Analysis_SAS

March 9, 2024

1 Import Libraries

```
[ ]: import numpy as np
     import matplotlib.pyplot as plt


     from __future__ import print_function
     import numpy as np
     import matplotlib.pyplot as plt
     from scipy.optimize import curve_fit
```

2 Useful Data

2.1 Doppler-Broadened Lines

```
[ ]: # Frequency differences in MHZ between the 87 and 85 transitions, for the GS
     f_87D2 = 384230484
     f_85D2 = 384230406
     f_87F1_rel = +4272
     f_87F2_rel = -2563
     f_85F2_rel = +1771
     f_85F3_rel = -1265
     df_87_F1 = f_87D2 + f_87F1_rel
     df_87_F2 = f_87D2 + f_87F2_rel
     df_85_F2 = f_85D2 + f_85F2_rel
     df_85_F3 = f_85D2 + f_85F3_rel

     # laser frequency, putting the zero to the resonance with 87_F1
     fL_87_F1 = df_87_F1 - df_87_F1
     fL_87_F2 = df_87_F2 - df_87_F1
     fL_85_F2 = df_85_F2 - df_87_F1
     fL_85_F3 = df_85_F3 - df_87_F1

     # array of the relative laser frequencies from the smallest to the largest
     fl_array = np.array([fL_87_F1,fL_85_F2,fL_85_F3,fL_87_F2])
```

3 Useful Functions

3.1 Fitting Functions

```
[ ]: # Gaussian functions for the fitting of the doppler-broadened lines
def Gauss(x, A, D,x0):
    y = A*np.exp(-(x-x0)**2/(2*D**2))
    return y

# Lorentzian function with amplitude A, width D, and center x0
def Lor(x,x0,D,A):
    return A/(1+(x-x0)**2/D**2)

# Fitting function for 6 SAS peaks, with a gaussian envelope
def SAS_fit_6(x,x1,D1,A1,x2,D2,A2,x3,D3,A3,x4,D4,A4,x5,D5,A5,x6,D6,A6,x0G,DG):
    'x is the independent variable, x0G, DG are the center and width of the␣
 ↪Gaussian envelope, the rest are the parameters for the 6 Lorentzian peaks.'
    v = 0
    v += Lor(x,x1,D1,A1)
    v += Lor(x,x2,D2,A2)
    v += Lor(x,x3,D3,A3)
    v += Lor(x,x4,D4,A4)
    v += Lor(x,x5,D5,A5)
    v += Lor(x,x6,D6,A6)
    v *= np.exp(-(x-x0G)**2/(2*DG**2))
    return v

def get_centers_6(params):
    'returns the centers of the 6 peaks'
    return np.
 ↪array([params[0],params[3],params[6],params[9],params[12],params[15]])

def get_widths_6(params):
    'returns the widths of the 6 peaks'
    return np.
 ↪array([params[1],params[4],params[7],params[10],params[13],params[16]])
```

