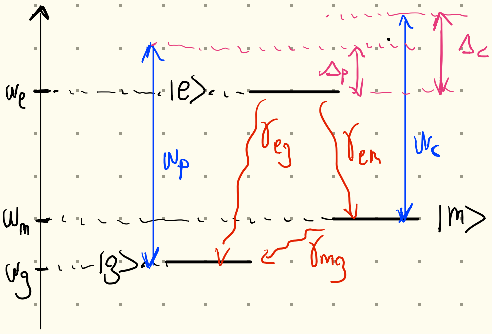
HARD THEORY



$\Delta_c = \omega_c - \omega_{em}$

$\Delta_p = \omega_p - \omega_{eg}$

The interaction is $\hat{V} = -\vec{d}\cdot\vec{E}(\vec{r},t)$

In the EDA we neglect the spatial dependence of $\vec{E}$, so

$$V_{ab} = \langle a|\hat{V}|b\rangle = -\vec{d}_{ab}\cdot\vec{E}(t) \qquad \vec{d}_{ab} = \langle a|\vec{d}|b\rangle$$

Notice that $|g\rangle, |m\rangle$ have the same F and therefore the same parity, so $\vec{d}_{gm} = 0$ (as $\vec{d}$ is odd)

Also

$$\vec{d}_{ab} = \int \langle a|q\vec{r}|b\rangle = \left(\int \langle b|q\vec{r}|a\rangle\right)^* = (d_{ba})^*$$

In our case $\vec{E} = \frac{1}{2}\left[\hat{\varepsilon}_p E_p e^{-i\omega_p t} + \hat{\varepsilon}_c E_c e^{-i\omega_c t} + cc\right]$

Considering $|\psi\rangle = C_g(t)|g\rangle + C_e(t)|e\rangle + C_m(t)|m\rangle$

Propagating $i\hbar|\dot{\psi}\rangle = (\hat{H}_0 + \hat{V})|\psi\rangle$ into the states I get:

$$i\hbar\dot{C}_g = \hbar\omega_g C_g(t) + V_{ge}C_e + \underbrace{V_{gm}}_{0} C_m$$

$$i\hbar\dot{C}_m = \hbar\omega_m C_m(t) + \underbrace{V_{mg}}_{0} C_g + V_{me} C_e$$

$$i\hbar\dot{C}_e = \hbar\omega_e C_e(t) + V_{em} C_m + V_{eg} C_g$$

with $\omega_e = \omega_{eg}$, $\omega_m = \omega_{eg} - \omega_{em}$

I go in the interaction picture as

$$a_g = C_g$$

$$a_m = e^{-i(\omega_p - \omega_c)t} C_m$$

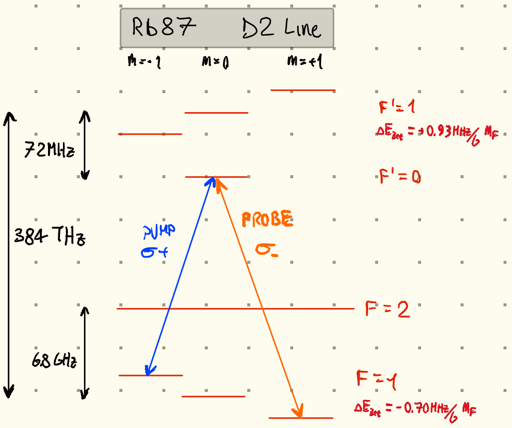
$$a_e = e^{-i\omega_p t} C_e$$

"False" three level system

We have population leakage in other levels, this can severely reduce visibility of the looked for feature, but it also reduces the linewidth of the feature itself

Precise values are unclear

Our System Rb87 D2 Line $F=1 \to F'=0$



Cell

We have a cell with Rb87 in natural abundance and 10 Torr of Ne as buffer gas

Our cell is 7.5 cm long and has N=40 coils wound around it

$$B = \mu_0 \frac{N}{L} I \qquad \mu_0 = 1.25664\cdot 10^{-6}\ N/A^2 \qquad 1G = 10^{-4}\ \frac{kg}{As^2}$$

$\Rightarrow B[G] \simeq 6.70208 \cdot I[A]$

So $\omega_{eg} - \omega_{em} = 9.38291\ MHz \cdot I[A]$

$\frac{1}{2} \Rightarrow 4.69146\ MHz \cdot I[A] = \zeta_{Zee}$

NB If you are not sure which beam is σ+ and which σ-, try everything inverting also the direction of the current, and hence of $\vec{B}$

