DEFINITION 12.1: Let $\{a_n\}_{n=1}^{\infty}$ be a sequence.

(1) We say $\{a_n\}_{n=1}^{\infty}$ is **increasing** if for all $n \in \mathbb{N}$ we have $a_n \leq a_{n+1}$.
(2) We say $\{a_n\}_{n=1}^{\infty}$ is **decreasing** if for all $n \in \mathbb{N}$, we have $a_n \geq a_{n+1}$.
(3) We say $\{a_n\}_{n=1}^{\infty}$ is **monotone** if it is either decreasing or increasing.
(4) We say $\{a_n\}_{n=1}^{\infty}$ is **strictly increasing** if for all $n \in \mathbb{N}$, $a_n < a_{n+1}$.

I leave the definition of **strictly decreasing** and **strictly monotone** to your imaginations.

(1) For each of the following sequences which of the following adjectives apply: bounded above, bounded below, bounded, (strictly) increasing, (strictly) decreasing, (strictly) monotone?
   (a) $\{\frac{1}{n}\}_{n=1}^{\infty}$
   (b) The Fibonacci sequence $\{f_n\}_{n=1}^{\infty}$ where $f_1 = f_2 = 1$ and $f_n = f_{n-1} + f_{n-2}$ for $n \geq 3$.
   (c) $\{(-1)^n\}_{n=1}^{\infty}$
   (d) $\{5 + (-1)^n \frac{1}{n}\}_{n=1}^{\infty}$.

(2) Prove or disprove: Every increasing sequence is bounded above.

(3) Prove or disprove: Every increasing sequence is bounded below.

(4) Prove or disprove: Every bounded sequence is convergent.

(5) Suppose that $\{a_n\}_{n=1}^{\infty}$ is a sequence that is bounded above by 1000 and below by -1000. Show that the sequence $\left\{\dfrac{a_n}{n}\right\}_{n=1}^{\infty}$ converges to 0.

   Suggestion: First start the way we always do when showing a sequence converges. Then see if you can use the hypothesis that $\{a_n\}_{n=1}^{\infty}$ is bounded in a useful way.