1)

$$f(x) = \begin{cases} Ae^{x}, & \text{when} \quad x < 0 \\ A, & \text{when} \quad 0 < x < 1 \\ \dfrac{A}{x^2}, & \text{when} \quad x > 1 \end{cases}$$

1a) $\displaystyle\int_{-\infty}^{\infty} f(x)\,dx = 1$

$$\int_{-\infty}^{0} Ae^{x}\,dx + \int_{0}^{1} A\,dx + \int_{1}^{\infty} A/x^2\,dx = 1$$

$$A\int_{-\infty}^{0} e^{x}\,dx + A\int_{0}^{1} dx + A\int_{1}^{\infty} \frac{1}{x^2}\,dx = 1$$

$$A + A + A = 1 \Rightarrow \boxed{A = \frac{1}{3}}$$

1b)

$$F(x) = \begin{cases} \displaystyle\int_{-\infty}^{x} Ae^{t}\,dt, & \text{when} \quad x < 0 \\ \displaystyle\int_{-\infty}^{0} Ae^{t}\,dt + \int_{0}^{x} A\,dt, & \text{when} \quad 0 < x < 1 \\ \displaystyle\int_{-\infty}^{0} Ae^{t}\,dt + \int_{0}^{1} A\,dt + \int_{1}^{x} \frac{A}{t^2}\,dt, & \text{when} \quad x > 1 \end{cases}$$

Simplifying,

$$F(x) = \begin{cases} Ae^x, & \text{when } x < 0 \\ A(x+1), & \text{when } 0 < x < 1 \\ A\left(3 - \frac{1}{x}\right), & \text{when } x > 1 \end{cases}$$

From part a, $A = \frac{1}{3}$:

$$F(x) = \begin{cases} \frac{1}{3}e^x, & \text{when } x < 0 \\ \frac{1}{3}(x+1), & \text{when } 0 < x < 1 \\ 1 - \frac{1}{3x}, & \text{when } x > 1 \end{cases}$$

1c) $Pr(-1 \le X \le 2) = F(2) - F(-1) = 1 - \frac{1}{3 \cdot 2} - \frac{1}{3}e^{-1}$

$$Pr(-1 \le X \le 2) \approx 0.71071$$

2) The discriminant of $f(x)=x^3-Ax+B$ is $4A^3-27B^2$

In calculus in order for there to be exactly one real root,

$4A^3-27B^2<0$.

We want to compute $\Pr(4A^3-27B^2<0)$

$$\Pr(4A^3-27B^2<0)=\Pr(4A^3<27B^2)$$

$$=\int_0^1 \Pr(4A^3<27x^2)f(x)\,dx$$

$$=\int_0^1\int_0^1 \underbrace{\Pr(4y^3<27x^2)}\,\underbrace{f(y)f(x)}_{1}\,dy\,dx$$

$$\Pr(4y^3<27x^2)=\Pr\left(x>\sqrt{\frac{4y^3}{27}}\right)=1-F_B\left(\sqrt{\frac{4y^3}{27}}\right)$$

Since $B\sim U[0,1)$ and integration occurs between $[0,1)$,

$$F_B(x)=x \qquad 0<x<1$$

Thus, $\Pr(4A^3-27B^2<0)=\int_0^1\int_0^1\left(1-\sqrt{\frac{4y^3}{27}}\right)dy\,dx=\boxed{0.84604}$

3a) Let X be the number of games played until one team has won 4 games.

Pascal distribution describes this scenario, i.e. what is the probability of obtaining r successes (wins) in just x trials (games).

$$\Pr(X=x) = \binom{x-1}{r-1} p^r (1-p)^{x-r}$$

So the probabily that the world series will last x games with $r=4$ since a team needs 4 wins to win the world series

$$\Pr(X=x) = \binom{x-1}{3} p^4 (1-p)^{x-4}$$

$$\Pr(X=4) = \binom{3}{3} p^4 = p^4$$

$$\Pr(X=5) = \binom{4}{3} p^4 (1-p) = 4p^4(1-p)$$

$$\Pr(X=6) = \binom{5}{3} p^4 (1-p)^2 = 10p^4(1-p)^2$$

$$\Pr(X=7) = \binom{6}{3} p^4 (1-p)^3 = 20p^4(1-p)^3$$

3b) More generally, if we need n games to win the series then

$$\Pr(X=x) = \binom{x-1}{n-1} p^n (1-p)^{x-n}$$