AOMs

My AOMs, when locking to f_0 with the laser, give me

$$f_p = f_0 \pm 2\cdot\Delta f_p \qquad f_c = f_0 \pm 2\cdot\Delta f_c$$

If it's "+", then I have $\delta = 0$ when

$$f_p - f_c = \omega_{eg} - \omega_{em} \Rightarrow 2\Delta f_p - 2\Delta f_c = \Delta\omega_{mg}$$

$$\Rightarrow \Delta f_p = \Delta f_c + \frac{1}{2}\Delta\omega_{mg}$$

$$i\dot{a}_g = -\frac{1}{\hbar}\vec{d}_{ge}\cdot\frac{1}{2}\left(\hat{\varepsilon}_p E_p e^{-i\omega_p t} + cc\right)\cdot\left(e^{-i\omega_p t} a_e\right)$$

$$\Rightarrow i\dot{a}_g = -\frac{1}{\hbar}\vec{d}_{ge}\cdot\frac{1}{2}\hat{\varepsilon}_p^* E_p^* a_e$$

$$\left(i\dot{a}_m + (\omega_p-\omega_c)a_m\right)e^{-i(\omega_p-\omega_c)t}$$

$$= (\omega_{eg}-\omega_{em})a_m e^{-i(\omega_p-\omega_c)t} - \frac{1}{\hbar}\vec{d}_{me}\cdot\frac{1}{2}\left(\hat{\varepsilon}_c E_c e^{-i\omega_c t} + cc\right)\cdot\left(e^{-i\omega_p t} a_e\right)$$

$$\Rightarrow i\dot{a}_m = (\Delta_c-\Delta_p)a_m - \frac{1}{\hbar}\vec{d}_{me}\cdot\frac{1}{2}\hat{\varepsilon}_c^* E_c^* a_e$$

$$(i\dot{a}_e + \omega_p a_e)e^{-i\omega_p t} = \omega_{eg} a_e e^{-i\omega_p t}$$

$$-\frac{1}{\hbar}\vec{d}_{eg}\cdot\frac{1}{2}\left(\hat{\varepsilon}_p E_p e^{-i\omega_p t} + cc\right) a_g$$

$$-\frac{1}{\hbar}\vec{d}_{em}\cdot\frac{1}{2}\left(\hat{\varepsilon}_c E_c e^{-i\omega_c t} + cc\right) a_m e^{-i(\omega_p-\omega_c)t}$$

$$\Rightarrow i\dot{a}_e = -\Delta_p a_e - \frac{1}{\hbar}\vec{d}_{eg}\cdot\frac{1}{2}\hat{\varepsilon}_p E_p a_g - \frac{1}{\hbar}\vec{d}_{em}\frac{1}{2}\hat{\varepsilon}_c E_c a_m$$

I call $\frac{\vec{d}_{eg}\cdot\hat{\varepsilon}_p E_p}{\hbar} = \Omega_p$ $\frac{\vec{d}_{em}\cdot\hat{\varepsilon}_c E_c}{\hbar} = \Omega_c$

So I have

$$\boxed{\begin{aligned} \dot{a}_g &= i\frac{\Omega_p^*}{2}a_e \\ \dot{a}_m &= i(\Delta_p-\Delta_c)a_m + i\frac{\Omega_c^*}{2}a_e \\ \dot{a}_e &= +i\Delta_p a_e + i\frac{\Omega_p}{2}a_g + i\frac{\Omega_c}{2}a_m \end{aligned}}$$

! Interaction picture breaks if there is no phase coherence between pump and probe $e^{-i(\omega_p t + \phi_p(t))}$, $e^{-i(\omega_c t + \phi_c(t))}$

I get
$$\begin{pmatrix}\dot{a}_g\\ \dot{a}_m\\ \dot{a}_e\end{pmatrix} = -\frac{i}{\hbar}\tilde{H}\begin{pmatrix}a_g\\ a_m\\ a_e\end{pmatrix} \Rightarrow \tilde{H} = \hbar\begin{pmatrix}0 & 0 & -\Omega_p^*/2\\ 0 & -(\Delta_p-\Delta_c) & -\Omega_c^*/2\\ -\Omega_p/2 & -\Omega_c/2 & -\Delta_p\end{pmatrix}$$

Going now to density matrix, in the interaction picture

$$\dot{\rho}_I = -\frac{i}{\hbar}[\tilde{H},\rho_I]$$, $\tilde{H}$ being the Hamiltonian just found

$$\dot{\rho}_{gg} = i\left(\frac{\Omega_p^*}{2}\rho_{eg} - \rho_{ge}\frac{\Omega_p}{2}\right)$$

$$\dot{\rho}_{ee} = i\left(\frac{\Omega_p}{2}\rho_{ge} + \frac{\Omega_c}{2}\rho_{me} + \Delta_p\rho_{ee} - \rho_{ee}\Delta_p - \rho_{eg}\frac{\Omega_p^*}{2} - \rho_{em}\frac{\Omega_c^*}{2}\right)$$

$$\dot{\rho}_{mm} = i\left((\Delta_p-\Delta_c)\rho_{mm} + \frac{\Omega_c^*}{2}\rho_{em} - \rho_{mm}(\Delta_p-\Delta_c) - \rho_{me}\frac{\Omega_c}{2}\right)$$

$$\dot{\rho}_{eg} = i\left(\frac{\Omega_p}{2}\rho_{gg} + \frac{\Omega_c}{2}\rho_{mg} + \Delta_p\rho_{eg} - \rho_{ee}\frac{\Omega_p}{2}\right)$$

$$\dot{\rho}_{em} = i\left(\frac{\Omega_p}{2}\rho_{gm} + \frac{\Omega_c}{2}\rho_{mm} + \Delta_p\rho_{em} - \rho_{em}(\Delta_p-\Delta_c) - \rho_{ee}\frac{\Omega_c}{2}\right)$$

$$\dot{\rho}_{gm} = i\left(\frac{\Omega_p^*}{2}\rho_{em} - \rho_{gm}(\Delta_p-\Delta_c) - \rho_{ge}\frac{\Omega_c}{2}\right)$$

