

- (1) Let $x, y \in \mathbb{R}$. The negation of the statement “If x and y are rational, then xy is rational” is “If x and y are rational, then xy is irrational”.
- (2) Let $x, y \in \mathbb{R}$. The contrapositive of the statement “If x and y are rational, then xy is rational” is “If xy is irrational, then x is irrational or y is irrational”.
- (3) The commutative property/axiom of addition says that $x + y = y + x$.
- (4) Every set of real numbers that is bounded above has a supremum.
- (5) There is a set S of real numbers such that $\sup(S)$ exists, but $\sup(S) \notin S$.
- (6) If $a < b$ are real numbers, there is an integer $n \in \mathbb{Z}$ such that $a < n < b$.
- (7) Every nonempty set of real numbers has a smallest element (i.e., a minimum element).
- (8) Every nonempty set of integers that is bounded below has a smallest element (i.e., a minimum element).
- (9) If $S \subseteq \mathbb{R}$ is bounded above, then there is a natural number b such that b is an upper bound for S .
- (10) Every set of real numbers satisfies the property that “for all $x \in S$, there exists a real number y such that $x < y^2$ ”.
- (11) Every set of real numbers satisfies the property that “for all $x \in S$, there exists a real number y such that $y^2 < x$ ”.
- (12) The supremum of the set $\{1/n \mid n \in \mathbb{N}\}$ is 1.
- (13) The supremum of the set $\{-1/n \mid n \in \mathbb{N}\}$ is -1 .
- (14) The negation of the statement “for all $x \in S$, there exists a real number y such that $x < y^2$ ” is “for all $x \in S$, there exists a real number y such that $x \geq y^2$ ”.
- (15) If a sequence $\{a_n\}_{n=1}^{\infty}$ converges to 5, then for all natural numbers n , $a_n > 4$.
- (16) If a sequence $\{a_n\}_{n=1}^{\infty}$ converges to L , then there is some $N \in \mathbb{R}$ such that for all natural numbers $n > N$, $a_n = L$.
- (17) For every real number L there is a sequence $\{a_n\}_{n=1}^{\infty}$ such that $a_n \neq L$ for all $n \in \mathbb{N}$ and converges to L .

- (18) To prove the formula $1 + \frac{1}{2} + \cdots + \frac{1}{2^n} = 2 - \frac{1}{2^n}$ for all natural numbers n , it suffices to show that $1 + \frac{1}{2} + \cdots + \frac{1}{2^k} = 2 - \frac{1}{2^k}$ implies $1 + \frac{1}{2} + \cdots + \frac{1}{2^{k+1}} = 2 - \frac{1}{2^{k+1}}$.
- (19) One can prove that $2^n \geq 1 + n$ for all integers n by showing that $2^1 \geq 1 + 1$ then assuming $2^k \geq 1 + k$ and deducing $2^{k+1} \geq 2 + k$.
- (20) A sequence of positive numbers can converge to zero.
- (21) A sequence of positive numbers can converge to a negative number.
- (22) There is a set S of irrational numbers such that $\sup(S) = 2$.
- (23) Every increasing sequence is convergent.
- (24) Every convergent sequence is either increasing or decreasing.
- (25) If $\{a_n\}_{n=1}^{\infty}$ and $\{b_n\}_{n=1}^{\infty}$ are convergent sequences, then $\{a_n + b_n\}_{n=1}^{\infty}$ is a convergent sequence.
- (26) The sequence $\left\{ \frac{3n^2 - 4n + 7}{6n^2 + 1} \right\}_{n=1}^{\infty}$ converges to $1/2$.
- (27) The negation of “ $\{a_n\}_{n=1}^{\infty}$ is a monotone sequence” is “there exists $n \in \mathbb{N}$ such that $a_n > a_{n+1}$ and $a_n < a_{n+1}$ ”.
- (28) Every convergent sequence of rational numbers converges to a rational number.
- (29) If a sequence is not bounded below, then it diverges to $-\infty$.
- (30) If $\{a_n\}_{n=1}^{\infty}$ diverges to $+\infty$ and $\{b_n\}_{n=1}^{\infty}$ diverges to $-\infty$, then $\{a_n + b_n\}_{n=1}^{\infty}$ converges to 0.
- (31) If $\{a_n\}_{n=1}^{\infty}$ diverges to $+\infty$ and $\{b_n\}_{n=1}^{\infty}$ converges, then $\{a_n + b_n\}_{n=1}^{\infty}$ diverges to $+\infty$.
- (32) If $\{a_n\}_{n=1}^{\infty}$ converges, then $\{\frac{a_n}{n} + 2\}_{n=1}^{\infty}$ converges to 2.
- (33) If $\{a_n\}_{n=1}^{\infty}$ diverges and $\{b_n\}_{n=1}^{\infty}$ converges, then $\{a_n b_n\}_{n=1}^{\infty}$ diverges.
- (34) If $\{a_n^2\}_{n=1}^{\infty}$ converges to 1, then $\{a_n\}_{n=1}^{\infty}$ converges.
- (35) A sequence of rational numbers can converge to an irrational number.
- (36) A sequence of integers can converge to an irrational number.