NUAA

Matrix Theory, Final,						Test Date: 2019 年 1 月 5 日	
矩阵论班号:		班内序号:		学号:		姓名:	
必做题（70 分）						选做题（30 分）	总分
题号	1	2	3	4	5		
得分							

Part I (必做题，共 5 题，70 分)

第 1 题（15 分）	得分

Let $\mathbf{P}_3$ be the vector space consisting of all real polynomials of degree less than 3. Suppose that $\sigma:\mathbf{P}_3 \to \mathbf{P}_3$ is a linear transformation on $\mathbf{P}_3$ defined by

$$\sigma(p(x)) = xp''(x) + p(0),$$

where $p''(x)$ is the second derivative (二阶导数) of the polynomial $p(x)$.

(a) Find the range and kernel of σ.

(b) Find the representing matrix A of σ with respect to the ordered basis $1, x, x^2$.

(c) Is matrix A in part (b) diagonalizable? Why?

第 2 题（10 分）	得分

Let $\mathbf{R}^4$ be the inner product space with the standard inner product and $\mathbf{S}$ be a subspace of $\mathbf{R}^4$ spanned by vectors $\mathbf{v}_1 = (1,1,1,1)^T$ and $\mathbf{v}_2 = (-1,4,4,-1)^T$. Suppose that $\sigma(\mathbf{x}) = A\mathbf{x}$ is the orthogonal projection from $\mathbf{R}^4$ to $\mathbf{S}$.

(a) Find the projection matrix A.

(b) What is the vector in $\mathbf{S}$ that is closest to vector $\mathbf{b} = (1,0,0,0)^T$?

第 3 题（15 分）	得分

Let $A=\begin{pmatrix} 2 & 0 & -1 \\ 1 & 1 & -1 \\ 1 & -1 & 2 \end{pmatrix}$. （Hint: Matrix A has an eigenvalue $\lambda=1$.）

(a) Find the elementary divisors and the minimal polynomial of A.

(b) Find a Jordan canonical form J of A and a nonsingular matrix P such that $P^{-1}AP=J$.

第 4 题（15 分）	得分

Let $A=\begin{pmatrix}1 & 1 & 1\\ 1 & 2 & 2\\ 1 & 2 & 6\end{pmatrix}$.

(a) Show that matrix A is positive definite.
(b) Find the LDU factorization of matrix A.
(c) Write matrix A as a product of the form $B^T B$, where matrix B is an upper triangular matrix with positive diagonal elements.

第 5 题（15 分）	得分

Let $A=\begin{pmatrix} 1 & -1 & 1 \\ -1 & 1 & 1 \\ 1 & -1 & 2 \end{pmatrix}$, $\mathbf{b}=\begin{pmatrix} 3 \\ 3 \\ 3 \end{pmatrix}$.

(a) Find A^{+}.

(b) Determine if $A\mathbf{x}=\mathbf{b}$ is consistent? If it is consistent, find the general solution of the system；if not, find all least-squares solutions of $A\mathbf{x}=\mathbf{b}$ and the least-squares solution with minimal norm.

Part II (选做题, 每题 10 分)

请在以下题目中（第 6 至第 9 题）选择三题解答. 如果你做了四题，请在题号上画圈标明需要批改的两题. 否则，阅卷者会随意挑选三题批改，这可能影响你的成绩.

第 6 题 Let $\mathbf{V}$ be a vector space over the real number field $\mathbf{R}$, $\boldsymbol{\varepsilon}_1,\boldsymbol{\varepsilon}_2,\cdots,\boldsymbol{\varepsilon}_n$ be an ordered-basis for $\mathbf{V}$. Suppose that $\mathbf{u}_1,\mathbf{u}_2,\cdots,\mathbf{u}_k$ are vectors in $\mathbf{V}$ with coordinate vector $\mathbf{x}_1,\mathbf{x}_2,\cdots,\mathbf{x}_k\in\mathbf{R}^n$, respectively. Show that $\mathbf{u}_1,\mathbf{u}_2,\cdots,\mathbf{u}_k$ are linearly independent in $\mathbf{V}$ if and only if $\mathbf{x}_1,\mathbf{x}_2,\cdots,\mathbf{x}_k$ are linearly independent in $\mathbf{R}^n$.

第 7 题 A matrix S is said to be anti-Hermitian if $S^H=-S$. Let A be a Hermitian matrix. Show that A is negative definite if and only if A can be written as $A=S^2$, where S is a nonsingular anti-Hermitian matrix.

第 8 题 For $A=(a_{ij})\in\mathbf{C}^{n\times n}$, define $\|A\|_{l_\infty}=\max\limits_{1\le i,j\le n}\{|a_{ij}|\}$. Show that $n\|A\|_{l_\infty}$ is a matrix norm on $\mathbf{C}^{n\times n}$.

第 9 题 Let A^- be a generalized inverse of matrix A. Show that AA^- is diagonalizable.

选做题得分	

若正面不够书写，请写在反面.

选做题解答：

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