

# Recommendation Systems

## Part I

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# Outline — Part I

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Introduction, simple methods

Module 1: background

Recommendation systems: why and what?

Example datasets

Module 2: problem statement

Recommendation systems: a prediction problem

Model: from caricature to extremely complex

Module 3: simple solutions

Solution I: averaging

Solution II: content-based

# Module 1: background

# Recommendations

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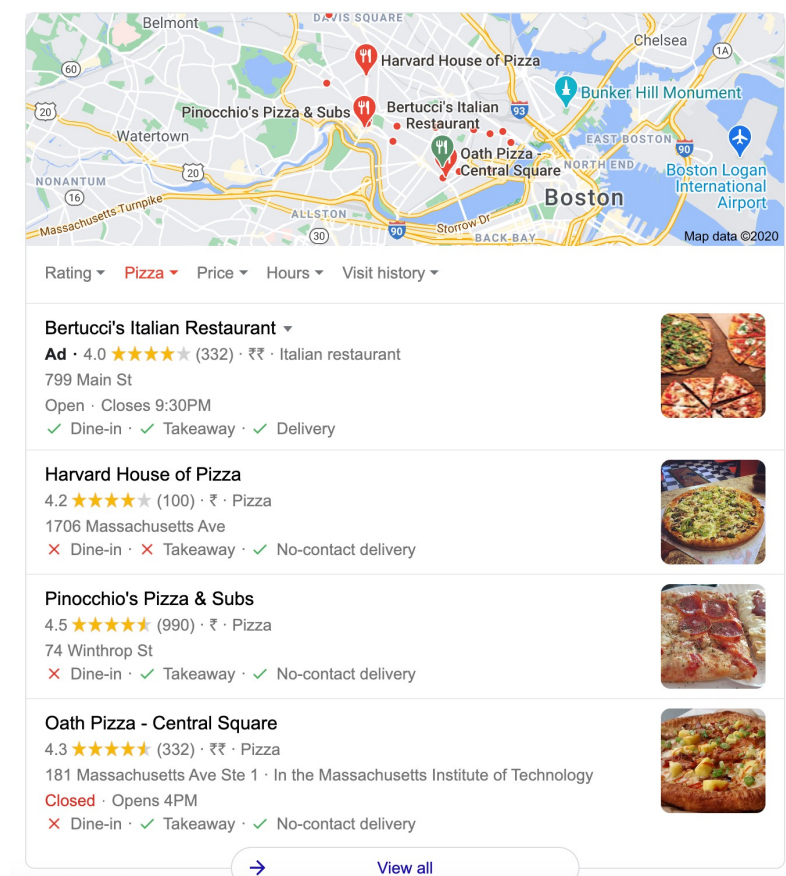
What is it, really?

What food should you eat today?

# Recommendations

Why not just search?

Want to eat Pizza, search for Pizza places near you



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But

Which of these *many* options  
should you be *recommended*?

Which of these *many* be  
*advertised*?

# Recommendations

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What is it, really?

What food should you eat today?

Which activities should you plan for the upcoming weekend?

Where should you plan your holidays?

Whom should you date (and marry)?

What are the professional connections of your interest?

Which advertisements should you be subjected to?

What content on YouTube or music on Spotify will fancy you?

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# Recommendations

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May be *data* can help. What data?

# Recommendations

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May be *data* can help. What data?

Example: Yelp data

Businesses: attributes (locations, category), hours

Users: attributes, friends

Reviews: rating, description, time

Check-ins: time

Tip

**Exercise:** go to link below, explore data and reproduce statistics reported

<https://www.kaggle.com/jagangupta/what-s-in-a-review-yelp-ratings-eda>



# Explore: Yelp Data

Businesses: 209393 (~200k) in total

	address	attributes	business_id	categories	city	hours	is_open	latitude	longitude	name	postal_code	review_count	stars	state
0	10913 Bailey Rd	{u'BusinessParking': u'garage': False, 'stre...	f9NumwFMBDn751xgFiRbNA	Active Life, Gun/Rifle Ranges, Guns & Ammo, Sh...	Cornelius	{u'Monday': u'10:0-18:0', u'Tuesday': u'11:0-2...	1	35.462724	-80.852612	The Range At Lake Norman	28031	36	3.5	NC
1	8880 E Via Linda, Ste 107	{u'GoodForKids': u'True', u'ByAppointmentOnly': ...	YzvJg0SayhoZgCljUJRF9Q	Health & Medical, Fitness & Instruction, Yoga,...	Scottsdale	None	1	33.569404	-111.890264	Carlos Santo, NMD	85258	4	5.0	AZ
2	3554 Rue Notre-Dame O	None	XNoUzKckATkOD1hP6vghZg	Pets, Pet Services, Pet Groomers	Montreal	None	1	45.479984	-73.580070	Felinus	H4C 1P4	5	5.0	QC
3	1015 Sharp Cir	{u'DogsAllowed': u'True', u'BusinessParking': ...	6OAZjbxqM5ol29BuHsil3w	Hardware Stores, Home Services, Building Suppl...	North Las Vegas	{u'Friday': u'7:0-16:0', u'Tuesday': u'7:0-16:...	0	36.219728	-115.127725	Nevada House of Hose	89030	3	2.5	NV
4	4827 E Downing Cir	{u'BusinessAcceptsCreditCards': u'True', u'ByA...	51M2Kk903DFYI6gnB5I6SQ	Home Services, Plumbing, Electricians, Handyma...	Mesa	{u'Friday': u'9:0-16:0', u'Tuesday': u'9:0-16:...	1	33.428065	-111.726648	USE MY GUY SERVICES LLC	85205	26	4.5	AZ

Data URL:

<https://www.kaggle.com/jagangupta/what-s-in-a-review-yelp-ratings-eda>

# Explore: Yelp Data

Users: 1968703 (~2M) in total

	average_stars	compliment_cool	compliment_cute	compliment_funny	compliment_hot	compliment_list	compliment_more	compliment_note
0	3.57	22	0	22	3	1	2	11
1	3.84	63	2	63	36	1	4	33
2	3.44	17	1	17	9	0	6	3
3	3.08	7	0	7	2	0	1	7
4	4.37	31	1	31	8	1	9	22

	elite	fans	friends	funny	name	review_count	useful	user_id	yelping_since
		14	oeMvJh94PiGQnx_6GIndPQ, wm1z1PaJKvHgSDRKfwhfDg...	225	Rafael	553	628	ntlvfPzc8eglqvk92iDIAw	2007-07-06 03:27:11
	2008,2009,2010,2011,2012,2013	27	ly7EnE8leJmyqyePVYFlug, pRIR63iDytsnnniPb3AOug...	316	Michelle	564	790	FOBRPIBHa3WPHFB5qYDIVg	2008-04-28 01:29:25
	2010	5	Uwlk0txjQBPw_JhHsQnyeg, Ybxr1tSCkv3IYA0I1qmnPQ...	125	Martin	60	151	zZUnPeh2hEp0WydbAZEEOg	2008-08-28 23:40:05
	2009	6	iog3Nyg1i4jeumiTVG_BSA, M92xWY2Vr9w0xoH8bPplfQ...	160	John	206	233	QaELAmRcDc5TfJEylaaP8g	2008-09-20 00:08:14
	2009,2010,2011,2012,2014,2015,2016,2017,2018	78	3W3ZMStjojCUirKEqAwGNw, eTlbuu23j9tOgmIa9POyLQ...	400	Anne	485	1265	xvu8G900tezTzbbfqmTKvA	2008-08-09 00:30:27

