

DERIVATIVES §4.1

DEFINITION 34.1: Let f be a function and r be a real number. We say that f is **differentiable at r** if f is defined at r and the limit

$$\lim_{x \rightarrow r} \frac{f(x) - f(r)}{x - r}$$

exists. In this case, we call the limit **the derivative of f at r** and write $f'(r)$ for this limit.

- (1) Use the definition of derivative to show that the function $f(x) = x$ is differentiable at any real number r and compute its derivative.
- (2) Use the definition of derivative to show that the function $f(x) = x$ is *not* differentiable at $x = 0$.
- (3) Prove that if f is differentiable at $x = r$ then f is continuous at $x = r$.
- (4) Prove or disprove the converse of the previous statement.
- (5) Prove or disprove: The function $g(x) = \begin{cases} x \sin(1/x) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$ is differentiable at $x = 0$.
- (6) Prove or disprove: The function $h(x) = \begin{cases} x^2 \sin(1/x) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$ is differentiable at $x = 0$.