CONVERGENCE OF A SEQUENCE §2.1

DEFINITION: A sequence $\{a_n\}_{n=1}^{\infty}$ **converges** to a real number L provided for every real number $\varepsilon > 0$, there is a real number N such that $|a_n - L| < \varepsilon$ for all natural numbers n such that $n > N$.

(1) Consider the sequence $\{a_n\}_{n=1}^{\infty} = \{2 - \frac{50}{n}\}_{n=1}^{\infty}$.
  (a) Write out the first ten terms of this sequence: use a calculator to write the decimal expansions for the numbers.
  (b) Can you find a point N such that every term in the sequence after N (that is, every a_n for $n > N$) is within 20 of 2 (that is, $|a_n - 2| < 20$)?
  (c) Can you find a point N such that every term in the sequence after N (that is, every a_n for $n > N$) is within 10 of 2 (that is, $|a_n - 2| < 10$)?
  (d) Can you find a point N such that every term in the sequence after N (that is, every a_n for $n > N$) is within 1 of 2 (that is, $|a_n - 2| < 1$)?
  (e) If ε is a positive number, can you find a point N such that every term in the sequence after N (that is, every a_n for $n > N$) is within ε of 2 (that is, $|a_n - 2| < \varepsilon$)?
  (f) Write a proof that the sequence $\{a_n\}_{n=1}^{\infty} = \{2 - \frac{50}{n}\}_{n=1}^{\infty}$ converges to 2.

(2) Prove that the sequence $\left\{\frac{1}{\sqrt{n}}\right\}_{n=1}^{\infty}$ converges to 0.

(3) Let $\{a_n\}_{n=1}^{\infty}$ be a sequence. Suppose we know that $\{a_n\}_{n=1}^{\infty}$ converges to 1. Prove[1] that there is a natural number $n \in \mathbb{N}$ such that $a_n > 0$.

(4) Prove or disprove: The sequence $\left\{\frac{n+1}{2n}\right\}_{n=1}^{\infty}$ converges to 0.

[1]Hint: If you know that a "for all" statement is true, you can choose any specific value for that variable and get a more specific true statement.