NUAA

Matrix Theory, Final,						Test Date: 2015 年 12 月 28 日	
矩阵论班号:		学号		姓名			
必做题（70 分）						选做题（30 分）	总分
题号	1	2	3	4	5		
得分							

Part I (必做题，共 5 题，70 分)

第 1 题（15 分）	得分

Let $\mathbf{P}_{[-1,1]}$ denote the set of all real polynomials of degree less than 3 with domain（定义域） $[-1,1]$. The addition and scalar multiplication are defined in the usual way. Define an inner product on $\mathbf{P}_{[-1,1]}$ by

$$< p,q >= \int_{-1}^{1} p(t)q(t)dt \ .$$

(1) Construct an orthonormal basis for $\mathbf{P}_{[-1,1]}$ from the basis $1, x, x^2$ by using the Gram-Schmidt orthogonalization process.

(2) Let $f(x) = x^2 - 1 \in \mathbf{P}_{[-1,1]}$. Find the orthogonal projection of f onto the subspace spanned by$\{1, x\}$.

Solution:

第 2 题（15 分）	得分

Let σ be the linear transformation on $\mathbf{P}_3$ (the vector space of real polynomials of degree less than 3) defined by

$$\sigma(p(x)) = xp'(x) + p''(x) .$$

(1) Find the matrix A representing σ with respect to the ordered basis $[1, x, x^2]$ for $\mathbf{P}_3$.

(2) Find a basis for $\mathbf{P}_3$ such that with respect to this basis, the matrix B representing σ is diagonal.

(3) Find the <u>kernel</u>（核） and <u>range</u> （值域）of this transformation.

Solution:

第 3 题（20 分）	得分

Let $A=\begin{pmatrix} 1 & -1 & 0 \\ 0 & 2 & 0 \\ 0 & -1 & 2 \end{pmatrix}$.

(1) Find all determinant divisors and elementary divisors of A.
(2) Find a Jordan canonical form of A.
(3) Compute e^{At}. (Give the details of your computations.)

Solution:

第 4 题（10 分）	得分

Suppose that $A \in \mathbf{R}^{3\times 3}$ and $A^2 - 5A - 6I = O$.

(1) What are the possible minimal polynomials of A? Explain.

(2) In each case of part (1), what are the possible characteristic polynomials of A? Explain.

Solution:

第 5 题（10 分）	得分

Let $A = \begin{pmatrix} 1 & 2 & 0 \\ 0 & 0 & 0 \end{pmatrix}$. Find the Moore-Penrose inverse A^{+} of A.

Solution:

Part II (选做题，每题 10 分)

请在以下题目中（第 6 至第 9 题）选择三题解答. 如果你做了四题，请在题号上画圈标明需要批改的三题. 否则，阅卷者会随意挑选三题批改，这可能影响你的成绩.

第 6 题 Let $\mathbf{P}_4$ be the vector space consisting of all real polynomials of degree less than 4 with usual addition and scalar multiplication. Let x_1, x_2, x_3 be three distinct real numbers. For each pair of polynomials f and g in $\mathbf{P}_4$, define

$$< f, g >= \sum_{i=1}^{3} f(x_i) g(x_i).$$

Determine whether $< f, g >$ defines an inner product on $\mathbf{P}_4$ or not. Explain.

第 7 题 Let $A \in \mathbf{R}^{n\times n}$. Show that if $\sigma(\mathbf{x}) = A\mathbf{x}$ is the orthogonal projection from $\mathbf{R}^n$ to $R(A)$, then A is symmetric and the eigenvalues of A are all 1's and 0's.

第 8 题 Let $A \in \mathbf{C}^{n\times n}$. Show that $\mathbf{x}^H A\mathbf{x}$ is real-valued for all $\mathbf{x} \in \mathbf{C}^n$ if and only if A is Hermitian.

第 9 题 Let A，$B \in \mathbf{C}^{n\times n}$ be Hermitian matrices, and A be positive definite. Show that AB is similar to BA, and is similar to a real diagonal matrix.

选做题得分	

若正面不够书写，请写在反面.

选做题解答：