南 京 航 空 航 天 大 学

2013～2014 学年 《 Matrix Theory 》Midterm

考试日期： 2013 年 11 月

学院		学号		姓名	

题号	1 (15)	2（20）	3（20）	4（15）	5（15）	6（15）	7（15）	8（15）	总分
得分									

Part I （70 分，必做题）

第 1 题	15 分
得分	

Let **S** be a subspace of $\mathbf{R}^4$ spanned by

$$\mathbf{u}_1 = (1,2,2,4)^T \quad \text{and} \quad \mathbf{u}_2 = (-2,0,-4,0)^T$$

(1) Find an orthonormal basis for the subspace **S**.

(2) Find the projection matrix P that projects vectors in $\mathbf{R}^4$ onto $\mathbf{S}$.

(3) Find the vector projection of $\mathbf{b} = (1,1,1,1)^T$ onto $\mathbf{S}$.

第 2 题	20 分
得分	

Let σ be a linear transformation on $\mathbf{P}_3$ defined by

$$\sigma(p(x)) = p(x) + p'(x) \quad (p'(x) \text{ 为 } p(x) \text{ 的导数})$$

where $\mathbf{P}_3$ is the vector space whose elements are real polynomials of degree less than 3.

(1) Find the kernel and range of σ.

(2) Find the matrix A representing σ with respect to the ordered basis $[1, x, x^2]$.

(3) Is matrix A diagonalizable? Why?

第 3 题	20 分
得分	

Consider the inner product space C[0,1] with inner product defined by

$$< f, g > = \int_{-\pi}^{\pi} f(x)g(x)dx .$$

Let **S** be the subspace spanned by vectors $\sin x$ and $\cos x$.

(1) Find $\|\sin x\|$, $\|\cos x\|$, $\|x\|$, and $< \cos x, \sin x >$.

(2) Find the projection of $h(x) = x$ onto the subspace **S**.

(3) Find the minimal distance from the vector $h(x) = x$ to the subspace **S**.

第 4 题	15 分
得分	

Given four points（−1, 0），（0, 1），（1, 3）and（2,9）on the plane, find <u>a linear function</u> (线性函数) $y = ax + b$ that best <u>fits</u>（拟合）the given data <u>in the "least squares" sense</u>（在最小二乘意义下）.

Part II (选做题, 30 分)

第五、第六题每题 15 分，请选择其中一题解答，并在所选的题号上划圈, 否则按得分最低的一题计分.

第五题 Definition: Let $\mathbf{V}_1, \mathbf{V}_2, \mathbf{V}_3$ be subspaces of vector space $\mathbf{V}$. $\mathbf{V}_1 + \mathbf{V}_2 + \mathbf{V}_3$ is a direct sum if each vector $\mathbf{x} \in \mathbf{V}_1 + \mathbf{V}_2 + \mathbf{V}_3$ can be uniquely represented as $\mathbf{x} = \mathbf{x}_1 + \mathbf{x}_2 + \mathbf{x}_3$, where $\mathbf{x}_k \in \mathbf{V}_k$ for $k = 1,2,3$.

Show that $\mathbf{V}_1 + \mathbf{V}_2 + \mathbf{V}_3$ is a direct sum if and only if

$$\dim(\mathbf{V}_1 + \mathbf{V}_2 + \mathbf{V}_3) = \dim(\mathbf{V}_1) + \dim(\mathbf{V}_2) + \dim(\mathbf{V}_3)$$

(注: 不可利用书中 36 页上的 Theorem 1.7.3)

第六题 Show that if $A \in C^{n\times n}$, then the column space of AA^H is the same as the column space of A. That is, $R(AA^H) = R(A)$.

第七、第八题每题 15 分，请选择其中一题解答，并在所选的题号上划圈, 否则按得分最低的一题计分.

第七题 Let $A \in C^{n\times n}$. Show that if $A = QDQ^T$, where $Q \in R^{n\times n}$ is a real orthogonal matrix and $D = \mathrm{diag}(\lambda_1, \lambda_2, \cdots, \lambda_n)$ with $|\lambda_k| = 1$ for $k = 1,2,\cdots,n$, then A is both symmetric and unitary.

第八题 Let $A \in C^{n\times n}$, and $AA^H = A^H A$. Show that $\|A\mathbf{x} - \lambda\mathbf{x}\| = \|A^H\mathbf{x} - \overline{\lambda}\mathbf{x}\|$ for any $\mathbf{x} \in C^n$ and $\lambda \in C$, where the inner product on C^n is the standard inner product.