THEOREMS ABOUT CONVERGENCE WARMUP

Which of the following implications about sequences hold in general? Either mention a relevant theorem or give a counterexample.

(a) monotone $\implies$ convergent
(b) convergent $\implies$ bounded
(c) bounded + decreasing $\implies$ convergent
(d) increasing + convergent $\implies$ bounded
(e) convergent $\implies$ monotone
(f) bounded $\implies$ convergent

DIVERGENCE TO $\pm\infty$

It is sometimes useful to distinguish between sequences like $\{(-1)^n\}_{n=1}^{\infty}$ that diverge because they "oscillate", and sequences like $\{n\}_{n=1}^{\infty}$ that diverge because they "head toward infinity".

(I) In intuitive language, a sequence converges to L if no matter how close we want or sequence to be to L, all values past some point are at least that close. Intuitively, a sequence *diverges to* $+\infty$ if no matter how large we want our sequence to be, all values past some point are at least that large. Write a precise definition for a sequence to diverge to $+\infty$.

(II) Write a precise definition for a sequence to diverge to $-\infty$.

CHECK ANSWERS TO I & II BEFORE CONTINUING

---

(1) Carefully write the logical negation of "$\{a_n\}_{n=1}^{\infty}$ diverges to $+\infty$" in simplified form.

(2) Use the definition to prove that the sequence $\{\sqrt{n}\}_{n=1}^{\infty}$ diverges to $+\infty$.

(3) Prove that if a sequence $\{a_n\}_{n=1}^{\infty}$ diverges to $+\infty$ then it is not bounded above.

(4) Use (3) to show that if a sequence diverges to $+\infty$ then it diverges.

(5) Prove or disprove: If a sequence is not bounded above, then it diverges to $+\infty$.

(6) Prove or disprove: If a sequence diverges to $+\infty$ then it is increasing.

(7) Prove or disprove: If a sequence is increasing and not bounded above, it diverges to ∞.