NUAA

Matrix Theory, Final	Test Date: 2017 年 1 月 5 日

矩阵论班号及班内序号:　　　　　　学号　　　　　　姓名

必做题（70 分）						选做题（30 分）	总分
题号	1	2	3	4	5		
得分							

Part I　(必做题，共 5 题，70 分)

第 1 题（10 分）	得分

Let $A = \begin{pmatrix} 1 & -1 & 4 \\ 1 & 4 & -2 \\ 1 & 4 & 2 \\ 1 & -1 & 0 \end{pmatrix}$.

(a) Find an orthonormal basis for the column space of A using Gram-Schmidt orthogonalization process.

(b) Find a matrix Q and R such that $A = QR$, where Q is a matrix whose column vectors form an orthonormal set and R is an upper triangular matrix with positive diagonal elements.

Solution:

第 2 题（10 分）	得分

Let $\mathbf{P}_3$ be the vector space consisting of all real polynomials of degree less than 3. Let σ be the linear mapping from $\mathbf{P}_3$ to $\mathbf{R}^2$ defined by

$$\sigma(p(x)) = \begin{pmatrix} \int_{-1}^{1} p(x)dx \\ p(0) \end{pmatrix}$$

(1) Find the range and kernel of this mapping.
(2) Find a matrix A such that

$$\sigma(a+bx+cx^2) = A\begin{pmatrix} a \\ b \\ c \end{pmatrix}$$

Solution:

第 3 题（20 分）	得分

Let $A=\begin{pmatrix} 2 & -6 & 2 \\ 0 & -1 & 1 \\ 0 & -4 & 3 \end{pmatrix}$

(1) Find the Smith normal form and all elementary divisors of A.

(2) Find a Jordan canonical form J of A, and find a nonsingular matrix P such that $P^{-1}AP = J$.

(3) Compute e^{Jt}. (Give the details of your computations.)

Solution:

第 4 题（10 分）	得分

Suppose that $A \in \mathrm{R}^{3\times 3}$ and the minimal polynomial of A is $m(\lambda) = (\lambda - 1)(\lambda - 2)$.

(1) What are the possible characteristic polynomials of A? Explain.

(2) What are the possible Jordan canonical forms of A? Explain.

Solution:

第 5 题（20 分）	得分

Let $A = \begin{pmatrix} 1 & 0 \\ 2 & 1 \\ 1 & 1 \end{pmatrix} \in \mathbf{R}^{3\times 2}$

(1) Find the Moore-Penrose inverse A^{+} of matrix A.

(2) Find all least-squares solutions of the system $A\mathbf{x} = \mathbf{b}$, where $\mathbf{b} = (0, 1, 0)^T$.

(3) Find the orthogonal projection matrix P from R^3 to the column space of A.

Solution:

Part II (选做题，每题 10 分)

请在以下题目中（第 6 至第 9 题）选择三题解答．如果你做了四题，请在题号上画圈标明需要批改的三题．否则，阅卷者会随意挑选三题批改，这可能影响你的成绩．

第 6 题 Let $\begin{pmatrix} A & O \\ O & B \end{pmatrix} \in \mathbf{R}^{n\times n}$, where A and B are square matrices. Show that if $\begin{pmatrix} A & O \\ O & B \end{pmatrix}$ is diagonalizable, then both A and B are diagonalizable.

第 7 题 Let A be a skew-Hermitian matrix, i.e., $A^H = -A$. Show that

(a) $I-A$ and $I+A$ are invertible.

(b) $(I-A)(I+A)^{-1}$ is a unitary matrix with eigenvalues not equal to -1

第 8 题 Let A and B be $n \times n$ Hermitian matrices . Show that A and B are similar if they have the same characteristic polynomial.

第 9 题 Let $A \in \mathbf{R}^{m\times n}$. Show that $A^{-}A = I_n$ if and only if $\operatorname{rank}(A) = n$

选做题得分	

若正面不够书写，请写在反面.

选做题解答：