True or False? Justify.

Side A

(1) Every bounded sequence is a convergent sequence.

(2) If a sequence has a divergent subsequence, then it diverges.

(3) The limit of $f(x) = \dfrac{x^2 - 2x + 3}{x - 7}$ as x approaches 3 is $-3/2$.

(4) The function $f(x) = \cos(1/x)$ has a limit as x approaches 0.

(5) If $\lim_{x \to -1} f(x)$ and $\lim_{x \to -1} g(x)$ both exist, then $\lim_{x \to -1} f(x)g(x)$ exists.

(6) If f is a function defined on $\mathbb{R}$, $\{f(1/n)\}_{n=1}^{\infty}$ converges to 2, and $\lim_{x \to 0} f(x) = L$, then $L = 2$.

(7) If f is continuous at 2, $f(2) = 3$, and $\lim_{x \to 1} g(x) = 2$, then $\lim_{x \to 1} (f \circ g)(x) = 3$.

(8) If $\{a_n\}_{n=1}^{\infty}$ converges to 1 and $\{b_n\}_{n=1}^{\infty}$ converges to -2, then $\{a_{3n-1}b_n - b_{n^2}/4\}_{n=1}^{\infty}$ converges to $-5 = (3 \cdot 1 - 1)(-2) - (-2)^2/4$.

(9) For a given sequence, there are at most two real numbers that occur as limits of subsequences of the sequence.

(10) The function $f(x) = \dfrac{x^2 - 2x + 3}{x - 7}$ is continuous on $(7, \infty)$.

(11) The function $f(x) = \sqrt{x^4 + 4x^2 + 5}$ is continuous on $\mathbb{R}$.

(12) If the domain of f is $\mathbb{R}$, then f is continuous at some point.

(13) If $\lim_{x \to a} f(x)$ exists and $\lim_{x \to a} g(x)$ does not exist, then $\lim_{x \to a} f(x) + g(x)$ does not exist.

(14) If f is continuous at a and $f(a) \geq 5$, then there is some $\delta > 0$ such that $f(x) \geq 5$ for all $x \in (a - \delta, a + \delta)$.

(15) There exists a sequence $\{a_n\}_{n=1}^{\infty}$ such that

$$\{a_n \mid n \in \mathbb{N}\} = (0, 3).$$

True or False? Justify.

Side B

(1) Every sequence has a bounded subsequence.

(2) There is a sequence without any monotone subsequence.

(3) The limit of $f(x) = \sqrt{4 - x^2}$ as x approaches 2 is 0.

(4) If $\lim_{x \to -1} f(x)/g(x) = 1$, then $\lim_{x \to -1} f(x) = \lim_{x \to -1} g(x)$.

(5) If $\lim_{x \to 2} f(x) = 3$ and $\lim_{x \to 1} g(x) = 2$, then $\lim_{x \to 1} (f \circ g)(x) = 3$.

(6) If f is a function defined on $\mathbb{R}$ and $\lim_{x \to 0} f(x) = 2$, then $\{f(1/n)\}_{n=1}^{\infty}$ converges to 2.

(7) If f is a function defined on $\mathbb{R}$ and $\{f(1/n)\}_{n=1}^{\infty}$ converges to 2, then $\lim_{x \to 0} f(x) = 2$.

(8) The sequence $a_n = \sqrt{\pi n - \lfloor \pi n \rfloor}$ has a convergent subsequence, where $\lfloor x \rfloor$ denotes the largest integer that is smaller than x.

(9) For a given convergent sequence, there are at most two real numbers that occur as limits of subsequences of the sequence.

(10) The function $f(x) = \dfrac{x^2 - 2x + 3}{x - 7}$ is continuous on $\mathbb{R}$.

(11) If $\lim_{x \to a} f(x)$ exists, then $f(x)$ is continuous at $x = a$.

(12) If f is continuous at a, then there exists some $\delta > 0$ such that f is continuous on $(a - \delta, a + \delta)$.

(13) If $\lim_{x \to a} f(x)$ exists and $\lim_{x \to a} g(x)$ does not exist, then $\lim_{x \to a} f(x)g(x)$ does not exist.

(14) If f is continuous at a and $f(a) > 5$, then there is some $\delta > 0$ such that $f(x) > 5$ for all $x \in (a - \delta, a + \delta)$.

(15) There exists a sequence $\{a_n\}_{n=1}^{\infty}$ such that

$$\{r \in \mathbb{R} \mid \text{there is a subsequence of } \{a_n\}_{n=1}^{\infty} \text{ that converges to } r\} = [0, 3].$$