LIMITS

(1) Use the definition to show that $\lim_{x\to 2} |2x-4| = 0$.

From last time:

Theorem 18.4: Let $f(x)$ be a function and let a be a real number. Let $r > 0$ be a positive real number such that f is defined at every point of $\{x \in \mathbb{R} \mid 0 < |x-a| < r\}$. Let L be any real number.

Then $\lim_{x\to a} f(x) = L$ if and only if for every sequence $\{x_n\}_{n=1}^{\infty}$ that converges to a and satisfies $0 < |x_n - a| < r$ for all n, we have that the sequence $\{f(x_n)\}_{n=1}^{\infty}$ converges to L.

Corollary 18.5: Let f be a function and a and L be real numbers. Suppose that the domain of f is all of $\mathbb{R}$ or $\mathbb{R} \setminus \{a\}$. Then $\lim_{x\to a} f(x) = L$ if and only if for every sequence $\{x_n\}_{n=1}^{\infty}$ that converges to a such that $x_n \neq a$ for all n, we have that the sequence $\{f(x_n)\}_{n=1}^{\infty}$ converges to L.

From the homework:

Example (HW): If $m, b, a \in \mathbb{R}$, then $\lim_{x\to a} mx + b = ma + b$. In particular, $\lim_{x\to a} b = b$ and $\lim_{x\to a} x = a$.

(2) Use Corollary 18.5 to show that $\displaystyle\lim_{x\to 0} \cos\left(\frac{1}{x}\right)$ does not exist.

Suggestion: Let $f(x) = \cos(\frac{1}{x})$ and suppose $\lim_{x\to 0} f(x) = L$. Find sequences $\{x_n\}_{n=1}$ and $\{y_n\}_{n=1}$ such that
- $\{x_n\}_{n=1}$ and $\{y_n\}_{n=1}$ both converge to 0,
- $f(x_n) = 1$ for all n, and
- $f(y_n) = -1$ for all n.

What does this say about L?

Theorem 19.1: Suppose f and g are two functions and that a is a real number, and assume that

$$\lim_{x\to a} f(x) = L \text{ and } \lim_{x\to a} g(x) = M$$

for some real numbers L and M. Then
(1) $\lim_{x\to a}(f(x) + g(x)) = L + M$.
(2) For any real number c, $\lim_{x\to a}(c \cdot f(x)) = c \cdot L$.
(3) $\lim_{x\to a}(f(x) \cdot g(x)) = L \cdot M$.
(4) If, in addition, we have that $M \neq 0$, then $\lim_{x\to a}(f(x)/g(x)) = L/M$.

(3) Use Theorem 19.1 plus Example HW to compute $\displaystyle\lim_{x\to 2} \frac{3x^2 - x + 2}{x+3}$.

(4) Use Theorem 18.4 to deduce Theorem 19.1 part (1) from our Theorem 10.2 on algebra and sequences.