3.2 Data Analysis Function

```
[ ]: # this function finds where the voltage ramp of the function generator starts␣
 ↪and ends averaging out the signal and finding the max

def find_extremes_ramp(x):
    t = np.arange(0,len(x))
    step = 5 # take the average every 5 points
    width = 25 # width of the averaging window
    l_d = int((len(x)-width)/step)+1
    pos_arr = np.zeros(l_d)
    avg_arr = np.zeros(l_d)
    for i in range(l_d):
        pos_arr[i] = i*step + width/2
        avg_arr[i] = np.mean(x[i*step:i*step+width])
    start = pos_arr[np.argmin(avg_arr)]
    end = pos_arr[np.argmax(avg_arr)]
    # if the start is after the end, swap them
    if start > end:
        start,end = end,start
    return int(start),int(end)

# this function identifies the regions where the spectroscopy signal is linear␣
 ↪(ie far from the resonances) with the second derivative of the signal

def find_where_linear(x, ax):
    t = np.arange(0,len(x))
    step = 10  # take the average every 10 points
    width = 40 # width of the fitting window
    l_dd = int((len(x)-width)/step)+1
    dd_arr = np.zeros(l_dd)
    pos_arr = np.zeros(l_dd)
    for i in range(l_dd):
        pfit = np.polyfit(t[i*step:i*step+width],x[i*step:i*step+width],2) #␣
 ↪fit a second order polynomial
        dd_arr[i] = pfit[0] # the second derivative is the first coefficient of␣
 ↪the polynomial
        pos_arr[i] = i*step + width/2
    lim = np.amax(abs(dd_arr))/15 # IMPORTANT threshold for the second␣
 ↪derivative to be basically considered 0, can be adjusted if needed

    # plot the second derivative and the threshold, useful for adjusting the␣
 ↪threshold
    ax.plot(pos_arr,abs(dd_arr))
    ax.axhline(y=lim, color='r', linestyle='-')
    ax.set_title('Second derivative, linear finding')

    # take the regions where the second derivative is below the threshold for␣
 ↪at least a fixed number of points (avoiding noise)
    thr_points = 5 # IMPORTANT minimum number of points below the threshold,␣
 ↪can be adjusted if needed
    dd_arr = abs(dd_arr)
    dd_arr[dd_arr<lim] = 0
    starts = np.zeros(4)
    ends = np.zeros(4)
    idx = 0
    running = False
    for i in range(len(dd_arr)-1):
        if (dd_arr[i] == 0) and (running == False):
            starts[idx] = pos_arr[i]
            running = True
            cnt = 1
        if (dd_arr[i] == 0) and (running == True):
            cnt += 1
        if (dd_arr[i] != 0) and (running == True):
            if cnt > thr_points:
                ends[idx] = pos_arr[i]
                idx += 1
                running = False
            if cnt<thr_points:
                running = False
    ends[-1] = pos_arr[-1]
    return starts,ends
```

3.3 Miscellaneous Functions

[ ]:
```
# takes a file with the data in columns of t,signal,FG and saves in arrays

def read_file(filename):
    data = np.loadtxt(filename,skiprows=2,delimiter=",",dtype=str)
    t = data[:,0].astype(float)
    V_pd = data[:,1].astype(float)
    V_fg = data[:,2].astype(float)
    return t,V_pd,V_fg
```

4 Hyperfine Analysis of Rb87, Fg=2 Line

4.1 Frequency Values

Here I save all the frequency values of the hyperfine states found in this line. The frequencies are relative to the center of the p-line

[ ]:
```
# we save all frequencies from the p line center

f_Rb87_Fe0 = - 302.074
f_Rb87_Fe1 = - 229.852
f_Rb87_Fe2 = - 72.911
f_Rb87_Fe3 = + 193.741
f_Rb87_CO_01 = (f_Rb87_Fe0 + f_Rb87_Fe1)/2
f_Rb87_CO_12 = (f_Rb87_Fe1 + f_Rb87_Fe2)/2
f_Rb87_CO_23 = (f_Rb87_Fe2 + f_Rb87_Fe3)/2
f_Rb87_CO_13 = (f_Rb87_Fe0 + f_Rb87_Fe3)/2
f_Rb87_CO_02 = (f_Rb87_Fe0 + f_Rb87_Fe2)/2
```

```
f_Rb87_CO_03 = (f_Rb87_Fe1 + f_Rb87_Fe3)/2
```

