Prelab 1

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1 Electrical Measurement Concepts

1a. An ideal voltage meter has **infinite** internal resistance and should be connected in **parallel** with the circuit or device being tested.

1b. An ideal current meter has **zero** internal resistance and should be connected in **series** with the circuit or device being tested.

2 Resistive Divider Accuracy

We know that the sensitivity of each resistor in a divider ratio is given by

$$R_n \cdot \frac{\partial}{\partial R_n} log\left(\frac{R_2}{R_1 + R_2}\right) \tag{1}$$

For the case of two resistors in series R_1 and R_2 we get that the sensitivities are:

$$S_{R_1} = 1 - \frac{R_1}{R_1 + R_2} \tag{2}$$

$$S_{R_2} = 1 - \frac{R_2}{R_1 + R_2} \tag{3}$$

We can now find the coefficient of variation for the ratio to be the square root of the sum of the squares of the individual sensitivities weighted by the square of the individual components' coefficient of variation:

$$CV = \sqrt{\left(1 - \frac{R_1}{R_1 + R_2}\right)^2 CV_{R_1}^2 + \left(1 - \frac{R_2}{R_1 + R_2}\right)^2 CV_{R_2}^2} \tag{4}$$

3 R-2R Ladder Network

We can approach the problem by finding a sequence of equivalent resistances. To begin, note that the whole network has an equivalent resistance of $2R$ (from right to left, resistors combine first in series, then parallel, then series, etc.).

We see that the equivalent resistance of the rightmost resistors (both of value R) is $2R$. The junction just before that where the branch splits to the $2R$ and the R resistor can be analyzed with Kirchhoff's Law: the current due to the voltage source I_{in} halves at each point, with half the current going through the $2R$ resistor and half the current going to the rest of the circuit.

So we know that the current going through any $2R$ resistor at index n is

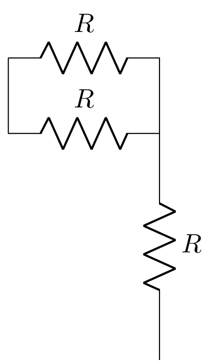
$$I_n = \sum_{i=1}^{N} \frac{1}{2}^n \cdot I_{in} \tag{5}$$

Substituting I_{in} using Ohm's Law ($I_{in} = \frac{V_{in}}{R_{eq}}$), and noting that $R_{eq} = 2R$, we are left with

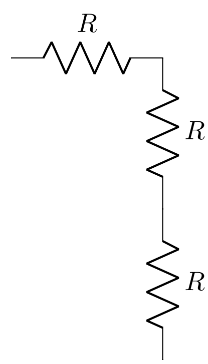
$$I_n = \left(\frac{1}{2}\right)^{n+1} \cdot \left(\frac{V_{in}}{R}\right) \tag{6}$$

where I_n is measured in amps, V_{in} in volts, and R in ohms.

4 Accurate 2:1 Resistor Ratios



Method 1 (parallel resistors)



Method 2 (series resistors)

Method 1 is *more efficient* because it uses a single unit resistor in its termination structure. In Method 1, each R is represented as two parallel unit resistors, and so one would think that the termination structure would have two sets of two parallel resistors in series, but this can be collapsed into a single equivalent unit resistor, so it's termination structure is a single unit resistor.

That contrasts with Method 2, where you still need two resistors in the termination structure.