GDM method for the 1D linear advection equation

1. Problem overview

The linear advection equation is of the following form:

$$\frac{\partial u}{\partial t} + a\frac{\partial u}{\partial x} = 0$$

where a is the advection speed and $u = u(x,t)$ is the unknown scalar function. This equation is hyperbolic.

For the initial conditions problem we have to specify the initial conditions $u(x,t) \mid_{t=0} = u_0(x)$, this can be solved analytically using the method of charcteristics.

2. Discontinous Galerkin Method

2.1. Spatial discretization

We split our domain (interval I) into elements $I_k = \langle x_i, x \rangle$

2.2. Time discretization