Math 325-002 — Problem Set #3
Due: Thursday, September 15 by 7 pm, on Canvas

Instructions: You are encouraged to work together on these problems, but each student should hand in their own final draft, written in a way that indicates their individual understanding of the solutions. Never submit something for grading that you do not completely understand.

If you do work with others, I ask that you write something along the top like "I collaborated with Steven Smale on problems 1 and 3". If you use a reference, indicate so clearly in your solutions. In short, be intellectually honest at all times.

Please write neatly, using complete sentences and correct punctuation. Label the problems clearly.

(1) Show that, for any positive number $\epsilon > 0$, there is a natural number n such that $0 < \epsilon < \frac{1}{n}$.

(2) Show that the supremum of the set $S = \left\{ 2 - \dfrac{3}{n} \mid n \in \mathbb{N} \right\}$ is 2.

(3) Let r be any real number. Consider the set

$$S_r = \{q \in \mathbb{Q} \mid q < r\}.$$

Prove that the supremum of S_r is r.

(4) Find all real numbers x that satisfy the given inequality, and express your answers as intervals or unions of intervals.
   (a) $|2x + 7| < 13$.
   (b) $|2x + 7| \leq 13$.
   (c) $|2x + 7| > 13$.

(5) (a) Use the Triangle Inequality and the Reverse Triangle Inequality[1] to show that for all real numbers x,

$$|2x - 3| - 6 \leq |2x + 3| \leq |2x - 3| + 6.$$

   (b) Find (with justification) a positive real number δ such that:
   If $|2x - 3| < \delta$, then $|4x^2 - 9| < \frac{1}{100}$.

[1] Hint: Write $2x + 3 = (2x - 3) + 6 = (2x - 3) - (-6)$