THEOREM 35.1 (DERIVATIVES AND ALGEBRA): Let f, g be functions that are differentiable at $x = r$, and c be a real number. Then,

(1) $f + g$ is differentiable at $x = r$ and $(f + g)'(r) = f'(r) + g'(r)$;
(2) cf is differentiable at $x = r$ and $(cf)'(r) = cf'(r)$;
(3) fg is differentiable at $x = r$ and $(fg)'(r) = f'(r)g(r) + f(r)g'(r)$.
(4) If in addition $f(r) \neq 0$, then $1/f$ is differentiable at $x = r$ and $\left(\frac{1}{f}\right)'(r) = \frac{-f'(r)}{f^2(r)}$.

(1) Use the Theorem and an appropriate proof technique to prove that, if $f(x) = x^n$, then f is differentiable at any value $x = r$, and $f'(r) = nr^{n-1}$.

(2) Use the Theorem plus the previous problem to compute the derivative of $f(x) = 5x^7 - \sqrt{19}\, x^4$.

(3) Use Parts (3) and (4) or the Theorem to deduce the Quotient Rule.

(4) True or False: If f is not differentiable at $x = 5$ and g is not differentiable at $x = 5$, then $f + g$ is not differentiable at $x = 5$.

(5) True or False: If f is differentiable at $x = 5$ and g is not differentiable at $x = 5$, then fg is not differentiable at $x = 5$.

(6) Prove Part (1) of the Theorem.

(7) Prove Part (2) of the Theorem.

(8) Prove Part (3) of the Theorem.

(9) Prove Part (4) of the Theorem.