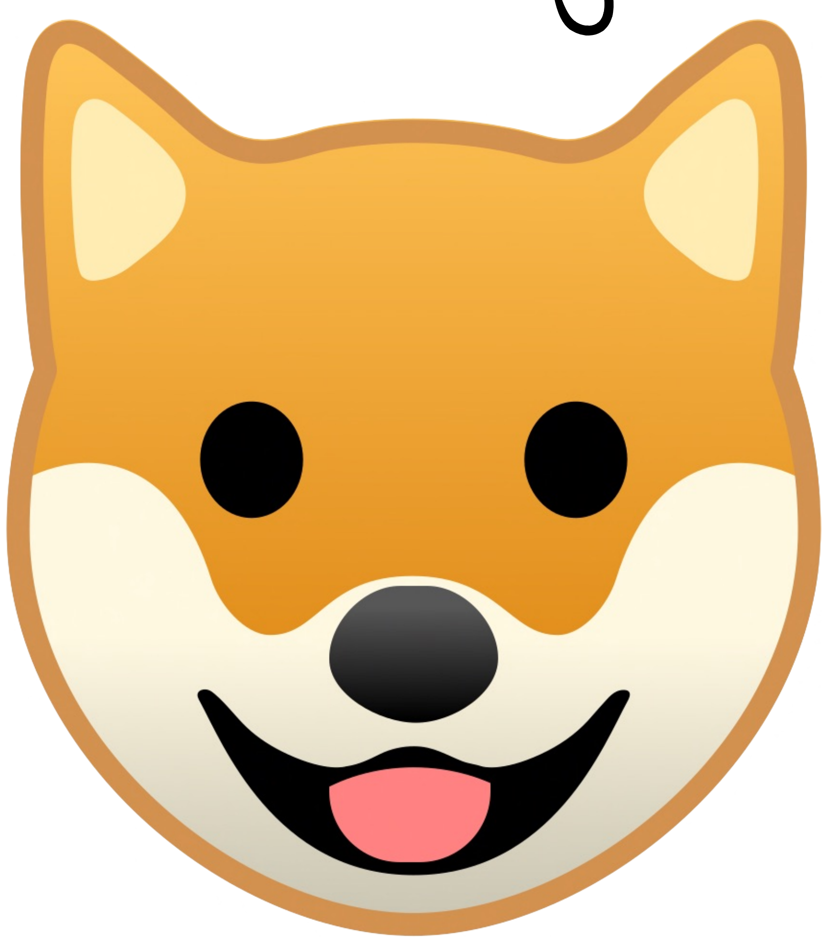
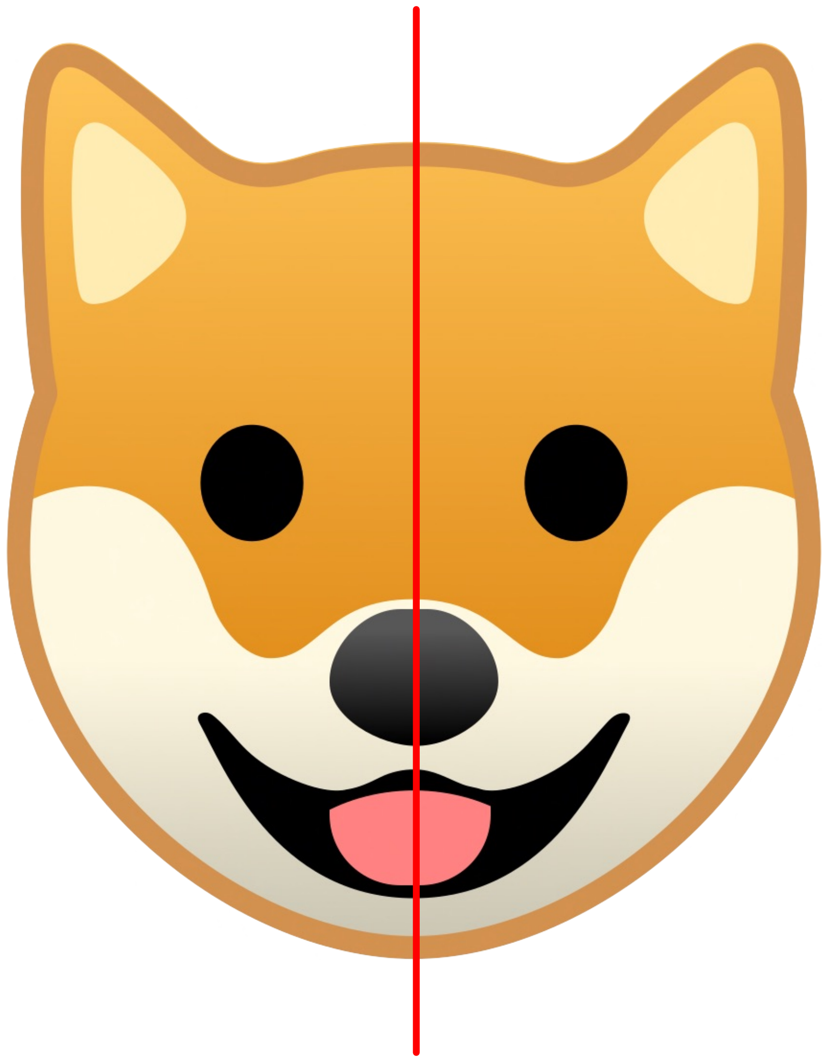
What do we mean when we say a shape is symmetric?