Data URL:

<https://www.kaggle.com/jagangupta/what-s-in-a-review-yelp-ratings-eda>

# Explore: Yelp Data

Reviews: 8021122 (~8M) in total

	business_id	cool	date	funny	review_id	stars	text	useful	user_id
0	-MhfebM0QIsKt87iDN-FNw	0	2015-04-15 05:21:16	0	xQY8N_XvtGbearJ5X4QryQ	2.0	As someone who has worked with many museums, I...	5	OwjRMXRC0KyPrllcjaXeFQ
1	lbrU8StCq3yDfr-QMnGrmQ	0	2013-12-07 03:16:52	1	UmFMZ8PyXZTY2QcwzsfQYA	1.0	I am actually horrified this place is still in...	1	nIJD_7ZXHq-FX8byPMOkMQ
2	HQI28KMwrEKHqhFrrDqVNQ	0	2015-12-05 03:18:11	0	LG2ZaYiOgpr2DK_90pYjNw	5.0	I love Deagan's. I do. I really do. The atmoosp...	1	V34qejxNsCbcgD8C0HVk-Q
3	5JxlZaqCnk1MnbgRirs40Q	0	2011-05-27 05:30:52	0	i6g_oA9Yf9Y31qt0wibXpw	1.0	Dismal, lukewarm, defrosted- tasting "TexMex" g...	0	ofKDkJKXSKZXu5xJNGiiBQ
4	IS4cv902ykd8wj1TR0N3-A	0	2017-01-14 21:56:57	0	6TdNDKywdbjoTkizeMce8A	4.0	Oh happy day, finally have a Canes near my cas...	0	UgMW8bLE0QMJDckQ1Ax5Mg

Data URL:

<https://www.kaggle.com/jagangupta/what-s-in-a-review-yelp-ratings-eda>

# Explore: Yelp Data

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Check-ins: 175187 (~175k) in total

	business_id	date
0	--1UhMGODdWsrMastO9DZw	2016-04-26 19:49:16, 2016-08-30 18:36:57, 2016...
1	--6MefnULPED_I942VcFNA	2011-06-04 18:22:23, 2011-07-23 23:51:33, 2012...
2	--7zmmkVg-IMGaXbuVd0SQ	2014-12-29 19:25:50, 2015-01-17 01:49:14, 2015...
3	--8LPVSo5i0Oo61X01sV9A	2016-07-08 16:43:30
4	--9QQLMTbFzLJ_oT-ON3Xw	2010-06-26 17:39:07, 2010-08-01 20:06:21, 2010...

Data URL:

<https://www.kaggle.com/jagangupta/what-s-in-a-review-yelp-ratings-eda>

# Explore: Yelp Data

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Tip: 1320761 (~1.3M) total

	business_id	compliment_count	date	text	user_id
0	UYX5zL_Xj9WEc_Wp-FrqHw	0	2013-11-26 18:20:08	Here for a quick mtg	hf27xTME3EiCp6NL6VtWZQ
1	Ch3HkwQYv1YKw_FO06vBWA	0	2014-06-15 22:26:45	Cucumber strawberry refresher	uEvusDwoSymbJJ0auR3muQ
2	rDoT-MgxGRiYqCmi0bG10g	0	2016-07-18 22:03:42	Very nice good service good food	AY-lalws3S7YXNI_f_D6rQ
3	OHXnDV01gLokiX1ELaQufA	0	2014-06-06 01:10:34	It's a small place. The staff is friendly.	Ue_7yUIkEbX4AhnYdUfL7g
4	GMrwDXRIAZU2zj5nH6I4vQ	0	2011-04-08 18:12:01	8 sandwiches, \$24 total...what a bargain!!! An...	LltbT_fUMqZ-ZJP-vJ84IQ

