

REVIEW (EXAM 1–2 MATERIAL): TRUE OR FALSE? JUSTIFY.

SIDE A

- (1) The sequence  $\left\{ \frac{3n-4}{2n+5} \right\}_{n=1}^{\infty}$  converges to  $\frac{3}{2}$ . (Use Theorems.)
- (2) If  $\{a_n\}_{n=1}^{\infty}$  converges to 5, then there is some  $N \in \mathbb{R}$  such that for all  $n > N$ ,  $2a_n^2 > 49$ .
- (3) The limit of  $f(x) = \begin{cases} 2x-1 & \text{if } x > 3 \\ x+2 & \text{if } x \leq 3 \end{cases}$  as  $x$  approaches 3 is 5. (Use only the definition.)
- (4) The function  $f(x) = \begin{cases} 2x \sin(1/x) & \text{if } x > 0 \\ 2 & \text{if } x = 0 \\ -2x \cos(1/x) & \text{if } x < 0 \end{cases}$  is continuous at  $x = 0$ .
- (5) Every nonempty set of real numbers that is bounded above has a maximum element.
- (6) The supremum of the set  $\{x \in \mathbb{Q} \mid x < \pi\}$  is  $\pi$ .
- (7) If the domain of  $f$  is  $\mathbb{R}$ , then  $f$  is continuous at some value of  $x$ .
- (8) Every decreasing sequence is convergent.
- (9) The sequence  $\left\{ \frac{\sin(n^2)}{n} \right\}_{n=1}^{\infty}$  is convergent.
- (10) We can prove that every polynomial  $p(x)$  has a property  $P$  by induction on degree by showing that every constant function has property  $P$  and then showing that if  $p(x)$  has property  $P$  then so does  $p'(x)$ .
- (11) For every pair of integers  $m, n \in \mathbb{Z}$ ,  $m^2 \neq 8n^2$ .
- (12) If  $\{b_n\}_{n=1}^{\infty}$  is not decreasing, then  $\{b_n\}_{n=1}^{\infty}$  has an increasing subsequence.
- (13) If  $f$  is continuous on  $[1, 3]$ , and  $y > f(1) > f(3)$ , then there is no  $c \in [1, 3]$  with  $f(c) = y$ .
- (14) If  $f$  is not bounded below, then  $f$  diverges to  $-\infty$ .
- (15) If  $f$  and  $g$  are continuous on  $(-7, 7)$  and  $g(4) = -1$ , then  $\lim_{x \rightarrow 4} (f \circ g)(x) = f(-1)$ .
- (16) If  $\{a_n\}_{n=1}^{\infty}$  and  $\{b_n\}_{n=1}^{\infty}$  both diverge, then so does  $\{a_n + b_n\}_{n=1}^{\infty}$ .
- (17) If  $f(x) > 5$  for all  $x \neq -7$  and  $\lim_{x \rightarrow -7} f(x) = L$ , then  $L > 5$ .

## DERIVATIVES REVIEW: TRUE OR FALSE? JUSTIFY.

### SIDE B

- (1) The sequence  $\left\{\frac{3n-4}{2n+5}\right\}_{n=1}^{\infty}$  converges to  $\frac{3}{2}$ . (Use only the definition.)
- (2) If  $\{a_n\}_{n=1}^{\infty}$  converges to  $L > 0$ , then there is some  $N \in \mathbb{R}$  such that for all  $n > N$ ,  $a_n > L/2$ .
- (3) The limit of  $f(x) = \begin{cases} 2x-1 & \text{if } x > 3 \\ x+3 & \text{if } x < 3 \end{cases}$  as  $x$  approaches 3 is 5. (Use only the definition.)
- (4) The function  $f(x) = \begin{cases} x^2 \sin(1/x^2) & \text{if } x > 0 \\ 0 & \text{if } x = 0 \\ -x^2 \cos(1/x^2) & \text{if } x < 0 \end{cases}$  is continuous at  $x = 0$ .
- (5) If  $S$  is a set of real numbers and  $\sup(S) \in S$ , then  $\sup(S)$  is the maximum element of  $S$ .
- (6) The maximum of the set  $\{x \in \mathbb{Q} \mid x < \pi\}$  is  $\pi$ .
- (7) Every convergent sequence is bounded.
- (8) Every monotone sequence has a convergent subsequence.
- (9) If  $f$  is differentiable at  $x = 2$  and  $f(2) = 5$ , then the sequence  $\left\{f\left(\frac{2n+1}{n+4}\right)\right\}_{n=1}^{\infty}$  converges to 5.
- (10) The sequence  $\{\sin(n^2)\}_{n=1}^{\infty}$  has a convergent subsequence.
- (11) For any rational numbers  $a < b$ , there is an irrational number  $c$  such that  $a < c < b$ .
- (12) If  $f$  does not diverge to  $-\infty$  and  $f$  does not diverge to  $+\infty$ , then  $f$  converges.
- (13) If  $f$  is differentiable on  $\mathbb{R}$ ,  $f'(x) \leq 0$  for all  $x > 0$ , and  $f(x) \geq -5$  for all  $x > 0$ , then the sequence  $\{f(n)\}_{n=1}^{\infty}$  converges.
- (14) Every sequence has a strictly increasing subsequence or a strictly decreasing subsequence.
- (15) We can prove that every polynomial  $p(x)$  has a property  $P$  by induction on degree by showing that every constant function has property  $P$  and then showing that if  $p'(x)$  has property  $P$  then so does  $p(x)$ .
- (16) If  $\{a_n\}_{n=1}^{\infty}$  and  $\{b_n\}_{n=1}^{\infty}$  both diverge to  $+\infty$ , then so does  $\{a_n + b_n\}_{n=1}^{\infty}$ .
- (17) If  $\lim_{x \rightarrow -3} f(x) > 5$ , then  $\exists \delta > 0$  such that  $f(x) > 5$  for all  $x \in (-3 - \delta, -3) \cup (-3, -3 + \delta)$ .