NUAA

Matrix Theory, Final							Test Date: 2015 年 1 月 14 日	
矩阵论班号:			学号		姓名			
题号	必做题（70 分）				选做题（30 分）			总分
	1	2	3	4	5	6	7	
得分								

Part I (必做题，共 4 题，70 分)

第 1 题（20 分）	得分

Let $A=\begin{pmatrix} 3 & -1 & 0 \\ 8 & -2 & -1 \\ 0 & -1 & 2 \end{pmatrix}$.

(1) Prove that matrix A has a unique elementary divisor $(\lambda-1)^3$ and find a Jordan canonical form of A.

(2) Find a nonsingular matrix P such that $P^{-1}AP$ is in Jordan canonical form.

(3) Compute e^{2A}.

第 2 题（15 分）	得分

Let $m(\lambda)=(\lambda-2)(\lambda-3)^2$ be the minimal polynomial for a 4×4 complex matrix A.
(1) What are the possibilities for the characteristic polynomial of matrix A. Explain.
(2) Find all possible Jordan canonical forms (up to similarity) of matrix A. Explain.
Hint: The degree of the characteristic polynomial of an $n\times n$ matrix is n.

第 3 题（15 分）	得分

Let $f(x_1,x_2) = -7\overline{x_1}x_1 + 2\overline{x_2}x_2 + 6\mathrm{i}\,\overline{x_1}x_2 - 6\mathrm{i}\,\overline{x_2}x_1$, where $\mathrm{i}=\sqrt{-1}$ is the imaginary unit.

(1) Find the Hermitian matrix A such that $f(x_1,x_2) = \mathbf{x}^H A\mathbf{x}$, where $\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$.

(2) Reduce this quadratic form to its standard form (标准形) by a unitary transformation $\mathbf{x} = U\mathbf{y}$. (需写出 U 和标准形的具体表达式)

(3) Find a matrix B such that $B^2 = A$. (只需利用已知矩阵和常数矩阵表示 B，不需要计算出 B 的最终结果. $A^{\frac{1}{2}}$ 或 $\sqrt{A}$ 这样的表达式不可使用.)

第 4 题（20 分）	得分

Let $A=\begin{pmatrix} 1 & -1 & 1 \\ -1 & 1 & 1 \\ 1 & -1 & 2 \end{pmatrix}$.

(1) Find a full-rank factorization of *A*.

(2) Find the Moore-Penrose inverse A^{+} of matrix *A*.

(3) Find the orthogonal projection of vector $\mathbf{b}=(1,2,0)^{T}\in\mathbf{R}^{3}$ onto the column space of *A*.

Part II (选做题，每题 15 分)

请在第 5、第 6、第 7 题中选择两题解答. 如果你做了三题，请在题号上画圈标明需要批改的两题. 否则，阅卷者会随意挑选两题批改，这可能影响你的成绩.

第 5 题 Let **V** be an inner product space of dimension *n*, and S be a subspace of $\mathbf{V}$. $\dim(\mathrm{S}) = k > 0$. Let σ be a linear transformation that orthogonally projects each vector in **V** onto the subspace **S**.

(1) Show that σ is diagonalizable.

(2) Show that the characteristic polynomial of the representing matrix of σ is $\lambda^{n-k}(\lambda-1)^k$.

第 6 题 Let *A* be an Hermitian matrix, $\mathrm{i} = \sqrt{-1}$ be the imaginary unit, and *t* be a nonzero real number.

(1) Show that $tI + \mathrm{i}A$ and $tI - \mathrm{i}A$ are both nonsingular.

(2) $(tI + \mathrm{i}A)(tI - \mathrm{i}A)$ is Hermitian and positive definite.

第 7 题 Let $A \in \mathbf{R}^{m\times n}$. Show that for each $\mathbf{b} \in \mathbf{R}^m$, $(A^T A)^{-} A^T \mathbf{b}$ is a least-squares solution to the system $A\mathbf{x} = \mathbf{b}$.

选做题得分	

若正面不够书写，请写在反面.