

Manipulation Tests in Regression Discontinuity Design: The Need for Equivalence Testing

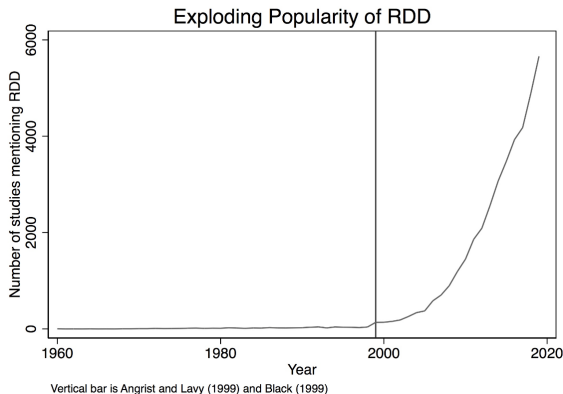
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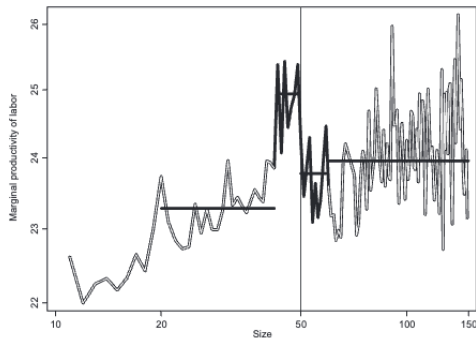
Regression Discontinuity Design (RDD)



Cunningham (2021) documents 5600 RDD papers published in 2019 alone

RDD's 'Experimental Appeal'

**Panel A: Value added per worker
relative to industry average**



In principle, when an agent's running variable (RV) crosses the assignment cutoff, the agent should be effectively randomized into or out of treatment

Source: Garicano, Lelarge, & van Reenen (2016)

RV Manipulation at the Cutoff

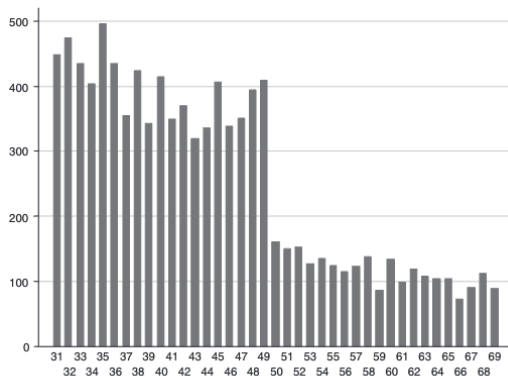


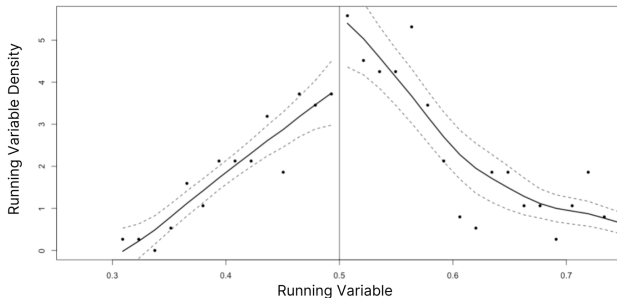
FIGURE 2. NUMBER OF FIRMS BY EMPLOYMENT SIZE IN FRANCE

Endogenous manipulation of RV values near the cutoff induces selection biases

- Agents can often effectively select into/out of treatment

Source: Garicano, Lelarge, & van Reenen (2016)

RV Manipulation Tests



RV manipulation tests estimate and assess discontinuities in the RV's density at the cutoff

- ▶ Well-known versions include `DCdensity` and `rddensity` (McCrary 2008; Cattaneo, Jansson, & Ma 2018; Cattaneo, Jansson, & Ma 2020)
- ▶ Per Web of Science, these tests have over 2100 citations between them

... and How They're Misused

[R Code](#)[Stata Code](#)

```
* If necessary, findit rddensity and install the rddensity package  
causaldata gov_transfers_density.dta, use clear download
```

```
* Limit to the bandwidth ourselves  
keep if abs(income_centered) < .02  
* Run the discontinuity check  
rddensity income_centered, c(0)
```

As expected, we find no statistically significant break in the distribution of income at the cutoff. Hooray!