So we would have $\Delta f_p \simeq \Delta f_c + 4.69\,\mathrm{MHz}\cdot I[A]$

If I call $F = f_{F'=0} - f_{F=1,m_F=0}$ then I have

$$\Delta_p = f_p - (F + C_{Zee}\cdot I) = f_c + 2\Delta f_p - F - C_{Zee}\cdot I$$

$$\Delta_c = f_c - (F - C_{Zee}\cdot I) = f_c + 2\Delta f_c - F + C_{Zee}\cdot I$$

$$\delta = \Delta_p - \Delta_c = 2\Delta f_p - 2\Delta f_c - 2C_{Zee}\cdot I = 2(\Delta f_p - \Delta f_c - C_{Zee}\cdot I)$$

If I call $f_c - F = \Delta_0$ then
$$\Delta_p = \Delta_0 + 2\Delta f_p - C_Z I$$
$$\Delta_c = \Delta_0 + 2\Delta f_c + C_Z I$$
$$\delta = 2(\Delta f_p - \Delta f_c - C_Z I)$$

Field Intensities

We have $\frac{|\Omega|^2}{\Gamma^2} = \frac{I}{2I_{sat}} \Rightarrow |\Omega| = \sqrt{\frac{I}{2I_{sat}}}\,\Gamma$

??

For Rb87 D2 $\Gamma \sim 6.0659$ MHz $I_{sat} \simeq 1.65\,\mathrm{mW/cm^2}$

For a Gaussian beam $I = I_0 e^{-2\frac{r^2}{W_0^2}}$ $I_0 = \frac{2P}{\pi W_0^2}$

So we get $\Omega \simeq \sqrt{\frac{I_0}{2I_{sat}}}\,\Gamma$

So considering the probe ($W_0 = 1$ mm) $\Omega_p \simeq 26.6\sqrt{P[mW]}$ MHz

For the pump ($W_c \simeq 2.5$ mm) $\Omega_c \simeq 10.6\cdot\sqrt{P[mW]}$ MHz

Alternatively $\Omega_p = 0.842\sqrt{P[\mu W]}$ MHz
$\Omega_c = 0.337\sqrt{P[\mu W]}$ MHz

NB I call $\delta = \Delta_p - \Delta_c$

$$\dot\rho_{gg} = -i\frac{\Omega_p}{2}\rho_{ge} + i\frac{\Omega_p^*}{2}\rho_{eg} + \Gamma_{eg}\rho_{ee}$$

$$\dot\rho_{mm} = -i\frac{\Omega_c}{2}\rho_{me} + i\frac{\Omega_c^*}{2}\rho_{em} + \Gamma_{em}\rho_{ee}$$

$$\dot\rho_{ee} = -\dot\rho_{gg} - \dot\rho_{mm}$$

$$\dot\rho_{mg} = -i(\Delta_c-\Delta_p)\rho_{mg} - i\frac{\Omega_p}{2}\rho_{me} + i\frac{\Omega_c^*}{2}\rho_{eg} - \gamma_{mg}\rho_{mg}$$

$$\dot\rho_{eg} = -i\frac{\Omega_p}{2}(\rho_{ee}-\rho_{gg}) + i\Delta_p\rho_{eg} + i\frac{\Omega_c}{2}\rho_{mg} - \gamma_{eg}\rho_{eg}$$

$$\dot\rho_{em} = -i\frac{\Omega_c}{2}(\rho_{ee}-\rho_{mm}) + i\Delta_c\rho_{em} + i\frac{\Omega_p}{2}\rho_{gm} - \gamma_{em}\rho_{em}$$

where γ_{mg}, γ_{eg}, γ_{em} are the decoherence rates

2-photon ; 1-photon

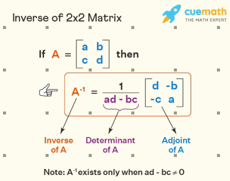
If $\Omega_p \ll \Omega_c$, keeping only linear terms in Ω_p, we get

$$\rho_{gg} = 1, \quad \rho_{ee} = \rho_{mm} = \rho_{mg} = 0$$

$$\Rightarrow 0 = i\frac{\Omega_p}{2} + i\frac{\Omega_c}{2}\rho_{mg} - \overbrace{(\gamma_{eg} - i\Delta_p)}^{\tilde\Gamma_{eg}}\rho_{eg}$$

$$0 = -\underbrace{(\gamma_{mg} - i\delta)}_{\tilde\Gamma_{mg}}\rho_{mg} + i\frac{\Omega_c^*}{2}\rho_{eg}$$

$$\Rightarrow \begin{pmatrix} \tilde\Gamma_{eg} & -i\frac{\Omega_c}{2} \\ -i\frac{\Omega_c^*}{2} & \tilde\Gamma_{mg} \end{pmatrix}\begin{pmatrix} \rho_{eg} \\ \rho_{mg} \end{pmatrix} = \begin{pmatrix} i\Omega_p/2 \\ 0 \end{pmatrix}$$



