Math 325-002 — Problem Set #5
Due: Thursday, September 29 by 7 pm, on Canvas

Instructions: You are encouraged to work together on these problems, but each student should hand in their own final draft, written in a way that indicates their individual understanding of the solutions. Never submit something for grading that you do not completely understand.

If you do work with others, I ask that you write something along the top like "I collaborated with Steven Smale on problems 1 and 3". If you use a reference, indicate so clearly in your solutions. In short, be intellectually honest at all times.

Please write neatly, using complete sentences and correct punctuation. Label the problems clearly.

(1) Consider the sequence $\left\{\dfrac{2n+1}{5n-2}\right\}_{n=1}^{\infty}$. Use the definition of converges (but not Theorem 10.2) to show that this sequence converges to $\dfrac{2}{5}$.

(2) Prove that the sequence
$$\left\{\frac{8n^2-5n+3}{4n^2+1}\right\}_{n=1}^{\infty}$$
converges. (This includes finding to what it converges.) You should use Theorem 10.2, but carefully justify every step using the Theorem.

(3) Let $\{a_n\}_{n=1}^{\infty}$ be a sequence, and K, L be real numbers. Suppose that for all $n \in \mathbb{N}$, $a_n \geq K$, and that $\{a_n\}_{n=1}^{\infty}$ converges to L. Prove that $L \geq K$.

(4) Prove that the sequence $\{\sqrt{n}\}_{n=1}^{\infty}$ diverges.

(5) Assume that $\{a_n\}_{n=1}^{\infty}$ converges to zero, and that $a_n \geq 0$ for all natural numbers n. Prove that $\{\sqrt{a_n}\}_{n=1}^{\infty}$ converges to zero also.