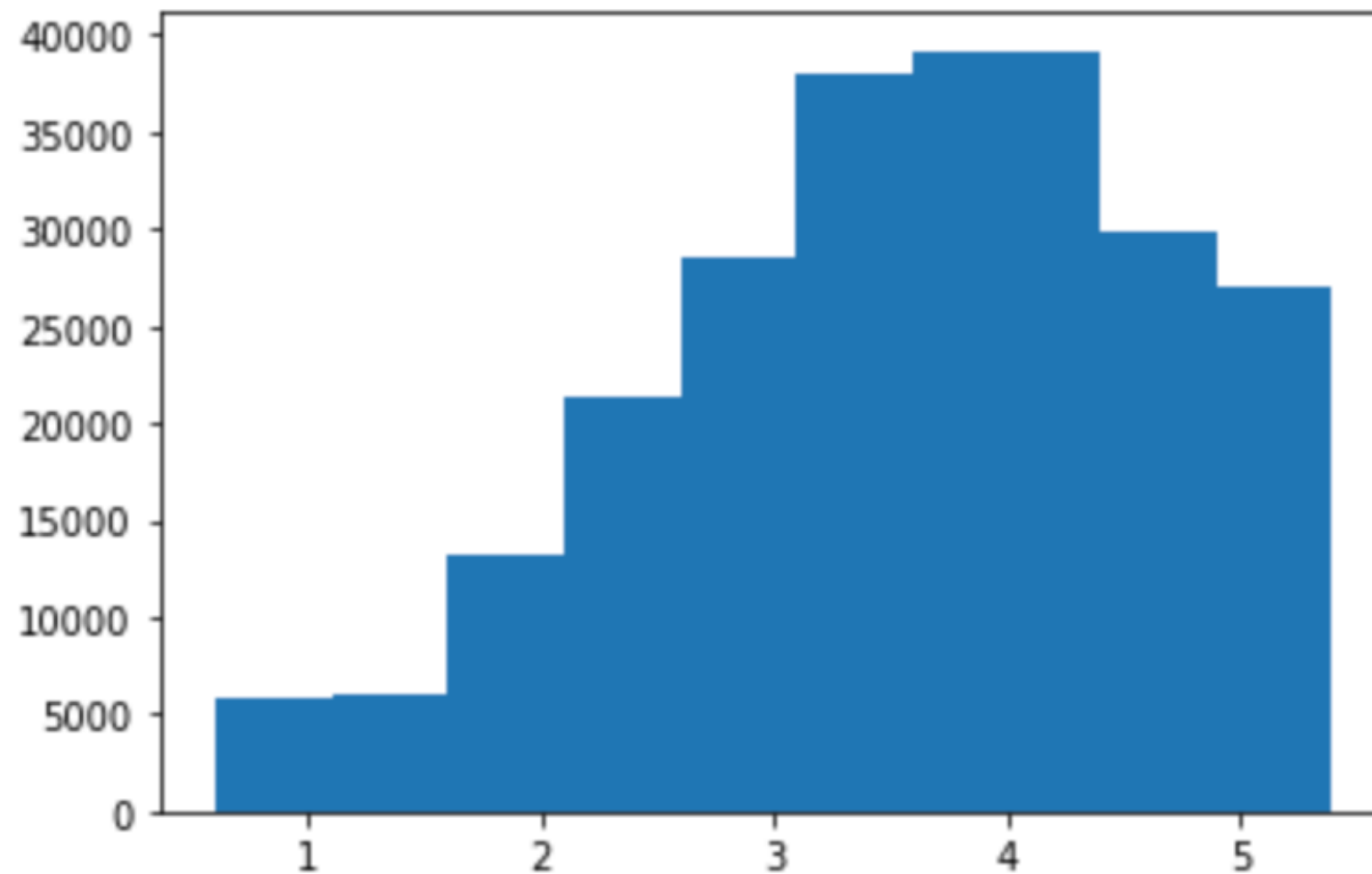
Data URL:

<https://www.kaggle.com/jagangupta/what-s-in-a-review-yelp-ratings-eda>

## Explore: Yelp Data

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(Aggregate) Star Rating Distribution



Data URL:

<https://www.kaggle.com/jagangupta/what-s-in-a-review-yelp-ratings-eda>

## Explore: Yelp Data

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What fraction of reviews are known?

Users =  $\sim 2\text{M}$

Businesses =  $\sim 200\text{k}$

Total possible reviews =  $\sim 2\text{M} \times \sim 200\text{k} = \sim 0.4\text{T}$

Known reviews =  $\sim 8\text{M}$

Fraction known =  $\sim 8\text{M} / 0.4\text{T} = 2 \times 10^{-5}$

i.e. 2 in every 100k reviews is known, rest are *unknown*

Finding these *unknown* reviews is the primary goal of Rec Sys

Data URL:

<https://www.kaggle.com/jagangupta/what-s-in-a-review-yelp-ratings-eda>

# Explore: MovieLens Data

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## MovieLens Data

Movies: attributes including title, release date, genre, actors, director

Users: demographics including age, gender, occupation, zip code

Reviews: ratings, timestamp

**Exercise:** go to link below, explore data and reproduce statistics reported

<https://grouplens.org/datasets/movielens/100k>



# Explore: MovieLens Data

---

Reviews: 100000 (100k) in total

	userId	movieId	rating	timestamp
0	1	31	2.5	1260759144
1	1	1029	3.0	1260759179
2	1	1061	3.0	1260759182
3	1	1129	2.0	1260759185
4	1	1172	4.0	1260759205

Data URL:

<https://grouplens.org/datasets/movielens/100k>

# Explore: MovieLens Data

---

Users: 1682 (~1.7k) in total

```
user id | age | gender | occupation | zip code
```

```
1|24|M|technician|85711
2|53|F|other|94043
3|23|M|writer|32067
4|24|M|technician|43537
5|33|F|other|15213
6|42|M|executive|98101
7|57|M|administrator|91344
8|36|M|administrator|05201
9|29|M|student|01002
10|53|M|lawyer|90703
```

Data URL:

<https://grouplens.org/datasets/movielens/100k>

# Explore: MovieLens Data

Movies: 943 (~1k) in total

movie id	movie title	release date	video release date	
IMDb URL	unknown	Action	Adventure	Animation
Children's	Comedy	Crime	Documentary	Drama
Film-Noir	Horror	Musical	Mystery	Romance
Thriller	War	Western		Sci-Fi

[illegible]

## Data URL:

<https://grouplens.org/datasets/movielens/100k>

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# Explore: MovieLens Data

---

Top rated movies

```
rating[ 'movieId' ].value_counts()
```

356	341
296	324
318	311
593	304
260	291
480	274
2571	259
1	247
527	244
589	237
1196	234

