HW1
Student ID: 112054320
Name: 黃章昱
Department: 隨班附讀
Date: 2023/10/16

1
(a)
It can be done by using Newton's method. Newton's method approximate square root of n through compute a guess repeatedly. The equation of the guess is: `new_guess = (old_guess + n / oldguess) / 2`, and the first guess is `n / 2`. We can repeat the guess untill `old_guess - new_guess < 0.1`, which means that the integer part is correct. To get the integer part, use a loop to subtract `new_guess` untill it becomes zero.

(b)

```
Algorithm roof_of_square_root(n):
    old_guess <- n / 2
    do:
        new_guess <- (old_guess + n / oldguess) / 2
    while old_guess - new_guess > 0.1

    result <- 0
    while new_guess >= 1:
        result <- result + 1
        new_guess <- new_guess - 1

    return result
```

2
(a)

```
Algorithm gcd(a,b):

    if a < b:
        swap(a,b)

    while b != 0:
        while a > b:
            a <- a - b
        swap(a,b)

    return a
```

(b)
Through Euclid's algorithm, we know that the smallest number this game can get is the greatest common divisor of a and b, gcd(a,b).

Assume that a > b. Then the greatest number in this game is a. Say that a = r * gcd(a,b). All numbers in the set `S = {a - i * gcd(a,b)| i from 0 to r - 1} - {a,b}` are an answer to the game. Hence, if |S| is odd, which means that r is odd, the first player will win; if |S| is even, the second player will win.

3

Proof of $(1+\frac{1}{n})^n < n$

1. $(1+\frac{1}{n})^n = (\frac{n+1}{n})^n = \frac{(n+1)^n}{n^n}$
2. Proving $(1+\frac{1}{n})^n < n$ equals to prove $(n+1)^n < n^{n+1}$
3. $(n+1)^n = n^n + C_1^n * n^{n-1} + ... + C_{n-1}^n * n + 1$
4. $n^{n+1} = n^n + n * n^{n-1} + ... + n^{n-1} * n$
5. The first n - 1 terms of step 4 are bigger than those in step 3. The last term in step 4: n^n, is absolutely bigger than the sum of nth and (n+1)th terms in step 3. In other words, $C_{n-1}^n * n + 1 = n^2 + 1 < n^n$ for $n \geq 3$.

Proof of that sequence $\{n^{\frac{1}{n}}\}_{n=3}^{\infty}$ is decreasing.

1. The statement equivalences to $\frac{1}{n} * \log n$ is decreasing, which equivalences to $\frac{1}{n+1} * \log(n+1) < \frac{1}{n} * \log n \equiv \log_n n + 1 < \frac{n+1}{n}$
2. Assume that $\log_n n + 1 \geq \frac{n+1}{n}$:
3. $\equiv \log_n (n+1) - 1 = \log_n \frac{n+1}{n} \geq \frac{1}{n}$
4. $\equiv n * \log_n \frac{n+1}{n} = \log_n (\frac{n+1}{n})^n \geq 1$
5. But in the previos proof, $\frac{(n+1)^n}{n^n} < n$. We can get $\log_n (\frac{n+1}{n})^n < \log_n n = 1$
6. In conclusion, the assumption in step 2 is wrong. Statement 1 is true.

4

Proof:

1. Basis step:
   When n = 1, $1 = f_1$
2. Induction step:
   Assume that for all $n \leq k$, n can be written as $n = f_a + f_b + f_c + ...$, which a, b, c... is not consecutive and $f_a > f_b > f_c > ...$.
   When $n = k+1$, $k+1 = f_a + (f_b + f_c + ... + 1)$. Because $(f_b + f_c + ... + 1) < k$, it can also be written as sum of nonconsecutive Fibonacci numbers, that say $f_b + f_c + ... + 1 = f_{b'} + f_{c'} + ...$ and $f_{b'} > f_{c'} ...$
   If f_a and $f_{b'}$ is consecutive, then we can use f_{a+1} to exchange them.