LIMITS §3.1

(1) Let $b \in \mathbb{R}$ be a real number. Use the $\varepsilon - \delta$ definition of limit to prove that for any $a \in \mathbb{R}$,

$$\lim_{x \to a} b = b.$$

(2) Let $m, b \in \mathbb{R}$ be real numbers. Use[1] the $\varepsilon - \delta$ definition of limit to prove that for any $a \in \mathbb{R}$,

$$\lim_{x \to a} mx + b = ma + b.$$

(3) In this problem we will prove that the function $f(x) = \dfrac{1}{x-3}$ does not have a limit as x approaches 3.

(a) What proof technique should we use? Write down the start of the proof.

(b) If $\lim_{x \to a} f(x) = L$ then for any positive number ε that we choose, we get a more specific true statement as a consequence of the definition. Write down what statement we get when $\varepsilon = 1$.

(c) Explain why there exists some real number x such that $3 < x < \min\{4, 3 + \delta\}$.

(d) Use the number x from the previous part to show that $L > 0$.

(e) Do something else to show that $L < 0$ and conclude the proof.

(4) Prove that the limit of f as x approaches a, if it exists, is unique.

(5) Let

$$f(x) = \begin{cases} 1 & \text{if } x \in \mathbb{Q} \\ 0 & \text{if } x \notin \mathbb{Q}. \end{cases}$$

Use the definition to show that $\lim_{x \to a} f(x)$ does not exist for any real number a.

[1] Suggestion: You may want to consider the case where $m = 0$ separately..