NUAA

Matrix Theory, Final						Test Date: 2017 年 1 月 5 日	
矩阵论班号及班内序号:			学号			姓名	
必做题（70 分）						选做题（30 分）	总分
题号	1	2	3	4	5		
得分							

Part I (必做题，共 5 题，70 分)

第 1 题（10 分）	得分

Let $\mathbf{C}_{[0,1]}$ be the vector space consisting of all continuous functions on the closed interval $[0, 1]$. The inner product on $\mathbf{C}_{[0,1]}$ is defined by $<f,g>=\int_0^1 f(t)g(t)dt$. Let $\mathrm{S}=\mathrm{span}\{1,x\}$.

(1) Find an orthonormal basis for the subspace **S**.

(2) Let $h\in\mathbf{C}_{[0,1]}$ be defined by $h(x)=e^x$. Find the orthogonal projection of h onto the subspace **S**.

Solution:

第 2 题（10 分）	得分

Let **S** be the subspace of $C[a,b]$ spanned by e^x, xe^x and x^2e^x. Let σ be the linear transformation on **S** defined by $\sigma(f)=f+f'$, where f' represents the derivative （导函数）of the function f.

(1) Find the matrix A representing σ with respect to the ordered basis $[e^x, xe^x, x^2e^x]$.

(2) Is the matrix A diagonalizable? Why?

Solution:

第 3 题（20 分）	得分

Let $A=\begin{pmatrix} 3 & 1 & -2 \\ -1 & -1 & 4 \\ 0 & -1 & 3 \end{pmatrix}$.

(1) Find all elementary divisors of A.

(2) Find a Jordan canonical form J of A, and find a nonsingular matrix P such that $P^{-1}AP = J$.

(3) Compute e^{Jt}. (Give the details of your computations.)

Solution:

第 4 题（15 分）	得分

Suppose that $A \in \mathrm{R}^{3\times 3}$ and $A^2 - 3A + 2I = O$.

(1) What are the possible minimal polynomials of A? Explain.

(2) In each case of part (1), what are the possible characteristic polynomials of A? Explain.

(3) In each case of part (1), what are the possible Jordan canonical forms of A? Explain.

Solution:

第 5 题（15 分）	得分

Let $A=\begin{pmatrix}1&2\\2&1\\1&1\end{pmatrix}\in\mathbf{R}^{3\times 2}$.

(1) Find the Moore-Penrose inverse A^{+} of matrix A.

(2) Find all least-squares solutions of the system $A\mathbf{x}=\mathbf{b}$, where $\mathbf{b}=(1,0,0)^{T}$.

(3) Find the orthogonal projection matrix P from R^3 to the column space of A.

Solution:

Part II (选做题，每题 10 分)

请在以下题目中（第 6 至第 9 题）选择三题解答. 如果你做了四题，请在题号上画圈标明需要批改的三题. 否则，阅卷者会随意挑选三题批改，这可能影响你的成绩.

第 6 题 Let $A \in \mathbf{R}^{n\times n}$. Show that the column space of A is the same as the column space of AA^T.

第 7 题 Let A be an $n\times n$ Hermitian matrix with eigenvalues $\lambda_1,\lambda_2,\cdots,\lambda_n$. Show that

(1) there exists an orthonormal set of vectors $\{\mathbf{u}_1,\mathbf{u}_2,\cdots,\mathbf{u}_n\}$ such that

$$A=\lambda_1\mathbf{u}_1\mathbf{u}_1^H+\lambda_2\mathbf{u}_2\mathbf{u}_2^H+\cdots+\lambda_n\mathbf{u}_n\mathbf{u}_n^H$$

(2) if $\mathbf{x}\in\mathbb{C}^n$ and $\|\mathbf{x}\|=1$, then $\min\{\lambda_1,\lambda_2,\cdots,\lambda_n\}\le\mathbf{x}^H A\mathbf{x}\le\max\{\lambda_1,\lambda_2,\cdots,\lambda_n\}$.

第 8 题 Let A , B be Hermitian matrices of order n. If A is positive definite and B is positive semi-definite, show that the eigenvalues of AB are nonnegative.

第 9 题 Let $A \in \mathbf{R}^{n\times n}$. Show that $\det(e^A)=e^{tr(A)}$.

选做题得分	

若正面不够书写，请写在反面.

选做题解答：