes$ |
| $\mathbf{x}_7$ | *No* | *Yes* | *No* | *No* | *None* | *$* | *Yes* | *No* | *Burger* | *0–10* | $y_7 = No$ |
| $\mathbf{x}_8$ | *No* | *No* | *No* | *Yes* | *Some* | *$$* | *Yes* | *Yes* | *Thai* | *0–10* | $y_8 = Yes$ |
| $\mathbf{x}_9$ | *No* | *Yes* | *Yes* | *No* | *Full* | *$* | *Yes* | *No* | *Burger* | *>60* | $y_9 = No$ |
| $\mathbf{x}_{10}$ | *Yes* | *Yes* | *Yes* | *Yes* | *Full* | *$$$* | *No* | *Yes* | *Italian* | *10–30* | $y_{10} = No$ |
| $\mathbf{x}_{11}$ | *No* | *No* | *No* | *No* | *None* | *$* | *No* | *No* | *Thai* | *0–10* | $y_{11} = No$ |
| $\mathbf{x}_{12}$ | *Yes* | *Yes* | *Yes* | *Yes* | *Full* | *$* | *No* | *No* | *Burger* | *30–60* | $y_{12} = Yes$ |

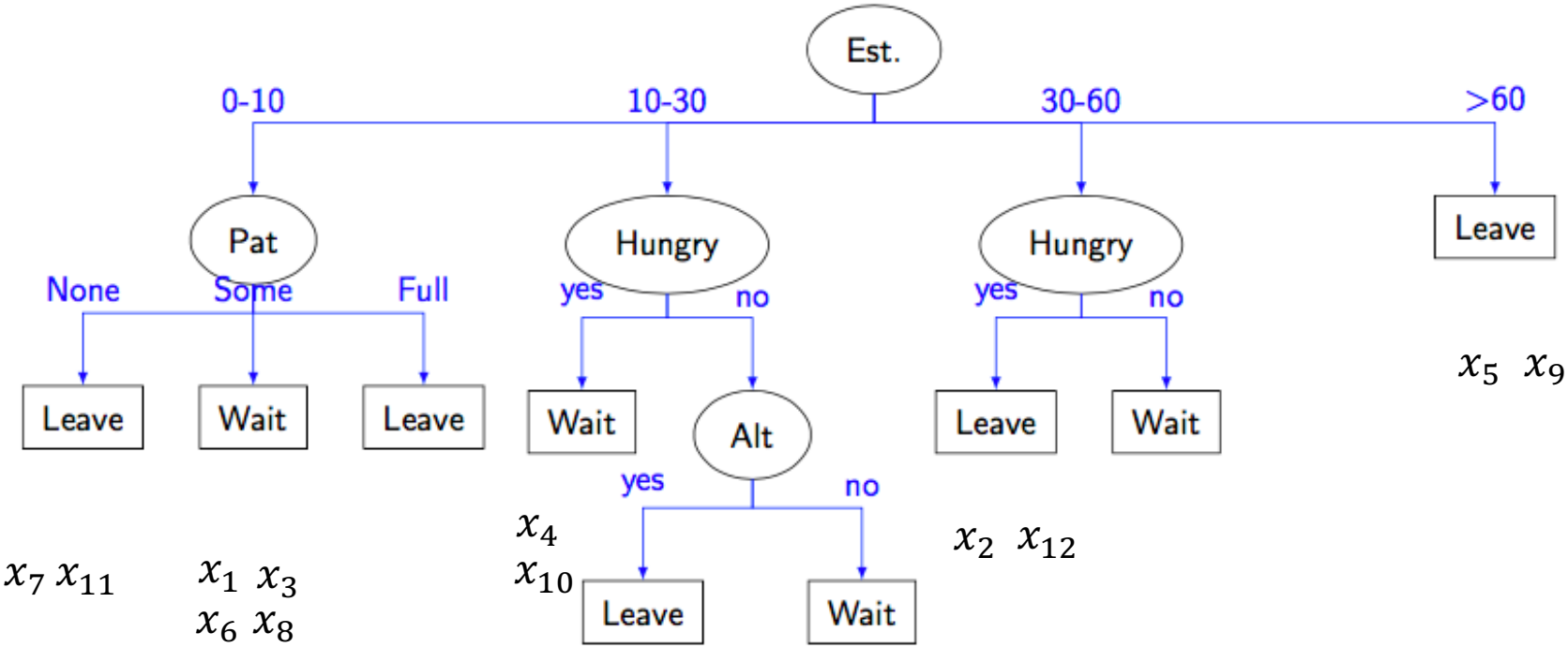
Attributes:

