ASSIGNMENT #9: DUE TUESDAY, NOVEMBER ?? AT MIDNIGHT

This problem set is to be turned in on Canvas. You may reference any result or problem from our worksheets or lectures, unless it is the fact to be proven! You are encouraged to work with others, but you should understand everything you write. Please consult the class website for acceptable/unacceptable resources for the problem sets.

(1) For each of the following, either give an example of a function with the indicated property or explain why none exists.
   (a) A function $f : (-1, 1) \to \mathbb{R}$ that is continuous on the open interval $(-1, 1)$ whose range is $[2, 3)$.
   (b) A function $f : [-1, 1] \to \mathbb{R}$ that is continuous on the closed interval $[-1, 1]$ whose range is $[0, \infty)$.
   (c) A function $f : [-1, 1] \to \mathbb{R}$ that is continuous on the closed interval $[-1, 1]$ whose range is $[2, 3)$.
   (d) A function $f : (-1, 1) \to \mathbb{R}$ that achieves a minimum value but not a maximum value on $(-1, 1)$.
   (e) A function $f : \mathbb{R} \to \mathbb{R}$ that is continuous on $\mathbb{R}$ with range $(0, \infty)$.

(2) Assume that f is continuous on the closed interval $[0, 1]$ and that $0 \leq f(x) \leq 1$ for all $x \in [0, 1]$. Show that there is some real number c such that $0 \leq c \leq 1$ and $f(c) = c$.

(3) Prove[1] that every real number has a unique cube root.

[1]Hint: For existence, use the Intermediate Value Theorem. For uniqueness, it may be useful to use the difference of cubes factorization $y^3 - x^3 = (y - x)(y^2 + xy + x^2)$ and apply completing the square to the latter factor.