

TRUE or FALSE. Justify.

- (1) Let  $x, y \in \mathbb{R}$ . The negation of the statement “If  $x$  and  $y$  are rational, then  $xy$  is rational” is “If  $x$  and  $y$  are rational, then  $xy$  is irrational”.
- (2) Let  $x, y \in \mathbb{R}$ . The contrapositive of the statement “If  $x$  and  $y$  are rational, then  $xy$  is rational” is “If  $xy$  is irrational, then  $x$  is irrational or  $y$  is irrational”.
- (3) The commutative property/axiom of addition says that  $x + y = y + x$ .
- (4) Every set of real numbers that is bounded above has a supremum.
- (5) There is a set  $S$  of real numbers such that  $\sup(S)$  exists, but  $\sup(S) \notin S$ .
- (6) If  $a < b$  are real numbers, there is an integer  $n \in \mathbb{Z}$  such that  $a < n < b$ .
- (7) Every nonempty set of real numbers has a smallest element (i.e., a minimum element).
- (8) Every nonempty set of integers that is bounded below has a smallest element (i.e., a minimum element).
- (9) If  $S \subseteq \mathbb{R}$  is bounded above, then there is a natural number  $b$  such that  $b$  is an upper bound for  $S$ .
- (10) Every set of real numbers satisfies the property that “for all  $x \in S$ , there exists a real number  $y$  such that  $x < y^2$ ”.
- (11) Every set of real numbers satisfies the property that “for all  $x \in S$ , there exists a real number  $y$  such that  $y^2 < x$ ”.
- (12) The supremum of the set  $\{1/n \mid n \in \mathbb{N}\}$  is 1.
- (13) The supremum of the set  $\{-1/n \mid n \in \mathbb{N}\}$  is  $-1$ .
- (14) The negation of the statement “for all  $x \in S$ , there exists a real number  $y$  such that  $x < y^2$ ” is “for all  $x \in S$ , there exists a real number  $y$  such that  $x \geq y^2$ ”.
- (15) If a sequence  $\{a_n\}_{n=1}^{\infty}$  converges to 5, then for all natural numbers  $n$ ,  $a_n > 4$ .

- (16) If a sequence  $\{a_n\}_{n=1}^{\infty}$  converges to  $L$ , then there is some  $N \in \mathbb{R}$  such that for all natural numbers  $n > N$ ,  $a_n = L$ .
- (17) For every real number  $L$  there is a sequence  $\{a_n\}_{n=1}^{\infty}$  such that  $a_n \neq L$  for all  $n \in \mathbb{N}$  and converges to  $L$ .
- (18) A sequence of positive numbers can converge to a negative number.
- (19) A sequence of positive numbers can converge to zero.
- (20) There is a set  $S$  of irrational numbers such that  $\sup(S) = 2$ .
- (21) Every increasing sequence is convergent.
- (22) Every convergent sequence is either increasing or decreasing.
- (23) If  $\{a_n\}_{n=1}^{\infty}$  and  $\{b_n\}_{n=1}^{\infty}$  are convergent sequences, then  $\{a_n + b_n\}_{n=1}^{\infty}$  is a convergent sequence.
- (24) If  $\{a_n\}_{n=1}^{\infty}$  and  $\{b_n\}_{n=1}^{\infty}$  are convergent sequences, and  $b_n \neq 0$  for all  $n \in \mathbb{N}$ , then  $\{a_n/b_n\}_{n=1}^{\infty}$  is a convergent sequence.
- (25) The sequence  $\left\{ \frac{3n^2 - 4n + 7}{6n^2 + 1} \right\}_{n=1}^{\infty}$  converges to  $1/2$ .
- (26) The negation of “ $\{a_n\}_{n=1}^{\infty}$  is a monotone sequence” is “there exists  $n \in \mathbb{N}$  such that  $a_n > a_{n+1}$  and  $a_n < a_{n+1}$ ”.
- (27) Every convergent sequence of rational numbers converges to a rational number.
- (28) If a sequence is not bounded below, then it diverges to  $-\infty$ .
- (29) If  $\{a_n\}_{n=1}^{\infty}$  diverges to  $\infty$  and  $\{b_n\}_{n=1}^{\infty}$  diverges to  $-\infty$ , then  $\{a_n + b_n\}_{n=1}^{\infty}$  converges to 0.
- (30) If  $\{a_n\}_{n=1}^{\infty}$  diverges to  $\infty$  and  $\{b_n\}_{n=1}^{\infty}$  converges, then  $\{a_n + b_n\}_{n=1}^{\infty}$  diverges to  $\infty$ .