Math 325. Quiz #7

(1) State the definition for the **limit** of $f(x)$ is L as x approaches a.

(2) TRUE OR FALSE, and *justify* with a short proof or example:
Every subsequence of a divergent sequence is divergent.

(3) TRUE OR FALSE, and *justify* with a short proof or example:
The[1] sequence $\{a_n\}_{n=1}^{\infty}$ where $a_n = n\sqrt{2} - \lfloor n\sqrt{2} \rfloor$ has a convergent subsequence.

[1]The notation $\lfloor x \rfloor$ means the smallest integer less than or equal to x, so $\lfloor x \rfloor \leq x < \lfloor x \rfloor + 1$. For example, $a_1 = (1.4142\ldots - 1) = 0.4142\ldots$, $a_2 = (2.8284\ldots - 2) = 0.8284\ldots$, and $a_3 = (4.2426\ldots - 4) = 0.2426\ldots$

Bonus. Prove or disprove: For the same sequence $\{a_n\}_{n=1}^{\infty}$ where $a_n = n\sqrt{2} - \lfloor n\sqrt{2} \rfloor$, there exists a subsequence that converges to 0.