

TRUE OR FALSE? JUSTIFY.

SIDE A

- (1) Every bounded sequence is a convergent sequence.
- (2) If a sequence has a divergent subsequence, then it diverges.
- (3) The limit of $f(x) = \frac{x^2 - 2x + 3}{x - 7}$ as x approaches 3 is $-3/2$.
- (4) The function $f(x) = \cos(1/x)$ has a limit as x approaches 0.
- (5) If $\lim_{x \rightarrow -1} f(x)$ and $\lim_{x \rightarrow -1} g(x)$ both exist, then $\lim_{x \rightarrow -1} f(x)g(x)$ exists.
- (6) If $\lim_{x \rightarrow -1} f(x)$ does not exist, then $\lim_{x \rightarrow -1} (f(x) + g(x))$ does not exist.
- (7) If f is a function defined on \mathbb{R} , $\{f(1/n)\}_{n=1}^{\infty}$ converges to 2, and $\lim_{x \rightarrow 0} f(x) = L$, then $L = 2$.
- (8) If f is continuous at 2, $f(2) = 3$, and $\lim_{x \rightarrow 1} g(x) = 2$, then $\lim_{x \rightarrow 1} (f \circ g)(x) = 3$.
- (9) If $\{a_n\}_{n=1}^{\infty}$ converges to 1 and $\{b_n\}_{n=1}^{\infty}$ converges to -2 , then $\{a_{3n-1}b_n - b_{n^2}/4\}_{n=1}^{\infty}$ converges to $-5 = (3 \cdot 1 - 1)(-2) - (-2)^2/4$.
- (10) For a given sequence, there are at most two real numbers that occur as limits of subsequences of the sequence.
- (11) The function $f(x) = \frac{x^2 - 2x + 3}{x - 7}$ is continuous on $(7, \infty)$.
- (12) The function $f(x) = \sqrt{x^4 + 4x^2 + 5}$ is continuous on \mathbb{R} .
- (13) If the domain of f is \mathbb{R} , then f is continuous at some point.
- (14) If $\lim_{x \rightarrow a} f(x)$ exists and $\lim_{x \rightarrow a} g(x)$ does not exist, then $\lim_{x \rightarrow a} f(x) + g(x)$ does not exist.
- (15) If f is continuous at a and $f(a) \geq 5$, then there is some $\delta > 0$ such that $f(x) \geq 5$ for all $x \in (a - \delta, a + \delta)$.
- (16) There exists a sequence $\{a_n\}_{n=1}^{\infty}$ such that
$$\{a_n \mid n \in \mathbb{N}\} = (0, 3).$$

TRUE OR FALSE? JUSTIFY.

SIDE B

- (1) Every sequence has a bounded subsequence.
- (2) There is a sequence without any monotone subsequence.
- (3) The limit of $f(x) = \sqrt{4 - x^2}$ as x approaches 2 is 0.
- (4) If $\lim_{x \rightarrow -1} f(x)/g(x) = 1$, then $\lim_{x \rightarrow -1} f(x) = \lim_{x \rightarrow -1} g(x)$.
- (5) If $\lim_{x \rightarrow 2} f(x) = 3$ and $\lim_{x \rightarrow 1} g(x) = 2$, then $\lim_{x \rightarrow 1} (f \circ g)(x) = 3$.
- (6) If f is a function defined on \mathbb{R} and $\lim_{x \rightarrow 0} f(x) = 2$, then $\{f(1/n)\}_{n=1}^{\infty}$ converges to 2.
- (7) If f is a function defined on \mathbb{R} and $\{f(1/n)\}_{n=1}^{\infty}$ converges to 2, then $\lim_{x \rightarrow 0} f(x) = 2$.
- (8) If f and g are functions defined on \mathbb{R} , and $f(x) = g(x)$ for all $x \neq a$, and f has a limit as x approaches a , then $\lim_{x \rightarrow a} f(x) = \lim_{x \rightarrow a} g(x)$.
- (9) The sequence $a_n = \sqrt{\pi n - \lfloor \pi n \rfloor}$ has a convergent subsequence, where $\lfloor x \rfloor$ denotes the largest integer that is smaller than x .
- (10) For a given convergent sequence, there are at most two real numbers that occur as limits of subsequences of the sequence.
- (11) The function $f(x) = \frac{x^2 - 2x + 3}{x - 7}$ is continuous on \mathbb{R} .
- (12) If $\lim_{x \rightarrow a} f(x)$ exists, then $f(x)$ is continuous at $x = a$.
- (13) If f is continuous at a , then there exists some $\delta > 0$ such that f is continuous on $(a - \delta, a + \delta)$.
- (14) If $\lim_{x \rightarrow a} f(x)$ exists and $\lim_{x \rightarrow a} g(x)$ does not exist, then $\lim_{x \rightarrow a} f(x)g(x)$ does not exist.
- (15) If f is continuous at a and $f(a) > 5$, then there is some $\delta > 0$ such that $f(x) > 5$ for all $x \in (a - \delta, a + \delta)$.
- (16) There exists a sequence $\{a_n\}_{n=1}^{\infty}$ such that
 $\{r \in \mathbb{R} \mid \text{there is a subsequence of } \{a_n\}_{n=1}^{\infty} \text{ that converges to } r\} = [0, 3]$.