True or False? Justify.

Side A

(1) Let $x, y \in \mathbb{R}$. The negation of the statement "If x and y are rational, then xy is rational" is "If x and y are rational, then xy is irrational".

(2) If a sequence $\{a_n\}_{n=1}^{\infty}$ converges to 5, then for all natural numbers n, $a_n > 4$.

(3) The sequence $\left\{\dfrac{3n^2 - 4n + 7}{6n^2 + 1}\right\}_{n=1}^{\infty}$ converges to $1/2$.

(4) The supremum of the set $\{1/n \mid n \in \mathbb{N}\}$ is 1.

(5) If a sequence $\{a_n\}_{n=1}^{\infty}$ converges to L, then there is some $N \in \mathbb{R}$ such that for all natural numbers $n > N$, $a_n = L$.

(6) Every increasing sequence is convergent.

(7) If a sequence is not bounded below, then it diverges to $-\infty$.

(8) If $\{a_n\}_{n=1}^{\infty}$ converges, then $\left\{\dfrac{a_n}{n} + 2\right\}_{n=1}^{\infty}$ converges to 2.

(9) Let $x, y \in \mathbb{R}$. The contrapositive of the statement "If x and y are rational, then xy is rational" is "If xy is irrational, then x is irrational or y is irrational".

(10) If $a < b$ are real numbers, there is an integer $n \in \mathbb{Z}$ such that $a < n < b$.

(11) Every set of real numbers that is bounded above has a supremum.

(12) There is a set S of rational numbers such that $\sup(S) = 5\sqrt{2}$.

(13) For every real number L there is a sequence $\{a_n\}_{n=1}^{\infty}$ such that $a_n \neq L$ for all $n \in \mathbb{N}$ and converges to L.

(14) The negation of "$\{a_n\}_{n=1}^{\infty}$ is a monotone sequence" is "there exists $n \in \mathbb{N}$ such that $a_n > a_{n+1}$ and $a_n < a_{n+1}$".

(15) If $\{a_n\}_{n=1}^{\infty}$ diverges to $+\infty$ and $\{b_n\}_{n=1}^{\infty}$ diverges to $-\infty$, then $\{a_n + b_n\}_{n=1}^{\infty}$ converges to 0.

(16) Every nonempty set of integers that is bounded below has a smallest element (i.e., a minimum element).

True or False? Justify.

Side B

(1) If $\{a_n\}_{n=1}^{\infty}$ and $\{b_n\}_{n=1}^{\infty}$ are convergent sequences, then $\{a_n + b_n\}_{n=1}^{\infty}$ is a convergent sequence.

(2) Every set of real numbers satisfies the property that "for all $x \in S$, there exists a real number y such that $y^2 < x$".

(3) If $S \subseteq \mathbb{R}$ is bounded above, then there is a natural number b such that b is an upper bound for S.

(4) The supremum of the set $\{-1/n \mid n \in \mathbb{N}\}$ is -1.

(5) Every convergent sequence is either increasing or decreasing.

(6) If $\{a_n^2\}_{n=1}^{\infty}$ converges to 1, then $\{a_n\}_{n=1}^{\infty}$ converges.

(7) There is a set S of real numbers such that $\sup(S)$ exists, but $\sup(S) \notin S$.

(8) Every set of real numbers satisfies the property that "for all $x \in S$, there exists a real number y such that $x < y^2$".

(9) The negation of the statement "for all $x \in S$, there exists a real number y such that $x < y^2$" is "for all $x \in S$, there exists a real number y such that $x \geq y^2$".

(10) If $\{a_n\}_{n=1}^{\infty}$ diverges and $\{b_n\}_{n=1}^{\infty}$ converges, then $\{a_n b_n\}_{n=1}^{\infty}$ diverges.

(11) A sequence of negative numbers can converge to zero.

(12) A sequence of negative numbers can converge to a positive number.

(13) Every nonempty set of real numbers has a smallest element (i.e., a minimum element).

(14) If $\{a_n\}_{n=1}^{\infty}$ diverges to $+\infty$ and $\{b_n\}_{n=1}^{\infty}$ converges, then $\{a_n + b_n\}_{n=1}^{\infty}$ diverges to $+\infty$.

(15) A sequence of rational numbers can converge to an irrational number.

(16) A sequence of integers can converge to an irrational number.