CLASSIFYING GROUPS UP TO ISOMORPHISM

(1) Classify all *abelian* groups of order 72 up to isomorphism. For each isomorphism class, give its expression in invariant factor form.

(2) Let $p < q$ be primes.

(a) Show that if p does not divide $q-1$, then any group of order pq is isomorphic to C_{pq} by the following steps:
- Use Sylow's Theorem to count the number of Sylow subgroups.
- Apply the Recognition Theorem for direct products.

(b) Show from that if p does divide $q-1$, then there are exactly two groups of order pq up to isomorphism by the following steps:
- Use Sylow's Theorem to count the number of Sylow subgroups.
- Apply the Recognition Theorem for semidirect products.
- Use an Exercise from class about when two semidirect products are isomorphic.

(3) Let p be a prime integer. Let G be a group of order p^2.

(a) Show[1] that G is abelian.

(b) Classify all groups of order p^2 up to isomorphism.

(4) Let p, q be primes such that $q = p + 2$ and $p \geq 5$. Show that any group of order p^2q^2 is either isomorphic to a cyclic group or a product of two cyclic groups.

[1]Hint: If not, what can you say about $Z(G)$ and $G/Z(G)$?