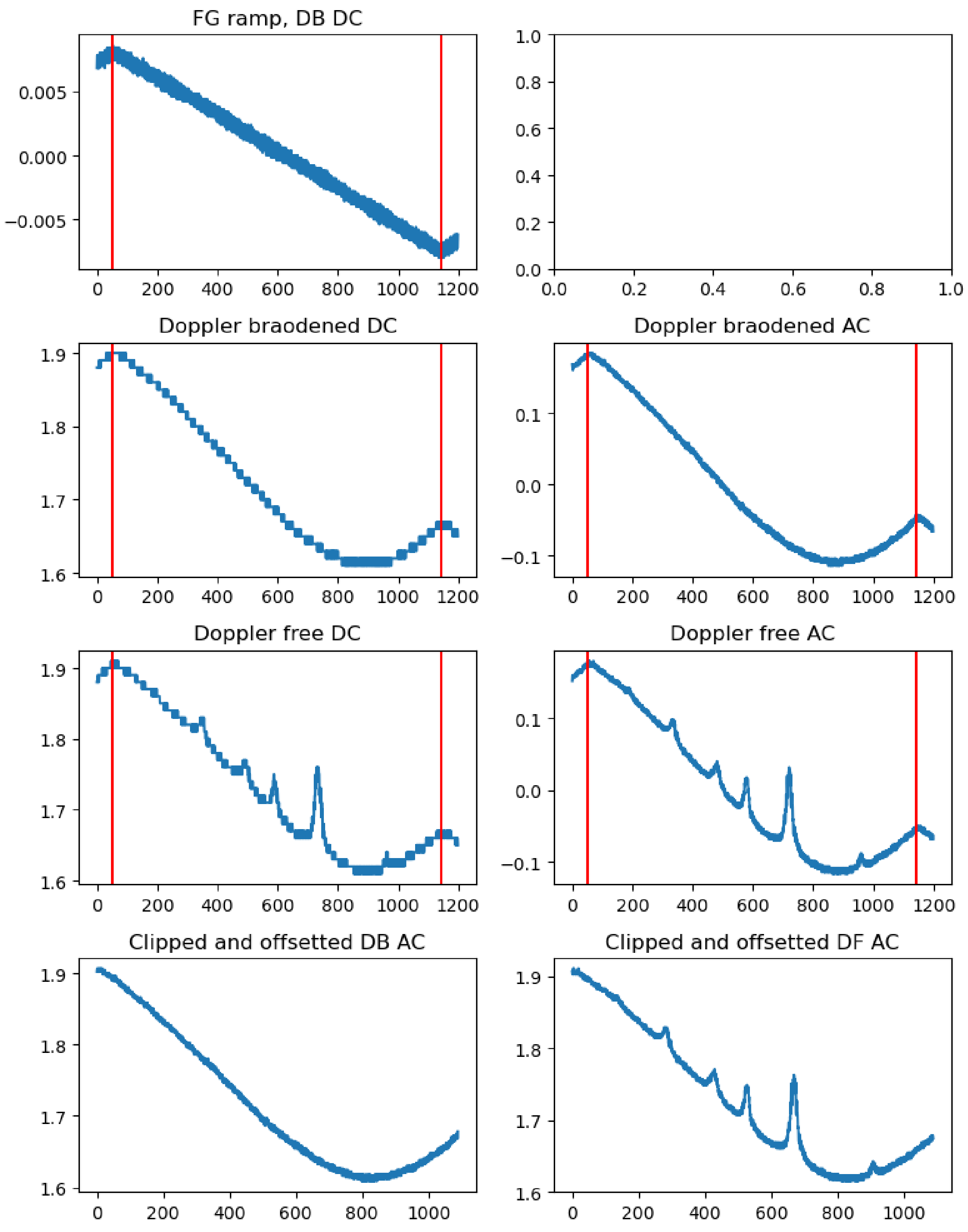
4.2 Post-Processing the data

I take the Doppler-broadened and Doppler-free data in AC and DC and process them to obtain the cleanest possible data

