THEOREM 37.1: Let f be a function that is differentiable at $x = r$.

(1) If $f'(r) > 0$, then there is some $\delta > 0$ such that
- if $x \in (r, r + \delta)$ then $f(r) < f(x)$;
- if $x \in (r - \delta, r)$ then $f(x) < f(r)$.

(2) If $f'(r) < 0$, then there is some $\delta > 0$ such that
- if $x \in (r, r + \delta)$ then $f(r) > f(x)$;
- if $x \in (r - \delta, r)$ then $f(x) > f(r)$.

COROLLARY 37.2 (DERIVATIVES AND OPTIMIZATION): Let f be a function that is continuous on a closed interval $[a, b]$. If f attains a maximum or minimum value on $[a, b]$ at $r \in (a, b)$, and f is differentiable at r, then $f'(r) = 0$.

(1) Find the values of x on $[0, 2]$ at which the function $f(x) = x^3 - x^2 - 2x$ achieves its minimum and maximum values. Justify your answer carefully using the results above.

(2) Explain why the Corollary follows from the Theorem.

(3) Give examples of continuous functions on $[0, 2]$ such that
  (a) $f(x)$ attains its maximum at $x = 0$;
  (b) $g(x)$ attains its maximum at $x = 2$;
  (c) $h(x)$ attains its maximum at $x = 1$ and h is differentiable at $x = 1$;
  (d) $j(x)$ attains its maximum at $x = 1$ and j is not differenitable at $x = 1$.

(4) Prove part (1) of the Theorem:
- Consider the function $h(x) = \dfrac{f(x) - f(r)}{x - r}$. Apply the definition of limit to this function with $\varepsilon = f'(r)$. What does the definition give you?
- If $h(x) > 0$ and $x > r$, what can you say about $f(x) - f(r)$?
- If $h(x) > 0$ and $x < r$, what can you say about $f(x) - f(r)$?

(5) Prove part (2) of the Theorem.

(6) True or false: If $f'(7) > 0$, then $f(7.0000001) > f(7)$.

(7) True or false: If $f'(7) > 0$, then there exists some $N \in \mathbb{N}$ such that for all natural numbers $n > N$, $f\left(7 + \frac{1}{10^n}\right) > f(7)$.