

## DIVERGENCE TO $+\infty$ AND DIVERGENCE TO $-\infty$ §2.2

DEFINITION 16.1: A sequence **diverges to**  $+\infty$  if for every real number  $M$ , there is some  $N \in \mathbb{R}$  such that for every natural number  $n > N$  we have  $a_n > M$ .

DEFINITION 16.2: A sequence **diverges to**  $-\infty$  if for every real number  $m$ , there is some  $N \in \mathbb{R}$  such that for every natural number  $n > N$  we have  $a_n < m$ .

- (1) Use the definition to prove that the sequence  $\{\sqrt{n}\}_{n=1}^{\infty}$  diverges to  $+\infty$ .
- (2) Prove that if a sequence  $\{a_n\}_{n=1}^{\infty}$  diverges to  $+\infty$  then it is not bounded above.
- (3) Use (2) to show that if a sequence diverges to  $+\infty$  then it diverges.
- (4) Prove or disprove: If a sequence diverges, then it diverges to  $+\infty$  or it diverges to  $-\infty$ .
- (5) Prove or disprove: If a sequence is not bounded above, then it diverges to  $+\infty$ .
- (6) Prove or disprove: If a sequence diverges to  $+\infty$  then it is increasing.
- (7) Prove or disprove: If a sequence is increasing and not bounded above, it diverges to  $\infty$ .