[ ]:
```
fig, ax = plt.subplots(4, 2, figsize=(8, 10), tight_layout=True)

# get the four files with the data
t_DB_DC,V_pd_DB_DC,V_fg_DB_DC = read_file("/media/jacklook/JL_USB/NewFile20.
 ↪csv")
t_DB_AC, V_pd_DB_AC, V_fg_DB_AC = read_file("/media/jacklook/JL_USB/NewFile21.
 ↪csv")
t_DF_DC, V_pd_DF_DC, V_fg_DF_DC = read_file("/media/jacklook/JL_USB/NewFile19.
 ↪csv")
t_DF_AC, V_pd_DF_AC, V_fg_DF_AC = read_file("/media/jacklook/JL_USB/NewFile18.
 ↪csv")

# I now find the extremes of the voltage ramp for the function generator
start,end = find_extremes_ramp(V_fg_DB_DC)
ax[0,0].plot(V_fg_DB_DC)
ax[0,0].set_title("FG ramp, DB DC")
ax[0,0].axvline(x=start, color='r', linestyle='-')
ax[0,0].axvline(x=end, color='r', linestyle='-')

# I now take the AC data, correct the DC offset and clip them
V_pd_DB_AC_c = V_pd_DB_AC - np.mean(V_pd_DB_AC) + np.mean(V_pd_DB_DC)
V_pd_DB_AC_c = V_pd_DB_AC_c[start:end]
V_pd_DF_AC_c = V_pd_DF_AC - np.mean(V_pd_DF_AC) + np.mean(V_pd_DF_DC)
V_pd_DF_AC_c = V_pd_DF_AC_c[start:end]

# plot everything to see the files are correct

ax[1,0].plot(t_DB_DC,V_pd_DB_DC)
ax[1,0].axvline(x=start, color='r', linestyle='-')
ax[1,0].axvline(x=end, color='r', linestyle='-')
ax[1,0].set_title("Doppler braodened DC")
ax[1,1].plot(t_DB_AC,V_pd_DB_AC)
ax[1,1].axvline(x=start, color='r', linestyle='-')
ax[1,1].axvline(x=end, color='r', linestyle='-')
ax[1,1].set_title("Doppler braodened AC")
ax[2,0].plot(t_DF_DC,V_pd_DF_DC)
ax[2,0].axvline(x=start, color='r', linestyle='-')
ax[2,0].axvline(x=end, color='r', linestyle='-')
ax[2,0].set_title("Doppler free DC")
ax[2,1].plot(t_DF_AC,V_pd_DF_AC)
```

```
ax[2,1].axvline(x=start, color='r', linestyle='-')
ax[2,1].axvline(x=end, color='r', linestyle='-')
ax[2,1].set_title("Doppler free AC")

ax[3,0].plot(V_pd_DB_AC_c)
ax[3,0].set_title("Clipped and offsetted DB AC")
ax[3,1].plot(V_pd_DF_AC_c)
ax[3,1].set_title("Clipped and offsetted DF AC")
```

```
[ ]: Text(0.5, 1.0, 'Clipped and offsetted DF AC')
```