$$A^{-1} = \frac{1}{\tilde\Gamma_{eg}\tilde\Gamma_{mg} + \frac{|\Omega_c|^2}{4}}\begin{pmatrix} \tilde\Gamma_{mg} & i\frac{\Omega_c}{2} \\ i\frac{\Omega_c^*}{2} & \tilde\Gamma_{eg} \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} \rho_{eg} \\ \rho_{mg} \end{pmatrix} = \frac{1}{\tilde\Gamma_{eg}\tilde\Gamma_{mg} + \frac{|\Omega_c|^2}{4}}\begin{pmatrix} \tilde\Gamma_{mg} & i\Omega_c/2 \\ i\Omega_c^*/2 & \tilde\Gamma_{eg} \end{pmatrix}\begin{pmatrix} i\Omega_p/2 \\ 0 \end{pmatrix}$$

$$\Rightarrow \rho_{eg} = \frac{i}{2}\frac{\Omega_p\tilde\Gamma_{mg}}{\tilde\Gamma_{eg}\tilde\Gamma_{mg} + \frac{|\Omega_c|^2}{4}}$$

Decoherence

1-photon

Doppler

$$f(v_z) \propto \exp\left(-\frac{mv_z^2}{2k_BT}\right) \Rightarrow \omega_0 = \omega_L - kv_z \Rightarrow \exp\left(-\frac{(\omega_L-\omega_0)^2}{2\frac{k_BT}{m}k^2}\right)$$

$$\Rightarrow \Delta f_{Dop} \simeq \frac{\sqrt{k_BT/m}}{\lambda} \simeq 220\ \text{MHz}$$

$\sqrt{\frac{k_BT}{m}} \simeq 170$ m/s @T_{room}

$\lambda \simeq 780$ nm

Dicke

$$\Delta f_{Dicke} \simeq 2\pi\frac{D}{\lambda^2}, \quad D = \frac{760}{P_{Ne}[Torr]}D_0$$

for $D_0 \simeq 4\cdot10^{-5}$ m²/s $P_{Ne} = 10$ Torr, $\lambda = 780$ nm

$$\Rightarrow \Delta f_{Dicke} \simeq 3.6\cdot10^4\ \text{MHz}$$

Probe Transit Time

I consider the transit length as $w_p/2$

For Doppler

$$p(v_\perp) = \frac{mv_\perp}{k_BT}\exp\left(-\frac{mv_\perp^2}{2k_BT}\right)$$

$$\Rightarrow \bar v_\perp = \frac{m}{k_BT}\int_0^{+\infty}dv\, v^2\exp\left(-\frac{mv^2}{2k_BT}\right) = \left[y = \sqrt{\frac{m}{2k_BT}}v\right] = \frac{m}{k_BT}\left(\frac{2k_BT}{m}\right)^{3/2}\int_0^{+\infty}dy\, y^2e^{-y^2}$$

Using $\int_0^{+\infty}dy\, y^2e^{-y^2} = \left[-\frac{1}{2}e^{-y^2}\cdot y\right]_0^{+\infty} - \int_0^{+\infty}dy\left(-\frac{1}{2}e^{-y^2}\right)\cdot 1 = \frac{\sqrt\pi}{4}$

$$\Rightarrow \bar v_\perp = \sqrt{\frac{\pi}{2}\frac{k_BT}{m}} \Rightarrow \Delta f_{tp} \simeq \frac{\bar v_\perp}{w_p/2} \simeq 4.3\cdot10^{-1}\ \text{MHz}$$

For diffusion, the 2D probability distribution evolves as

$$p(r,t) \propto \exp\left(-\frac{r^2}{2(2Dt)}\right) \Rightarrow 2D\,T_{transit} \simeq \left(\frac{w_p}{2}\right)^2$$

$$\Rightarrow \frac{1}{T_{transit}} \simeq \frac{D}{\left(\frac{1}{\sqrt2}\frac{w_p}{2}\right)^2}$$

so I get a $\frac{1}{\sqrt2}$ in front of the characteristic length compared to the 1D case

$$\Rightarrow \Delta f_{Dicke} \simeq 2\pi\frac{D}{\left(\frac{1}{\sqrt2}\frac{w_p}{2}\right)^2} \simeq 1.7\cdot10^{-1}\ \text{MHz}$$

Collisions

I have $\Delta f_{coll} \simeq P[Torr]\cdot 10\ \text{MHz} \simeq 100\ \text{MHz}$

TOTAL

$$\gamma_{13} \simeq \overbrace{\underbrace{\Gamma_{tot}/2}_{\sim 3\,\text{MHz}} + \underbrace{\gamma_{coll}}_{\sim 100\,\text{MHz}}}^{Hom} + \overbrace{\underbrace{\gamma_{motion}}_{\sim 220\,\text{MHz}} + \underbrace{\gamma_{finite\ beam}}_{\sim 0.17\,\text{MHz}}}^{Inhom} \simeq 323\ \text{MHz}$$

