## IF-THEN STATEMENTS

(1) For each of the following statements, write its contrapositive and its converse. Is the original/contrapositive/converse true or false? Explain why (but don't bother proving). For all of the statements below, $a, b$ are real numbers.
   (a) If $a$ and $b$ are irrational, then $ab$ is irrational.
   (b) If $a > 3$, then $a^2 > 9$.

(2) Let $x$ and $y$ be real numbers. Use the axioms of $\mathbb{R}$ to prove[1] that $x \geq y$ if and only if $-x \leq -y$.

(3) Let $x$ be a real number. Show that if $x^2$ is irrational, then $x$ is irrational.

(4) Let $x$ be a real number. Use the axioms of $\mathbb{R}$ and facts we have proven in class to show that if there exists a real number $y$ such that $xy = 1$, then $x \neq 0$.

## QUANTIFIER STATEMENTS

(1) Let $x$ be a real number. Prove that there exists a real number $y$ such that $xy = 1$ if and only if $x \neq 0$.

(2) Let $S \subseteq \mathbb{R}$ be a set of real numbers. Apply your results above to prove that if for every $x \in S$, $x^2$ is irrational, then for every $y \in S$, $-y$ is irrational.

[1] Hint: You may want to add something to both sides.