FILL IN THE BLANK RING REVIEW

- The kernel of a ring homomorphism is a(n) <u>ideal</u>.
- The image of a ring homomorphism is a(n) <u>subring</u>.
- Use the candidates below to fill in the following:
  <u>field</u> $\implies$ <u>Euclidean domain</u> $\implies$ <u>PID</u> $\implies$ <u>UFD</u> $\implies$ <u>domain</u>.
  - domain
  - Euclidean domain
  - field
  - PID
  - UFD
- In a ring, unit <u>$\implies$ not</u> zerodivisor.
- A commutative ring has (exact) division by nonzero elements if it is a <u>field</u>.
- A commutative ring has cancellation by nonzero elements if it is a <u>domain</u>.
- A commutative ring has division with remainder by nonzero elements if it is a <u>Euclidean domain</u>.
- In[1] a commutative ring, $(a) \subseteq (b) \iff \underline{b \mid a}$.
- In[1] a commutative ring, $(a) = (b) \iff \underline{a \mid b \text{ and } b \mid a}$.
- In[2] a <u>domain</u>, $(a) = (b) \iff$ <u>associates</u>.
- In a <u>UFD</u>, GCDs exist.
- In a <u>PID</u>, the GCD of two elements is a linear combination of them.
- In a <u>domain</u>, GCDs are unique <u>up to associates</u>.
- In a <u>commutative ring</u>, maximal ideal $\implies$ prime ideal.
- In a <u>PID</u>, (nonzero) prime ideal $\implies$ maximal ideal.
- In a commutative ring R, I is a maximal ideal $\iff R/I$ <u>is a field</u>.
- In a commutative ring R, I is a prime ideal $\iff R/I$ <u>is a domain</u>.
- In a <u>domain</u>, prime element $\implies$ irreducible element.
- In a <u>UFD</u>, irreducible element $\implies$ prime element.

[1] Express in terms of divides.
[2] Express in terms of a word.