1. Alternate: whether there is a suitable alternative restaurant nearby.
2. Bar: whether the restaurant has a comfortable bar area to wait in.
3. Fri/Sat: true on Fridays and Saturdays.
4. Hungry: whether we are hungry.
5. Patrons: how many people are in the restaurant (values are None, Some, and Full).
6. Price: the restaurant's price range ($, $$, $$$).
7. Raining: whether it is raining outside.
8. Reservation: whether we made a reservation.
9. Type: the kind of restaurant (French, Italian, Thai or Burger).
10. WaitEstimate: the wait estimated by the host (0-10 minutes, 10-30, 30-60, >60).

[from: Russell & Norvig]



# Misclassification Error




*Misclassification: 2/12*

# Probabilistic Description

**Feature Space- A vector of categorical data**: $\mathcal{X}$.

**Outcome Class (categorical)**: $\mathcal{Y}$

**A Decision Rule**: $f : \mathcal{X} \to \mathcal{Y}$

**A Probabilistic Model**: $X \in \mathcal{X}$ and $Y \in \mathcal{Y}$ are Random Variables

**Error of a decision Rule** $f : R^*(f) = \mathbf{P}\{(X, Y) : f(X) \neq Y\}$

# Empirical Estimates

**Data Set:** $\{(x_i, y_i) \in \mathcal{X} \times \mathcal{Y}, i = 1, \ldots N\}$

**Empirical Error of a decision Rule** $f$ :

$$R(f) = \frac{1}{N} \sum_i^N \mathbf{I}(f(x_i) \neq y_i)$$

$\mathbf{I}(x) = 1 \; if \; x \neq 0, otherwise \; it \; is \; 0$

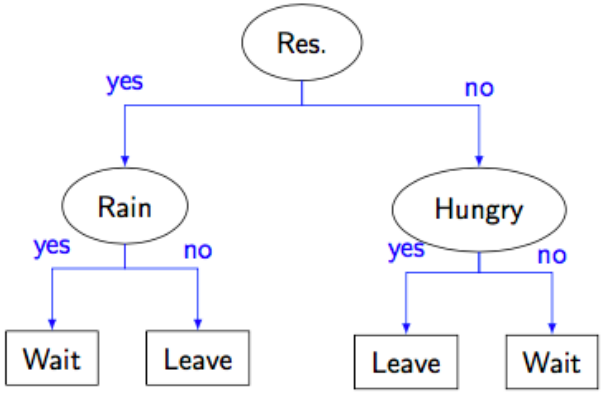
# Learning a DT from Data

- Split the data
  - Training Data
  - Cross Validation data
- Train the model using the “Training Data”
- Evaluate the performance on the “Cross Validation Data”
- 80-20 ratio generally works!

# Learning a DT from Data

- How do we learn:
  - Pick a feature
  - Split the data based on that feature
  - Define new classes
  - Repeat
  - Label the final clusters using majority rule
- The model does not have to split all the features at each level
- The model does not have to use all the features
- Combinatorial explosion
- Order matters!

# Trees as Decision Rules




- Define a sub-class as the outcome of a decision rule:

