Reivew (Exam 1–2 material): True or False? Justify.

Side A

(1) The sequence $\left\{\dfrac{3n-4}{2n+5}\right\}_{n=1}^{\infty}$ converges to $\frac{3}{2}$. (Use Theorems.)

(2) If $\{a_n\}_{n=1}^{\infty}$ converges to 5, then there is some $N \in \mathbb{R}$ such that for all $n > N$, $2a_n^2 > 49$.

(3) The limit of $f(x) = \begin{cases} 2x-1 & \text{if } x > 3 \\ x+2 & \text{if } x \leq 2 \end{cases}$ as x approaches 3 is 5. (Use only the definition.)

(4) The function $f(x) = \begin{cases} 2x\sin(1/x) & \text{if } x > 0 \\ 2 & \text{if } x = 0 \\ -2x\cos(1/x) & \text{if } x < 0 \end{cases}$ is continuous at $x = 0$.

(5) Every nonempty set of real numbers that is bounded above has a maximum element.

(6) The supremum of the set $\{x \in \mathbb{Q} \mid x < \pi\}$ is π.

(7) If the domain of f is $\mathbb{R}$, then f is continuous at some value of x.

(8) Every decreasing sequence is convergent.

(9) The sequence $\left\{\dfrac{\sin(n^2)}{n}\right\}_{n=1}^{\infty}$ is convergent.

(10) We can prove that every polynomial $p(x)$ has a property P by induction on degree by showing that every constant function has property P and then showing that if $p(x)$ has property P then so does $p'(x)$.

(11) For every pair of integers $m, n \in \mathbb{Z}$, $m^2 \neq 8n^2$.

(12) If $\{b_n\}_{n=1}^{\infty}$ is not decreasing, then $\{b_n\}_{n=1}^{\infty}$ has an increasing subsequence.

(13) If f is continuous on $[1, 3]$, and $y > f(1) > f(3)$, then there is no $c \in [1, 3]$ with $f(c) = y$.

(14) If f is not bounded below, then f diverges to $-\infty$.

(15) If f and g are continuous on $(-7, 7)$ and $g(4) = -1$, then $\lim_{x\to 4}(f \circ g)(x) = f(-1)$.

(16) If $\{a_n\}_{n=1}^{\infty}$ and $\{b_n\}_{n=1}^{\infty}$ both diverge, then so does $\{a_n + b_n\}_{n=1}^{\infty}$.

(17) If $f(x) > 5$ for all $x \neq -7$ and $\lim_{x\to -7} f(x) = L$, then $L > 5$.

Reivew (Exam 1–2 material): True or False? Justify.

Side B

(1) The sequence $\left\{\dfrac{3n-4}{2n+5}\right\}_{n=1}^{\infty}$ converges to $\frac{3}{2}$. (Use only the definition.)

(2) If $\{a_n\}_{n=1}^{\infty}$ converges to $L > 0$, then there is some $N \in \mathbb{R}$ such that for all $n > N$, $a_n > L/2$.

(3) The limit of $f(x) = \begin{cases} 2x-1 & \text{if } x > 3 \\ x+3 & \text{if } x < 3 \end{cases}$ as x approaches 3 is 5. (Use only the definition.)

(4) The function $f(x) = \begin{cases} x^2\sin(1/x^2) & \text{if } x > 0 \\ 0 & \text{if } x = 0 \\ -x^2\cos(1/x^2) & \text{if } x < 0 \end{cases}$ is continuous at $x = 0$.

(5) If S is a set of real numbers and $\sup(S) \in S$, then $\sup(S)$ is the maximum element of S.

(6) The maximum of the set $\{x \in \mathbb{Q} \mid x < \pi\}$ is π.

(7) Every convergent sequence is bounded.

(8) Every monotone sequence has a convergent subsequence.

(9) If f is differentiable at $x = 2$ and $f(2) = 5$, then the sequence $\left\{f\left(\dfrac{2n+1}{n+4}\right)\right\}_{n=1}^{\infty}$ converges to 5.

(10) The sequence $\left\{\sin(n^2)\right\}_{n=1}^{\infty}$ has a convergent subsequence.

(11) For any rational numbers $a < b$, there is an irrational number c such that $a < c < b$.

(12) If f does not diverge to $-\infty$ and f does not diverge to $+\infty$, then f converges.

(13) If f is differentiable on $\mathbb{R}$, $f'(x) \leq 0$ for all $x > 0$, and $f(x) \geq -5$ for all $x > 0$, then the sequence $\{f(n)\}_{n=1}^{\infty}$ converges.

(14) Every sequence has a strictly increasing subsequence or a strictly decreasing subsequence.

(15) We can prove that every polynomial $p(x)$ has a property P by induction on degree by showing that every constant function has property P and then showing that if $p'(x)$ has property P then so does $p(x)$.

(16) If $\{a_n\}_{n=1}^{\infty}$ and $\{b_n\}_{n=1}^{\infty}$ both diverge to $+\infty$, then so does $\{a_n + b_n\}_{n=1}^{\infty}$.

(17) If $\lim\limits_{x\to-3} f(x) > 5$, then $\exists\delta > 0$ such that $f(x) > 5$ for all $x \in (-3-\delta, -3) \cup (-3, -3+\delta)$.