2-photon

First I need to compute the effective wavelength

$$\lambda_{2p} = \frac{2\pi}{|\vec k_p - \vec k_c|}, \qquad k = \frac{2\pi}{\lambda} = \frac{2\pi}{c/f} = \frac{2\pi}{c}f$$

$$|\vec k_p - \vec k_c| = \sqrt{k_p^2 + k_c^2 - 2k_pk_c\cos\theta} \simeq \sqrt{k_p^2 + k_c^2 - 2k_pk_c + k_pk_c\theta^2}$$

$$\simeq \frac{2\pi}{c}\sqrt{(f_p-f_c)^2 - 2f_pf_c\theta^2}$$

PHENOMENON THEORY

We are interested now in understanding how the medium interacts with light, in particular the probe. We need to find the induced dipole moment:

$$\langle \vec{d} \rangle_\rho = Tr(\vec{d}\rho) = \vec{d}_{ge}\rho_{eg} + \vec{d}_{me}\rho_{em} + \vec{d}_{eg}\rho_{ge} + \vec{d}_{em}\rho_{me}$$
$$= \vec{d}_{ge}\rho_{eg} + \vec{d}_{me}\rho_{em} + cc$$

But this is ρ not in the interaction picture, so calling it ρ'

$$\rho'_{eg} = c_e c_g^* = e^{-i\omega_p t} a_e d_g^* = e^{-i\omega_p t}\rho_{eg}$$

$$\rho'_{em} = e^{-i\omega_p t} a_e e^{+i(\omega_p-\omega_c)t} d_m^* = e^{-i\omega_c t}\rho_{em}$$

So I get

$$\langle \vec{d} \rangle = \underbrace{\rho_{eg} e^{-i\omega_p t}\vec{d}_{ge}}_{\text{probe induced dipole moment}} + \rho_{em} e^{-i\omega_c t}\vec{d}_{me} + cc$$

The probe-induced dipole moment, in term of polarizability, can also be written

$$\langle \vec{d} \rangle_{probe} = \alpha_p\left(\frac{1}{2}\hat{\varepsilon}_p E_p e^{-i\omega_p t} + cc\right)$$

$$\hat{\varepsilon}_p^* \cdot \alpha_p \frac{1}{2}\hat{\varepsilon}_p E_p e^{-i\omega_p t} = \rho_{eg}\vec{d}_{ge} e^{-i\omega_p t}\cdot\hat{\varepsilon}_p^*$$

$$\Rightarrow \alpha_p = \frac{2\rho_{ge}\vec{d}_{eg}\cdot\hat{\varepsilon}_p^*}{E_p}$$

We now want to find the susceptibility χ

$$\vec{P} = \varepsilon_0\chi\vec{E}_{tot} \qquad \vec{P}_{at} = \alpha_p\vec{E}_{ext}$$

For a charge distribution in spherical volume of radius R and dipole moment $\vec{p}$ the average field inside the sphere is

$$\vec{E}_{at} = -\frac{1}{4\pi\varepsilon_0 R^3}\vec{P}_{at}$$

$$\text{So } \vec{E}_{tot} = \vec{E}_{ext} + \vec{E}_{at} = \vec{E}_{ext} - \frac{1}{4\pi\varepsilon_0 R^3}\alpha_p\vec{E}_{ext} = \left(1-\frac{\alpha_p}{4\pi\varepsilon_0 R^3}\right)\vec{E}_{ext}$$

If one atom occupies volume $\frac{4}{3}\pi R^3$ and we have an atom density n then

$$n = \frac{1}{V_{at}} = \frac{3}{4\pi R^3} \Rightarrow \vec{E}_{tot} = \left(1-\frac{\alpha_p n}{3\varepsilon_0}\right)\vec{E}_{ext}$$

We now have $\varepsilon_0\chi\vec{E}_{tot} = n\alpha_p\vec{E}_{ext} \Rightarrow \varepsilon_0\chi\left(1-\frac{\alpha_p n}{3\varepsilon_0}\right) = n\alpha_p$

$$\Rightarrow \chi = \frac{3n\alpha_p}{3\varepsilon_0-\alpha_p} \Rightarrow \chi = \frac{n\alpha_p/\varepsilon_0}{1-\frac{n}{3}\alpha_p/\varepsilon_0}$$

Notice that $3\varepsilon_0\chi - \alpha_p\chi = 3n\alpha_p \Rightarrow \alpha_p(3n+\chi) = 3\varepsilon_0\chi \Rightarrow \alpha_p = \frac{\frac{\varepsilon_0}{n}\chi}{1+\frac{\chi}{3n}}$

If $\chi \ll 1 \Rightarrow \alpha_p \ll 1$

We have $f_p \simeq f_c \simeq 4\cdot 10^{14}$ Hz, $\Delta f \simeq 10^7$ Hz

So considering a 1mm misalignment in 10 m $\Rightarrow \theta \simeq 10^{-4}$ I still get

$$\Delta f^2 \simeq 10^{14} \qquad 2f_pf_c\theta^2 \simeq 10^{21}$$

I then approximate

$$\lambda_{2p} \simeq \frac{c}{\sqrt{2f_pf_c}}\frac{1}{\theta} \simeq 5.5\cdot 10^{-7}\,{}^{1}/_{\theta}, \text{ for } \theta \gtrsim 10^{-6}$$

