Discussion Questions

Consider the differential equations

(♣) $$y'' + \sin(t)y' + e^{x^2}y = 0$$

(♢) $$y'' + \sin(t)y' + e^{x^2}y = \tan(t)$$

(1) What is the order of these equations? Are they linear? Are the homogeneous?

(2) Say that we have solutions $f(t)$ and $g(t)$ to equation (♣), and a solution $h(y)$ to equation (♢). Which of the following definitely are solutions to (♣)? Which definitely are solutions to (♢)?

(a) $y = 2f$	(c) $y = 3f - g$	(e) $y = 0$	(g) $y = tg$
(b) $y = 2h$	(d) $y = f^2$	(f) $y = g + h$	(h) $y = h - 4f$

(3) What can you say about existence and uniqueness of the following initial value problems? Are they true on some interval? If so, what's the biggest such interval?

(a)
$$\begin{cases} y'' + \sin(t)y' + e^{x^2}y = 0 \\ y(0.2) = 4 \\ y'(-0.1) = \pi \end{cases}$$

(b)
$$\begin{cases} y'' + \sin(t)y' + e^{x^2}y = \tan(t) \\ y(0.2) = 4 \\ y'(-0.1) = \pi \end{cases}$$

(c)
$$\begin{cases} y'' + \sin(t)y' + e^{x^2}y = 0 \\ y(0.3) = 7 \end{cases}$$

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Existence and uniqueness theorem for linear IVPs: Given a linear ODE

$$a_n(x)y^{(n)} + a_{n-1}(x)y^{(n-1)} + \cdots + a_1(x)y' + a_0(x)y = g(x)$$

where $g(x), a_0(x), \ldots, a_n(x)$ are continuous and $a_n(x) \neq 0$ for all x, then there exists a unique solution

$$\begin{cases} a_n(x)y^{(n)} + a_{n-1}(x)y^{(n-1)} + \cdots + a_1(x)y' + a_0(x)y = g(x) \\ y(t_0) = y_0 \\ y'(t_0) = y_1 \\ \quad \vdots \\ y^{(n-1)}(t_0) = y_{n-1} \end{cases}$$

on some interval containing t_0.

Superposition principle for linear ODEs:

(1) Given solutions $y_1, \ldots, y_t$ to a *homogeneous* linear ODE, any superposition

$$c_1y_1 + \cdots + c_ty_t$$

(for constants $c_1, \ldots, c_t$) is also a solution.

(2) Give a solution y_p to a *nonhomogeneous* linear ODE and solutions $y_1, \ldots, y_t$ to the corresponding *homogeneous* equation, y_p plus any superposition of $y_1, \ldots, y_t$, i.e., a function like

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