4.3 Fitting

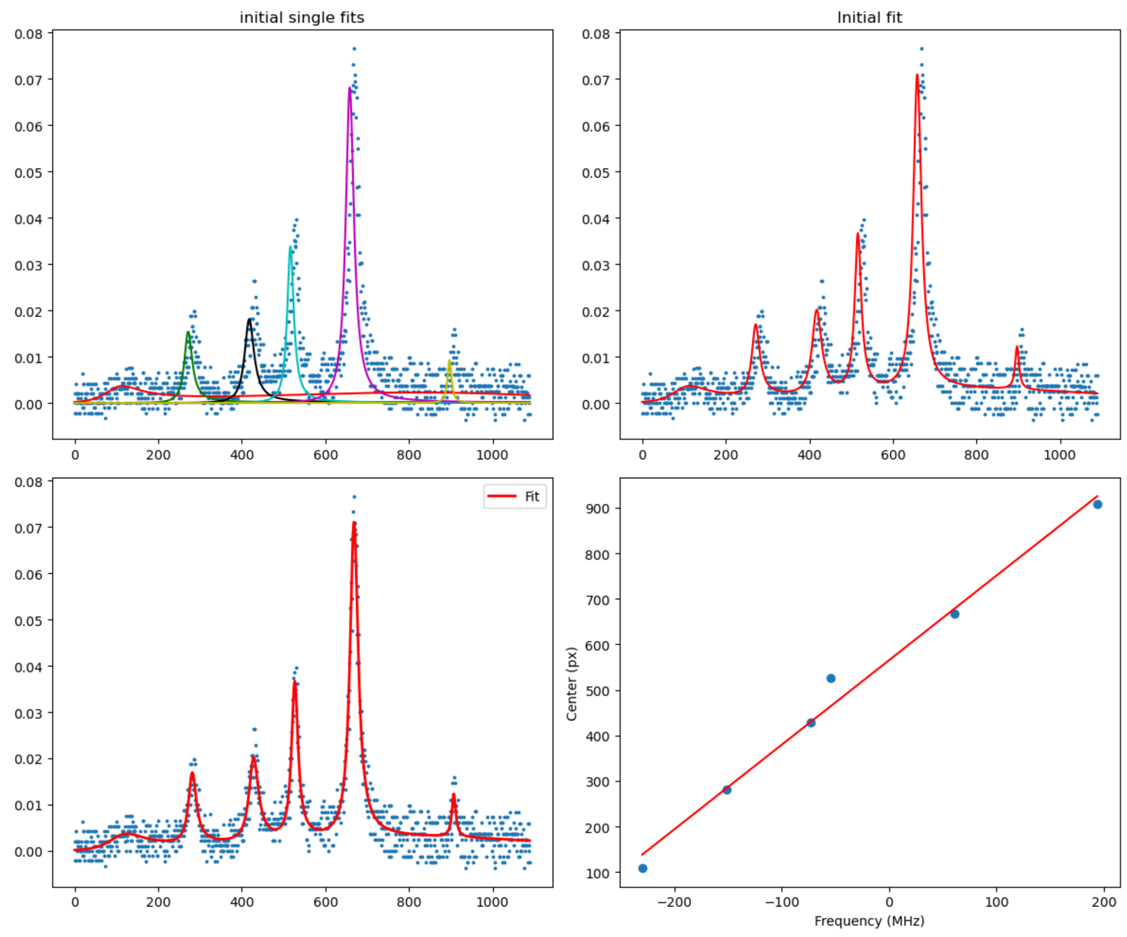
I now compute the Doppler-free absorption profile and fit it, I then fit the centers of the Lorentzian with the true frequencies

```
[ ]: fig,ax = plt.subplots(2,2,figsize=(12,10),tight_layout=True)

     # I now compute the absorption signal
     K_sg = np.log(V_pd_DF_AC_c/V_pd_DB_AC_c)
     x = np.arange(len(K_sg))


     # array of six colours
     colors = ['r','g','k','c','m','y']

     lor_1 = np.array([99,51.23,0.639])
     lor_2 = np.array([271,11.35,0.529])
     lor_3 = np.array([417,13.6,0.191])
     lor_4 = np.array([516,9.39,0.184])
     lor_5 = np.array([658,11.87,0.173])
     lor_6 = np.array([897,5.54,0.0109])
     lor_s = np.array([lor_1,lor_2,lor_3,lor_4,lor_5,lor_6])
     gauss = np.array([1066,298.9,0.01])

     ax[0,0].scatter(x,K_sg,s=3)
     for i in range(6):
         #ax[0,0].axhline(y=lor_s[i,2]*np.exp(-(lor_s[i,0]-gauss[0])**2/2/
       ↪gauss[1]**2), color=colors[i], linestyle='-')
         #ax[0,0].axvline(x=lor_s[i,0], color=colors[i], linestyle='-')
         #ax[0,0].axvspan(lor_s[i,0]-lor_s[i,1],lor_s[i,0]+lor_s[i,1], alpha=0.2,␣
       ↪color=colors[i])