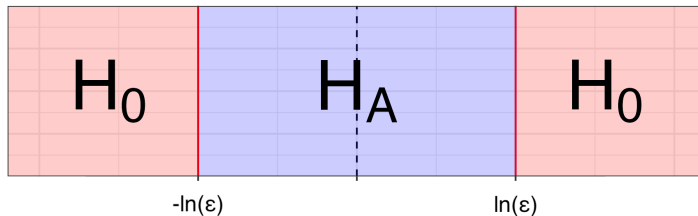
Source: Huntington-Klein (2022)

Unfortunately, researchers (mis)interpret *stat. insig.* manipulation as evidence of *negligible* manipulation

- This is a well-known fallacy (Altman & Bland 1995; Imai, King, & Stuart 2008; Wasserstein & Lazar 2016)

Meaningful manipulation may go undetected if these tests are underpowered

An Alternative Testing Framework



Ideal: Stat. sig. evidence that RV manipulation ≈ 0 . We can get this using **equivalence testing**:

1. Define the smallest practically/economically significant RV density discontinuities at the cutoff for our given research setting
2. Use interval tests to assess whether the RV density discontinuity at the cutoff is bounded beneath this effect size

This Project

Novel equivalence testing procedure for RV manipulation tests

- ▶ Can provide sig. evidence that RV manipulation ≈ 0 , which is what applied researchers usually want to show

Empirical evidence of its necessity in applied RDD research

- ▶ Replicating 36 published RDD papers shows that $> 44\%$ of RV density discontinuity magnitudes can't be stat. sig. bounded beneath a 50% upward jump

Guidelines and statistical software commands for credible implementation

- ▶ `lddtest` command in Stata and in the `eqtesting` R package

Setup (1/2)

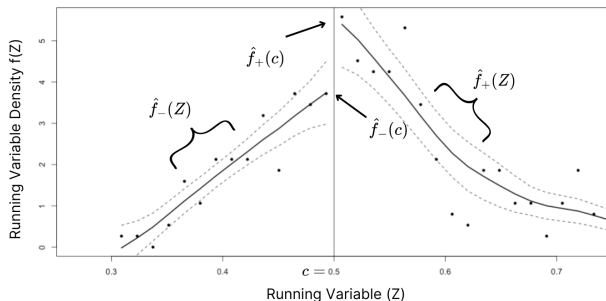
Standard cross-sectional RDD setup (panel setup possible via bootstrap)

- ▶ Agents i have some running variable Z_i
- ▶ Agents are assigned to treatment if Z_i crosses cutoff c :

$$D_i = \begin{cases} 1 & \text{if } Z_i \geq c \\ 0 & \text{if } Z_i < c \end{cases} \quad \text{or} \quad D_i = \begin{cases} 1 & \text{if } Z_i \leq c \\ 0 & \text{if } Z_i > c \end{cases}$$

- ▶ Z_i exhibits probability density function $f(Z_i)$

Setup (2/2)



We'll test for RV manipulation by testing a continuity assumption: $\lim_{Z_i \rightarrow c^-} f(Z_i) = \lim_{Z_i \rightarrow c^+} f(Z_i)$

- ▶ RV manipulation tests estimate density functions on each side of the cutoff, $\hat{f}_-(Z_i)$ and $\hat{f}_+(Z_i)$
- ▶ Our estimates of the LHS and RHS density limits are respectively $\hat{f}_-(c)$ and $\hat{f}_+(c)$

The Wrong Hypotheses: Standard NHST

Standard RV manipulation tests effectively assess the hypotheses

$$H_0 : \lim_{Z_i \rightarrow c^-} f(Z_i) = \lim_{Z_i \rightarrow c^+} f(Z_i)$$

$$H_A : \lim_{Z_i \rightarrow c^-} f(Z_i) \neq \lim_{Z_i \rightarrow c^+} f(Z_i).$$

There are many problems with this standard NHST approach

- ▶ **No burden of proof**: Researchers assume in the null hypotheses that what they want to show is true
- ▶ For most researchers, **imprecision is 'good'**
- ▶ **Negligible manipulation can be 'significant'** in high-powered research settings

