Making sense of if then statements and quantifier statements.

(1) For each of the following statements, write its contrapositive and its converse. Is the original/contrapositive/converse true or false? Explain why (but don't prove them). For all of the statements below, a, b are real numbers.
  (a) If a is irrational, then $1/a$ is irrational.
  (b) If a and b are irrational, then ab is irrational.
  (c) If $a > 3$, then $a^2 > 9$.

(2) Rewrite each statement with symbols in place of quantifiers, and write its negation. Is the original statement true or false? Explain why (but don't prove them).
  (a) There exists $x \in \mathbb{Q}$ such that $x^2 = 2$.
  (b) For all $x \in \mathbb{R}$, $x^2 > 0$.
  (c) For all $x \in \mathbb{R}$ such that $x \neq 0$, $x^2 > 0$.
  (d) For all $x \in \mathbb{R}$, there exists $y \in \mathbb{R}$ such that $x < y$.
  (e) There exists $x \in \mathbb{R}$ such that for all $y \in \mathbb{R}$, $x < y$.

Proving if then statements and quantifier statements.

(3) Let x and y be real numbers. Use the axioms of $\mathbb{R}$ to prove[1] that $x \geq y$ if and only if $-y \geq -x$.

(4) Let x be a real number. Show that if x^2 is irrational, then x is irrational.

(5) Let x be a real number. Use the axioms of $\mathbb{R}$ and facts we have proven in class to show that if there exists a real number y such that $xy = 1$, then $x \neq 0$.

(6) Prove that[2] for all $x \in \mathbb{R}$ such that $x \neq 0$, we have $x^2 \neq 0$.

(7) Prove that there exists some $x \in \mathbb{R}$ such that for every $y \in \mathbb{R}$, $xy = x$.

(8) Prove that (2d) is true and (2e) is false. (You are free to use (10) below in your proof, even if you didn't prove it.)

(9) Let $S \subseteq \mathbb{R}$ be a set of real numbers. Apply your results above to prove that if for every $x \in S$, x^2 is irrational, then for every $y \in S$, y is irrational.

(10) Prove that $1 > 0$.

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[1] Hint: You may want to add something to both sides.

[2] Hint: Use (**??**).