         ax[0,0].plot(x,Lor(x,lor_s[i,0],lor_s[i,1],lor_s[i,2])*np.
       ↪exp(-(x-gauss[0])**2/2/gauss[1]**2),color=colors[i])

     ax[0,0].set_title("initial single fits")

     p0 =␣
       ↪[lor_1[0],lor_1[1],lor_1[2],lor_2[0],lor_2[1],lor_2[2],lor_3[0],lor_3[1],lor_3[2],lor_4[0],

     # get the fit with the initial parameters
     fit_0 = SAS_fit_6(x,*p0)

     ax[0,1].scatter(x,K_sg,s=3)
     ax[0,1].plot(x,fit_0,color='r',label='Initial fit')
     ax[0,1].set_title("Initial fit")

     popt, pcov = curve_fit(SAS_fit_6, x, K_sg, p0)
     ax[1,0].scatter(x,K_sg,s=3)
     ax[1,0].plot(x,SAS_fit_6(x,*popt),lw=2,c='r',label='Fit')
     ax[1,0].legend()
```

```
fs = np.
 ↪array([f_Rb87_Fe1,f_Rb87_CO_12,f_Rb87_Fe2,f_Rb87_CO_13,f_Rb87_CO_23,f_Rb87_Fe3])
centers = get_centers_6(popt)
ind = np.argsort(centers)
centers = centers[ind]
lin = np.polyfit(fs,centers,1)
ax[1,1].scatter(fs,centers)
ax[1,1].plot(fs,lin[0]*fs+lin[1],c='r',label='Linear fit')
print("Widths: ",get_widths_6(popt)[ind]/lin[0])
ax[1,1].set_xlabel("Frequency (MHz)")
ax[1,1].set_ylabel("Center (px)")
```

```
Widths:  [28.45958868  6.1062959   7.36637206  5.06193008  6.39115573
2.96641457]
```

```
[ ]: Text(0, 0.5, 'Center (px)')
```