$$C = \{(x,\, y) \,|\, x(k) = v_k,\; k \text{ subset of all feature indices}\}$$

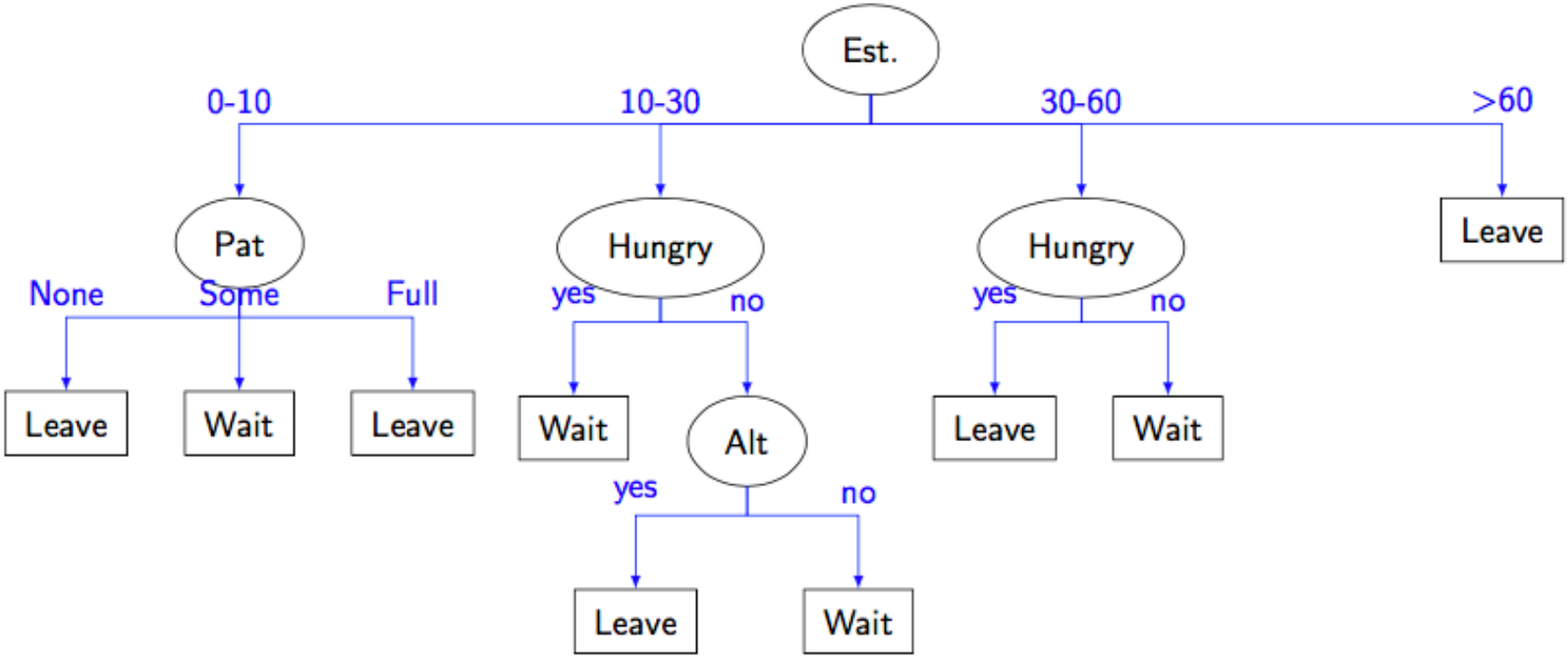
- The assignment mapping (label) is given by

$$f(C) = y, \quad y = \max_y \mathbf{P}(y|C)$$

- Assignment from Data

$$\text{Majority of } \{y_i,\, (x_i, y_i) \in C\}$$

# Back to Waiting for a Table




- How do we pick the feature at each level
- Greedy Algorithm
- Define the cost!

# Optimizing Splits

- Motivate Entropy as an 'Impurity' measure
- Greedy algorithm for splits minimize impurity?

# Introduce Entropy as a measure of uncertainty

- Review Entropy; conditional entropy
- Impurity measure
- Define greedy algorithm
- Show how it works

