SUBSEQUENCES §2.3

Use any of the trig facts below to answer the following questions:

(1) **Prove or disprove:** Is $\{\cos(\pi n)\}_{n=1}^{\infty}$ a subsequence of $\{\cos(n)\}_{n=1}^{\infty}$?

(2) **Prove or disprove:** The sequence $\{\cos(n)\}_{n=1}^{\infty}$ has a convergent subsequence.

(3) **Prove or disprove:** The sequence $\{\cos(n)\}_{n=1}^{\infty}$ has a constant subsequence.

(4) **Prove or disprove:** The sequence $\{\cos(n)\}_{n=1}^{\infty}$ has a subsequence that converges to some $x > 1$.

- $-1 \leq \cos(x) \leq 1$ for all $x \in \mathbb{R}$
- $\cos(x) = 1 \iff x \in 2\pi\mathbb{Z}$
- $\cos(x) = 0 \iff x \in \frac{\pi}{2} + \pi\mathbb{Z}$
- $\cos(x) = -1 \iff x \in \pi + 2\pi\mathbb{Z}$
- $\pi \notin \mathbb{Q}$
- $\cos(x) = \cos(y) \iff x - y \in 2\pi\mathbb{Z}$ or $x + y \in 2\pi\mathbb{Z}$