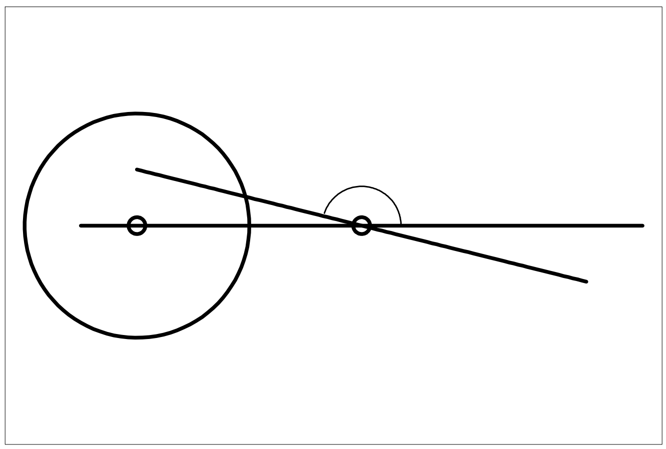
ESE503 - Simulation Modeling & Analysis (Exam #1)

Summer Semester, 2020 M. Carchidi

Problem #1 (20 points) - A Probability Problem

The figure below shows a uniform horizontal interval from 0 to 1. A point $X \sim U[0,1)$ is chosen at random on this interval and it serves as the center of a circle whose radius (R) which is also chosen at random so that $R \sim U[0,1)$. A second point $Y \sim U[0,1)$ is next chosen at random on this horizontal unit interval and it serves as the center of a line segment whose length (L) is also chosen at random so that $L \sim U[0,1)$. The counterclockwise angle (Θ) that this line segment makes with the positive horizontal unit interval is also chosen at random so that $\Theta \sim U[0,2\pi)$. Use 5000 Monte-Carlo simulations to estimate the probability that the line segment intersects the circle, as illustrated in the figure below.



The line segment of length L
intersects the circle of radius R

Problem #2 (20 points) - A Reliability Problem

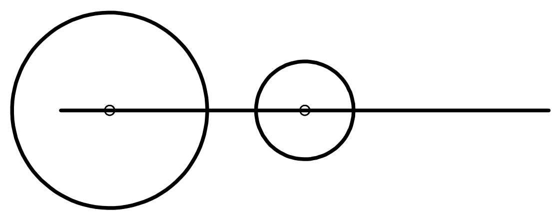
A film supplier produces R rolls of a specially sensitized film each day. Past experience indicates that the daily demand for this film is uniformly *continuous* with $X \sim U[0,17)$, so that it is possible to demand a *fractional roll* of film, and any film that is not sold at the end of the day must be discarded. A *profit* of $P = \$7$ is made on every roll which is sold, while a *lost* of $L = \$3$ is incurred on every roll which must be discarded.

a.) (**10 points**) Using a method much like the newspaper-seller problem done in class, determine an equation using P and L and the cdf of the X, to get the value of R which *maximizes* the average daily profit and determine this maximum average daily profit using the values of $P = \$7$ and $L = \$3$. Be sure to include lost profits.

b.) (**10 points**) Run a simulation using Excel (or any other program) to check your results in part (a).

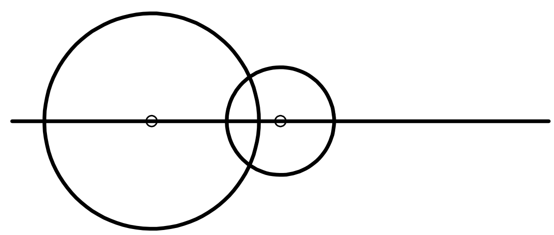
Problem #3 (20 points) - Another Probability Problem

Two coins (having radius R_1 and R_2) are uniformly dropped onto a horizontal line (along the *x* axis) of length L so that their centers lie somewhere on this line, as shown in the figure below.



The two circles do not overlap here

If the center of the one circle is at $(X, 0)$ and the center of the other circle is at $(Y, 0)$ with $X \sim U[0, L)$ and $Y \sim U[0, L)$ and if $R_1 \sim U[0, a)$ and $R_2 \sim U[0, b)$, construct a 1000-sample Monte-Carlo simulation (using different values of a, b and L) to estimate the probability that the two circles will overlap as shown in the next figure.



The two circles overlap here

For 5 extra-credit points, compute the exact value of this probability in terms of a, b and L, and compare your simulation results (using $a = 1$, $b = 2$ and $L = 4$) to this. *Hint*: You may begin with the theoretical result

$$\Pr(\text{Overlap}|R_1, R_2) = \begin{cases} \rho(2-\rho), & \text{when} \quad \rho = (R_1 + R_2)/L \le 1 \\ 1, & \text{when} \quad 1 \le \rho = (R_1 + R_2)/L \end{cases}$$

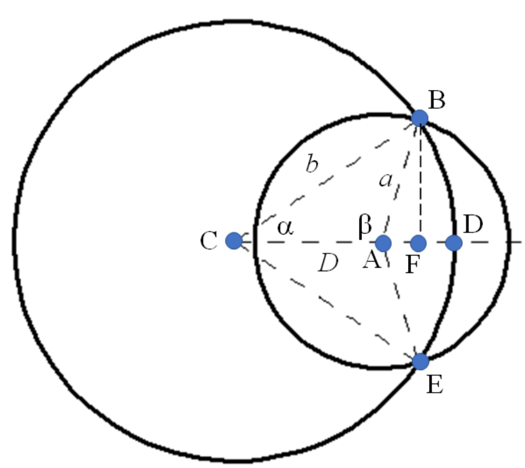
given in the two-coin overlap problem provided on Canvas.

Problem #4 (20 points) - A Queueing Problem

Suppose that customers enter a store so that the morning (from 8:00 AM - 12:00 PM) interarrival times (in minutes) are $U[2,6)$ and the afternoon (from 12:00 PM - 4:00 PM) interarrival times (in minutes) are $U[4,8)$. There are two servers, a morning server (from 8:00 AM - 12:00 PM) with a service time (in minutes) distribution that is $U[1,5)$ and an afternoon server (from 12:00 PM - 4:00 PM) with a service time (in minutes) distribution that is $U[3,7)$. Run a single-channel queue simulation of 100 customers to estimate: (a) the average time a customer must wait in line, (b) the probability that a customer has to wait, (c) the average waiting time for those customers that wait and (d) the average time a customer spends in the system. All times are in minutes.

Problem #5 (20 points) - Average Overlap Areas

Consider a fixed circle of radius b with its center at $(0,0)$. A coin, having radius $a < b$, is thrown onto this circle so that the center of the coin is a distance $D \geq 0$ from the center of the circle with $D \sim U[0,b)$. Use 5000 Monte-Carlo samples to estimate the *average* overlap area between the coin and the circle when $a = 2$ and $b = 3$. *Hint*: Of course when $0 \leq D \leq b - a$, all of the coin lies in the circle so that the overlap area is $A = \pi a^2$, which is the area of the coin. However, when $b - a \leq D \leq b$, then the overlap area of less than πa^2 and you should use a figure like the one below



The Coin and the Circle

to show that the overlap area is

$$A = \beta a^2 - bD\sin(\alpha) + \alpha b^2$$

when $b - a \leq D \leq b$.