# *Questions and Break*

# *Part III: Information Gain*

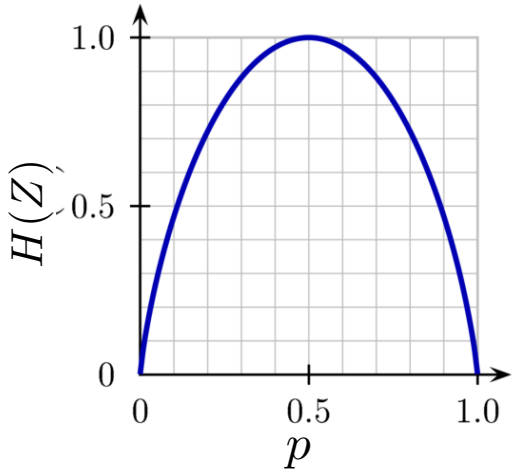
# Entropy

$Z \in \mathcal{Z}$ is a random variable with probability mass function $P(z)$

**Entropy**: $H(Z) = -\sum_{z \in \mathcal{Z}} P(z) \log P(z)$

*Example:* Coin flip with $P(head) = p$

**Entropy**: $-p \log p - (1-p) log(1-p)$




**Measures uncertainty in a random variable**

Maximum at $p$= 0.5

# Example: Conditional Entropy

*Example:* if $\mathcal{Z} = \mathcal{X} \times \mathcal{Y}$, then

$P(X,Y) =$

| | $Y=0$ | $Y=1$ |
|---|---|---|
| $X=0$ | 1/8 | 3/8 |
| $X=1$ | 3/8 | 1/8 |

$$H(Y) = H(1/2, 1/2) = 1$$

Verify!

If X is observed, do we reduce the entropy of Y?

# Example: Conditional Entropy

$P(X,Y) =$

| | $Y=0$ | $Y=1$ |
|---|---|---|
| $X=0$ | 1/8 | 3/8 |
| $X=1$ | 3/8 | 1/8 |

**Conditional Entropy**:

$$H(Y|X=0) = -\sum_{y\in\mathcal{Y}} P(y|x=0)\log P(y|x=0) = -1/4\log 1/4 - 3/4\log 3/4$$

$$H(Y|X=1) = -\sum_{y\in\mathcal{Y}} P(y|x=1)\log P(y|x=1) = -3/4\log 3/4 - 1/4\log 1/4$$

$$H(Y|X) = -\sum_{x\in\mathcal{X}} P(x)H(Y|X=x) = 1/2H(Y|X=0) + 1/2H(Y|X=1)$$

$$= .812$$

**Observe**: $H(Y|X) \leq H(Y)$

# Information Gain

**Information Gain**: $IG(Y|X) = H(Y) - H(Y|X)$

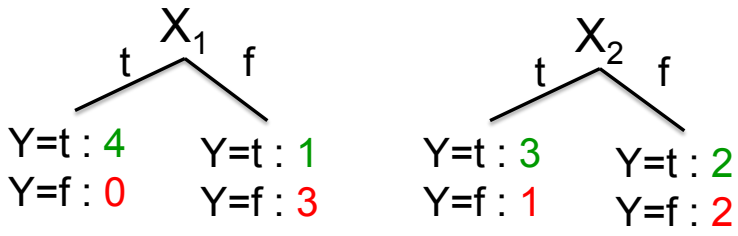
$$\text{If } X \perp Y \text{ then } IG = 0 \; X \text{ is not informative}$$

$$\text{If } IG(Y|X) = H(Y) \text{ then } Y \text{ is most informative}$$

# Simple Illustration

## Splitting: choosing a good attribute

Would we prefer to split on $X_1$ or $X_2$?

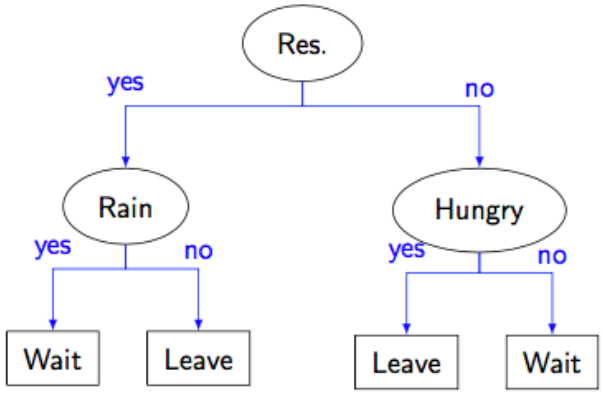



| $X_1$ | $X_2$ | Y |
|---|---|---|
| T | T | T |
| T | F | T |
| T | T | T |
| T | F | T |
| F | T | T |
| F | F | F |
| F | T | F |
| F | F | F |

Idea: use counts at leaves to define probability distributions, so we can measure uncertainty!

