ESE503 - Simulation Modeling & Analysis (Exam #2)

Summer Semester, 2020 M. Carchidi

Problem #1 (20 points) - Computing A Moment Generating Function

Suppose that X is a random variable with pdf

$$f_X(x) = \begin{cases} 0, & \text{for} \quad x < 0 \\ A(e^{-\alpha x} + e^{-\beta x}), & \text{for} \quad x > 0 \end{cases}$$

where A, α and β are positive constants.

a.) (**5 points**) Determine A in terms of α and β.

b.) (**7 points**) Determine the moment generating function, $M_X(t)$, of X in terms of α, β and t.

c.) (**8 points**) Use your result in part (b) to determine $E(X)$ and $V(X)$ in terms of α and β.

Hint: You may use the fact that

$$\int_0^\infty e^{-au} du = \frac{1}{a} \qquad \text{for} \qquad a > 0.$$

Problem #2 (10 points) - A Non-Stationary Poisson Process

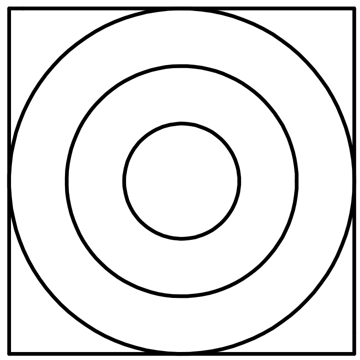
Suppose that customer's enter a store that is open for 8 hours, starting at 9:00:00 AM and suppose that rate function of arrivals is given by

$$\lambda(t) = \begin{cases} 6t, & \text{for} \quad 0 \le t \le 4 \\ 48 - 6t, & \text{for} \quad 4 \le t \le 8 \end{cases}$$

with time in units of hours. Compute the probability that *more that* 60 people enter the store between 12 PM and 3 PM.

Problem #3 (20 points) - Playing With Darts

Consider the dart board in the figure below which has three concentric circles of radii, R, $2R$ and $3R$, respectively, inside a square of side length $L = 6R$.



The Dart Board in the problem

Getting the dart in the inner circle scores 30 points, getting the dart in the middle circle (but not in the inner circle) scores 20 points, getting the dart in the outer circle (but not in the middle or inner circles) scores 10 points and getting the dart outside the outer circle scores 0 points. Suppose that the random variables X and Y in throwing the dart at point (X, Y) are both independent and uniform between 0 and $L = 6R$.

a.) (**8 points**) Fill in the table below which gives the pmf of the score random variable from the throw of a single dart.

Score (S)	0	10	20	30
$P(S)$				

b.) (**12 points**) If two darts are independently thrown and their scores are added to form a total score, compute the probability distribution on this score and then check your results using a 5000 sample Monte-Carlo Simulation.

Problem #4 (30 points) - Computing a PDF

Suppose that $X_j \sim U[0,1)$ for $j = 1,2,3,4$ and that

$$X = \max\{X_1, X_2\} \qquad \text{and} \qquad Y = \min\{X_3, X_4\}.$$

a.) (**7 points**) Compute the pdf for $Z = X + Y$.

b.) (**7 points**) Compute the cdf for $Z = X + Y$ and use this to fill in the following table

Interval	Probability	Interval	Probability
$[0.0, 0.2)$		$[1.0, 1.2)$	
$[0.2, 0.4)$		$[1.2, 1.4)$	
$[0.4, 0.6)$		$[1.4, 1.6)$	
$[0.6, 0.8)$		$[1.6, 1.8)$	
$[0.8, 1.0)$		$[1.8, 2.0)$	

where the Probability entries in the table for $[a,b)$ are $P(a \leq Z < b)$.

c.) (**7 points**) Using your result in part (a), compute $E(Z)$ and $\sigma(Z)$ and then run a 5000 sample Monte-Carlo simulation to check your computed values of $E(Z)$ and $\sigma(Z)$ and your table from part (b).

d.) (**9 points**) Suppose that $X_1 \sim U[0,1)$ and $X_2 \sim U[0,1)$ and that

$$X = \max\{X_1, X_2\} \qquad \text{and} \qquad Y = \min\{X_1, X_2\}.$$

Compute the pdf for $W = X + Y$. Then compute $E(Z)$ and $\sigma(Z)$ and then run a 5000 sample Monte-Carlo simulation to check your computed values of $E(Z)$ and $\sigma(Z)$.

Problem #5 (20 points) - Computing Two Probabilities

a.) (**10 points**) A professor finds that the time to grade each problem on an exam is a random variable with an *exponential* distribution having a mean grading time of 10 minutes per problem. Determine the probability that the time it takes for the professor to grade an exam containing 4 independent problems is more than 50 minutes.

b.) (**10 points**) Suppose that the travel time from your home to your office is *normally distributed* with a mean of 40 minutes and a standard deviation of 5 minutes. If you want to be 97.5% certain that you will not be late for an office appointment exactly at 9:00:00 AM, what is the latest time (to the second) that you should leave your home?