



# **Divide21: The Impact of One-Step Lookahead on Greedy Strategy**

Jacinto Quimua

## Background/Review

How do we know that a number is divisible by:

- 2
- 3
- 4
- ...
- 9

# Background

- **Divide21 (Divide to One)** is a game designed to foster logical and strategic thinking, improved memory and pattern recognition through digit manipulation and division.
- It can help students, especially those in foundational math courses, to better understand and master core concepts such as divisibility, prime numbers, factors and multiples, through a fun and interactive play.
- It can be used as a Logical Reasoning (Symbolic AI) benchmark.

# Rules

## 1 - Initial Setup

A random positive integer **X** is generated, such that:

- It has more than one digit, which is determined before the game starts.
- All of its divisors (except 1) must also have more than one digit.

## 2 - Player Actions

Players alternate turns, and on each turn may:

- Change a single digit of **X**, such that:
  - **X** does not become 0 or 1
  - All ten digits (0-9) must be used at a given index before any of them is reused.
  - Leading zeros are not allowed; if they appear, they cannot be changed.
- Attempt division of **X** by a one-digit number (2-9):
  - If successful, the divisor is added to the player's score.
  - If unsuccessful, the divisor is subtracted from the score.
  - If a player misses a valid division opportunity, the largest one-digit factor of **X** is subtracted from the player's score.

## 3 - After Division

- The quotient becomes the new **X**, and the player may keep dividing it to get extra points.
- Digits in the quotient cannot be reused at their respective index, unless all ten one-digit numbers (2-9) have been used.
- The timer does not reset.

## 4 - Win and Loss Conditions

- Win: Reach a quotient 1 or accumulate 9x the number of digits points or more.
- Loss: Run out of time or drop to -9x the number of digits points or less.

# Example

**Player1:** 493 → change digit 9 to 0 → 403

**Player2:** 403 → change digit 3 to 9 → 409

**Player1:** 409 → change digit 4 to 3 → 309

**Player2:** 309 ÷ 3 = 103 (+3 points) → 103

**Player2:** 103 → change digit 0 to 7 → 173

**Player1:** 173 → change digit 3 to 2 → 172

**Player2:** 172 ÷ 4 = 43 (+4 points) → 43

**Player2:** 43 → change digit 4 to 1 → 13

**Player1:** 13 → change digit 3 to 7 → 17

**Player2:** 17 → change digit 7 to 4 → 14

**Player1:** 14 ÷ 2 = 7 (+2 points) → 7

**Player1:** 7 ÷ 7 = 1 (+7 points) → 1 (**Player1 wins by Quotient 1**)

# Methodology

**Greedy Agent**

VS

**Greedy Agent**

+

One Move Lookahead  
**(Selfish)**

# Experiment Design

- 10 Game Simulations
  - Per digit
- Simple Greedy vs. Selfish Greedy
  - Each agent gets to make the first move

# Metric Collection

- Win Rate
  - by Quotient 1
  - by points
  - by Quotient 1 and points
- Average Game Length
  - per Digit
- Turn Advantage Effect
- Average index chosen
- Average digit usage



# Expected Outcomes

- The Selfish Greedy Agent outperforms the Simple Greedy Agent.
- More wins by points than by quotient 1 or both.
- No turn advantage effect.

# Preliminary Results

- The Selfish Greedy Agent outperforms the Simple Greedy Agent.
  - True
  - Total games (per digit): 10, with 2-9 digits
    - Selfish Greedy Agent: 77-3
- More wins by points than by quotient 1 or both.
  - Not Clear
    - The results seem balanced (need more data)
- No turn advantage effect.
  - True
  - Each agent made the first move 50% of the time

Play Online!

[divide21.com](https://divide21.com)