Discussion Questions

The coins that are used by a country are made of a low quality metal and become defective on a regular basis. Suppose that right now there are 20 million coins in circulation, of which half are defective. Every week, about one third of the good coins in circulation become bad coins, and one fifth of the bad coins fall apart and disappear from circulation. To make sure there are enough coins, for every (bad) coin that gets destroyed, it issues a *two* good replacement coins.

The government wants to know: Does the number of bad coins tend to zero? If not, what happens to the ratio of bad coins?

(1) We want to set up a linear system to model the number of good coins and the number of defective coins as time progresses. Introduce variables to keep track of the quantities we are interested in. What is the independent variable and what are the dependent variables? What are the units for each?
(2) What factors in the story affect the number of good coins in circulation? What factors in the story affect the number of bad coins in circulation? Can you express each in terms of the number of good coins / bad coins at a given time?
(3) Express the previous part as a system of differential equations.
(4) We also need an initial condition. Write it down.

Let's do an experiment to test our model. Each coin you've been given represents a million coins. Some represent good coins and some represent bad. Every day, take a third of the coins, and replace the good ones you collected with bad ones. Then take a fifth of the coins, throw away the bad ones you collected, and add back in twice as many of good coins.

(1) Discuss whether your model for the previous situation is relevant to this experiment. What aspects fit the story well, and what ones don't?
(2) Run the experiment, keeping track of the number of good coins and bad coins each day. How well does your data fit the model?

Now let's solve this.

(1) Solve the linear system.
(2) Solve the initial value problem. Use your solution to answer the questions we started with.
(3) Draw a phase portrait for the linear system. Use it to address the questions we started with as initial conditions vary.

Now consider the linear system

$$\begin{cases} x' = \frac{1}{4}x - \frac{1}{4}y \\ y' = \frac{1}{2}x - \frac{1}{4}y \end{cases} .$$

First use the coins to do an experiment to test what happens with the linear system starting with $x = 10$ and $y = 10$.

Now try to solve the system using our techniques. What do you observe?