How is a face symmetric?





flipping over the red line
keeps the face the same

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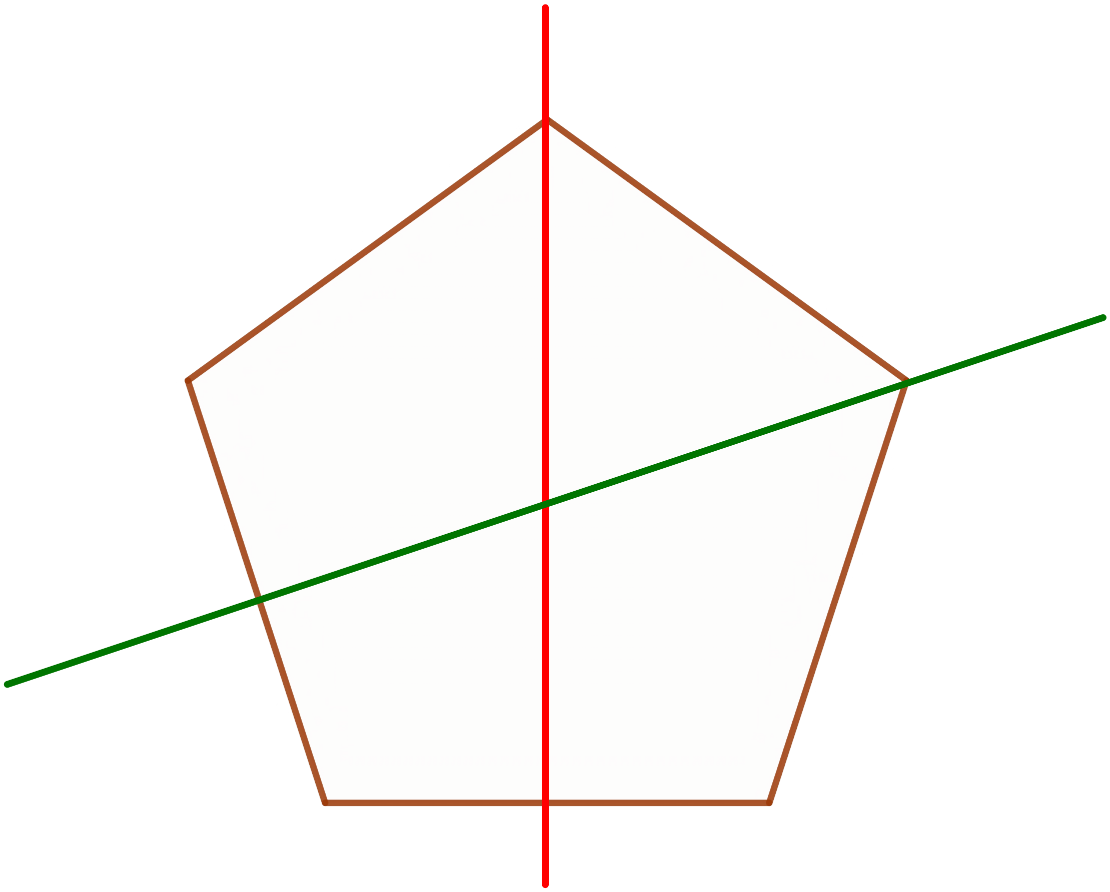
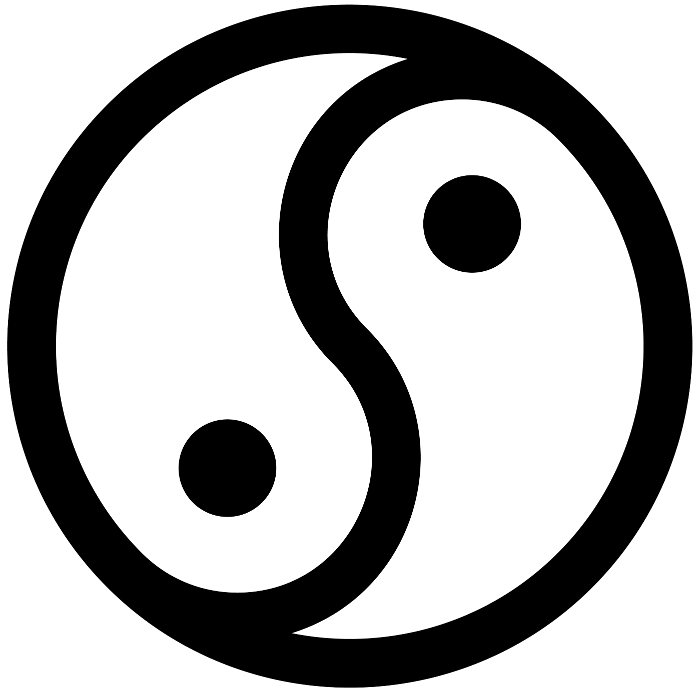
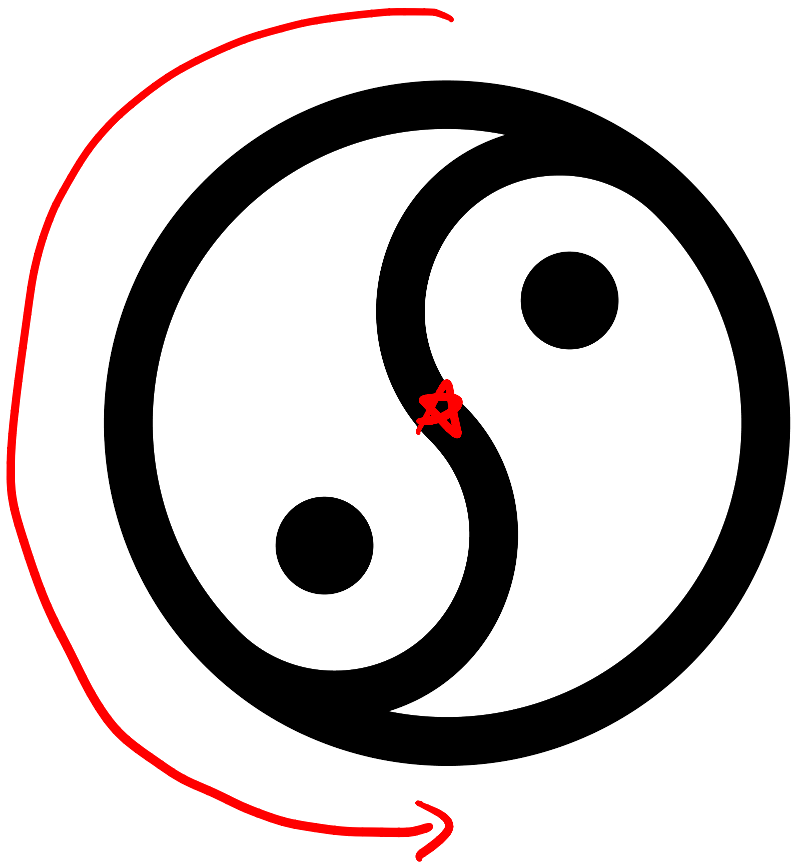
flipping over a line is a <u>reflection</u>
that returns the shape to the
same place



