Problem Set 7

Due Thursday, March 5

Instructions: You are encouraged to work together on these problems, but each student should hand in their own final draft, written in a way that indicates their individual understanding of the solutions. Never submit something for grading that you do not completely understand. You cannot use any resources besides me, your classmates, and our course notes.

I will post the .tex code for these problems for you to use if you wish to type your homework. If you prefer not to type, please *write neatly.* As a matter of good proof writing style, please use complete sentences and correct grammar. You may use any result stated or proven in class or in a homework problem, provided you reference it appropriately by either stating the result or stating its name (e.g. the definition of ring or Lagrange's Theorem). Please do not refer to theorems by their number in the course notes, as that can change.

Problem 1. Let F be a field, let V and W be vector spaces over F, let $a\colon V \to V$ and $b\colon W \to W$ be linear transformations and let V_a and W_b be the $F[x]$-modules they determine.

a) Show that a function $g : V_a \to W_b$ is an $F[x]$-module homomorphism if and only if

   (1) $g : V \to W$ is a linear transformation and

   (2) $g \circ a = b \circ g$.

b) Suppose that $V = F^m = W$, and let $A, B \in \mathrm{M}_m(F)$ be the matrices representing the linear transformations a and b, respectively, in the standard basis of F^m. Show that there is an $F[x]$-module isomorphism $V_a \cong W_b$ if and only if the matrices A and B are similar.

Problem 2. Determine, with justification, if the following two matrices with complex entries are similar.

$$A = \begin{bmatrix} 0 & -4 & 0 & 0 \\ 1 & 4 & 0 & 0 \\ 0 & 0 & 0 & -4 \\ 0 & 0 & 1 & 4 \end{bmatrix} \text{ and } B = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 1 & 2 \end{bmatrix}.$$