# Greedy Algorithm

- Start with the complete data s
- Pick a feature (formally:$X(m)$)




- Describe the data based on this feature

$$\{(x_i(m),\, y_i),\, i = 1, \ldots N)\}$$

- Split the outcome data based on the classes

$$S_1 = \{(y_i \,|\, x_i(m) = 0\}$$

$$S_2 = \{(y_i \,|\, x_i(m) = 1\}$$

# Empirical Computation of Entropy

- Empirically compute the conditional entropy

$$H(Y|X(m))$$

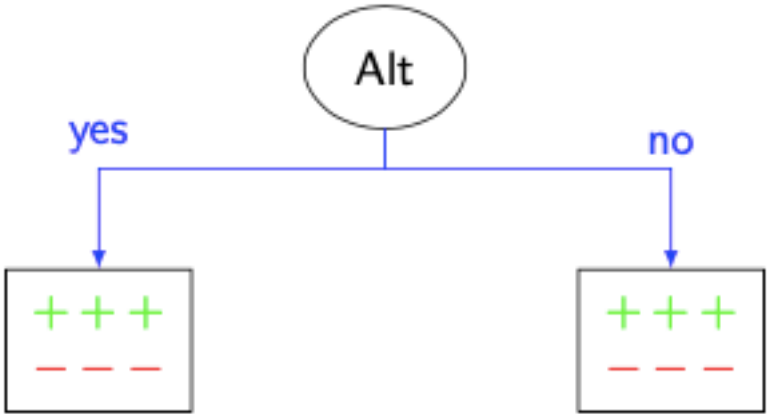
- Estimate:

$$\mathbf{P}(S_1)H(S_1) + \mathbf{P}(S_2)H(S_2)$$

- Pick $m$ to minimize this (maximize information gain)

Feature splitting = Conditioning

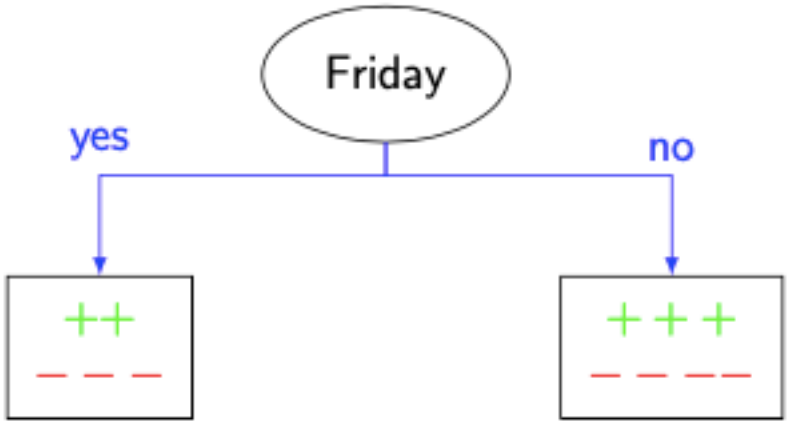
# Splitting on Alt




Loss: 1.

$$IG = 0$$

# Back to Example: Splitting on Friday




Loss: 0.97928

# Computation

- $H(S_1) = -\frac{2}{5}\log\frac{2}{5} - \frac{3}{5}\log\frac{3}{5} = .97$
- $H(S_2) = -\frac{3}{7}\log\frac{3}{7} - \frac{4}{7}\log\frac{4}{7} = .99$

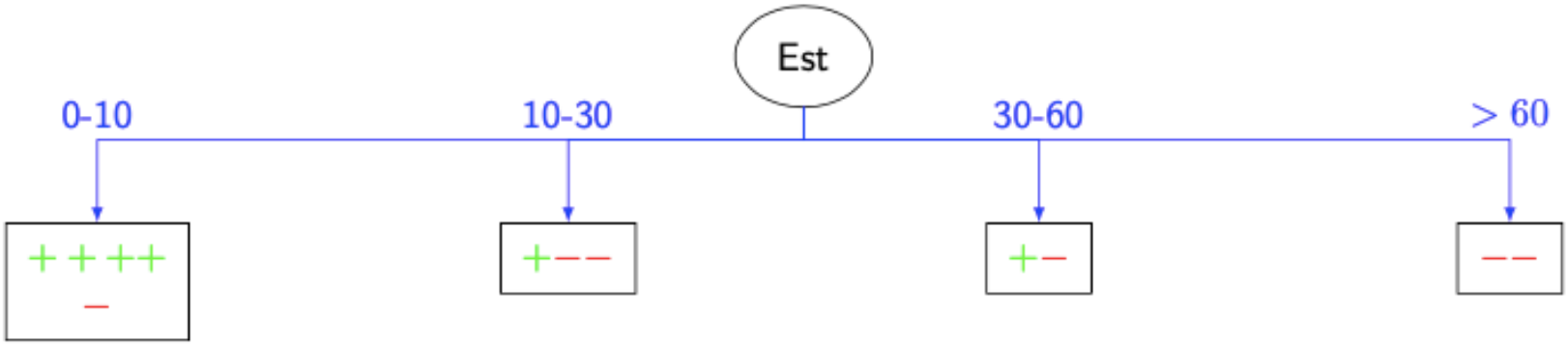
- Entropy of Split: .

