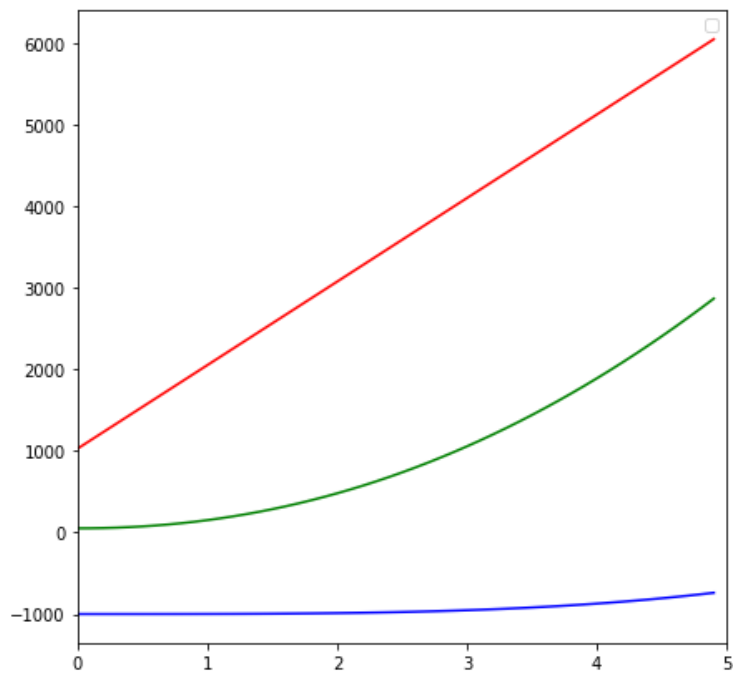


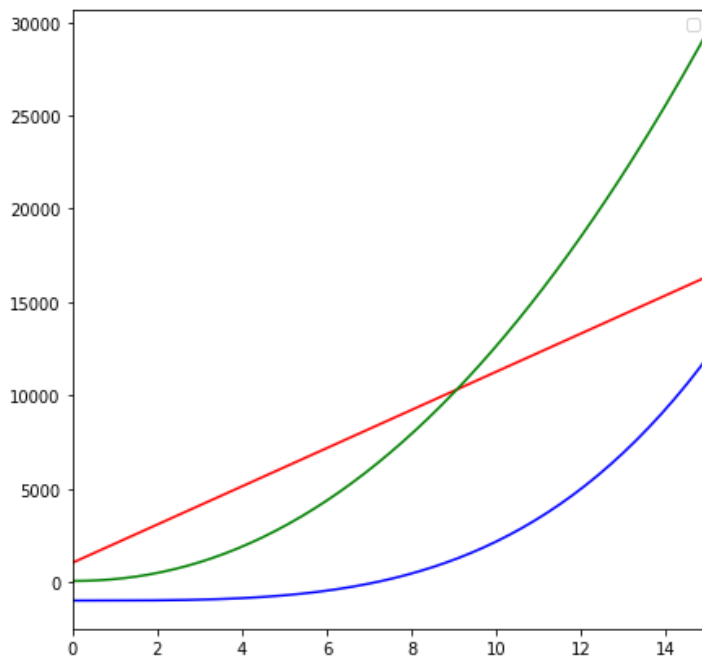
Jacey (Jixing) Man CS 566

Question 1

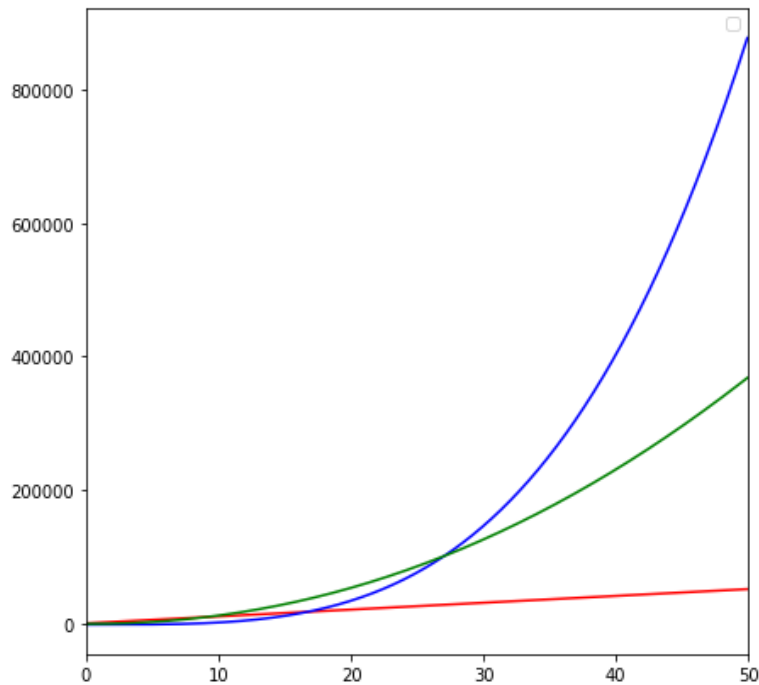
X-axis =5



x-axis = 15.



x-axis = 50.