Motional

So I get $\Delta f_{\alpha_p} \simeq \frac{\sqrt{u_{at}^2}}{\lambda_{2p}} \simeq 3.1\cdot 10^2\,\theta$ MHz

$$\Delta f_{Dicke} \simeq 2\pi\frac{D}{\lambda_{2p}^2} \simeq 6.3\cdot 10^4\,\theta^2 \text{ MHz}$$

Collisional

I have spin decoherence $\gamma_{12} \simeq 3$ kHz

TOTAL

Consider 5mm misalignment in 1m $\Rightarrow \theta \simeq 5\cdot 10^{-3}$

$\Rightarrow \Delta f_{\alpha_p} \simeq 1.5$ MHz $\quad \Delta f_{Dicke} \simeq 1.6$ MHz

$$\gamma_{12} \simeq \overbrace{\underset{\simeq 3\text{kHz}}{\gamma_{12}^{cell}}}^{Hom} + \overbrace{\underset{\simeq 1.5\text{MHz}}{\gamma_{motion}} + \underset{\simeq 0.17\text{MHz}}{\gamma_{finite\ beam}}}^{Inhom} \simeq 1.7 \text{ MHz}$$

We now want to get the absorption α

$$e^{i(kx-\omega t)} = e^{ik(x-\frac{\omega}{k}t)} = e^{ik(x-vt)}$$

In free space $\frac{\omega}{k}=c$, $n=\frac{c}{v}$ => $v=\frac{c}{n}$ => $k = n\frac{2\pi}{\lambda_0}$

But n is complex in general, so $e^{i(nk_0x-\omega t)} = e^{i(n'kx-\omega t)}e^{-n''kx}$

$$\Rightarrow I = |\vec{E}|^2 = e^{-2n''\frac{2\pi}{\lambda_0}x} = e^{-\alpha x}$$

Moreover $n = \sqrt{\varepsilon_r \underset{\sim 1}{\mu_r}} = \sqrt{1+\chi} = 1+\frac{1}{2}(\chi'+i\chi'') \Rightarrow n'' = \frac{\chi''}{2}$

$$\Rightarrow \alpha = \frac{2\pi}{\lambda_0}\chi''$$

$$\rho_{eg} = \frac{i}{2}\frac{\Omega_p\tilde{\Gamma}_{mg}}{\tilde{\Gamma}_{eg}\tilde{\Gamma}_{mg}+\frac{|\Omega_c|^2}{4}} \qquad \Omega_p = \frac{\vec{d}_{eg}\cdot\hat{\varepsilon}_p E_p}{\hbar} \qquad \Omega_c = \frac{\vec{d}_{em}\cdot\hat{\varepsilon}_c E_c}{\hbar}$$

$$\alpha_p = \frac{2\rho_{eg}\vec{d}_{ge}\cdot\hat{\varepsilon}_p^*}{E_p} = \frac{2\left(\frac{i}{2}\frac{\Omega_p\tilde{\Gamma}_{mg}}{\tilde{\Gamma}_{eg}\tilde{\Gamma}_{mg}+\frac{|\Omega_c|^2}{4}}\right)\overbrace{\vec{d}_{ge}\cdot\hat{\varepsilon}_p^*}^{=\frac{\hbar\Omega_p^*}{E_p^*}}}{E_p}$$

$$= i\hbar\frac{|\Omega_p|^2\tilde{\Gamma}_{mg}}{|E_p|^2\left(\tilde{\Gamma}_{eg}\tilde{\Gamma}_{mg}+\frac{|\Omega_c|^2}{4}\right)} = \frac{i}{\hbar}|\mu_{eg}|^2\frac{\tilde{\Gamma}_{mg}}{\tilde{\Gamma}_{mg}\tilde{\Gamma}_{eg}+\frac{|\Omega_c|^2}{4}}$$

$$|\Omega_p|^2 = \frac{|E_p|^2|\vec{d}_{eg}\cdot\hat{\varepsilon}_p|^2}{\hbar^2} = \frac{|E_p|^2|\mu_{eg}|^2}{\hbar^2}$$

$$\chi = \frac{n\frac{\alpha_p}{\varepsilon_0}}{1-\frac{n}{3}\frac{\alpha_p}{\varepsilon_0}} \sim \frac{n}{\varepsilon_0}\alpha_p = \frac{n|\mu_{eg}|^2}{\hbar\varepsilon_0}\frac{i\tilde{\Gamma}_{mg}}{\tilde{\Gamma}_{mg}\tilde{\Gamma}_{eg}+\frac{|\Omega_c|^2}{4}}$$

$$\tilde{\Gamma}_{mg} = \gamma_{mg}-i\delta \qquad \Gamma_{eg} = \gamma_{eg}-i\Delta_p$$

$$\Rightarrow \chi = \frac{n|\mu_{eg}|^2}{\hbar\varepsilon_0}\frac{i(\gamma_{mg}-i\delta)}{(\gamma_{mg}-i\delta)(\gamma_{eg}-i\Delta_p)+\frac{|\Omega_c|^2}{4}}$$

