DEFINITION 34.1: Let f be a function and r be a real number. We say that f is **differentiable at** r if f is defined at r and the limit

$$\lim_{x\to r}\frac{f(x)-f(r)}{x-r}$$

exists. In this case, we call the limit **the derivative of** f **at** r and write $f'(r)$ for this limit.

(1) Use the definition of derivative to show that the function $f(x) = x$ is differentiable at any real number r and compute its derivative.

(2) Use the definition of derivative to show that the function $f(x) = x$ is *not* differentiable at $x = 0$.

(3) Prove[1] that if f is differentiable at $x = r$ then f is continuous at $x = r$.

(4) Prove or disprove the converse of the previous statement.

(5) Prove or disprove: The function $g(x) = \begin{cases} x\sin(1/x) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$ is differentiable at $x = 0$.

(6) Prove or disprove: The function $h(x) = \begin{cases} x^2\sin(1/x) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$ is differentiable at $x = 0$.

[1]Hint: If $g(x) = \dfrac{f(x)-f(r)}{x-r}$, consider $\lim_{x\to r}(x-r)g(x)$.