ESE 503 - Simulation Modeling & Analysis (Assignment #4)

Summer Semester, 2020 M. Carchidi

Problem #1 (20 points) - A Queueing Problem

Suppose that customers enter a store as a stationary Poisson process with an average rate of 1.5 customers per minute. In the store, there are c identical servers all working in parallel with service time distribution S that is a continuous random variable with pdf

$$f(s) = \frac{2s(3-s)}{9}$$

for $0 < s < 3$, in units of minutes.

a.) (**5 points**) Determine the *smallest* number of servers (*i.e.*, the smallest value of c) so that steady-state conditions can exist. *Hint*: You may use the fact that

$$\int_0^a s^n(a-s)ds = \frac{a^{n+2}}{(n+1)(n+2)}$$

for $n \geq 0$ and $a > 0$.

b.) (**15 points**) Using the value of c determined in part (a), compute the steady-state values of: ρ, L_Q, w_Q, w and L.

Problem #2 (15 points) - A Problem With Four Life Bulbs

Suppose that four identical light bulbs B1, B2, B3 and B4, have lifetime distributions, X_1, X_2, X_3 and X_4, that are all exponential with the same parameter λ. A person takes all four of these into a room and turns on B1 and B2 at the same time. When both of these burn out, then the person turns on only B3 and when this burns out the person turns on B4. Compute the probability that the person will be in the dark two weeks (336 hours) later, if the average lifetime of each bulb is 150 hours.

Problem #3 (15 points) - Sampling a No Name Probability Distribution

Suppose that a continuous random variable X has a pdf given by

$$f(x) = \frac{1}{3} \times \begin{cases} e^x, & \text{when} \quad x < 0 \\ 1, & \text{when} \quad 0 < x < 1 \\ 1/x^2, & \text{when} \quad 1 < x \end{cases} .$$

Construct a method for providing a sample of X from a random number $R \sim U[0,1)$ and then compute $\Pr(a < X < b)$ as a function of a and b and then run a 5000-sample Monte-Carlo simulation to test your result for $\Pr(a < X < b)$ for different values of $a < b$.

Problem #4 (10 points) - Computing a PDF for a Random Variable

Suppose that X is exponential with parameter λ and that $Y = X^2 + 2X$. Determine the pdf for Y.