Creates perverse incentives for **'reverse p -hacking'** by setting restrictive bandwidths or not reporting RV manipulation tests (see Dreber, Johanneson, & Yang 2024)

The Right Hypotheses: Equivalence Testing

We'll fix these problems by 1) flipping the hypotheses and 2) relaxing the constraints. As a reminder, **standard NHST hypotheses**:

$$H_0 : \lim_{Z_i \rightarrow c^-} f(Z_i) = \lim_{Z_i \rightarrow c^+} f(Z_i)$$

$$H_A : \lim_{Z_i \rightarrow c^-} f(Z_i) \neq \lim_{Z_i \rightarrow c^+} f(Z_i).$$

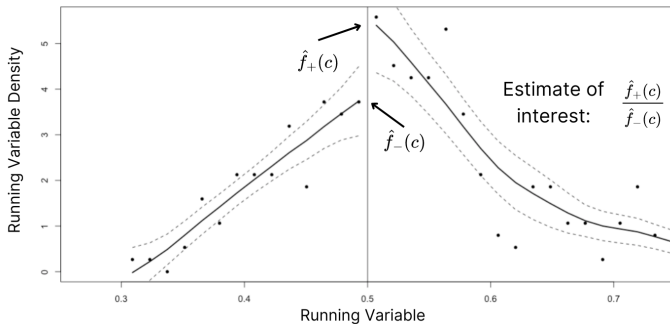
And now **equivalence testing hypotheses**:

$$H_0 : \lim_{Z_i \rightarrow c^-} f(Z_i) \not\approx \lim_{Z_i \rightarrow c^+} f(Z_i)$$

$$H_A : \lim_{Z_i \rightarrow c^-} f(Z_i) \approx \lim_{Z_i \rightarrow c^+} f(Z_i).$$

If we can set a range of values wherein the RV's density jump at the cutoff ≈ 0 , then we can get stat sig. evidence for H_A with a simple interval test

Step 1: Set the Effect Size Threshold



Set largest practically/economically insignificant RTL density ratio $\epsilon > 1$ for our research setting

- ▶ RTL density ratios are useful effect sizes because they are always comparable across datasets
- ▶ This threshold can be credibly set by surveying other researchers for their judgments [Details](#)

Step 2: Estimate the Logarithmic Density Discontinuity

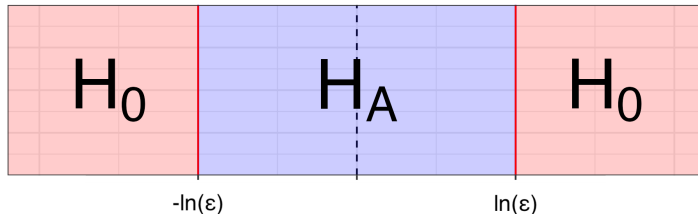
McCrary's (2008) DCdensity procedure estimates **logarithmic density discontinuities**:

$$\begin{aligned}\hat{\theta} &\equiv \ln \left(\hat{f}_+(c) \right) - \ln \left(\hat{f}_-(c) \right) \\ &= \ln \left(\frac{\hat{f}_+(c)}{\hat{f}_-(c)} \right)\end{aligned}$$

McCrary (2008) also shows that $\hat{\theta}$ is consistent and asymptotically normal

- We can thus use $\hat{\theta}$ and $\text{SE}(\hat{\theta})$ from DCdensity for standard Gaussian inference

Step 3: Equivalence Testing



We'll test whether $\hat{\theta}$ is stat. sig. bounded between $-\ln(\epsilon)$ and $\ln(\epsilon)$ w/ two one-sided tests of the form

$$H_0 : \theta < -\ln(\epsilon)$$

$$H_0 : \theta > \ln(\epsilon)$$

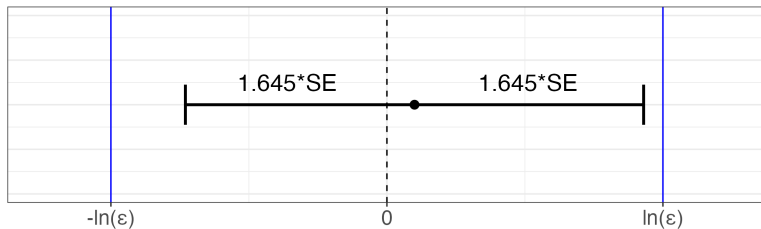
$$H_A : \theta \geq -\ln(\epsilon)$$

$$H_A : \theta \leq \ln(\epsilon)$$

If both tests are stat. sig. at level α , then there's size- α stat. sig. evidence that RV manipulation at the cutoff is practically equal to zero (see Schuirmann 1987; Berger & Hsu 1996)