a pentagon is symmetric in multiple ways:
there are multiple <u>reflection symmetries</u>



How is a yinyang symmetric?



spinning halfway around keeps the yinyang the same

---

spinning halfway around is a **rotation** by 180°

---

the yinyang has a 180° **rotation symmetry**

For us, a symmetry of a shape is a transformation (rotation, reflection, dilation, ...) that keeps the shape in place.

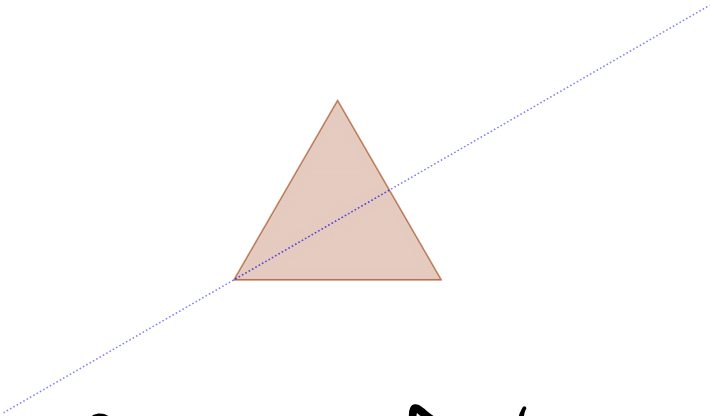
Questions

- Which of the shapes seem most symmetric to you?
  Which would you guess have rotational symmetries? reflection symmetries?