$$.97 * \frac{5}{12} + .99 * \frac{7}{12} = .979$$

- $IG = H(Y) - .979 = 1 - .979 = .021$

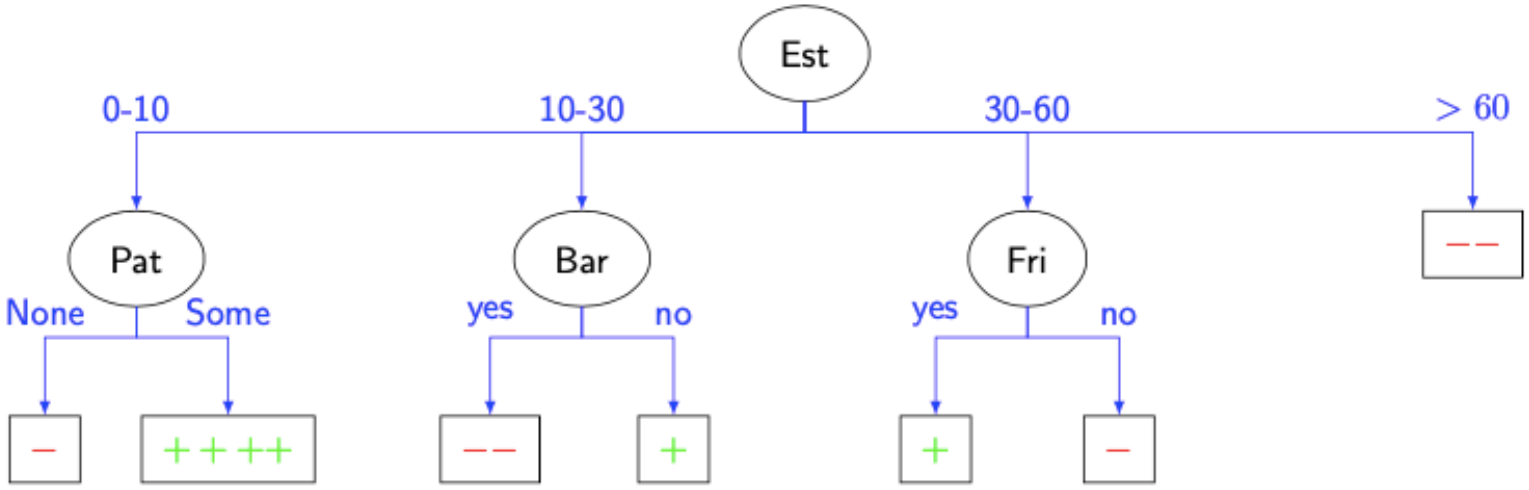
- *Not a great split.*

# Back to Example: Splitting on Est




Loss: 0.69704

# Back to Example: Multiple Splits




Loss: 0.0

# Gini Index

Replace Entropy with $\sum_x p(x)\left(1-p(x)\right) = 1 - \sum_x p(x)^2$

Gini Index: Favors larger partitions

# Summary

- Interpreting a decision tree

- Learning a Decision Tree
  - ✓ Mis-classification error
  - ✓ Probabilistic interpretation
  - ✓ Formal definition of a Decision Tree

- Impurity minimization
  - ✓ Entropy minimization
  - ✓ Information Gain

# *Questions*

# Thank You