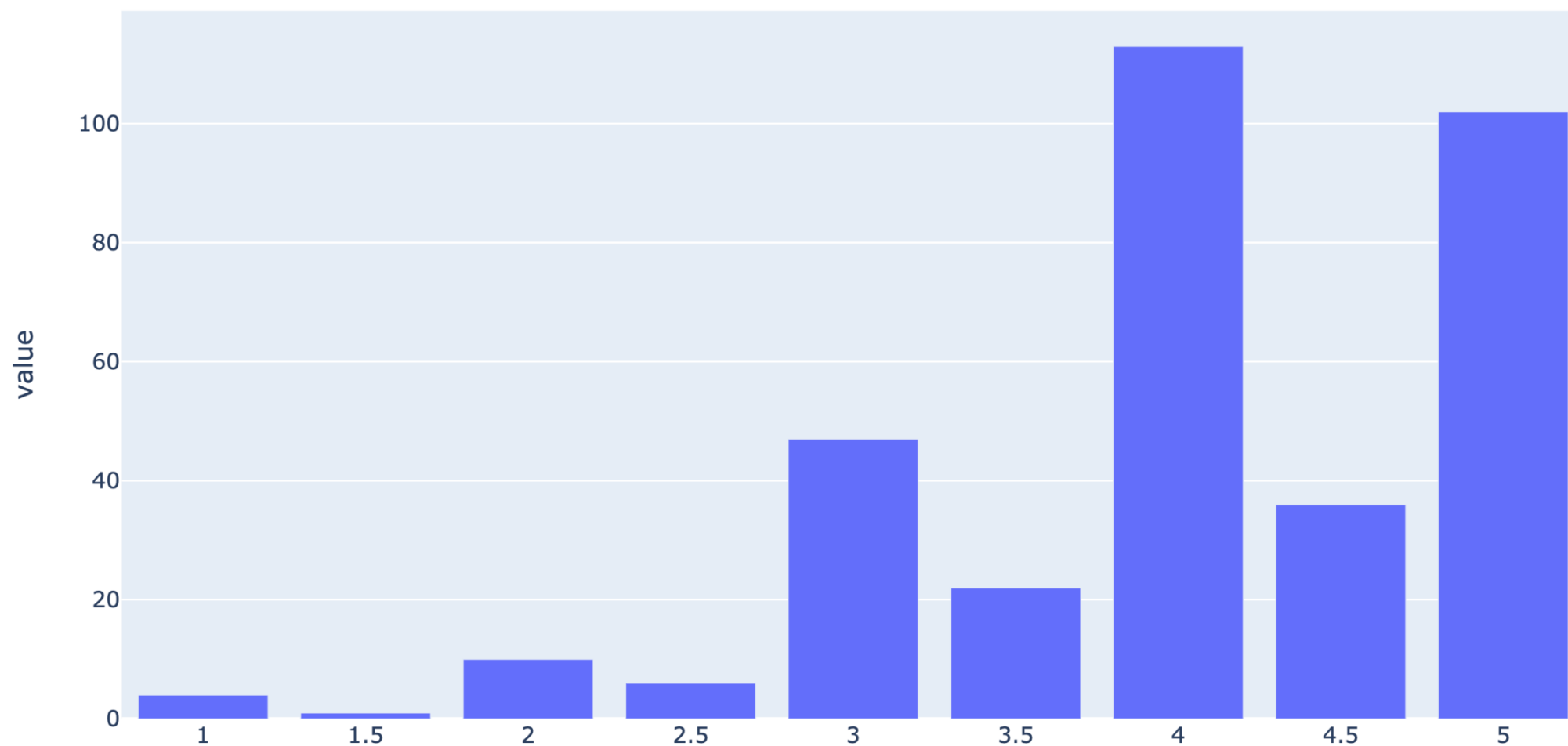
Data URL:

<https://grouplens.org/datasets/movielens/100k>

# Explore: MovieLens Data

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Distribution of top-rated movie (356)



Data URL:

<https://grouplens.org/datasets/movielens/100k>

## Explore: MovieLens Data

---

What fraction of reviews are known?

Users =  $\sim 1.7k$

Movies =  $\sim 1k$

Total possible reviews =  $\sim 1.7k \times \sim 1k = \sim 1.7M$

Known reviews =  $\sim 100k$

Fraction known =  $\sim 100k / 1.7M = \sim 0.058$  or  $\sim 6\%$

i.e. 6 in every 100 reviews is known, rest are *unknown*

Finding these *unknown* reviews is the primary goal of Rec Sys

Data URL:

<https://www.kaggle.com/jagangupta/what-s-in-a-review-yelp-ratings-eda>

## Module 2: problem statement

## Recommendation: Problem statement

---

What does Yelp want to do?

Provide list of businesses

that satisfy your search criteria

*ordered* as per

user's interest or preference list at that moment

advertisement revenue considerations

so that ultimately *matching* happens within *few* clicks

that provides instant gratification to both user and Yelp

and continues bringing user back to Yelp



## Recommendation: Problem statement

---

What about?

Linked In: connect people professionally

Facebook: filter friends' feed

Poshmark, Etsy: organize content of display

Amazon, Retail: display products and sellers

Tinder, Match: find suitable partner

Netflix, YouTube, Spotify: entertainment of interest

o o o o

So that ultimately *matching* happens within *few* clicks

providing instant gratification to *user, provider* and *platform*

and *users* and *providers* continue engaging with the *platform*

# Recommendation: Problem statement

---

Prediction problem

what is the likelihood of *matching*

user

provider

at a given time

in a given *context*

Accurate solution to this prediction problem

provides essential ingredient for connecting users, providers on the platform

while respecting interests of users, providers and the platform

# Recommendation: Problem statement

---

Prediction problem

N users

M providers or items (or other users)