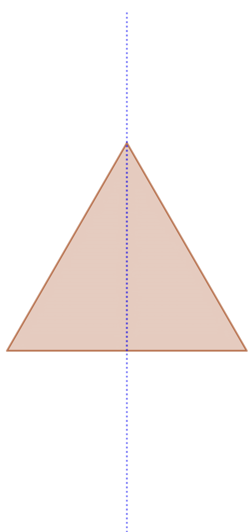
- Find as many symmetries as possible for the shapes using the applet.

- Which shape is most symmetric? Which shape has the most unexpected symmetry?

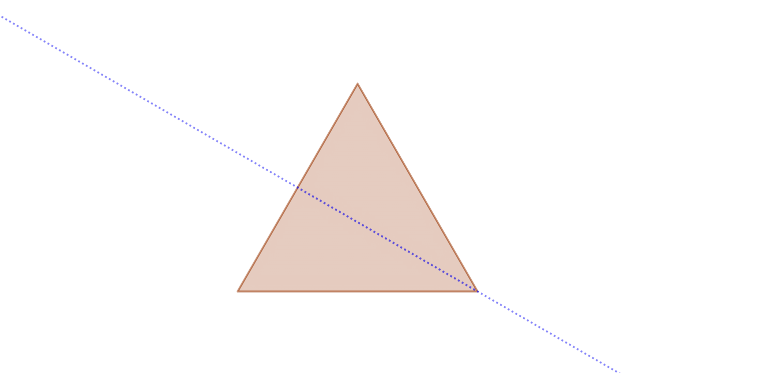
Six symmetries of the triangle



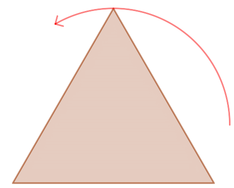
30°-line reflection



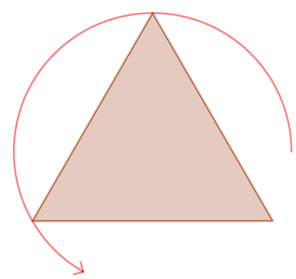
90°-line reflection



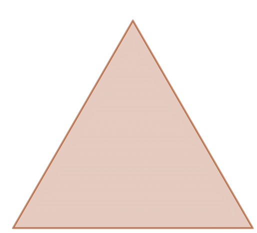
150°-line reflection



120° rotation



240° rotation



do nothing!
(trivial symmetry)

Six symmetries of the hexagoose

60° rotation

120° rotation

180° rotation

240° rotation

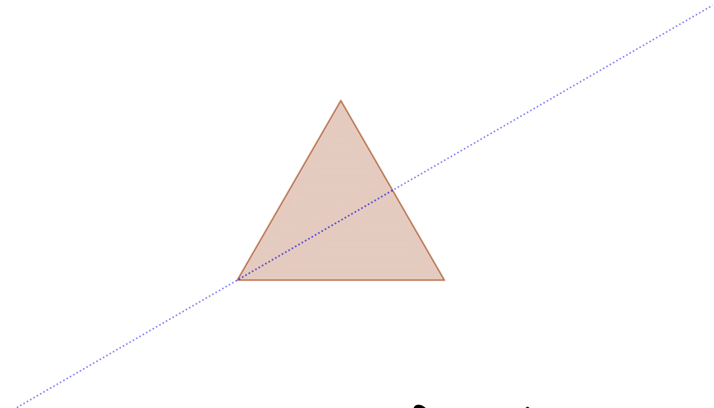
300° rotation

do nothing

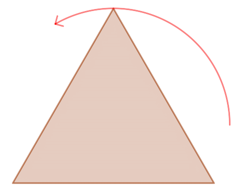
Can you find a shape with:

- a rotational symmetry but no reflection symmetry?
- a reflection symmetry but no rotational symmetry?
- the exact same six symmetries as our triangle that is not the triangle?
- infinitely many rotational symmetries?
- a $60°$-line reflection symmetry and no other nontrivial symmetries?
- a $120°$ rotation symmetry and no other nontrivial symmetry?
- exactly two reflection symmetries and no other nontrivial symmetry?