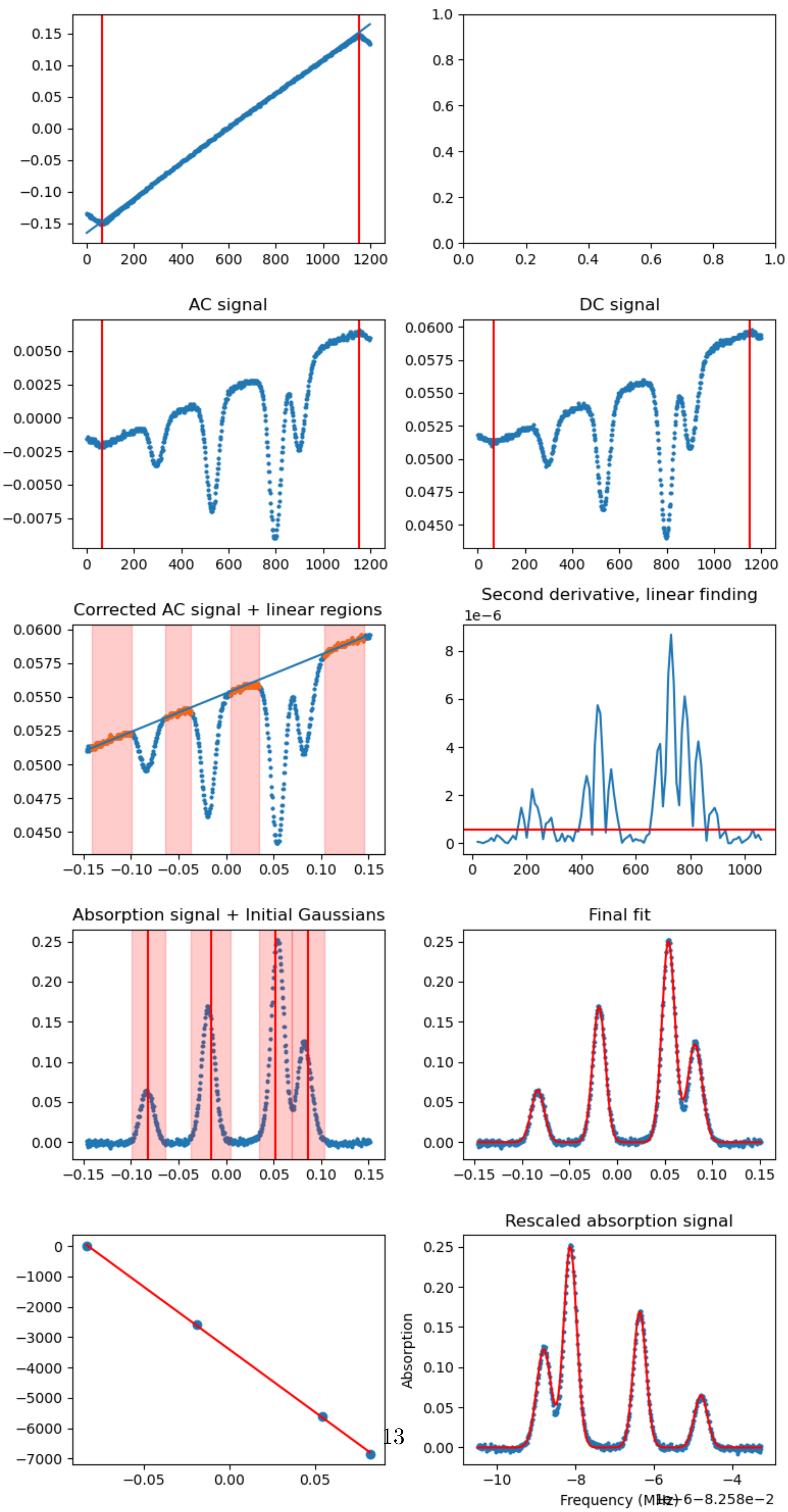


5 Doppler-broadened spectroscopy

Here I do all the analysis for the Doppler-broadened data, as explained in "Theory_Spectroscopy.pdf"

