CONVERGENCE OF A SEQUENCE §2.1

DEFINITION: A sequence $\{a_n\}_{n=1}^{\infty}$ **converges** to a real number L provided for every real number $\varepsilon > 0$, there is a real number N such that $|a_n - L| < \varepsilon$ for all natural numbers n such that $n > N$.

(1) THE ε VS N GAME:
   (a) Player 0 starts by graphing a sequence $\{a_n\}_{n=1}^{\infty}$, and specifies y-value L for which they believe $\{a_n\}_{n=1}^{\infty}$ converge to L. [You can't graph all the values, but make a large graph with a generous number of values.]
   (b) Player 1 choses an ε. This is how close we would like our sequence to be to L. Thus, ε goes up and down from L (corresponding to $|a_n - L| < \varepsilon$). Draw horizontal dotted lines with y-values $L - \varepsilon$ and $L + \varepsilon$. [The ε should be large enough for people to see and have room to work in the picture.]
   (c) Player 2 must find an N such that all of the values of the sequence to the right of N are in $(L - \varepsilon, L + \varepsilon)$. Draw a vertical line for $x = N$. [Everyone in the team can assist player 2!]
   (d) Repeat with the same graph, players 1& 2 switching roles (and a new ε).
   (e) Now do a couple of rounds with a sequence $\{a_n\}_{n=1}^{\infty}$ and an L for which you believe $\{a_n\}_{n=1}^{\infty}$ *does not* converge to L.

(2) Prove that the sequence $\left\{\frac{1}{\sqrt{n}}\right\}_{n=1}^{\infty}$ converges to 0.

(3) Let $\{a_n\}_{n=1}^{\infty}$ be a sequence. Suppose we know that $\{a_n\}_{n=1}^{\infty}$ converges to 1. Prove[1] that there is a natural number $n \in \mathbb{N}$ such that $a_n > 0$.

(4) Prove or disprove: The sequence $\left\{\frac{n+1}{2n}\right\}_{n=1}^{\infty}$ converges to 0.

[1]Hint: If you know that a "for all" statement is true, you can choose any specific value for that variable and get a more specific true statement.