TRUE or FALSE. Justify.

F (1) Let $\{a_n\}_{n=1}^{\infty}$, $\{b_n\}_{n=1}^{\infty}$, and $\{c_n\}_{n=1}^{\infty}$ be sequences. The negation of the statement "If $\{a_n\}_{n=1}^{\infty}$ and $\{b_n\}_{n=1}^{\infty}$ converge, then $\{c_n\}_{n=1}^{\infty}$ converges" is "If $\{a_n\}_{n=1}^{\infty}$ and $\{b_n\}_{n=1}^{\infty}$ converge, then $\{c_n\}_{n=1}^{\infty}$ diverges".

F (2) The commutative property/axiom of addition says that $(x+y)+z = x+(y+z)$.

F (3) Every nonempty set of real numbers that is bounded above has a maximum element.

T (4) If S is a set of real numbers and $\sup(S) \in S$ then $\sup(S)$ is the maximum element of S.

T (5) Every nonempty set of natural numbers that is bounded below has a minimum element.

T (6) The supremum of the set $\{x \in \mathbb{Q} \mid x < \pi\}$ is π.

F (7) A sequence of negative numbers can converge to a positive number.

F (8) Every decreasing sequence is convergent.

T (9) Every convergent sequence is bounded.

T (10) If $\{a_n\}_{n=1}^{\infty}$ and $\{b_n\}_{n=1}^{\infty}$ are convergent sequences, then $\{3a_n^2 + a_n b_n - 6b_n\}_{n=1}^{\infty}$ is a convergent sequence.

T (11) If f is differentiable on $\mathbb{R}$, $f'(x) \leq 0$ for all $x > 0$, and $f(x) \geq -100$ for all $x > 0$, then the sequence $\{f(n)\}_{n=1}^{\infty}$ converges.

F (12) If for every $\varepsilon > 0$, there is some $N \in \mathbb{R}$ such that $|a_n - a_{n+1}| < \varepsilon$ for all $n > N$, then $\{a_n\}_{n=1}^{\infty}$ is a Cauchy sequence.

F (13) If $\lim_{x \to -1} f(x) > 5$, then $\exists \delta > 0$ such that $f(x) > 5$ for all $x \in (-1-\delta, -1+\delta) \setminus \{-1\}$.

T (14) If $\lim_{x \to -1} f(x) \geq 5$, then $\exists \delta > 0$ such that $f(x) \geq 5$ for all $x \in (-1-\delta, -1+\delta) \setminus \{-1\}$.

F (15) If $\{a_n\}_{n=1}^{\infty}$ and $\{b_n\}_{n=1}^{\infty}$ diverge, then so does $\{a_n + b_n\}_{n=1}^{\infty}$.

T (16) If $\{a_n\}_{n=1}^{\infty}$ converges and $\{b_n\}_{n=1}^{\infty}$ diverges, then $\{a_n + b_n\}_{n=1}^{\infty}$ diverges.

F (17) We can prove that every polynomial $p(x)$ has a property P by induction on the degree of $p(x)$ by showing that every constant function has property P and then showing that if $p(x)$ has property P then $p'(x)$ has property P.

T (18) If $f'(x) < 0$ for all $x \in (a, b)$, then f is strictly decreasing on (a, b).

F (19) If f is strictly decreasing and differentiable on (a, b), then $f'(x) < 0 \ \forall x \in (a, b)$.

T (20) If f, g are continuous on $(-7, 7)$, and $g(4) = -1$, then $\lim_{x \to 4}(f \circ g)(x) = f(-1)$.

F (21) The supremum of the set $\{x \in \mathbb{Q} \mid x > \pi\}$ is π.

T (22) The sequence $\{a_n\}_{n=1}^{\infty}$ where

$$a_n = \quad n \cdot \sqrt{2} \; - \; \text{the largest integer that is less than } n \cdot \sqrt{2}$$

has a convergent subsequence.

F (23) The sequence $\{a_n\}_{n=1}^{\infty}$ where

$$a_n = \quad n \cdot \sqrt{2} \; - \; \text{the largest integer that is less than } n \cdot \sqrt{2}$$

has a constant subsequence.

F (24) If the domain of f is $\mathbb{R}$, then f is continuous at some point.

T (25) The function $f(x) = |x^2 - |x^3 - 3||$ is continuous on $\mathbb{R}$.

T (26) If $f(x)$ is continuous at $x = a$ then $\lim_{x \to a} f(x)$ exists.

F (27) If f is continuous on $\mathbb{R}$ and $a < b$, and $y > f(a) > f(b)$, then there is no $c \in [a, b]$ such that $f(c) = y$.

T (28) There is a continuous function $f : [2, 3] \to \mathbb{R}$ with range $[3, 5]$.

F (29) There is a continuous function $f : [2, 3] \to \mathbb{R}$ with range $(3, 5]$.

F (30) There is a continuous function $f : [2, 3] \to \mathbb{R}$ with range $[3, 4] \cup [5, 6]$.

F (31) The function $f(x) = x^3 - 2x^2 + 5$ is increasing or decreasing on the interval $(1, 2)$.

T (32) If $f(x)$ is defined and bounded on some interval $(-\delta, \delta)$, then $x^2 f(x)$ is differentiable at $x = 0$.