Design Analysis Of Algorithm - Analytical Problems.

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1) Solve the following recurrence relations

a) $x(n) = x(n-1) + 5$ for $n > 1$ $x(1) = 0$

Solution

$$x(n) = x(n-1) + 5 \rightarrow (1)$$

$$x(n-1) = x(n-1-1) + 5$$

$$x(n-1) = x(n-2) + 5 \rightarrow (2)$$

$$x(n-2) = x(n-2-1) + 5$$

$$= x(n-3) + 5 \rightarrow (3)$$

sub (3) in (2)

$$x(n-1) = x(n-3) + 5 + 5$$

$$x(n-1) = x(n-3) + 10 \rightarrow (4)$$

Sub (4) in (1)

$$x(n) = x(n-3) + 10 + 5$$

$$x(n) = x(n-3) + 15 \rightarrow (5)$$

for k,

$$x(n) = x(n-k) + 5k \rightarrow (6)$$

$n-k=1$, $n-1=k$

Eq ⑤ => $x(n) = x(1) + 5(n-1)$.

$= 0+5n+5$

$x(n) = 5n-5$

Time Complexity $= O(n)$

b) $x(n) = 3x(n-2)$ for $n>2$., $x(1) = 4$

Solution

$x(n) = 3x(n-1) \rightarrow ①$

$x(n-1) = 3x(n-1-1) = 3x(n-2) \rightarrow ②$

$x(n-2) = 3x(n-3) \rightarrow ③$

Sub ③ in ②

$x(n-1) = 3[3x(n-3)]$.

$x(n-1) = 9x(n-3) \rightarrow ④$

Sub ④ in ①.

$x(n) = 3[9x(n-3)]$

$x(n) = 27x(n-3)$

At some k

$x(n) = 3^k x(n-k) \rightarrow ⑤$

$n-k=1$

$k=n-1$

Eq ⑤ => $x(n) = 3^{n-1} x(1)$

$= 3^{n-1} \cdot 4 = 3^n \cdot 3^{-1} \cdot 4$

$= 3^n$

Time Complexity $= O(3^n)$

c) $x(n) = x(n/2) + n$ for $n > 1$ $x(1) = 1$

(Solve for $n = 2^k$)

Solution;

$$x(n) = x(n/2) + c \rightarrow ①$$

$$x(n/2) = x(n/4) + c \rightarrow ②$$

$$x(n/4) = x(n/8) + c \rightarrow ③$$

sub ② in ①

$$x(n) = x(n/4) + c + c$$

$$x(n) = x(n/4) + 2c \rightarrow ④$$

sub ③ in ④

$$x(n) = x(n/8) + c + 2c$$

$$x(n) = x(n/2^3) + 3c.$$

$$x(n) = x(n/2^k) + kc.$$

$\boxed{n = 2^k}$; $\boxed{x(1) = 1}$

$$x(n) = x(k/k) + kc$$

$$x(n) = 1 + kc$$

$$x(n) = 1 + \log n \cdot c.$$

$n/2^k = 1$

$n = 2^k$

$\log n = k \log 2$

$k = \log n$

$n = 2k$

$\boxed{k = n/2}$

Time
Complexity $= O(\log n)$

d) $x(n) = x(n/3) + 1$ for $n > 1$, $x(1) = 1$

(solve for $n = 3^k$)

Solution

$x(n) = x(n/3) + 1 \longrightarrow$ ①

$x(n/3) = x(n/9) + 1 \longrightarrow$ ②

$x(n/9) = x(n/27) + 1 \longrightarrow$ ③

Sub ② in ①,

$x(n) = x(n/9) + 2 \longrightarrow$ ④

$= x(n/3^2) + 2$

Sub ③ in ④

$x(n) = x(n/27) + 3 \longrightarrow$ ⑤

$= x(n/3^3) + 3$

$x(n) = x(n/3^k) + k$

$\frac{n}{3^k} = 1$

$\boxed{n = 3^k}$

$\log n = \log 3^k$

$\boxed{k = \log n}$

$x(n) = x(n/3^k) + k$

$= x(n/n) + k$

$= x(1) + k$

$= 1 + k$

$x(n) \simeq \log n$

Time Complexity $= O(\log n)$

Q) Evalulate following recurrences Completely

i) $T(n) = T(n/2) + 1$ where $n = 2^k$ for all $k \geq 0$.

