

Math 325-002 — Problem Set #5
Due: Thursday, September 29 by 7 pm, on Canvas

Instructions: You are encouraged to work together on these problems, but each student should hand in their own final draft, written in a way that indicates their individual understanding of the solutions. Never submit something for grading that you do not completely understand.

If you do work with others, I ask that you write something along the top like “I collaborated with Steven Smale on problems 1 and 3”. If you use a reference, indicate so clearly in your solutions. In short, be intellectually honest at all times.

Please write neatly, using complete sentences and correct punctuation. Label the problems clearly.

- (1) Consider the sequence $\left\{ \frac{2n+1}{5n-2} \right\}_{n=1}^{\infty}$. Use the definition of converges (but not Theorem 10.2) to show that this sequence converges to $\frac{2}{5}$.

- (2) Prove that the sequence

$$\left\{ \frac{8n^2 - 5n + 3}{4n^2 + 1} \right\}_{n=1}^{\infty}$$

converges. (This includes finding to what it converges.) You should use Theorem 10.2, but carefully justify every step using the Theorem.

- (3) Prove part (1) of Theorem 10.2 in the special case $c = 2$.

- (4) Prove that the sequence $\{\sqrt{n}\}_{n=1}^{\infty}$ diverges.

- (5) Assume that $\{a_n\}_{n=1}^{\infty}$ converges to zero, and that $a_n \geq 0$ for all natural numbers n . Prove that $\{\sqrt{a_n}\}_{n=1}^{\infty}$ converges to zero also.