Visualization

Equivalence Confidence Interval (ECI) Approach



■ θ & Exact 95% ECI ■ ROPE = $[-\ln(\epsilon), \ln(\epsilon)]$

$\hat{\theta}$'s $(1 - \alpha)$ **equivalence confidence interval (ECI)** is just its $(1 - 2\alpha)$ CI

- If $\hat{\theta}$'s $(1 - \alpha)$ ECI is entirely bounded in $[-\ln(\epsilon), \ln(\epsilon)]$, then we have size- α evidence under the TOST procedure that RV manipulation at the cutoff ≈ 0 (Berger & Hsu 1996)

Replication Data

I leverage replication data from Stommes, Aronow, & Sävje (2023), who run robustness checks on 36 published RDD papers in *AJPS*, *APSR*, and *JOP* from 2009-2018

- Some papers use multiple datasets; I run RV manipulation tests in each dataset (45 in total)

Designs in this dataset include close election designs, spatial discontinuities, and age discontinuities

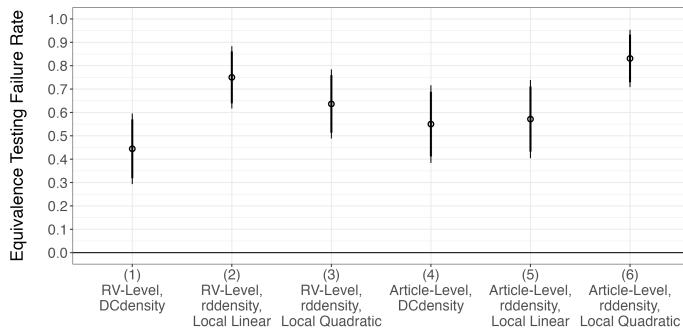
Equivalence Testing Performance

I re-examine these papers with my equivalence-based RV manipulation test, using a lenient threshold of $\epsilon = 1.5$ [Why?](#)

- ▶ I.e., each test asks: *Can we significantly bound RV manipulation at the cutoff beneath a 50% upward jump/33.3% downward jump?*
- ▶ Given the caliber of journals, these RVs should 'pass' this lenient equivalence test

I then compute **equivalence testing failure rates** – the proportion of these equivalence tests that are *not* significant at a 5% level

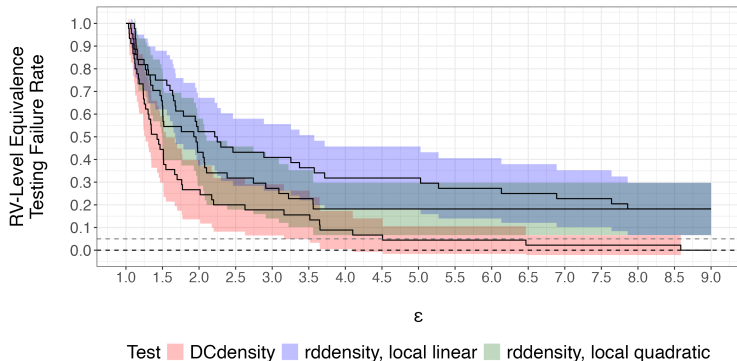
Main Equivalence Testing Failure Rate Estimates



Failure rates for my equivalence-based RV manipulation test range from 44-83%

- **Interpretation:** Over 44% of RV density discontinuity magnitudes at the cutoff cannot be significantly bounded beneath a 50% upward jump

