September 13, 2025

To the hiring committee,

It is my pleasure to write on behalf of Pedro Ramírez-Moreno (CIMAT) for your postdoctoral position. I have known Pedro for four years, and would say that I know him well as a student and scholar. Pedro is a creative and capable researcher with expertise in a range of areas in commutative algebra with connections to other areas, as well as a clear and enthusiastic speaker. In my opinion, Pedro is one of the strongest students in commutative algebra graduating this year, and one of the two strongest on the boundary of combinatorics and commutative algebra. In particular, I would compare Pedro to recent Michigan postdoc Daniel Smolkin. I recommend him very highly for your position!

I first came to know Pedro when I was a faculty member at CIMAT in the 2019–2020 academic year; at the time Pedro was a masters student working with my then-colleague Luis Núñez-Betancourt, who is one of the top leaders in our field and the recipient of numerous international awards. I had Pedro in two masters level classes as a student: Commutative Algebra 1 and a topics course on differential operators in commutative algebra. Pedro was the ideal student in these courses—he would ask a number of interesting questions, and his homework assignments were exceptional in terms of clarity and thoroughness. In the latter topics course, we covered a great deal of material, and Pedro followed astutely, even as the class transitioned to zoom in the middle due to the pandemic.

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At that time, Pedro was masters student at UANL visiting CIMAT for the year to work with Luis as his masters advisor. In the next year, Pedro began his Ph.D. at CIMAT with Luis. In case the Ph.D. programs in Mexico are less familiar, I might give some context for the CIMAT program. CIMAT is one of the top math departments in the country of Mexico, arguably in the top two with UNAM/CINVESTAV. A large number of international conferences are hosted there, and many recent undergraduate recipients of the national top undergraduate thesis award are advised at CIMAT. My Commutative Algebra 1 class went at a similar pace to the same class I taught at the University of Michigan, and many of the peers in the classes Pedro took from me went to large commutative algebra centers in the US (University of Utah, Notre Dame, University of Nebraska). The number of topics classes and related international conferences available to students at CIMAT is rivaled only by the top institutions in the US. I would say that the quality of Pedro's graduate education at CIMAT is comparable to many of the strongest Ph.D. institutions in the US.

More recently, Pedro came to visit my institution, the University of Nebraska, to meet with our group and speak in our seminars. Pedro gave a very entertaining and mathematically stimulating talk on his work on our seminar. He also gave a colloquium-style talk in our department's Spanish language math seminar, and gave a highly engaging talk there as well. Overall, I find him to be a great speaker and mathematical communicator. He also has extensive teaching experience and I am confident he would excel in any teaching role.

Let me now write in some more detail about Pedro's work. Pedro's research is in commutative algebra and connects a range of vibrant subfields in the discipline, including combinatorially-defined ideals, positive characteristic techniques, Gröbner degeneration, connectedness of varieties, and symbolic powers.

His first project, which was the subject of his masters thesis, was joint work with Lilia Alanís-López and Luis Núñez-Betancourt. This paper established connectedness results for varieties in comparison with their initial ideals. A general construction assigns to any ideal in a polynomial ring an ideal generated by monomials, called its initial ideal. The main result of the paper compares the connectedness dimension of a variety, i.e., the dimension of a subvariety that must be removed to disconnect it, with the connectedness dimension of the variety cut out by its initial ideal. In particular, the main result asserts that if the initial ideal is reduced, then these numbers are in fact equal. This is a very useful result for two reasons: first, computing the connectedness dimension of an ideal generated by monomials is relatively easy; second, many interesting naturally occurring examples of varieties satisfy the hypothesis that the initial ideal is reduced. A consequence of this result is that, under the same hypotheses, connectedness dimension is preserved under extension of scalars. This is quite striking: consider the simple example of $\mathbb{R}[x]/(x^2+1) \cong \mathbb{C}$, which geometrically corresponds to a (connected) point, while $\mathbb{C} \otimes_{\mathbb{R}} \mathbb{R}[x]/(x^2+1) \cong \mathbb{C} \times \mathbb{C}$, which geometrically corresponds to a (disconnected) pair of points; the aformentioned result says that no such pathologies can occur when the initial ideal is reduced. The results in this paper build on a famous and highly-cited result of Hochster and Huneke, and add to other exciting recent results relating various algebraic and geometric aspects of ideals to their initial ideals, including a major breakthrough on extremal Betti numbers by Conca and Varbaro (Invent. Math. 2020). I think the results in this paper are very compelling, and I believe that the ideas used in this work are likely to produce additional interesting results; in particular, Pedro's in-progress work in this direction is very promising.

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His second project is on positive characteristic singularities of binomial edge ideals. This work appears in a solo paper that is the core of his Ph.D. thesis. A binomial edge ideal is a combinatorially defined ideal in a polynomial ring that is associated to a graph. These have been much studied in the approximately fifteen years since they were originally defined, partially motivated by algebraic statistics; one recent big result is a combinatorial description of their Betti numbers by Peeva (Crelle, 2023). Pedro's work studies the behavior of the powers and symbolic powers of these ideals under the Frobenius map in positive characteristic. One of the main results of this work shows that, for a large family of graphs, the symbolic powers of the binomial edge ideals agree with the ordinary powers, and the Rees algebra is strongly F-regular. This result is an analogue of a famous result on ideals of maximal minors of a generic matrix. He also establishes that various families of binomial edge ideals are symbolic F-split, a notion recently defined by De Stefani, Montaño, and Núñez-Betancourt with various strong consequences for asymptotics for symbolic powers. I expect that the consequences of this work will be quite interesting to the substantial community of researchers on binomial edge ideals. Pedro has also earlier used the techniques of diagonal F-splitting to study symbolic powers of determinantal ideals, though the main result of his work in progress was unluckily subsumed by independent work of Smolkin (Epijournal Algebraic Geometry 2024). Pedro is currently working with Carvajal-Rojas on follow-up questions to this project, and

I anticipate very interesting results to arise from this work.

Overall Pedro's work skillfully uses a balance of combinatorial techniques, positive characteristic F-singularity techniques, and Gröbner degeneration. I believe that he has a sound research plan that should produce interesting results; his broad foundations make him well-suited to collaborate on a wide range of problems with potential postdoctoral mentors and future colleagues.

Finally, on a personal note, Pedro always has an excellent attitude and is quite pleasant to have around. On all accounts, he would be an excellent member of your department, and I recommend most highly with no reservations.

Sincerely,

Jack Jeffries
Associate Professor
Department of Mathematics
University of Nebraska–Lincoln

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