

THE $\varepsilon - \delta$ GAME. §3.1

(1) Play the $\varepsilon - \delta$ game. Rules:

- Player 0 starts by graphing a function f (like a familiar one from calculus) and specifies an x -value a and a y -value L that (based on previous calculus knowledge) they think makes $\lim_{x \rightarrow a} f(x) = L$ **true**.

The graph should be large.

- Player 1 choses an ε . This is how close we would like our function to be to L . Thus, ε goes up and down from L . Draw horizontal dotted lines with y -values $L - \varepsilon$ and $L + \varepsilon$. The ε should be large enough for people to see and have room to work in the picture.

- Player 2 must find a δ such that every $x \in (a - \delta, a) \cup (a, a + \delta)$ is

- in the domain of f , and
- has an output $f(x)$ within $(L - \varepsilon, L + \varepsilon)$.

Draw vertical dotted lines for the x -values $a - \delta$ and $a + \delta$.

Everyone in the team can assist player 2!

(2) Repeat with another function.

(3) Now play except player 0 graphs an f with an L that makes $\lim_{x \rightarrow a} f(x) = L$ **false**.

(4) Now play except player 0 “chooses” $f(x) = \frac{2x - 2}{x - 1}$, $a = 1$, and $L = 2$. Play with actual numbers ε and δ .