ESE 503 - Simulation Modeling & Analysis (Assignment #2)

Summer Semester I, 2020 M. Carchidi

Problem #1 (15 points) - A No Name Probability Distribution

Suppose that a continuous random variable has a pdf given by

$$f(x) = \begin{cases} Ae^x, & \text{when} \quad x < 0 \\ A, & \text{when} \quad 0 < x < 1 \\ A/x^2, & \text{when} \quad 1 < x \end{cases} .$$

a.) (**5 points**) Determine the value of A.
b.) (**5 points**) Construct the cdf for X.
c.) (**5 points**) Determine the probability that $-1 \leq X \leq +2$.

Problem #2 (15 points) - A Probability Problem

Suppose that $A \sim U[0,1)$, $B \sim U[0,1)$. Compute the probability that the cubic equation

$$x^3 - Ax + B = 0$$

has only one real root. You should use your knowledge of Calculus to help with determining conditions on A and B which insure that $x^3 - Ax + Bb = 0$ has only one real root. Hint: Look at the local maximum and minimum values for the function $f(x) = x^3 - Ax + B$.

Problem #3 (20 points) - The World Series of Baseball

a.) (**15 points**) In the World series, two teams play until one team has won four games. Suppose that team #1 has a probability of p to win each game (so that team #2 has a probability of $1 - p$ of winning). Determine (in terms of p) the probability that the World series will last (a) 4 games, (b) 5 games, (c) 6 games, or (d) 7 games. Then run a Monte-Carlo simulation using 1000 trials to numerically check your results for $p = 1/4$, 1/2, and 3/4.
b.) (**5 points**) Generalize your results in part (a) by assuming that in the World series, two teams play until one team has won n games for some fixed value of n, by finding the probability (in terms of p, n and x) that the World series will last exactly x games for $x = n, n+1, n+2, \ldots, 2n-1$.
