DIVERGENCE TO $+\infty$ AND DIVERGENCE TO $-\infty$ §2.2

DEFINITION 16.1: A sequence **diverges to** $+\infty$ if for every real number M, there is some $N \in \mathbb{R}$ such that for every natural number $n > N$ we have $a_n > M$.

DEFINITION 16.2: A sequence **diverges to** $-\infty$ if for every real number m, there is some $N \in \mathbb{R}$ such that for every natural number $n > N$ we have $a_n < m$.

(1) Use the definition to prove that the sequence $\{\sqrt{n}\}_{n=1}^{\infty}$ diverges to $+\infty$.

(2) Prove that if a sequence $\{a_n\}_{n=1}^{\infty}$ diverges to $+\infty$ then it is not bounded above.

(3) Use (2) to show that if a sequence diverges to $+\infty$ then it diverges.

(4) Prove or disprove: If a sequence diverges, then it diverges to $+\infty$ or it diverges to $-\infty$.

(5) Prove or disprove: If a sequence is not bounded above, then it diverges to $+\infty$.

(6) Prove or disprove: If a sequence diverges to $+\infty$ then it is increasing.

(7) Prove or disprove: If a sequence is increasing and not bounded above, it diverges to ∞.