(Axiom 1) There are operations $+$ and $\cdot$ defined on $\mathbb{R}$:

$$\text{for all } p, q \in \mathbb{R}, \quad p + q \in \mathbb{R} \text{ and } p \cdot q \in \mathbb{R}.$$

(Axiom 2) Each of $+$ and $\cdot$ is a commutative operation:

$$\text{for all } p, q \in \mathbb{R}, \quad p + q = q + p \text{ and } p \cdot q = q \cdot p.$$

(Axiom 3) Each of $+$ and $\cdot$ is an associative operation:

$$\text{for all } p, q, r \in \mathbb{R}, \quad (p + q) + r = p + (q + r) \text{ and } (p \cdot q) \cdot r = p \cdot (q \cdot r).$$

(Axiom 4) The number 0 is an identity element for addition and the number 1 ($\neq 0$) is an identity element for multiplication:

$$\text{for all } p \in \mathbb{R}, \quad 0 + p = p \text{ and } 1 \cdot p = p.$$

(Axiom 5) The distributive law holds:

$$\text{for all } p, q, r \in \mathbb{R}, \quad p \cdot (q + r) = p \cdot q + p \cdot r.$$

(Axiom 6) Every number has an additive inverse:

$$\text{for each } p \in \mathbb{R}, \text{ there is some “}-p\text{”} \in \mathbb{R} \text{ such that } \quad p + (-p) = 0.$$

(Axiom 7) Every nonzero number has a multiplicative inverse:

$$\text{for each } p \in \mathbb{R}, p \neq 0, \text{ there is some “}p^{-1}\text{”} \in \mathbb{R} \text{ such that } \quad p \cdot p^{-1} = 1.$$

(Axiom 8) There is a “total ordering” $\leq$ on $\mathbb{R}$. This means that
(a) for all $p, q \in \mathbb{R}$, either $p \leq q$ or $q \leq p$.
(b) for all $p, q \in \mathbb{R}$, if $p \leq q$ and $q \leq p$, then $p = q$.
(c) for all $p, q, r \in \mathbb{R}$, if $p \leq q$ and $q \leq r$, then $p \leq r$.

(Axiom 9) The total ordering $\leq$ is compatible with addition:

$$\text{for all } p, q, r \in \mathbb{R}, \quad \text{if } p \leq q \text{ then } p + r \leq q + r.$$

(Axiom 10) The total ordering $\leq$ is compatible with multiplication by nonnegative numbers:

$$\text{for all } p, q, r \in \mathbb{R}, \quad \text{if } p \leq q \text{ and } r \geq 0 \text{ then } pr \leq qr.$$

(Axiom 11) The COMPLETENESS AXIOM