DIVERGENCE TO $\pm\infty$

It is sometimes useful to distinguish between sequences like $\{(-1)^n\}_{n=1}^{\infty}$ that diverge because they "oscillate", and sequences like $\{n\}_{n=1}^{\infty}$ that diverge because they "head toward infinity".

(1) In intuitive language, a sequence converges to L if no matter how close we want or sequence to be to L, all values past some point are at least that close. Intuitively, a sequence *diverges to* $+\infty$ if no matter how large we want our sequence to be, all values past some point are at least that large. Write a precise definition for a sequence to diverge to $+\infty$.

(2) Write a precise definition for a sequence to diverge to $-\infty$.

(3) Stop and check your definitions with Jack, Uyen, or a group that has been checked before proceeding.

(4) Carefully write the logical negation of "$\{a_n\}_{n=1}^{\infty}$ diverges to $+\infty$" in simplified form.

(5) Use the definition to prove that the sequence $\{\sqrt{n}\}_{n=1}^{\infty}$ diverges to $+\infty$.

(6) Use the definition to prove that the sequence $\{(-1)^n\}_{n=1}^{\infty}$ does not diverge to $+\infty$.

(7) Prove that if a sequence $\{a_n\}_{n=1}^{\infty}$ diverges to $+\infty$ then it is not bounded above.

(8) Use (7) to show that if a sequence diverges to $+\infty$ then it diverges.

(9) Disprove the following: If a sequence is not bounded above, then it diverges to $+\infty$.

(10) Disprove the following: If a sequence diverges to $+\infty$ then it is increasing.