[ ]:
```
fig,ax = plt.subplots(5,2,figsize=(8,15),tight_layout=True)

# load the data
t_DC,V_fg_DC,V_pd_DC = read_file("/media/jacklook/JL_USB/NewFile13.csv")
t_AC,V_fg_AC,V_pd_AC = read_file("/media/jacklook/JL_USB/NewFile12.csv")

# find the extremes of the voltage ramp
start,end = find_extremes_ramp(V_fg_DC)

# linearize the FG signal
Vf_vs_t_linfit = np.polyfit(t_DC[start:end],V_fg_DC[start:end],1)
Vf_vs_t_a = Vf_vs_t_linfit[0]
Vf_vs_t_b = Vf_vs_t_linfit[1]
Vf_better = Vf_vs_t_a*t_DC+Vf_vs_t_b

ax[0,0].scatter(t_DC,V_fg_DC,s=3)
ax[0,0].axvline(x=start, color='r', linestyle='-')
ax[0,0].axvline(x=end, color='r', linestyle='-')
ax[0,0].plot(t_DC,t_DC*Vf_vs_t_a+Vf_vs_t_b,label='linear fit')

ax[1,0].scatter(t_AC,V_pd_AC,s=3)
ax[1,0].axvline(x=start, color='r', linestyle='-')
ax[1,0].axvline(x=end, color='r', linestyle='-')
ax[1,0].set_title("AC signal")
ax[1,1].scatter(t_DC,V_pd_DC,s=3)
ax[1,1].axvline(x=start, color='r', linestyle='-')
ax[1,1].axvline(x=end, color='r', linestyle='-')
ax[1,1].set_title("DC signal")

# correct the Vfg vs t for clipping
Vf_vs_t_b = Vf_vs_t_a*start+Vf_vs_t_b

# correct the AC signal for the DC offset and clip it
V_pd_AC_c = V_pd_AC - np.mean(V_pd_AC) + np.mean(V_pd_DC)
V_pd_AC_c = V_pd_AC_c[start:end]
Vf_c = Vf_better[start:end]

# find where the signal is linear
starts,ends = find_where_linear(V_pd_AC_c,ax[2,1])

# collect the indexes of the linear regions
lin_indexes = np.arange(int(starts[0]),int(ends[0]))
for i in range(1,4):
    lin_indexes = np.concatenate((lin_indexes,np.
↪arange(int(starts[i]),int(ends[i]))))
lin_indexes = lin_indexes.astype(int)

# fit the linear part of the signal, to find the relation Vfg vs Vpd out of␣
↪resonance
Vf_lin = Vf_c[lin_indexes]
Vpd_lin = V_pd_AC_c[lin_indexes]
Vpd_vs_Vf_linfit = np.polyfit(Vf_lin,Vpd_lin,1)
Vpd_vs_Vf_a = Vpd_vs_Vf_linfit[0]
Vpd_vs_Vf_b = Vpd_vs_Vf_linfit[1]

# transform regions from t to Vf
starts = starts*Vf_vs_t_a+Vf_vs_t_b
ends = ends*Vf_vs_t_a+Vf_vs_t_b

ax[2,0].scatter(Vf_c,V_pd_AC_c,s=3)
ax[2,0].scatter(Vf_lin,Vpd_lin,s=3)
ax[2,0].plot(Vf_c,Vf_c*Vpd_vs_Vf_a+Vpd_vs_Vf_b,label='linear fit')
for i in range(4):
    ax[2,0].axvspan(starts[i],ends[i], alpha=0.2, color='r')
ax[2,0].set_title("Corrected AC signal + linear regions")

# compute the absorption signal
K_sg = -np.log(V_pd_AC_c/(Vf_c*Vpd_vs_Vf_a+Vpd_vs_Vf_b))

# fit the absorption signal
G_s1 = ends[0]
G_e1 = starts[1]
G_s2 = ends[1]
G_e2 = starts[2]
G_s3 = ends[2]
G_e3 = 0.5*ends[2]+0.5*starts[3]
G_s4 = 0.5*ends[2]+0.5*starts[3]
G_e4 = starts[3]

G_s = np.array([G_s1,G_s2,G_s3,G_s4])
Gs_ind = (G_s-Vf_vs_t_b)/Vf_vs_t_a
G_e = np.array([G_e1,G_e2,G_e3,G_e4])
Ge_ind = (G_e-Vf_vs_t_b)/Vf_vs_t_a
G_c = (G_e+G_s)/2
G_d = np.zeros(4)


ax[3,0].scatter(Vf_c,K_sg,s=3)

for i in range(4):
    ax[3,0].axvspan(G_s[i],G_e[i], alpha=0.2, color='r')
    ax[3,0].axvline(x=G_c[i], color='r', linestyle='-')

ax[3,0].set_title("Absorption signal + Initial Gaussians")

# now fit absorption signal
gauss_tot = np.zeros(len(Vf_c))
for i in range(4):
    popt,pcov = curve_fit(Gauss,Vf_c[int(Gs_ind[i]):
 ↪int(Ge_ind[i])],K_sg[int(Gs_ind[i]):int(Ge_ind[i])],p0=[0.1,0.1,G_c[i]])
    gauss_tot += Gauss(Vf_c,*popt)
    G_c[i] = popt[2]
    G_d[i] = popt[1]

ax[3,1].plot(Vf_c,gauss_tot,c='r',label='Fit')
ax[3,1].scatter(Vf_c,K_sg,s=3)
ax[3,1].set_title("Final fit")

# now fit frequency vs center
Vf_vs_fl_linfit = np.polyfit(G_c,fl_array,1)
Vf_vs_fl_a = Vf_vs_fl_linfit[0]
Vf_vs_fl_b = Vf_vs_fl_linfit[1]

ax[4,0].scatter(G_c,fl_array)
ax[4,0].plot(G_c,Vf_vs_fl_a*G_c+Vf_vs_fl_b,c='r',label='Fit')

# now rescale absorption signal
fl_vals = (Vf_c-Vf_vs_fl_b)/Vf_vs_fl_a
ax[4,1].plot(fl_vals,gauss_tot,c='r',label='Fit')
ax[4,1].scatter(fl_vals,K_sg,s=3)
ax[4,1].set_xlabel("Frequency (MHz)")
ax[4,1].set_ylabel("Absorption")
ax[4,1].set_title("Rescaled absorption signal")
```

```
[ ]: Text(0.5, 1.0, 'Rescaled absorption signal')
```


AC signal
DC signal
Corrected AC signal + linear regions
Second derivative, linear finding
1e−6
Absorption signal + Initial Gaussians
Final fit
Rescaled absorption signal
Absorption
Frequency (MHz)
1e−6−8.258e−2