

## SUBSEQUENCES §2.3

Use any of the trig facts below to answer the following questions:

- (1) **Prove or disprove:** Is  $\{\cos(\pi n)\}_{n=1}^{\infty}$  a subsequence of  $\{\cos(n)\}_{n=1}^{\infty}$ ?
- (2) **Prove or disprove:** The sequence  $\{\cos(n)\}_{n=1}^{\infty}$  has a convergent subsequence.
- (3) **Prove or disprove:** The sequence  $\{\cos(n)\}_{n=1}^{\infty}$  has a constant subsequence.
- (4) **Prove or disprove:** The sequence  $\{\cos(n)\}_{n=1}^{\infty}$  has a subsequence that converges to some  $x > 1$ .

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- $-1 \leq \cos(x) \leq 1$  for all  $x \in \mathbb{R}$
  - $\cos(x) = 1 \iff x \in 2\pi\mathbb{Z}$
  - $\cos(x) = 0 \iff x \in \frac{\pi}{2} + \pi\mathbb{Z}$

- $\cos(x) = -1 \iff x \in \pi + 2\pi\mathbb{Z}$
- $\pi \notin \mathbb{Q}$
- $\cos(x) = \cos(y) \iff x - y \in 2\pi\mathbb{Z} \text{ or } x + y \in 2\pi\mathbb{Z}$