$$= \frac{n|\mu_{eg}|^2}{\hbar\varepsilon_0}\frac{(\delta+i\gamma_{mg})\left(\gamma_{mg}\gamma_{eg}-\delta\Delta_p+\frac{|\Omega_c|^2}{4}+i(\delta\gamma_{eg}+\Delta_p\gamma_{mg})\right)}{\left(\gamma_{mg}\gamma_{eg}-\delta\Delta_p+\frac{|\Omega_c|^2}{4}\right)^2+(\delta\gamma_{eg}+\Delta_p\gamma_{mg})^2}$$

$$\Rightarrow \alpha = \frac{k_p n|\mu_{eg}|^2}{\hbar\varepsilon_0}\frac{\gamma_{mg}\left(\gamma_{mg}\gamma_{eg}-\delta\Delta_p+\frac{|\Omega_c|^2}{4}\right)+\delta(\delta\gamma_{eg}+\Delta_p\gamma_{mg})}{\left(\gamma_{mg}\gamma_{eg}-\delta\Delta_p+\frac{|\Omega_c|^2}{4}\right)^2+(\delta\gamma_{eg}+\Delta_p\gamma_{mg})^2}$$

Notice, if there is no coupling beam $\gamma_{mg}=0$, $\Omega_c=0$, $\delta=\Delta_p$

$$\alpha_{2lev} = \frac{k_p n|\mu_{eg}|^2}{\hbar\varepsilon_0}\frac{\Delta_p^2\gamma_{eg}}{\Delta_p^4+\Delta_p^2\gamma_{eg}^2} = \frac{k_p n|\mu_{eg}|^2}{\hbar\varepsilon_0}\frac{\gamma_{eg}}{\Delta_p^2+\gamma_{eg}^2}$$

In the absence of EIT, the resonant absorption is

$$\alpha_{2lev}(\Delta_p=0) = \alpha_0 = \frac{k_p n|\mu_{eg}|^2}{\hbar\varepsilon_0\gamma_{eg}}$$

Resonant Absorption

$\lambda_p \simeq 780\,\text{nm}$ $\quad \mu_{eg} \simeq 4{,}2275\, e a_0$ $\quad a_0 \simeq 5.9\cdot 10^{-11}\,\text{m}$

$\hbar \simeq 1{,}0546\cdot 10^{-34}\,\text{m}^2\,\text{kg}\,\text{s}^{-1}$ $\quad \varepsilon_0 = \frac{e^2}{2\alpha h c}$ $\quad \alpha = \frac{1}{137}$, $c = 3\cdot 10^8\,\text{m/s}$, $h = 2\pi\hbar$

I rewrite

$$\alpha = \alpha_0 \frac{\gamma_{mg}\left(\gamma_{mg} + \frac{|\Omega_c|^2}{4\gamma_{eg}} - \frac{\delta\Delta_p}{\gamma_{eg}}\right) + \delta^2\left(1 + \frac{\Delta_p\gamma_{mg}}{\gamma_{eg}\delta}\right)}{\left(\gamma_{mg} + \frac{|\Omega_c|^2}{4\gamma_{eg}} - \frac{\delta\Delta_p}{\gamma_{eg}}\right)^2 + \delta^2\left(1 + \frac{\Delta_p\gamma_{mg}}{\gamma_{eg}\delta}\right)^2}$$

If $|\Omega_c|, \gamma_{eg} \gg \delta, \Delta_p, \gamma_{mg}$ then

$$\frac{\Delta_p\gamma_{mg}\delta}{\gamma_{eg}} \ll 1$$

$$\frac{\Delta_p\gamma_{mg}}{\gamma_{eg}} \ll 1$$

and I get

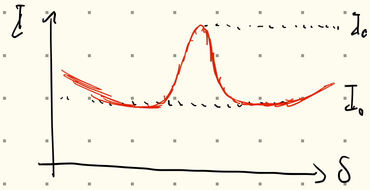
$$\alpha \simeq \alpha_0 \frac{\gamma_{mg}\gamma_{EIT} + \delta^2}{\gamma_{EIT}^2 + \delta^2} \qquad \gamma_{EIT} = \gamma_{mg} + \frac{|\Omega_c|^2}{4\gamma_{eg}}$$

and if $|\Omega_c| = 0$

$$\alpha \simeq \alpha_0$$

and @ $\delta = 0$ $\quad \alpha \simeq \alpha_0 \frac{\gamma_{mg}}{\gamma_{EIT}} = \alpha_0 \frac{1}{1 + \frac{|\Omega_c|^2}{4\gamma_{eg}\gamma_{mg}}} = \alpha_0/\gamma_{EIT}$

Remember that we measure $I_{out} = I_{in}\, e^{-\alpha\Delta z}$



$I_0 e^{-\alpha_0/\gamma\,\Delta z} = I_0 e^{-OD + OD_{EIT}}$

$I_0 e^{-\alpha_0\Delta z} = I_0 e^{-OD}$

We look at $R = I_{EIT}/I_{norm} = e^{-(\alpha(\delta) - \alpha_0)\Delta z}$

@ $\delta = 0$ $\quad R_{max} = e^{-\alpha_0\Delta z\left(\frac{\gamma_{mg}}{\gamma_{EIT}} - 1\right)} = e^{OD_{EIT}}$ $\qquad OD_{EIT} = \alpha_0\Delta z\left(1 - \frac{\gamma_{mg}}{\gamma_{EIT}}\right)$