Given user  $i$  *and* item  $j$

find the likelihood of  $i$  matching with  $j$

$$L_{ij} = ?$$

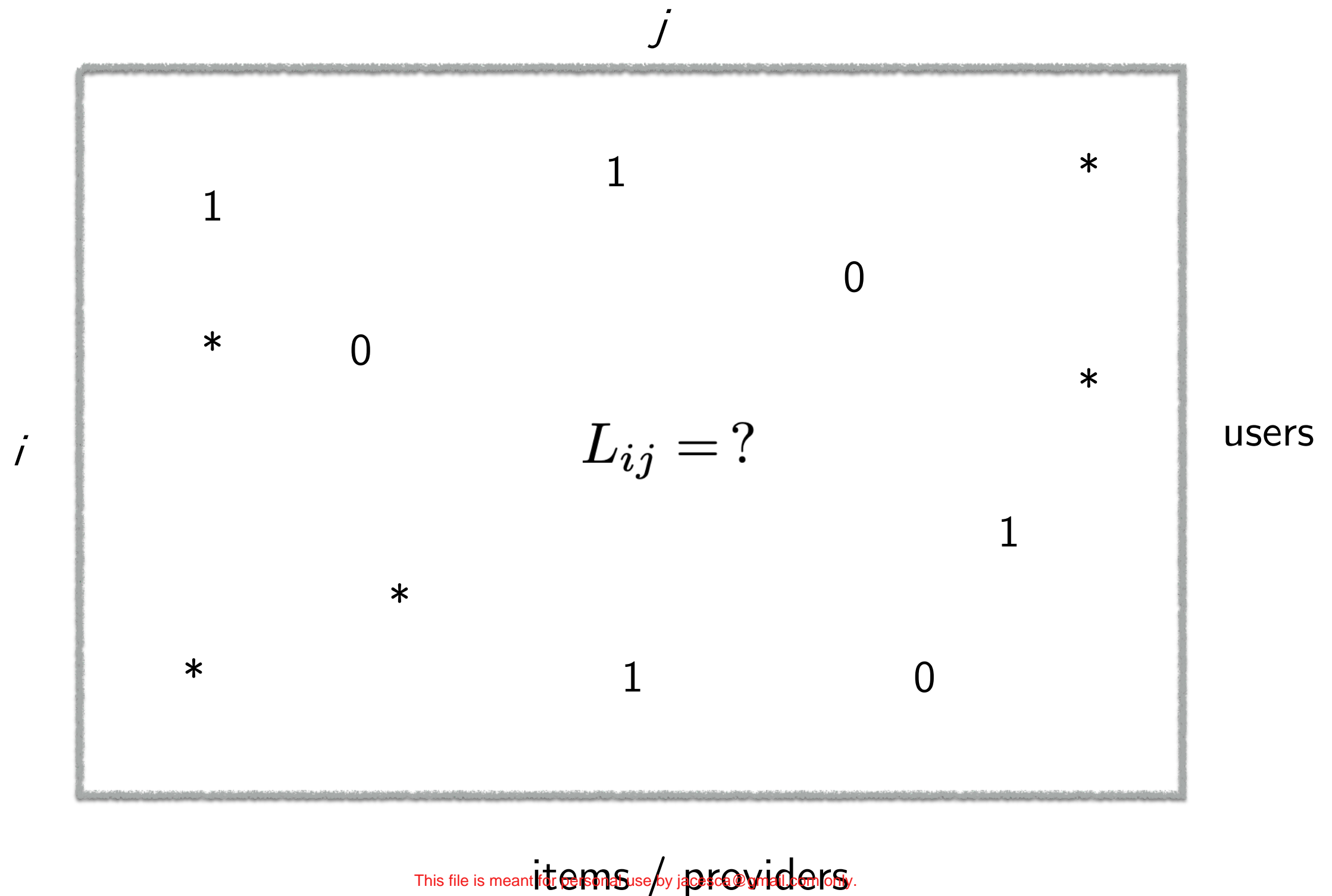
Using data

Historical matchings or preferences

# Recommendation: Problem statement

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Prediction problem: complete the matrix



# Recommendation: Problem statement

Complete the matrix

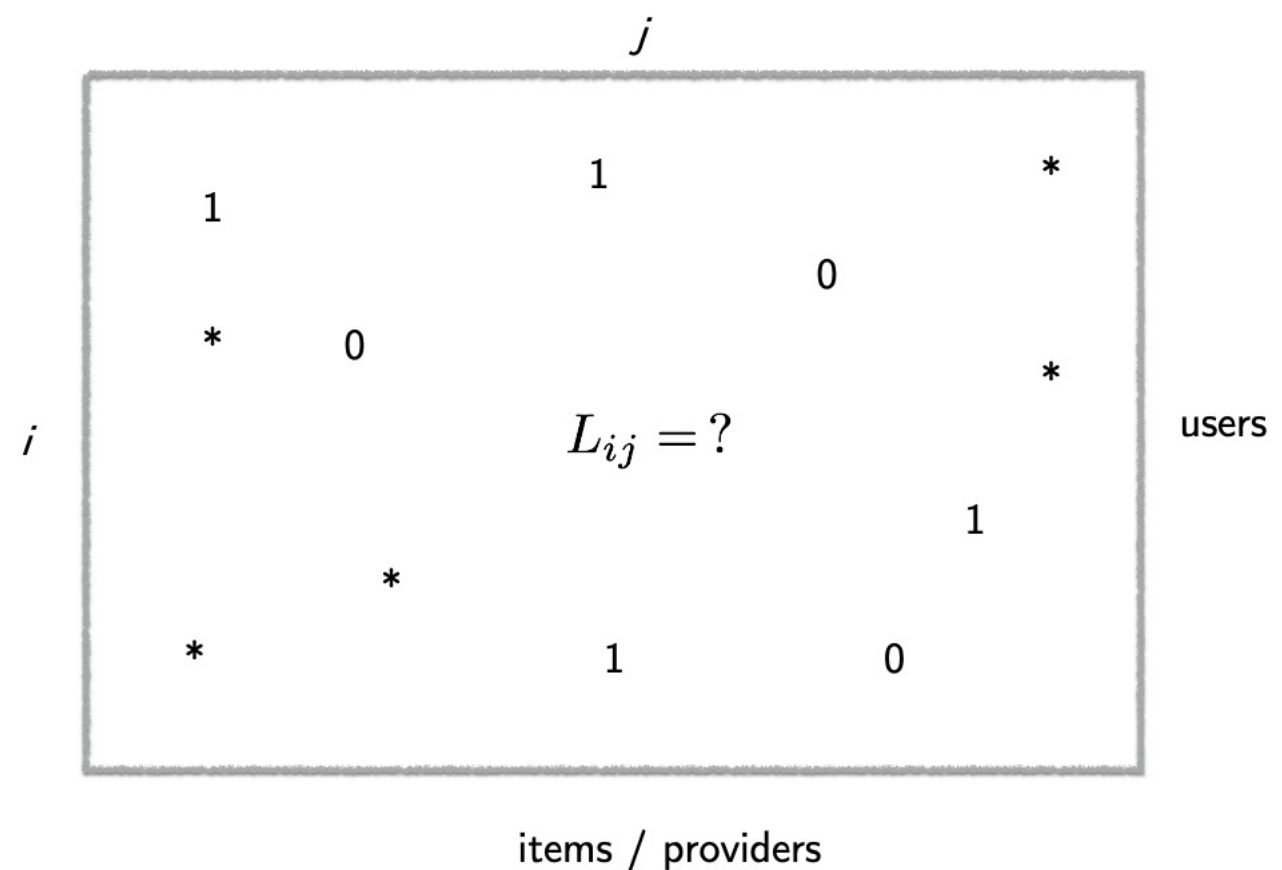
Observations:  $Y_{ij}$  over  $i$  in users,  $j$  in items

