

You can use any of the trig facts below to answer the following questions.

- (1) **Explain but don't prove:** Is $\{\cos(\pi n)\}_{n=1}^{\infty}$ a subsequence of $\{\cos(n)\}_{n=1}^{\infty}$?
- (2) **Prove or disprove:** The sequence $\{\cos(n)\}_{n=1}^{\infty}$ has a convergent subsequence.
- (3) **Prove or disprove:** The sequence $\{\cos(n)\}_{n=1}^{\infty}$ has a constant subsequence.
- (4) **Prove or disprove:** The sequence $\{\cos(n)\}_{n=1}^{\infty}$ has a subsequence that converges to some $x > 1$.

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- $-1 \leq \cos(x) \leq 1$ for all $x \in \mathbb{R}$
 - $\cos(x) = 1 \iff x \in 2\pi\mathbb{Z}$
 - $\cos(x) = 0 \iff x \in \frac{\pi}{2} + \pi\mathbb{Z}$

- $\cos(x) = -1 \iff x \in \pi + 2\pi\mathbb{Z}$
- $\pi \notin \mathbb{Q}$
- $\cos(x) = \cos(y) \iff x - y \in 2\pi\mathbb{Z} \text{ or } x + y \in 2\pi\mathbb{Z}$