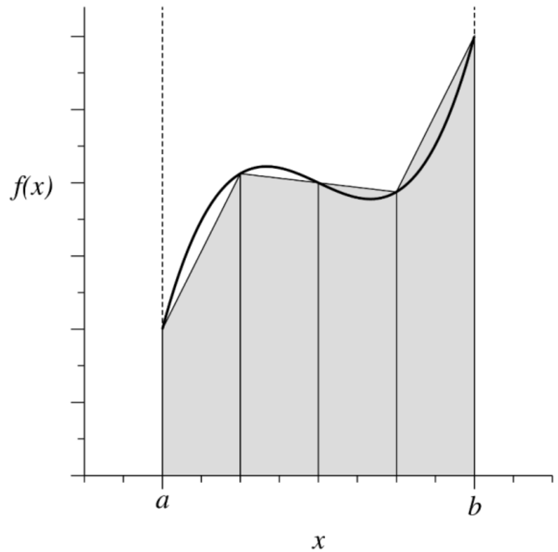
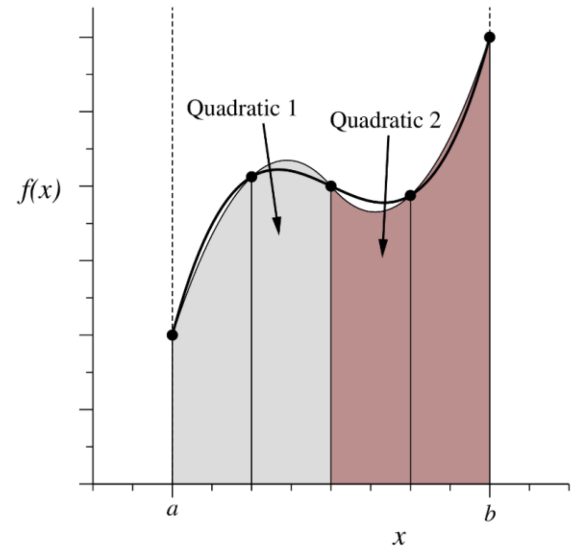
DEPARTAMENTO de FÍSICA TEÓRICA e EXPERIMENTAL

UNIVERSIDADE FEDERAL DO RIO GRANDE DO NORTE

Física Computacional I

Prof.: Leonardo D. Machado





- Regra do trapézio:

$$I(a,b) \approx \frac{1}{2}h\sum_{k=1}^{N}[f(a+(k-1)h)+f(a+kh)]$$

$$I(a,b) \approx h\left[\frac{1}{2}f(a)+\frac{1}{2}f(b)+\sum_{k=1}^{N-1}f(a+kh)\right]$$

$$h = \frac{(b-a)}{N}$$

- Nova notação:

$$x_k = a + kh$$
$$x_0 = a$$
$$x_N = a + Nh = b$$

$$I(a,b) \approx \frac{1}{2} h \sum_{k=1}^{N} [f(x_{k-1}) + f(x_k)]$$

- Erro de arredondamento para um número x:

$$\varepsilon_{ar} \approx Cx$$

Onde $C \approx 10^{-16}$

- Erro de aproximação?

- Série de Taylor:

$$f(x) = f(a) + f'(a)(x-a) + \frac{1}{2}f''(a)(x-a)^2 + \cdots$$

- Equação A:

$$\int_{x_{k-1}}^{x_k} f(x)dx = hf(x_{k-1}) + \frac{h^2}{2} f'(x_{k-1}) + \frac{h^3}{6} f''(x_{k-1}) + \cdots$$

- Equação B:

$$\int_{x_{k-1}}^{x_k} f(x)dx = hf(x_k) - \frac{h^2}{2} f'(x_k) + \frac{h^3}{6} f''(x_k) + \cdots$$

- Equação C:

$$\int_a^b f(x)dx = \frac{h}{2}\sum_{k=1}^{N}[f(x_{k-1}) + f(x_k)] + \frac{h^2}{4}[f'(a) - f'(b)] + \frac{h^3}{12}\sum_{k=1}^{N}[f''(x_{k-1}) + f''(x_k)] + O(h^4)$$

Resultado

$$\int_a^b f(x)dx = \frac{h}{2}\sum_{k=1}^{N}[f(x_{k-1}) + f(x_k)] + \frac{h^2}{12}[f'(a) - f'(b)]$$
$$+ O(h^4)$$

5.18 Rearranging Eq. (5.19) into a slightly more conventional form, we have:

$$\int_a^b f(x)\,\mathrm{d}x = h\left[\tfrac{1}{2}f(a) + \tfrac{1}{2}f(b) + \sum_{k=1}^{N-1} f(a+kh)\right] + \tfrac{1}{12}h^2\big[f'(a) - f'(b)\big] + \mathrm{O}(h^4).$$

This result gives a value for the integral on the left which has an error of order h^4—a factor of h^2 better than the error on the trapezoidal rule and as good as Simpson's rule. We can use this formula as a new rule for evaluating integrals, distinct from any of the others we have seen in this chapter. We might call it the "Euler–Maclaurin rule."