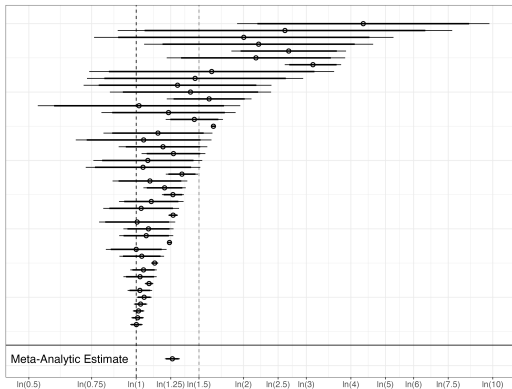
Failure Curves



To obtain 'equivalence testing failure rates' beneath 5%, we'd have to be willing to argue that a 350% upward density jump is practically equal to zero

- **Takeaway:** Meaningful RV manipulation at the cutoff is still a serious problem in RDD research

Meta-Analytic Results



Absolute Logarithmic RV Density Discontinuity at the Cutoff

The meta-analytic average $|\hat{\theta}| = 0.234$

$\left[SE\left(|\hat{\theta}|\right) = 0.05\right]$ Model

- Equivalent to a 26% upward jump in RV density at the cutoff

This is statistically significantly smaller than a 50% upward jump...

- ... but I picked that threshold b/c it's huge!

The practical significance of a 26% upward jump in RV density depends on our research setting

Practical Considerations

How do you set the threshold ϵ ?

- ▶ If we set it ourselves, we'll likely get (reasonable) accusations of p -hacking
- ▶ But if others set it for us, the threshold is credibly independent of our data

I recommend setting ϵ by **surveying other researchers for their judgments** of the smallest practically/economically significant RV density jump at the cutoff

- ▶ Practical using online resources such as the Social Science Prediction Platform (DellaVigna, Pope, & Vivaldi 2019)
- ▶ Data from these researcher surveys can be useful for reasons beyond this test

If this is not feasible (or an RV fails my manipulation test), consider the `rdbounds` partial identification procedure (Gerard, Rokkanen, & Rothe 2020)

- ▶ But for most projects, the procedures I'm proposing will be feasible

Step 1

Paper, Software Commands, & More Information



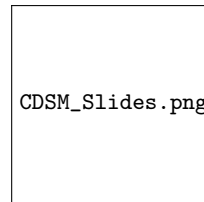
lddtest Stata command
(QR: Github repository)



eqtesting R package,
containing lddtest
command (QR: Github
repository)




I4R Discussion
Paper Working paper
(QR: PDF link, I4R DP)





SlidesWorking paper
(QR: PDF link, I4R DP)


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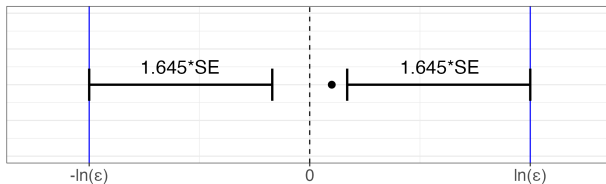
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Two One-Sided Tests (TOST) Procedure



■ θ w/ t-Tests from Above and Below ■ ROPE = $[-\ln(\epsilon), \ln(\epsilon)]$

In other words, we have stat. sig. evidence at the 5% level that $\theta \approx 0$ if

1. $\hat{\theta}$ is 1.645 SEs above $-\ln(\epsilon)$, **and**
2. $\hat{\theta}$ is 1.645 SEs below $\ln(\epsilon)$

Step 3

Why $\epsilon = 1.5$?

- ▶ Chen, Cohen, & Chen (2010) show that an odds ratio of 1.5 corresponds closely w/ a Cohen's (1988) $d = 0.2$, the classic small effect size benchmark
- ▶ Same effect size proposed by Hartman (2021)
- ▶ Practically large in many research-relevant RDD settings (e.g., elections)

[Back](#)

Meta-Analytic Model

I use an unrestricted weighted least squares model of the form

$$\frac{|\hat{\theta}_i|}{\text{SE}(\hat{\theta}_i)} = \beta \frac{1}{\text{SE}(\hat{\theta}_i)} + \mu_i,$$

where β is the meta-analytic estimate (Stanley, Doucouliagos, & Ioannidis 2022)

- ▶ Equivalent to a weighted average of $|\hat{\theta}_i|$ with weights $(\text{SE}(\hat{\theta}_i))^{-2}$ (Stanley et al. 2023)
- ▶ Reflects the fact that unrestricted weighted least squares gives more weight to more precise estimates

[Back](#)