WORKSHEET PREVIEWS FOR MATH 905

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INTRODUCTION

What am I? The majority of this document consists of the 1–2 page daily quick summaries that you should read before each class. These will include some reminders of things from previous algebra courses that we will use, as well as the statements of definitions and theorems we will encounter in class, so that we aren't just wasting class time reading a definition or theorem for the first time. We will not follow any textbook directly, but most of the material will overlap with the recommended text Atiyah-MacDonald and Grifo's Fall 2022 905 notes, the latter of which is available here:

`https://eloisagrifo.github.io/Teaching/ca1/CA1notes.pdf`

Each course preview references the relevant sections of the sources in this case. Some previews also have a "Just for fun" at the end: this is either an open question or easily stated fact requiring deeper techniques. This part of the reading is optional and can be skipped if you don't like fun.

Mathematical ground rules. In this class, all rings are commutative with $1 \neq 0$, and all modules are unital, meaning $1m = m$ for all $m \in M$. We are assuming as background knowledge the content covered in the first year algebra sequence Math 817–818.

Using these worksheets.

- To complete a problem on a worksheet means to discuss as a group until every member of the group understands the solution. I envision solving a "Prove" or "Show that" problem as meaning to know how to fill in all of the details of a proof (though you might not find it practical to write out a full proof of everything starting from ZFC), whereas an "Explain" or "Discuss" might not require as rigorous a solution or might not even be a completely precise question. If you do not understand your solution or are unsure of something, let your group know: they are probably missing something or could understand some detail better. Conversely, if someone in your group doesn't understand the solution, you should thank them for the opportunity to understand the problem better, as you may have missed something, or you might understand better by explaining your thoughts if you think you haven't.
- The worksheets have some problems numbered in bold **(1)**, some in standard font (2), and some in italics *(3)*. Those marked in bold **(1)** you should think of as mandatory, either in class, or after class if you didn't get to them. Those in standard font (2) are recommended. Those in italics *(3)* are somewhat more for adventure seekers.
- As noted above, the assumed background is Math 817–818. If you've taken a Homological Algebra or Commutative Algebra 2 course or a reading on related topics like Gröbner bases, you might find that some questions are an easy consequence of some fact about faithfully flat modules, Ext-modules, regular sequences, or regular rings. You should feel free to enjoy your knowledge in such cases, but every problem has a solution only using material the background sequence, and you should find a solution of that type: this is both so that you develop mastery of the notions of basic commutative algebra and to avoid any logical circularities!

Why are you doing this to me? Math is learned by working through proofs and examples, not by watching someone else do the work. I could tell you about all of the interesting commutative algebra I know, and I could mix it in with funny anecdotes and obscure puns, but my algebra will never be your own until you do it. So we will just skip the step where I read to you: you know how to read anyway. This style of class may stretch our comfort zone more than a conventional lecture, but it's a much better approximation of doing research and writing a thesis than the latter.