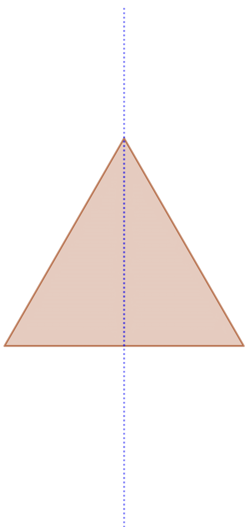
six symmetries of the triangle



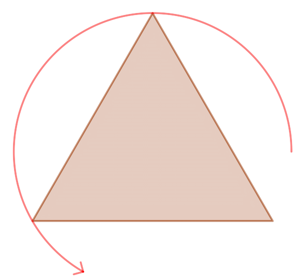
R_1 30°-line reflection



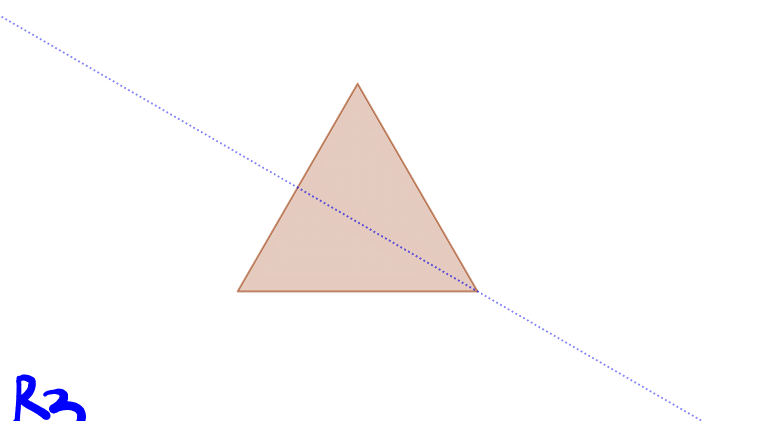
120° rotation T_1



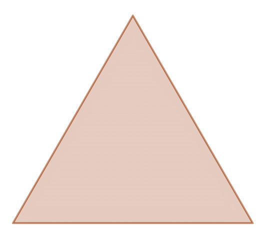
R_2 90°-line reflection



240° rotation T_2



R_3 150°-line reflection



N

do nothing! (trivial symmetry)

If we "do" one symmetry and then "do" another, our shape stays in the same place, so the result is a symmetry of our shape!

Call this *composing* symmetries.

"Times table" for symmetries of hexagoose

G_0 = do nothing

G_1 = rotate 60°

G_2 = rotate 120°

G_3 = rotate 180°

G_4 = rotate 240°

G_5 = rotate 300°

then this

do this

	G_0	G_1	G_2	G_3	G_4	G_5
G_0						
G_1						
G_2						
G_3						
G_4						
G_5						

"Times table" for symmetries of hexagoose

G_0 = do nothing

G_1 = rotate 60°

G_2 = rotate 120°

G_3 = rotate 180°

G_4 = rotate 240°

G_5 = rotate 300°

then this

do this

	G_0	G_1	G_2	G_3	G_4	G_5
G_0	G_0	G_1	G_2	G_3	G_4	G_5
G_1	G_1	G_2	G_3	G_4	G_5	G_0
G_2	G_2	G_3	G_4	G_5	G_0	G_1
G_3	G_3	G_4	G_5	G_0	G_1	G_2
G_4	G_4	G_5	G_0	G_1	G_2	G_3
G_5	G_5	G_0	G_1	G_2	G_3	G_4

"Times table" for symmetries of triangle

N = do nothing

T_1 = rotate $120°$

T_2 = rotate $240°$

R_1 = reflect over $30°$-line

R_2 = reflect over $90°$-line

R_3 = reflect over $150°$-line

then this

do this

	N	T_1	T_2	R_1	R_2	R_3
N						
T_1						
T_2						
R_1						
R_2						
R_3						

"Times table" for symmetries of triangle

N = do nothing
T_1 = rotate $120°$
T_2 = rotate $240°$
R_1 = reflect over $30°$-line
R_2 = reflect over $90°$-line
R_3 = reflect over $150°$-line

then this

do this	N	T_1	T_2	R_1	R_2	R_3
N	N	T_1	T_2	R_1	R_2	R_3
T_1	T_1	T_2	N	R_3	R_1	R_2
T_2	T_2	N	T_1	R_2	R_3	R_1
R_1	R_1	R_2	R_3	N	T_1	T_2
R_2	R_2	R_3	R_1	T_2	N	T_1
R_3	R_3	R_1	R_2	T_1	T_2	N