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1) Air

$$\alpha = \sqrt{\frac{0.0001\times10^{9}\ N/m^{2} + \frac{4}{3}(0)}{1\ kg/m^{3}}} = \boxed{316.22\ m/s}$$

$$\beta = \sqrt{\frac{0}{1}} = \boxed{0\ m/s}$$

$$r = \frac{3(0.0001) - 2(0)}{6(0.0001) + 2(0)} = \boxed{0.5}$$

water

$$\alpha = \sqrt{\frac{2.2\times10^{9} + \frac{4}{3}(0)}{1000}} = \boxed{1{,}483.24\ m/s}$$

$$\beta = \sqrt{\frac{0}{1000}} = \boxed{0\ m/s}$$

$$r = \frac{3(2.2) - 2(0)}{6(2.2) + 2(0)} = \boxed{0.5}$$

ICE

$$\alpha = \sqrt{\frac{8\times10^{9} + \frac{4}{3}(3.9\times10^{9})}{920}} = \boxed{3{,}787.85\ m/s}$$

$$\beta = \sqrt{\frac{3.9\times10^{9}}{920}} = \boxed{2{,}058.91\ m/s}$$

$$r = \frac{3(8.0\times10^{9}) - 2(3.9\times10^{9})}{6(8.0\times10^{9}) + 2(3.9\times10^{9})} = \frac{16.2}{55.8} = \boxed{0.29}$$

SANDSTONE

$$\alpha = \sqrt{\frac{24\times10^9 + \frac{4}{3}(17\times10^9)}{2500}} = \boxed{4{,}320.50\ m/s}$$

$$\beta = \sqrt{\frac{17\times10^9}{2500}} = \boxed{2{,}607.68\ m/s}$$

$$r = \frac{3(24\times10^9) - 2(17\times10^9)}{6(24\times10^9) + 2(17\times10^9)} = \frac{38}{178} = \boxed{0.21}$$

Limestone

$$\alpha = \sqrt{\frac{38\times10^9 + \frac{4}{3}(22\times10^9)}{2700}} = \boxed{4{,}993.82\ m/s}$$

$$\beta = \sqrt{\frac{22\times10^9}{2700}} = \boxed{2{,}854.50\ m/s}$$

$$r = \frac{3(38\times10^9) - 2(22\times10^9)}{6(38\times10^9) + 2(22\times10^9)} = \frac{70}{272} = \boxed{0.26}$$

GRANITE

$$\alpha = \sqrt{\frac{88\times10^9 + \frac{4}{3}(22\times10^9)}{2600}} = \boxed{6{,}717.75\ m/s}$$

$$\beta = \sqrt{\frac{22\times10^9}{2600}} = \boxed{2{,}908.87\ m/s}$$

$$r = \frac{3(88\times10^9) - 2(22\times10^9)}{6(88\times10^9) + 2(22\times10^9)} = \frac{220}{572} = \boxed{0.38}$$

PERIODITITE

$$\alpha = \sqrt{\frac{140\times10^9 + \frac{4}{3}(58\times10^9)}{3300}} = \boxed{8,115.33 \ m/s}$$

$$\beta = \sqrt{\frac{58\times10^9}{3300}} = \boxed{4,192.34 \ m/s}$$

$$\nu = \frac{3(140\times10^9) - 2(58\times10^9)}{6(140\times10^9) + 2(58\times10^9)} = \frac{304}{956} = \boxed{0.32}$$

MATERIAL	K $(10^9 N/m^2)$	μ $(10^9 N/m^2)$	ρ (kg/m^3)	$\alpha = \sqrt{\frac{k+\frac{4}{3}\mu}{\rho}}$ m/s	$\beta = \sqrt{\frac{\mu}{\rho}}$ m/s	$\nu = \frac{3k-2\mu}{6k+2\mu}$
AIR	0.00010	0	1.0	316.22	0	0.5
WATER	2.2	0	1000	1483.24	0	0.5
ICE	8.0	3.9	920	3782.85	2058.91	0.29
SANDSTONE	24	17	2500	4320.50	2607.68	0.21
LIMESTONE	38	22	2700	4993.82	2854.5	0.26
GRANITE	88	22	2600	6712.75	2908.87	0.38
PERIODITITE	140	58	3300	8115.33	4192.34	0.32

2) $\nu = \dfrac{3k - 2\mu}{6k + 2\mu} \Rightarrow \mu = \dfrac{3k - 6\nu k}{2 + 2\nu}$