Given,

$$T(n) = T(n/2 + 1)$$

$$n = 2^k$$

$$T(2^k) = T(2^k/2) + 1 = T(2^{k-1}) + 1$$

$$T(2^{k-1}) = T\left(\frac{2^{k-1}}{2}\right) + 1$$

$$= T(2^{k-2}) + 1$$

Again

$$T(2^{k-2}) = T(2^{k-2}/2) + 1 = T(2^{k-3}) + 1$$

$$T(2^1) = T(2^0) + 1$$

Now,

$$n = 2^k \Rightarrow k = \log_2 n.$$

$$T(2^k) = T(2^{k-1}) + 1 = T(2^{k-2}) + 1 + 1 \ldots T(2^0) + k$$

Since,

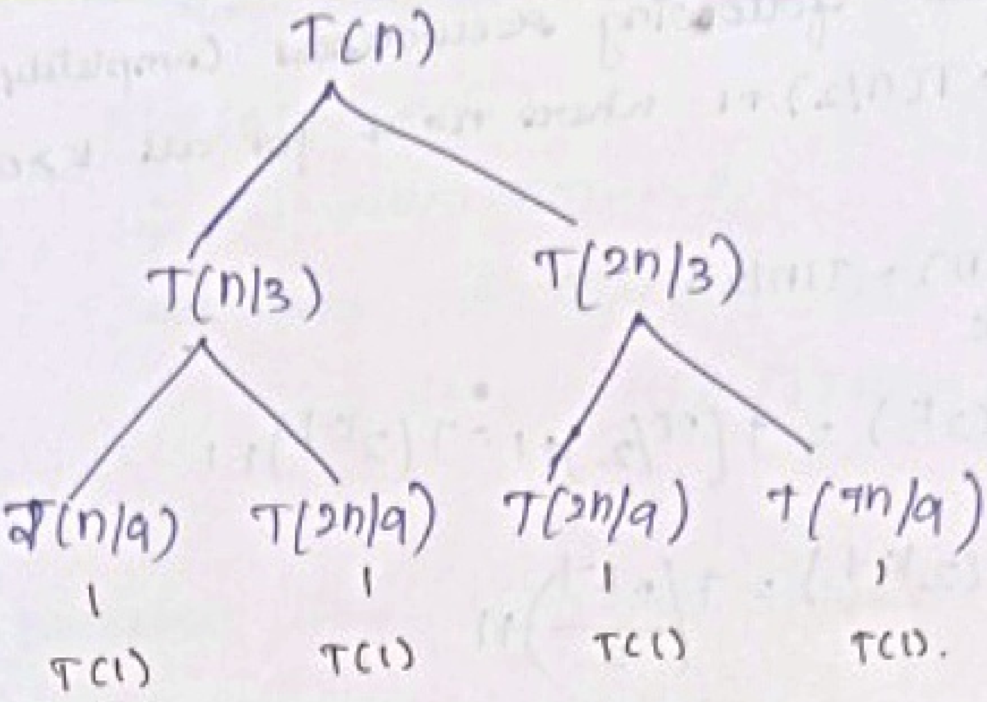
$$2^0 = 1, \; T(2^0) = T(1). \quad \boxed{T(1) = 1}$$

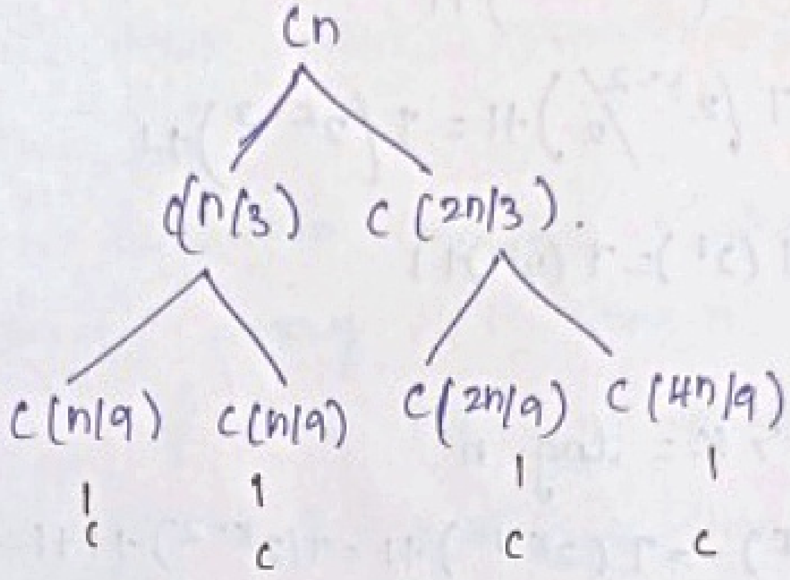
$$T(2^k) = 1 + k. \quad \boxed{O(\log n)}$$

$$T(n) = 1 + \log_2 n.$$

ii) $T(n) = T(n/3) + T(2n/3) + cn$

Using Recursion tree Method.





Length $= \log_3 n$ (divided by 3)

$T(n) = cn \log_3 n \Rightarrow \omega(n \log n)$

3) consider following algorithm

Mini (A[0 ... n-1])

if n-1 return A[0] $\longrightarrow$ 1

else temp = min 1[A[0 ... n-2])

if temp $\leq$ A[n-1] return A[n-1] $\longrightarrow$ n-1

a) What does this algorithm compute?

b) Setup a recurrence relation for algorithm basic operation count and. Slove it

(a) This algorithm computes minimum element in an array A of size n.

if i < n A[i] is smaller than all elements then A[j] = j = i+1 to n-1 then it returns A[i], it also returns the left most Minimal element

(b) Main to Computer Occurs during recursion

So, $T(n) = T(n-1) + 1$ when $n > 1$

$T(1) = 0$ (no Compare when $n=1$)

$T(n) = T(1) + (n-1) * 1$

$= 0 + (n-1)$

$= n-1$

Time Complexity: $O(n)$

4) Analyze order of Growth

i) $f(n) = 2n^2+5$ and $g(n) = 7n$ use Ω $(g(n))$ notation

Given;

$$f(n) = 2n^2+5$$

$$c.g(n) = 7n$$

$$f(n) \geq c.g(n)$$

$n=1$

$f(1) = 2(1)^2+5 = 7$

$g(1) = 7$

$n=2$

$f(2) = 2(2^2+5$

$= 8+5 = 13$

$g(2) = 7 \times 2 = 14$

$n=3$ $f(3) = 2(3)^2+5$

$= 23$

$g(3) = 21$

$n=1$, $7=7$

$n=2$, $13=14$

$n=3$, $23=21$

$n \geq 3$ $f(n) \geq g(n).c$

$f(n)$ is always greater than or equal to $c.g(n)$ when, n value is greater or equal to 3

$$f(n) = \Omega(g(n))$$

f(n) grows more than g(n)