If  $(i, j)$  is observed

$$E[Y_{ij}] = L_{ij}$$

If  $(i, j)$  is *not* observed

$$Y_{ij} = \star \text{ or } ?$$



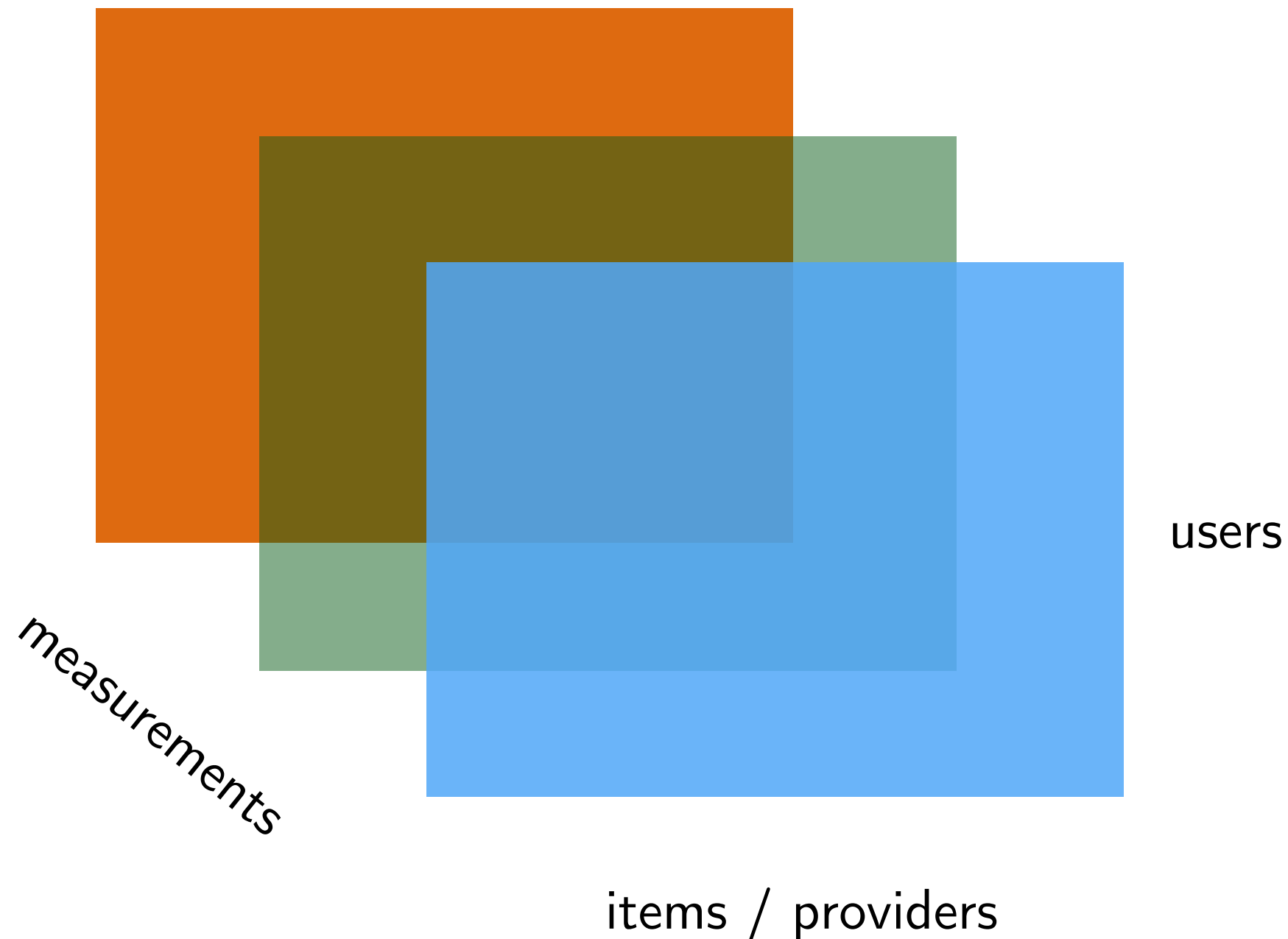
Goal: produce estimation  $\hat{L}_{ij}$  for all  $i, j$

so that  $\hat{L}_{ij} \approx L_{ij}$  for all  $i, j$

# Recommendation: Problem statement

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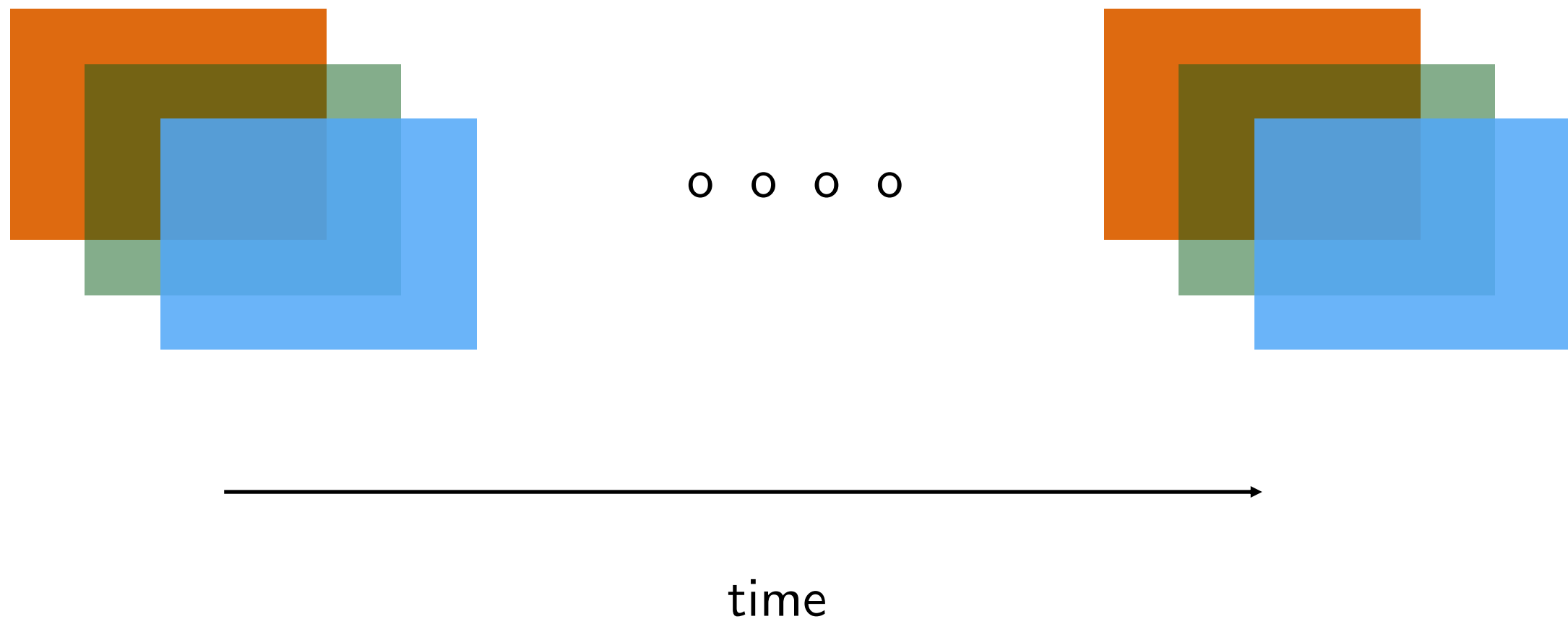
Prediction problem: complete the tensor



# Recommendation: Problem statement

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Prediction problem: complete the time varying tensor



