COURSE ANNOUNCEMENT
MATH 918: TOPICS IN COMMUTATIVE ALGEBRA — SPRING 2023
KÄHLER DIFFERENTIALS

Time: Tuesdays and Thursdays 9:30 am – 10:45 am

Room: Burnett 121

Instructor: Jack Jeffries

What are Kähler differentials? The content of this class is based on the observation that, given a polynomial, if you want to take its derivative, you don't actually need to deal with limits at all: you just need to know the product rule. This means that we can do calculus entirely algebraically... and that we can do calculus over any ring, including not-so-calculus-y ones like $\mathbb{Z}[i]$. Kähler differentials are fundamental modules that arise from, and govern, this notion of calculus.

In this class, we will study these modules, and apply them to understand crinkles and wrinkles in high-dimensional spaces. The surprising thing is that this is all algebra! Along the way, we will find many connections with multivariable calculus and topology, and take the excuse to remember what we have learned elsewhere.

Likely topics include: derivations, basic properties of Kähler differentials, more field theory (transcendence bases, p-bases, separably generated extensions), complete local rings, coefficient fields and Cohen's structure theorem, regular rings, Jacobian criterion, smooth / étale / unramified extensions, Zariski's main theorem, structure theory of smooth / étale / unramified extensions, reduction to positive characteristic, regular sequences and complete intersections, Ferrand-Vasconcelos theorem, differential graded algebras, and André-Quillen cohomology. It's not necessary to know what any of these things mean now; we will learn about them during the course.

Prerequisites: This class will assume knowledge of basic Commutative Algebra. Anyone who has taken Math 905 or Math 902 should be prepared for the course. On occasion, we will use basic tools from Homological Algebra covered in Math 901; in these cases, we will either give a quick overview in class and/or provide supplementary resources for those who haven't encountered the material. Students attending this course may also benefit from taking Math 935 concurrently to obtain additional geometric motivation and intuition for the material.

Textbook: There will be no assigned textbook for the course. I will provide lecture notes instead. Some recommended complementary sources include:

- *Commutative Ring Theory* by Hideyuki Matsumura
- *Kähler Differentials* by Ernst Kunz
- *Commutative Algebra* with a View Towards Algebraic Geometry by David Eisenbud
- *Math 615 Lecture Notes, Winter 2017* by Melvin Hochster

Structure of the course: We will have four or five problem sets throughout the semester, as well as a few short quizzes, to help us make sure we stay up-to-date on our definitions and key theorems. If you are worried about the workload for the course, please contact me before being scared away from enrolling.