TRUE or FALSE. Justify.

(1) Every bounded sequence is a convergent sequence.

(2) To prove that the formula
$$1 + \frac{1}{2} + \frac{1}{4} + \cdots + \frac{1}{2^{n-1}} = 2 - \frac{1}{2^{n-1}}$$
is true for every natural number $n \in \mathbb{N}$ by the Principle of Mathematical Induction, it is logically sufficient to show that

- $1 = 2 - \frac{1}{2^{1-1}}$, and
- $1 + \frac{1}{2} + \frac{1}{4} + \cdots + \frac{1}{2^{k-1}} \neq 2 - \frac{1}{2^{k-1}}$ for some natural number k.

(3) To prove that a sequence $\{a_n\}_{n=1}^{\infty}$ is bounded above by 10 by the Principle of Mathematical Induction, it is logically sufficient to show that

- $a_1 < 10$, and
- if $a_k < 10$ for some natural number k, then $a_{k+1} < 10$.

(4) Every sequence has a bounded subsequence.

(5) If a sequence has a divergent subsequence, then it diverges.

(6) Every Cauchy sequence converges.

(7) Every convergent sequence is Cauchy.

(8) There is a sequence without any monotone subsequence.

(9) If $\{a_n\}_{n=1}^{\infty}$ is Cauchy, then the sequence $\{a_n - a_{2n}\}_{n=1}^{\infty}$ converges to 0.

(10) The limit of $f(x) = \frac{x^2-2x+3}{x-7}$ as x approaches 3 is $-3/2$.

(11) The function $f(x) = \cos(1/x)$ has a limit as x approaches 0.

(12) If $\lim_{x\to-1} f(x)/g(x) = 1$, then $\lim_{x\to-1} f(x) = \lim_{x\to-1} g(x)$.

(13) If $\lim_{x\to-1} f(x)$ and $\lim_{x\to-1} g(x)$ both exist, then $\lim_{x\to-1} f(x)g(x)$ exists.

(14) If $\lim_{x\to-1} f(x)$ and $\lim_{x\to-1} g(x)$ both exist, then $\lim_{x\to-1} f(x)/g(x)$ exists.

(15) If $\lim_{x\to2} f(x) = 3$ and $\lim_{x\to1} g(x) = 2$, then $\lim_{x\to1}(f \circ g)(x) = 3$.

(16) If $\lim_{x\to 0} f(x) = 3$, then the sequence $\{f(1/n)\}_{n=1}^{\infty}$ converges to 3.

(17) If f is a function defined on $\mathbb{R}$ and $\{f(1/n)\}_{n=1}^{\infty}$ converges to 3, then $\lim_{x\to 0} f(x) = 3$.

(18) If f is a function defined on $\mathbb{R}$, $\{f(1/n)\}_{n=1}^{\infty}$ converges to 3, and $\lim_{x\to 0} f(x) = L$, then $L = 3$.

(19) If $\{a_n\}_{n=1}^{\infty}$ converges to 1 and $\{b_n\}_{n=1}^{\infty}$ converges to -2, then $\{a_{3n-1}b_n - b_{n^2}/4\}_{n=1}^{\infty}$ converges to $-5 = (3 \cdot 1 - 1)(-2) - (-2)^2/4$.

(20) The sequence $a_n = \sqrt{\pi n - \lfloor \pi n \rfloor}$ has a convergent subsequence, where $\lfloor x \rfloor$ denotes the largest integer that is smaller than x.

(21) For a given sequence, there are at most two real numbers that occur as limits of subsequences of the sequence.

(22) For a given convergent sequence, there are at most two real numbers that occur as limits of subsequences of the sequence.

(23) The function $f(x) = \frac{x^2-2x+3}{x-7}$ is continuous on $(7, \infty)$.

(24) The function $f(x) = \frac{x^2-2x+3}{x-7}$ is continuous on $\mathbb{R}$.

(25) The function $f(x) = \sqrt{|x^3 - 7x + 1|}$ is continuous on $\mathbb{R}$.

(26) If $\lim_{x\to a} f(x)$ exists, then $f(x)$ is continuous at $x = a$.

(27) There is some $c \in [-1, 0]$ such that $c^5 + c^3 + 1 = 0$.

(28) There is some $c \in (-1, 0)$ such that $c^5 + c^3 + 1 = 0$.

(29) If f is continuous on $\mathbb{R}$ and $a < b$, and y is between $f(a)$ and $f(b)$, then there is exactly one $c \in [a, b]$ such that $f(c) = y$.

(30) If f is defined on $\mathbb{R}$ and f has the property that for every $a < b$ if y is between $f(a)$ and $f(b)$ then there is some $c \in [a, b]$ such that $f(c) = y$, then f is continuous on $\mathbb{R}$.