EXAMPLE 13.1:

(1) A constant sequence $\{c\}_{n=1}^{\infty}$ converges to $c$.
(2) The sequence $\{\frac{1}{n}\}_{n=1}^{\infty}$ converges to $0$.

THEOREM 13.2 (LIMITS AND ALGEBRA):
Let $\{a_n\}_{n=1}^{\infty}$ be a sequence that converges to $L$, and $\{b_n\}_{n=1}^{\infty}$ be a sequence that converges to $M$.

(1) If $c$ is any real number, then $\{ca_n\}_{n=1}^{\infty}$ converges to $cL$.
(2) The sequence $\{a_n + b_n\}_{n=1}^{\infty}$ converges to $L + M$.
(3) The sequence $\{a_n b_n\}_{n=1}^{\infty}$ converges to $LM$.
(4) If $L \neq 0$ and $a_n \neq 0$ for all $n \in \mathbb{N}$, then $\left\{\dfrac{1}{a_n}\right\}_{n=1}^{\infty}$ converges to $\dfrac{1}{L}$.
(5) If $M \neq 0$ and $b_n \neq 0$ for all $n \in \mathbb{N}$, then $\left\{\dfrac{a_n}{b_n}\right\}_{n=1}^{\infty}$ converges to $\dfrac{L}{M}$.

(1) Use Theorem 13.2 and Example 13.1 to show that the sequence $\{2 + 5/n - 7/n^2\}_{n=1}^{\infty}$ converges to 2. Show every step in your argument.

(2) Use Theorem 13.2 and Example 13.1 to show that the sequence $\left\{\dfrac{2n+3}{3n-4}\right\}_{n=1}^{\infty}$ converges to $\dfrac{2}{3}$.

(3) Use Theorem 13.2 to show that if $\{a_n\}_{n=1}^{\infty}$ converges to $L$, and $\{b_n\}_{n=1}^{\infty}$ converges to $M$, then $\{a_n - b_n\}_{n=1}^{\infty}$ converges to $L - M$.

(4) Prove or disprove the following converse to part (2): If $\{a_n + b_n\}_{n=1}^{\infty}$ converges to $L + M$ then $\{a_n\}_{n=1}^{\infty}$ converges to $L$ and $\{b_n\}_{n=1}^{\infty}$ converges to $M$.

(5) Prove or disprove: If $\{a_n\}_{n=1}^{\infty}$ is a convergent sequence and $\{b_n\}_{n=1}^{\infty}$ is a divergent sequence, then $\{a_n + b_n\}_{n=1}^{\infty}$ is divergent.

(6) Prove part (1) of Theorem 10.2 in the special case $c = 2$ by following the following steps:
   - Assume that $\{a_n\}_{n=1}^{\infty}$ converges to $L$.
   - We now want to show that $\{2a_n\}_{n=1}^{\infty}$ converges to something. You know what goes next!
   - Now we do some scratchwork: we want an $N$ such that for $n > N$ we have $|2a_n - 2L| < \varepsilon$. Factor this to get some inequality with $a_n$. How can we use our assumption to get an $N$ that "works"?
   - Complete the proof.

(7) Prove part (1) of Theorem 10.2.

(8) Prove part (2) of Theorem 10.2.

(9) Prove part (3) of Theorem 10.2.