Code:

```

"""
Created on Sun Sep 13 14:50:19 2020

@author: jixingman
"""

import math
import numpy as np
import matplotlib.pyplot as plt

msize = 5

t = np.arange(0, msize, 0.1)
plt.plot(t, 2**10*t + 2**10, 'red', t, t**3.5 - 1000, 'blue', t, 100*t**2.1 + 50, 'green')

plt.xlim(0, msize)
plt.rcParams["figure.figsize"] = (7,7)
plt.legend()
plt.show()

#####
msize = 15

t = np.arange(0, msize, 0.1)
plt.plot(t, 2**10*t + 2**10, 'red', t, t**3.5 - 1000, 'blue', t, 100*t**2.1 + 50, 'green')

plt.xlim(0, msize)
plt.rcParams["figure.figsize"] = (7,7)
plt.legend()
plt.show()

####
msize = 50

t = np.arange(0, msize, 0.1)
plt.plot(t, 2**10*t + 2**10, 'red', t, t**3.5 - 1000, 'blue', t, 100*t**2.1 + 50, 'green')

plt.xlim(0, msize)
plt.rcParams["figure.figsize"] = (7,7)
plt.legend()
plt.show()

```

Description:

F1 is a linear function, F2 and F3 are both exponential growth function, therefore when the x-axis is small, such as 5, it will show F1 as more steep, with the other two line be more flat. However as the x-axis increased, both F2 and F3 will have higher growth and have much higher y-value on the graph compare to linear function F1

Question 2:

Rule: $O(g(n)) = \{f(n) \text{ there exists constants } c, n_0 > 0 \text{ such that } 0 \leq f(n) \leq c \cdot g(n) \text{ for all } n \geq n_0\}$ f grows no faster than g

$$1. 2^{(n+1.3)} = O(2^n)$$

Answer: Yes

$$f(n) = 2^{1.3} * 2^n = 2.46 * 2^n$$

$$n_0 = 1 \text{ and } c = 2.46$$

$$0 \leq 2.46 \leq 2.46$$

#####

$$2. 3^{(2*n)} = O(3^n)$$

Answer: NO

$f(n) = (3^2)^n = 9^n$, I can not find the constants c to make this possible, and f does grow faster than g

Question 3: Rule: (holds true if there exist positive constants n_0, c_1 and c_2), f grows as the same rate as g

$\Theta(g(n)) = \{f(n) : \text{there exist positive constants } c_1, c_2 \text{ and } n_0 \text{ such that } 0 \leq c_1 g(n) \leq f(n) \leq c_2 g(n) \text{ for all } n \geq n_0\}$

$$1. f(n) = (4^n)^{150} + (2^n + 1024)^{400} = 4^{150n} + n^{150} + 2^{400n} + 1024^{400}$$

$$g(n) = 20^n * 400 + (n + 1024)^{200} = 20^n * 400 + n^{200} + 1024^{200}$$

Answer: Yes, $f(n)$ does equal to $\Theta(g(n))$, because f grow rate is same as g

$$2. f(n) = n^{1.4} * 4^n$$

$$g(n) = n^{200} * 3.99^n$$

Answer: No, $f(n)$ does not equal $\Theta(g(n))$, because f grows rate is faster than g . The exponent 4^n of $f(n)$ will grow faster than $g(n) 3.99^n$

$$3. f(n) = 2^{\log(n)}$$

$$g(n) = n^{1024}$$

Answer: No, $f(n)$ does not equal $\Theta(g(n))$, because f grows rate is slower than g . The $f(n)$ formula even as a exponent, its growth is very slow

Question 4: Start with inner while loop at line 8 is with two constant, so it is $\log(n)+2$, line 6's for loop, is the bigger inner loop, so $n*(\log(n)+2)$, with line 7 as another constant, so $n*((\log(n)+2))+1$, the while loop at line 2, is not part of the for loop in line 6, it is $\log(n)+2$, line 1: $i=1$ is also constant, so the whole code is:

$$\begin{aligned} &1 + \log(n) + 2 + n * ((\log(n) + 2) + 1) \\ &= 1 + \log(n) + 2 + n * \log(n) + 2n + n * 1 \\ &= n * \log(n) + 3n + 3 \end{aligned}$$

so the tight Theta of this pseudocode is $f(n) = \Theta(n^2)$

Question 5:

Start with inner loop at line 3, 1 for loop with a constant, it is $n+1$, with the bigger loop at line 2, so it is $n(n+1)$, with line 1 as another constant :

$$\begin{aligned} &n(n+1) + 1 \\ &= n^2 + n + 1 \\ &\text{with } f(n) = \Theta(n^2) \end{aligned}$$

so the big O of this pseudocode is $f(n) = O(n^3)$