The width of $R = R_{max}\, e^{-1} \Rightarrow -(\alpha(\delta) - \alpha_0)\Delta z = \alpha_0\Delta z - \alpha_0\Delta z\frac{\gamma_{mg}}{\gamma_{EIT}} - 1$

$$\Rightarrow -\alpha_0\frac{\gamma_{mg}\gamma_{EIT} + \delta^2}{\gamma_{EIT}^2 + \delta^2}\Delta z = -\alpha_0\Delta z\frac{\gamma_{mg}}{\gamma_{EIT}} - 1$$

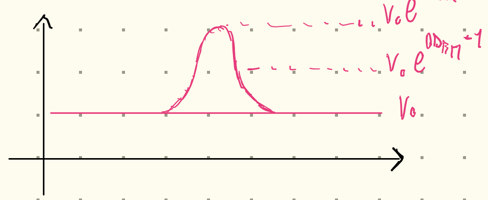
$$\Rightarrow \gamma_{mg}\gamma_{EIT} + \delta^2 = \frac{\gamma_{mg}}{\gamma_{EIT}}\left(\gamma_{EIT}^2 + \delta^2\right) + \frac{1}{\alpha_0\Delta z}\left(\gamma_{EIT}^2 + \delta^2\right)$$

$$\Rightarrow \delta^2\left(1 - \frac{\gamma_{mg}}{\gamma_{EIT}} - \frac{1}{\alpha_0\Delta z}\right) = \frac{\gamma_{EIT}^2}{\alpha_0\Delta z} \Rightarrow \delta^2 = \frac{\gamma_{EIT}^2}{OD_{EIT} - 1}$$

$$\Rightarrow \delta_{1/2} = \frac{\gamma_{EIT}}{\sqrt{OD_{EIT} - 1}}$$

The peak value vs bckgnd value is

$$\frac{I_0\, e^{-OD + OD_{EIT}}}{I_0\, e^{-OD}} = \boxed{e^{OD_{EIT}}} \quad @\ \delta_{1/2} \quad e^{OD_{EIT} - 1}$$



$$\alpha_0 = \frac{n}{\gamma_{eg}} \frac{\frac{2\pi}{7.8\cdot 10^{-7}}\cdot\left(4{,}2275\cdot \cancel{e}\cdot 5{,}9\cdot 10^{-11}\right)^2}{\cancel{\hbar}\,\frac{\cancel{e^2}}{2\frac{1}{137}2\pi\cancel{\hbar}c}} = \frac{n}{\gamma_{eg}}\frac{8\pi^2}{137}(4{,}2275\cdot 5{,}9)^2\cdot 3\cdot 10^{-7}$$

$$\simeq \frac{n[m^{-3}]}{\gamma_{eg}[MHz]}\, 1{,}075\cdot 10^{-13}\ m^{-1}$$

with $L_{cell} \sim 7{,}5$ cm, $\gamma_{eg} \simeq 326$ MHz

$$OD = \alpha_0\Delta z \simeq 2.5\cdot 10^{-17}\cdot n[m^{-3}]$$

@ $T = 65°C$ $\quad n \simeq 3\cdot 10^{17} \Rightarrow OD \simeq 7.5$

@ $T = 20°C$ $\quad n \simeq 5.7\cdot 10^{15} \Rightarrow OD \simeq 0.14$

Also $\gamma_{EIT} = \gamma_{mg} + \frac{|\Omega_c|^2}{\gamma_{eg}} \simeq 1{,}7 + \frac{|\Omega_c|^2}{326}$

$$\gamma_{mg}/\gamma_{EIT} = \frac{1}{1 + \frac{|\Omega_c|^2}{\gamma_{mg}\gamma_{eg}}} \qquad \frac{10}{3{,}14\cdot 2{,}5^2} \qquad \frac{10}{3{,}14\cdot 6{,}25}$$

If I want $\frac{|\Omega_c|^2}{\gamma_{mg}\gamma_{eg}} = 1 \Rightarrow P_c \simeq 5$ mW $\left(I_{0,c} \simeq 50\ mW/cm^2 \sim 15\text{-}20\ I_{sat}\right)$

With $\frac{|\Omega_c|^2}{\gamma_{mg}\gamma_{eg}} = 1 \Rightarrow OD_{EIT} = OD\left(1 - \frac{1}{2}\right) = \frac{1}{2}OD$

But if $I_{0,c} \simeq 20\, I_{sat} \Rightarrow OD \to OD/4$

But Already for $P_p \simeq 20\ \mu W \Rightarrow \Omega_p \simeq 3{,}8$ MHz !

@ 65°C with $P_c \simeq 5$ mW, $P_p \simeq 50\ \mu W$ $\quad \Omega_c \simeq 24$ MHz, $\Omega_p \simeq 6$ MHz

$\hookrightarrow OD \simeq \frac{7{,}5}{5{,}75} \simeq 1{,}3 \qquad OD_{EIT} = OD\left(1 - \frac{1}{2}\right) \simeq 0{,}65$!

$$B \simeq \frac{\gamma_{EIT}}{\sqrt{0{,}65}} \simeq \frac{3{,}47}{0{,}65} \simeq 4{,}3\ MHz\ !$$