UNIVERSAL MAPPING THEOREM FOR CYCLIC GROUPS: Let $G = \langle x \rangle$ be a cyclic group and H be an arbitrary group.

(1) If $|x| = n < \infty$ and $y \in H$ is such that $y^n = e$, then there is a unique homomorphism $f : G \to H$ such that $f(x) = y$.
(2) If $|x| = \infty$ and $y \in H$ is arbitrary, then there is a unique homomorphism $f : G \to H$ such that $f(x) = y$.

DEFINITION:

- The **infinite cyclic group** is the group $C_\infty = \{a^j \mid j \in \mathbb{Z}\}$ with operation $a^j a^k = a^{j+k}$. Its presentation[1]is $\langle a \mid \varnothing \rangle$.
- For any $n \in \mathbb{Z}_{\geq 1}$, the cyclic group of order n is the group $C_n = \{a^j \mid j \in \{0, 1, \ldots, n-1\}\}$ with operation $a^j a^k = a^{j+k \,(\mathrm{mod}\, n)}$. Its presentation is $\langle a \mid a^n = e \rangle$.

CLASSIFICATION OF CYCLIC GROUPS: Every infinite cyclic group is isomorphic to C_∞. Every cyclic group of order n is isomorphic to C_n.

(1) Use the Universal Mapping Theorem for cyclic groups to prove the classification of cyclic groups.

(2) Prove the Universal mapping theorem for cyclic groups.

(3) Classify all subgroups of C_∞ and describe the subgroup lattice.

[1]We write the empty set in the relations spot to indicate that there are no defining relations.