# Recommendation: Problem statement

---

Challenges that we will not discuss

- user provided data

  - not generated at random

  - can be strategic

  - can be driven by innate preference

- provider / item

  - can be systematically manipulated

  - convenient location

  - strategically modified content



# Recommendation: Problem statement

---

Challenges that we will not discuss

Side effects of recommendation systems

Information bubble

Feedback loops

Too powerful platforms

Regulation of platforms

Interplay with recommendation systems

## Module 3: simple solutions

## Recommendation: Problem statement

We will start with simple problem statement: complete the matrix

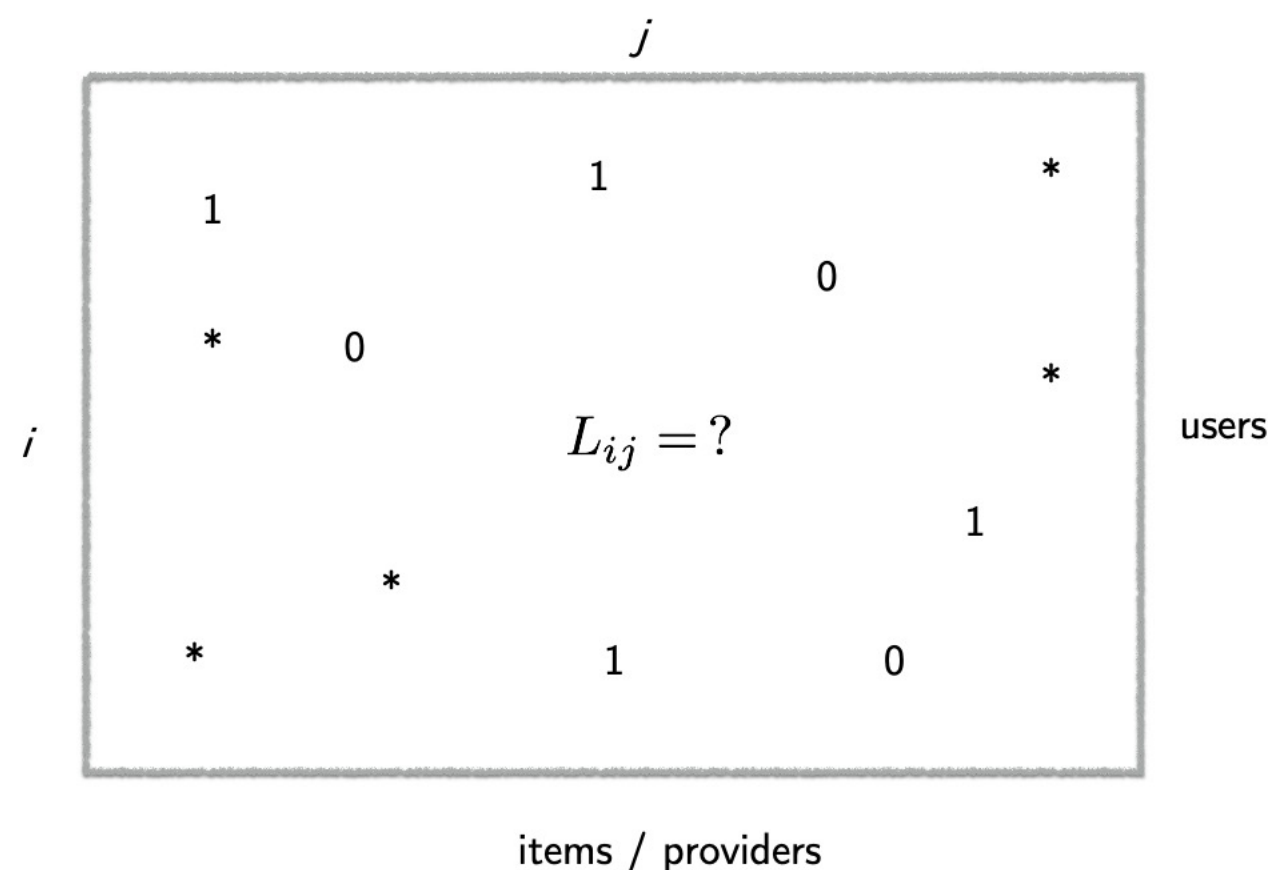
Observations:  $Y_{ij}$  over  $i$  in users,  $j$  in items

If  $(i, j)$  is observed

$$E[Y_{ij}] = L_{ij}$$

If  $(i, j)$  is *not* observed

$$Y_{ij} = \star \text{ or } ?$$



Goal: produce estimation  $\hat{L}_{ij}$  for all  $i, j$

so that  $\hat{L}_{ij} \approx L_{ij}$  for all  $i, j$

## Solution 1: Averaging

---

A simple assumption to get started

All users are identical

That is, all rows of the matrix are the *same*

We wish to predict  $L_{ij}$

All rows in the column  $j$  are the same

Each observation in column  $j$  is outcome of a coin toss with bias  $L_{ij}$

Solution

Estimate  $L_{ij}$  as the average of all observations in column  $j$

## Solution 1: Averaging

---

How accurate?

By law of large numbers

as number of observations in column  $j$  grow  
the estimate converges to the true likelihood

But how large should it be?

one (or few) good rating does not mean the place is excellent

By central limit theorem

the estimation error scale as  $\frac{1}{\sqrt{n}}$  with  $n$  observations

An improved estimate: average  $+\frac{1}{\sqrt{n}}$

**Exercise:** why correction is  $+\frac{1}{\sqrt{n}}$  and not  $-\frac{1}{\sqrt{n}}$

## Solution 1: Averaging

---

What if, instead we assume

All items or providers are identical

Then estimate: row average (+ correction for number of observations)

How to put these two simple estimators together?

$$2L_{ij} = L_{i.} + \frac{1}{\sqrt{n_{i.}}} + L_{.j} + \frac{1}{\sqrt{n_{.j}}}$$

where  $L_{i.}$  is average of observed entries in row  $i$

$n_{i.}$  is number of observed entries in row  $i$

$L_{.j}$  is average of observed entries in column  $j$

$n_{.j}$  is number of observed entries in column  $j$

## Solution 2: Content Based

---

A little more involved assumption

Users and items have features

that are observed and can predict the likelihood

Let features of user  $i$  be  $x_i$

MovieLens: demographics (age, gender, occupation, zip) of users

Let features of item  $j$  be  $y_j$

MovieLens: attributes (genre, director, actors, year, title) of movies

Then, goal is to learn  $f$  where  $L_{ij} = f(x_i, y_j)$

**Exercise:** how to utilize features of MovieLens operationally?

## Solution 2: Content Based

---

This is *supervised learning* problem we have already seen

Labeled data:

each observed entry in matrix  $(i, j)$  corresponds to labeled data

$$((x_i, y_j); L_{ij})$$

Learning problem:

learn the model / function that maps features to label

For likelihood setting with observations being 0 or 1, it is *classification*

### Exercise:

What method would you use for classification?

What if observations were not 0/1 but continuous numbers?



