Física Computacional I

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- Regra do trapézio:

$$I(a,b) \approx h\left[\frac{1}{2}f(a) + \frac{1}{2}f(b) + \sum_{k=1}^{N-1} f(a+kh)\right]$$

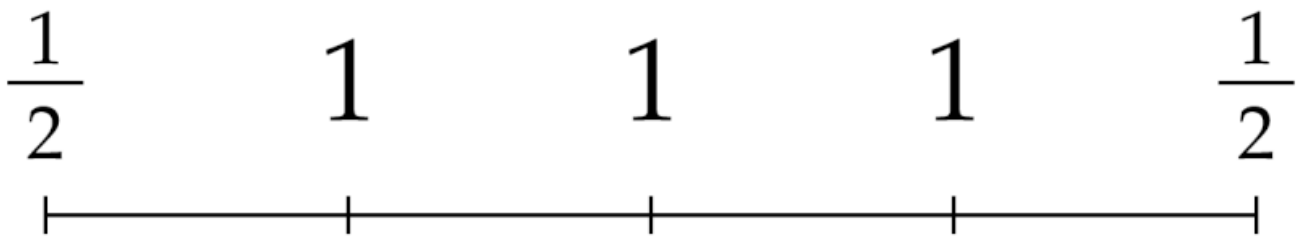
- Erro:

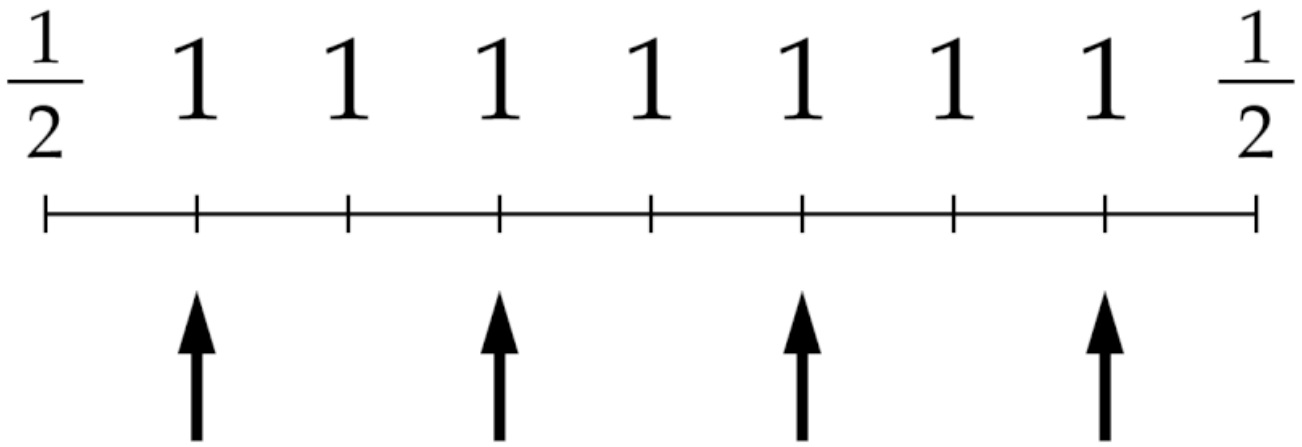
$$\int_a^b f(x)dx = \frac{h}{2}\sum_{k=1}^{N}[f(x_{k-1}) + f(x_k)] + \frac{h^2}{12}[f'(a) - f'(b)] + O(h^4)$$

- Regra prática para erro:

$$\varepsilon_2 = \frac{1}{3}(I_2 - I_1)$$

Regra do trapézio adaptativa





- Erro da regra do trapézio:

$$I = I_i + c_1 h_i^2 + c_2 h_i^4 + c_3 h_i^6 + \cdots$$

- Ordem 2:

$$c_1 h_i^2 = \frac{1}{3}(I_i - I_{i-1})$$

$$
\begin{array}{ccccccc}
I_1 \equiv R_{1,1} & & & & & & \\
 & \searrow & & & & & \\
I_2 \equiv R_{2,1} & \rightarrow & R_{2,2} & & & & \\
 & \searrow & & \searrow & & & \\
I_3 \equiv R_{3,1} & \rightarrow & R_{3,2} & \rightarrow & R_{3,3} & & \\
 & \searrow & & \searrow & & \searrow & \\
I_4 \equiv R_{4,1} & \rightarrow & R_{4,2} & \rightarrow & R_{4,3} & \rightarrow & R_{4,4} \\
 & \searrow & & \searrow & & \searrow & & \searrow
\end{array}
$$

- Erro da regra do trapézio:

$$I = I_i + c_1 h_i^2 + c_2 h_i^4 + c_3 h_i^6 + \cdots$$

- Ordem 2:

$$c_1 h_i^2 = \frac{1}{3}(I_i - I_{i-1})$$

- Ordem 4:

$$c_2 h_i^4 = \frac{1}{15}(R_{i,2} - R_{i-1,2})$$

- Ordem 2m:

$$c_m h_i^{2m} = \frac{1}{4^m - 1}(R_{i,m} - R_{i-1,m})$$

$$
\begin{array}{ccccccc}
I_1 \equiv R_{1,1} & & & & & & \\
 & \searrow & & & & & \\
I_2 \equiv R_{2,1} & \rightarrow & R_{2,2} & & & & \\
 & \searrow & & \searrow & & & \\
I_3 \equiv R_{3,1} & \rightarrow & R_{3,2} & \rightarrow & R_{3,3} & & \\
 & \searrow & & \searrow & & \searrow & \\
I_4 \equiv R_{4,1} & \rightarrow & R_{4,2} & \rightarrow & R_{4,3} & \rightarrow & R_{4,4} \\
 & \searrow & & \searrow & & \searrow & \searrow
\end{array}
$$

Para m = 1: Regra do trapézio

no cóigo
RHS: m + 1 --> m
LHS: m --> m-1

Para m ≠ 1: $R_{i,m+1} = R_{i,m} + \frac{1}{4^m - 1}(R_{i,m} - R_{i-1,m})$