$\beta = \sqrt{\dfrac{\mu}{\rho}}$

$\alpha = \sqrt{\dfrac{k + \frac{4}{3}\mu}{\rho}}$

$\Rightarrow \beta/\alpha = \dfrac{\sqrt{\frac{\mu}{\rho}}}{\sqrt{\frac{k + 4/3\,\mu}{\rho}}} = \sqrt{\dfrac{\mu}{k + \frac{4}{3}\mu}}$

$= \sqrt{\dfrac{\frac{3k - 6\nu k}{2 + 2\nu}}{k + \frac{4}{3}\left(\frac{3k - 6\nu k}{2 + 2\nu}\right)}} = \sqrt{\dfrac{3k - 6\nu k}{2 + 2\nu\left(k + \frac{4}{3}\left(\frac{3k - 6\nu k}{2 + 2\nu}\right)\right)}}$

$= \sqrt{\dfrac{3k - 6\nu k}{k(2 + 2\nu) + \frac{4}{3}(3k - 6\nu k)}} = \sqrt{\dfrac{3k - 6\nu k}{2k + 2\nu k + 4k - 8\nu k}}$

$= \sqrt{\dfrac{3k - 6\nu k}{6k - 6\nu k}} = \sqrt{\dfrac{\cancel{3k}(1 - 2\nu)}{\cancel{6k}(1 - \nu)}} = \sqrt{\dfrac{1 - 2\nu}{2 - 2\nu}}$

$\dfrac{\beta}{\alpha} = \sqrt{\dfrac{1 - 2\nu}{2 - 2\nu}}$ $(0 \le \nu \le 0.5)$

$\boxed{\beta_{max}} = \sqrt{\dfrac{1 - 2(0)}{2 - 2(0)}} = \dfrac{\sqrt{2}}{2} \approx \boxed{0.707\,\alpha}$

$\boxed{\beta_{min}} = \sqrt{\dfrac{1 - 2(\frac{1}{2})}{2 - 2(\frac{1}{2})}} = \sqrt{\dfrac{1 - 1}{2 - 1}} = \boxed{0}$

S-wave reaches max of 0.707 of P-wave speed and minimum when P-wave doesn't exist.

3) $\phi = A e^{i(\vec{k_p} \cdot \vec{r} - wt)}$

(a) $= A e^{i(k_{p_x} x + k_{p_y} y + k_{p_z} z - wt)}$

$\nabla = \frac{\partial}{\partial x}\hat{x} + \frac{\partial}{\partial y}\hat{y} + \frac{\partial}{\partial z}\hat{z}$

$\Rightarrow \frac{\partial}{\partial x}\left(A e^{i(k_{p_x} x + k_{p_y} y + k_{p_z} z - wt)}\right)\hat{x} = i k_{p_x} \hat{x} A e^{i(\vec{k_p} \cdot \vec{r} - wt)}$

$\Rightarrow \frac{\partial}{\partial y}\left(A e^{i(k_{p_x} x + k_{p_y} y + k_{p_z} z - wt)}\right)\hat{y} = i k_{p_y} \hat{y} A e^{i(\vec{k_p} \cdot \vec{r} - wt)}$

$\Rightarrow \frac{\partial}{\partial z}\left(A e^{i(k_{p_x} x + k_{p_y} y + k_{p_z} z - wt)}\right)\hat{z} = i k_{p_z} \hat{z} A e^{i(\vec{k_p} \cdot \vec{r} - wt)}$

$\Rightarrow \nabla\phi = \left(A i k_{p_x}\hat{x} + A i k_{p_y}\hat{y} + A i k_{p_z}\hat{z}\right) e^{i(\vec{k_p} \cdot \vec{r} - wt)}$

$\Rightarrow \nabla\phi = iA\left(k_{p_x}\hat{x} + k_{p_y}\hat{y} + k_{p_z}\hat{z}\right) e^{i(\vec{k_p} \cdot \vec{r} - wt)}$

$\Rightarrow \boxed{\nabla\phi = iA\vec{k_p}\, e^{i(\vec{k_p} \cdot \vec{r} - wt)}}$

(3) (b) $\vec{\Psi} = \vec{B} e^{i(\vec{k_s} \cdot \vec{r} - \omega t)} = \hat{x} B_x e^{i(\vec{k_s} \cdot \vec{r} - \omega t)} + B_y \hat{y} e^{i(\vec{k_s} \cdot \vec{r} - \omega t)} + B_z \hat{z} e^{i(\vec{k_s} \cdot \vec{r} - \omega t)}$

