

CS566 HW2 Jixing Jacey Man

Question 1:

$$C = \begin{pmatrix} 1 & 3 \\ 7 & 5 \end{pmatrix} * \begin{pmatrix} 6 & 8 \\ 4 & 2 \end{pmatrix}$$

$$C = \begin{pmatrix} 1*6 + 3*4 & 1*8 + 3*2 \\ 7*6 + 5*4 & 7*8 + 5*2 \end{pmatrix}$$

$$C = \begin{pmatrix} 6 + 12 & 8 + 6 \\ 42 + 20 & 56 + 10 \end{pmatrix}$$

$$C = \begin{pmatrix} 18 & 14 \\ 62 & 66 \end{pmatrix}$$

Question 2:

Pesudocode for Strassen:

SquareMatrixMultipy(A,B)

N = A.rows

Let C be a new n * n matrix

If n == 1 then

$$C_{11} = A_{11} * B_{11}$$

Else Partition A,B, and C

#getting sums

$$\text{sum1} = A_{11} + A_{12}$$

$$\text{sum2} = A_{21} + A_{22}$$

$$\text{sum3} = B_{12} - B_{22}$$

$$\text{sum4} = A_{11} + A_{22}$$

$$\text{sum5} = B_{21} - B_{11}$$

$$\text{sum6} = A_{12} - A_{22}$$

$$\text{sum7} = B_{11} + B_{22}$$

$$\text{sum8} = A_{11} - A_{21}$$

$$\text{sum9} = B_{21} + B_{22}$$

$$\text{sum10} = B_{11} + B_{12}$$

#getting products

$$p1 = \text{SquareMatrixMultipy}(A_{11}, \text{sum3})$$

$$p2 = \text{SquareMatrixMultipy}(\text{sum3}, B_{22})$$

$$p3 = \text{SquareMatrixMultipy}(\text{sum2}, B_{11})$$

$$p4 = \text{SquareMatrixMultipy}(A_{22}, \text{sum5})$$

$$p5 = \text{SquareMatrixMultipy}(\text{sum4}, \text{sum7})$$

$$p6 = \text{SquareMatrixMultipy}(\text{sum6}, \text{sum9})$$

$$p7 = \text{SquareMatrixMultipy}(\text{sum8}, \text{sum10})$$

$$C_{11} = p4 + p5 - p2 + p6$$

$$C_{12} = p1 + p2$$

$$C_{21} = p3 + p4$$

$C_{22} = p_1 - p_7 + p_5 - p_3$

end if
return C
#EndFunction

Question 3:

Answer: Yes, it is possible to use Strassen's algorithm to compute this following 3×3 matrix.

Is it possible to use Strassen's algorithm for any matrix multiplication?

Answer: Yes

Computation Step: I would first get products using cross multiplication, then get the sum of the products to get the final $C(3 \times 3)$ matrix.

1. So divided the input matrices A and B and output matrix C into $n/3 \times n/3$, this takes $\Theta(1)$
2. Create the Sum and Product equation to get all the individual number.
By cross – multiplying A and B there should be 9×9 from A and B, total 18 number from Products.
3. Then adding all 18 number into 9 numbers matrix would results the final C.

Is it possible to use Strassen's Algorithm for any matrix multiplication?

Answer: Yes

What is the resulting algorithm run time in Theta notation?

Answer: The running Theta time for 3×3 would be $\Theta(n^3)$ time, so for any matrix multiplication would be $\Theta(n^n)$ for running time.

Question 4:

1. Show that the solution of $T(n) = T(n-1) + n$ is $O(n^2)$

Solution: solution guess is : $T(n) \leq cn^2$

Assuming c and n_0 are positive constants > 0 , and prove $T(n) \leq cn^2$

$$T(n) \leq c(n-1)^2 + n$$

$$= cn^2 - 2cn + c + n$$

$$= cn^2 - c(1-2c) + c$$

$$\leq cn^2 \text{ holds as long as } c \geq 1$$

2. Show that the solution of

$T(n) = T(\lfloor n/2 \rfloor) + 1$ is $O(\lg n)$.

Solution: solution guess is $T(n) \leq c \lg(n-2)$

Assuming c and n_0 are positive constants > 0 , and prove $T(n) \leq c \lg(n-2)$

$$T(n) \leq \lg(\lfloor n/2 \rfloor - 2) + 1$$

$$\leq \lg(n/2 + 1 - 2) + 1$$

$$= \lg((n-2)/2) + 1$$

$$= \lg(n-2) - \lg 2 + 1$$

$$\leq \lg(n-2) \text{ holds as long as } c \geq 1$$

Question 5:

If $a = b^d$ is case 1

If $a < b^d$ is case 2

If $a > b^d$ is case 3

For all the recurrences below, $a = 2$, and $b = 4$, therefore, $n^{(\log \text{base}^a)} = \sqrt{n}$

1. $T(n) = 2T(n/4) + 1$

$$F(n) = O(1) = O(n^{(0.5-0.5)}) = \text{case 1}$$

$$\text{Answer: } \Theta(n^{(\log \text{base } 4^2)}) = T(n) = \Theta(n^{0.5})$$

2. $T(n) = 2T(n/4) + \sqrt{n}$

$$F(n) = O(n^{0.5}) = \text{case 2}$$

$$\text{Answer: } \Theta(n^{(\log \text{base } 4^2) * \log n}) = T(n) = \Theta(\sqrt{n^{0.5} \log n})$$

3. $T(n) = 2T(n/4) + n^2$

$$F(n) = O(n^2) = O(n^{(0.5+1.5)}) = \text{case 3}$$

$$\text{Answer: } T(n) = \Theta(n^2)$$