ASSIGNMENT #2: DUE THURSDAY, SEPTEMBER 12 AT 7PM

This problem set is to be turned in on Canvas. You may reference any result or problem from our worksheets or lectures, unless it is the fact to be proven! You are encouraged to work with others, but you should understand everything you write. Please consult the class website for acceptable/unacceptable resources for the problem sets.

(1) Let S be a subset of $\mathbb{R}$ and T be a subset of S. Prove that if S is bounded above then T is also bounded above.

(2) Prove that if S is a subset of $\mathbb{R}$ that is bounded above, then S has infinitely many upper bounds.

(3) Given a subset S of $\mathbb{R}$, a **lower bound** for S is a real number z such that $z \leq s$ for all $s \in S$. We say S is **bounded below** if S has at least one lower bound. Given a subset S of $\mathbb{R}$, define a new subset $-S$ by

$$-S = \{x \in \mathbb{R} \mid x = -s \text{ for some } s \in S\}.$$

For example, $-\{-2, -1, 1, 3\} = \{-3, -1, 1, 2\}$. Prove[1] that S is bounded below if and only if $-S$ is bounded above.

DEFINITION: Suppose S is a subset of $\mathbb{R}$. A real number y is called the **infimum** (also known as greatest lower bound) of S if

- y is a lower bound for S, and
- if z is any lower bound for S then $z \leq y$.

(4) Let S be a subset of $\mathbb{R}$.
  (a) Show that the open interval $(1, 2)$ does not have a minimum element.
  (b) Show that if y is the minimum of S, then y is the infimum of S.
  (c) Show that if ℓ is the infimum of S and $\ell \in S$, then ℓ is the minimum of S.

[1] Warning: It is easy to get things out of order here. For each "if then" direction, unpackage the hypothesis, and use that to establish the conclusion.