$\Rightarrow \vec{u_s} = \nabla \times \vec{\Psi}$

$$= \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \partial / \partial z \\ B_x e^{i(\vec{k_s} \cdot \vec{r} - \omega t)} & B_y e^{i(\vec{k_s} \cdot \vec{r} - \omega t)} & B_z e^{i(\vec{k_s} \cdot \vec{r} - \omega t)} \end{vmatrix}$$

$$= \left[\frac{\partial}{\partial y} \left(B_z e^{i(\vec{k_s} \cdot \vec{r} - \omega t)} \right) - \frac{\partial}{\partial z} \left(B_y e^{i(\vec{k_s} \cdot \vec{r} - \omega t)} \right) \right] \hat{x}$$

$$- \left[\frac{\partial}{\partial x} \left(B_z e^{i(\vec{k_s} \cdot \vec{r} - \omega t)} \right) - \frac{\partial}{\partial z} \left(B_x e^{i(\vec{k_s} \cdot \vec{r} - \omega t)} \right) \right] \hat{y}$$

$$+ \left[\frac{\partial}{\partial x} \left[B_y e^{i(\vec{k_s} \cdot \vec{r} - \omega t)} \right] - \frac{\partial}{\partial y} \left(B_x e^{i(\vec{k_s} \cdot \vec{r} - \omega t)} \right) \right] \hat{z}$$

NB: $\frac{\partial}{\partial x} \left(B_z e^{i(\vec{k_s} \cdot \vec{r} - \omega t)} \right) = B_z i k_{sx} e^{i(\vec{k_s} \cdot \vec{r} - \omega t)}$

$\frac{\partial}{\partial x} \left(B_y e^{i(\vec{k_s} \cdot \vec{r} - \omega t)} \right) = B_y i k_{sx} e^{i(\vec{k_s} \cdot \vec{r} - \omega t)}$

$\frac{\partial}{\partial y} \left(B_z e^{i(\vec{k_s} \cdot \vec{r} - \omega t)} \right) = B_z i k_{sy} e^{i(\vec{k_s} \cdot \vec{r} - \omega t)}$

$\frac{\partial}{\partial y} \left(B_x e^{i(\vec{k_s} \cdot \vec{r} - \omega t)} \right) = B_x i k_{sy} e^{i(\vec{k_s} \cdot \vec{r} - \omega t)}$

$\frac{\partial}{\partial z} \left(B_y e^{i(\vec{k_s} \cdot \vec{r} - \omega t)} \right) = B_y i k_{sz} e^{i(\vec{k_s} \cdot \vec{r} - \omega t)}$

$\frac{\partial}{\partial z} \left(B_x e^{i(\vec{k_s} \cdot \vec{r} - \omega t)} \right) = B_x i k_{sz} e^{i(\vec{k_s} \cdot \vec{r} - \omega t)}$

$$\Rightarrow \left[B_z i k_{s_y} e^{i(\vec{k_s}\cdot\vec{r}-\omega t)} - B_y i k_{s_z} e^{i(\vec{k_s}\cdot\vec{r}-\omega t)} \right]\hat{x}$$

$$- \left[B_z i k_{s_x} e^{i(\vec{k_s}\cdot\vec{r}-\omega t)} - B_x i k_{s_z} e^{i(\vec{k_s}\cdot\vec{r}-\omega t)} \right]\hat{y}$$

$$+ \left[B_y i k_{s_x} e^{i(\vec{k_s}\cdot\vec{r}-\omega t)} - B_x i k_{s_y} e^{i(\vec{k_s}\cdot\vec{r}-\omega t)} \right]\hat{z}$$

$$= i e^{i(\vec{k_s}\cdot\vec{r}-\omega t)} \Big[\left[B_z k_{s_y} - B_y k_{s_z} \right]\hat{x} - \left[B_z k_{s_x} - B_x k_{s_z} \right]\hat{y}$$

$$+ \left[B_y k_{s_x} - B_x k_{s_y} \right]\hat{z} \Big]$$

$$= \left[\vec{k_s} \times \vec{B} \right] i e^{i(\vec{k_s}\cdot\vec{r}-\omega t)}$$

$$\boxed{\therefore \quad \vec{u_s} = i\vec{k_s} \times \vec{B} e^{i(\vec{k_s}\cdot\vec{r}-\omega t)}}$$