# Appendix: Converting Content to Features

---

Challenge: Content is *not* structured

e.g. recall user information from MovieLens data

```
user id | age | gender | occupation | zip code
```

```
1|24|M|technician|85711
2|53|F|other|94043
3|23|M|writer|32067
4|24|M|technician|43537
5|33|F|other|15213
6|42|M|executive|98101
7|57|M|administrator|91344
8|36|M|administrator|05201
9|29|M|student|01002
10|53|M|lawyer|90703
```

How do we convert these “attributes” or “content” to features

## Appendix: Converting Content to Features

---

Challenge: Content is *not* structured

How do we convert these “attributes” or “content” to features

Age: It's a number. That's easy.

Gender: Two classes or binary. Convert into 0 / 1.

Occupation:

Treat as a class. Use one-hot encoding.

```
user id | age | gender | occupation | zip code
```

```
1|24|M|technician|85711
2|53|F|other|94043
3|23|M|writer|32067
4|24|M|technician|43537
5|33|F|other|15213
6|42|M|executive|98101
7|57|M|administrator|91344
8|36|M|administrator|05201
9|29|M|student|01002
10|53|M|lawyer|90703
```

# Appendix: Converting Content to Features

---

Challenge: Content is *not* structured

What about “Tip” data. It has free-form text.

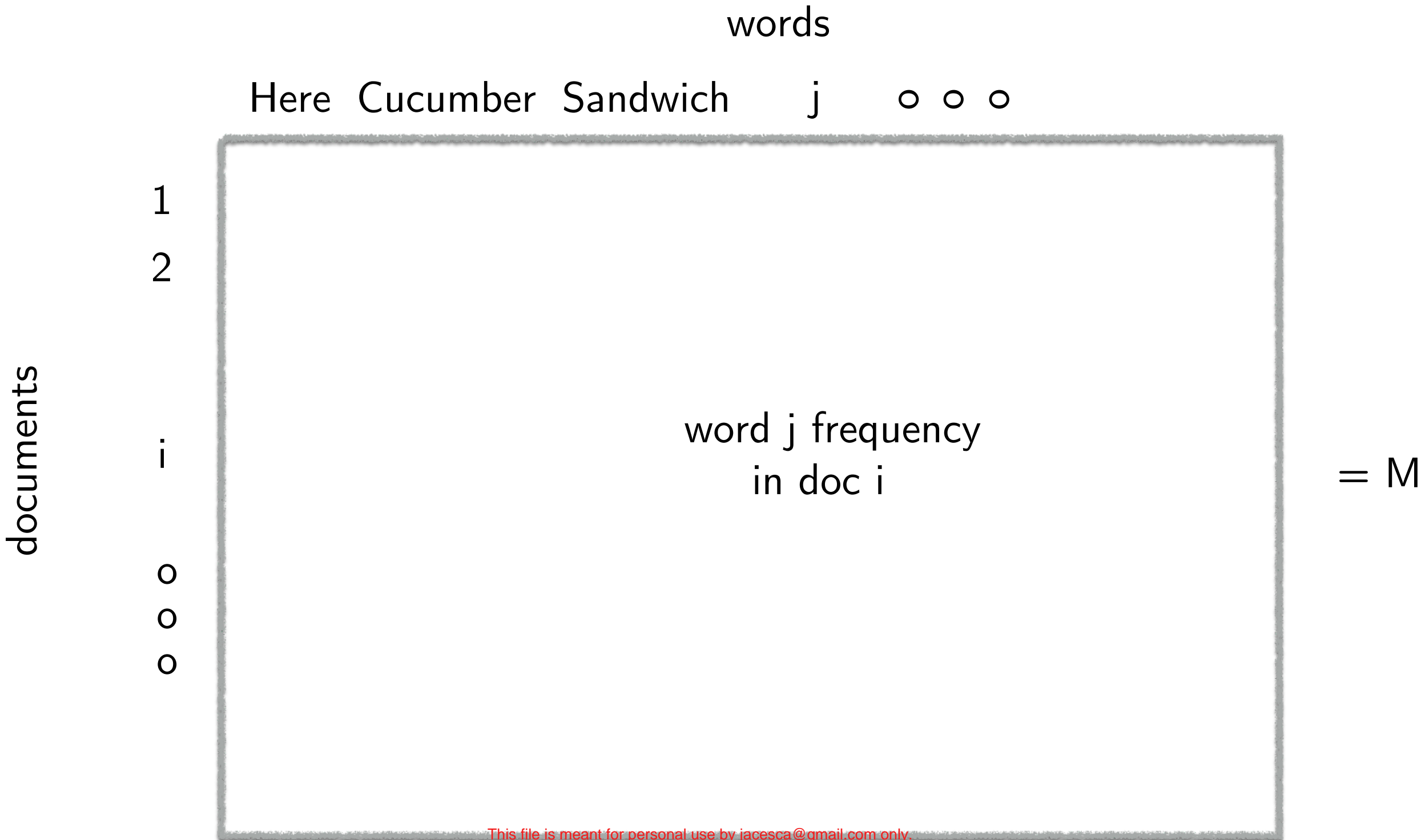
	business_id	compliment_count	date	text	user_id
0	UYX5zL_Xj9WEc_Wp-FrqHw	0	2013-11-26 18:20:08	Here for a quick mtg	hf27xTME3EiCp6NL6VtWZQ
1	Ch3HkwQYv1YKw_FO06vBWA	0	2014-06-15 22:26:45	Cucumber strawberry refresher	uEvusDwoSymbJJ0auR3muQ
2	rDoT-MgxGRiYqCmi0bG10g	0	2016-07-18 22:03:42	Very nice good service good food	AY-lalws3S7YXNI_f_D6rQ
3	OHXnDV01gLokiX1ELaQufA	0	2014-06-06 01:10:34	It's a small place. The staff is friendly.	Ue_7yUlKEbX4AhnYdUfL7g
4	GMrwDXRIAzu2zj5nH6l4vQ	0	2011-04-08 18:12:01	8 sandwiches, \$24 total...what a bargain!!! An...	LltbT_fUMqZ-ZJP-vJ84IQ

Need an approach to convert text into number or vector of numbers

# Appendix: Converting Content to Features

Text to vector of number:

Create word-frequency in documents matrix  $M$



## Appendix: Converting Content to Features

---

Text to vector of number:

Create word-frequency in documents matrix  $M$

Perform Principal Component Analysis of  $M$

Each document receives  $k$  co-ordinates  
via  $k$  principal components

This is the vector representing the text features (restricted to data)

Another (more classical) option:

TF-IDF vector

But it can be very large