

TRUE OR FALSE? JUSTIFY.

SIDE A

- (1) Every bounded sequence is a convergent sequence.
- (2) If a sequence has a divergent subsequence, then it diverges.
- (3) The limit of  $f(x) = \frac{x^2 - 2x + 3}{x - 7}$  as  $x$  approaches 3 is  $-3/2$ .
- (4) The function  $f(x) = \cos(1/x)$  has a limit as  $x$  approaches 0.
- (5) If  $\lim_{x \rightarrow -1} f(x)$  and  $\lim_{x \rightarrow -1} g(x)$  both exist, then  $\lim_{x \rightarrow -1} f(x)g(x)$  exists.
- (6) If  $f$  is a function defined on  $\mathbb{R}$ ,  $\{f(1/n)\}_{n=1}^{\infty}$  converges to 2, and  $\lim_{x \rightarrow 0} f(x) = L$ , then  $L = 2$ .
- (7) If  $f$  is continuous at 2,  $f(2) = 3$ , and  $\lim_{x \rightarrow 1} g(x) = 2$ , then  $\lim_{x \rightarrow 1} (f \circ g)(x) = 3$ .
- (8) If  $\{a_n\}_{n=1}^{\infty}$  converges to 1 and  $\{b_n\}_{n=1}^{\infty}$  converges to  $-2$ , then  $\{a_{3n-1}b_n - b_{n^2}/4\}_{n=1}^{\infty}$  converges to  $-5 = (3 \cdot 1 - 1)(-2) - (-2)^2/4$ .
- (9) For a given sequence, there are at most two real numbers that occur as limits of subsequences of the sequence.
- (10) The function  $f(x) = \frac{x^2 - 2x + 3}{x - 7}$  is continuous on  $(7, \infty)$ .
- (11) The function  $f(x) = \sqrt{x^4 + 4x^2 + 5}$  is continuous on  $\mathbb{R}$ .
- (12) If the domain of  $f$  is  $\mathbb{R}$ , then  $f$  is continuous at some point.
- (13) If  $\lim_{x \rightarrow a} f(x)$  exists and  $\lim_{x \rightarrow a} g(x)$  does not exist, then  $\lim_{x \rightarrow a} f(x) + g(x)$  does not exist.
- (14) If  $f$  is continuous at  $a$  and  $f(a) \geq 5$ , then there is some  $\delta > 0$  such that  $f(x) \geq 5$  for all  $x \in (a - \delta, a + \delta)$ .
- (15) There exists a sequence  $\{a_n\}_{n=1}^{\infty}$  such that
$$\{a_n \mid n \in \mathbb{N}\} = (0, 3).$$

TRUE OR FALSE? JUSTIFY.

SIDE B

- (1) Every sequence has a bounded subsequence.
- (2) There is a sequence without any monotone subsequence.
- (3) The limit of  $f(x) = \sqrt{4 - x^2}$  as  $x$  approaches 2 is 0.
- (4) If  $\lim_{x \rightarrow -1} f(x)/g(x) = 1$ , then  $\lim_{x \rightarrow -1} f(x) = \lim_{x \rightarrow -1} g(x)$ .
- (5) If  $\lim_{x \rightarrow 2} f(x) = 3$  and  $\lim_{x \rightarrow 1} g(x) = 2$ , then  $\lim_{x \rightarrow 1} (f \circ g)(x) = 3$ .
- (6) If  $f$  is a function defined on  $\mathbb{R}$  and  $\lim_{x \rightarrow 0} f(x) = 2$ , then  $\{f(1/n)\}_{n=1}^{\infty}$  converges to 2.
- (7) If  $f$  is a function defined on  $\mathbb{R}$  and  $\{f(1/n)\}_{n=1}^{\infty}$  converges to 2, then  $\lim_{x \rightarrow 0} f(x) = 2$ .
- (8) The sequence  $a_n = \sqrt{\pi n - \lfloor \pi n \rfloor}$  has a convergent subsequence, where  $\lfloor x \rfloor$  denotes the largest integer that is smaller than  $x$ .
- (9) For a given convergent sequence, there are at most two real numbers that occur as limits of subsequences of the sequence.
- (10) The function  $f(x) = \frac{x^2 - 2x + 3}{x - 7}$  is continuous on  $\mathbb{R}$ .
- (11) If  $\lim_{x \rightarrow a} f(x)$  exists, then  $f(x)$  is continuous at  $x = a$ .
- (12) If  $f$  is continuous at  $a$ , then there exists some  $\delta > 0$  such that  $f$  is continuous on  $(a - \delta, a + \delta)$ .
- (13) If  $\lim_{x \rightarrow a} f(x)$  exists and  $\lim_{x \rightarrow a} g(x)$  does not exist, then  $\lim_{x \rightarrow a} f(x)g(x)$  does not exist.
- (14) If  $f$  is continuous at  $a$  and  $f(a) > 5$ , then there is some  $\delta > 0$  such that  $f(x) > 5$  for all  $x \in (a - \delta, a + \delta)$ .
- (15) There exists a sequence  $\{a_n\}_{n=1}^{\infty}$  such that  
 $\{r \in \mathbb{R} \mid \text{there is a subsequence of } \{a_n\}_{n=1}^{\infty} \text{ that converges to } r\} = [0, 3]$ .