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Homework 2 - Serial-Link Manipulator Kinematics

Assigned - Nov 07, 2017, **Due - Nov 19, 2017**

1. Consider the three-DOF planar robot arm given in Figure 1, which consists of three links with lengths a_1, a_2 and a_3 connected with three revolute joints θ_1, θ_2 and θ_3. Derive the planar (2D) forward kinematics for this arm to compute the position and orientation of the end effector frame E relative to the fixed inertial frame O as a function of the joint angles θ_1, θ_2 and θ_3 as defined in the figure. In other words, find a function $f : S^1 \times S^1 \times S^1 \to SE(2)$ such that ${}^{O}T_E = f(\theta_1, \theta_2, \theta_3)$, where the homogeneous transform ${}^{O}T_E$ maps points in frame E to points in O. Explain your work.

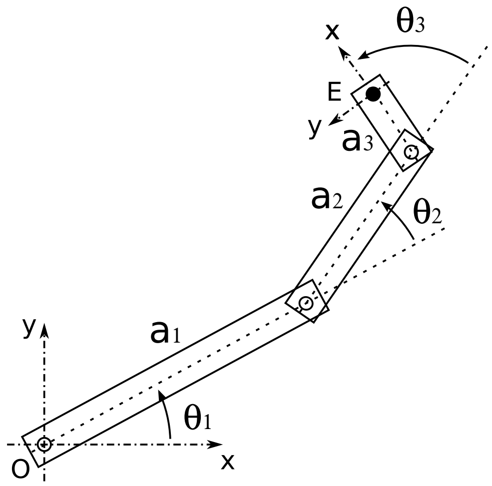

Figure 1: Three joint planar robot arm.

2. For the same robot arm, derive the planar inverse kinematic equations to achieve a desired end effector pose ${}^{O}T_E^*$. In other words, find a function $f^{-1} : SE(2) \to S^1 \times S^1 \times S^1$ such that ${}^{O}T_E^* = f(f^{-1}({}^{O}T_E^*))$. Explain your work.

3. Suppose that a parametric curve $\mathbf{c} : [0, 1] \to \mathbb{R}^2$ in the workspace of the robot arm described above is given in the form

$$\mathbf{c}(\gamma) := (1 + 0.2\sin(12\pi\gamma)) \begin{bmatrix} \cos(2\pi\gamma) \\ \sin(2\pi\gamma) \end{bmatrix}, \qquad (1)$$

corresponding to the path shown in the left plot of Figure 2.

Our goal is to have the end effector follow this path, while keeping the x axis of the frame E perpendicular (pointing outwards) to this curve at all times. Find an expression for the end effector pose ${}^{O}T_E(\gamma)$ that realizes this goal a given $\gamma \in [0, 1]$.

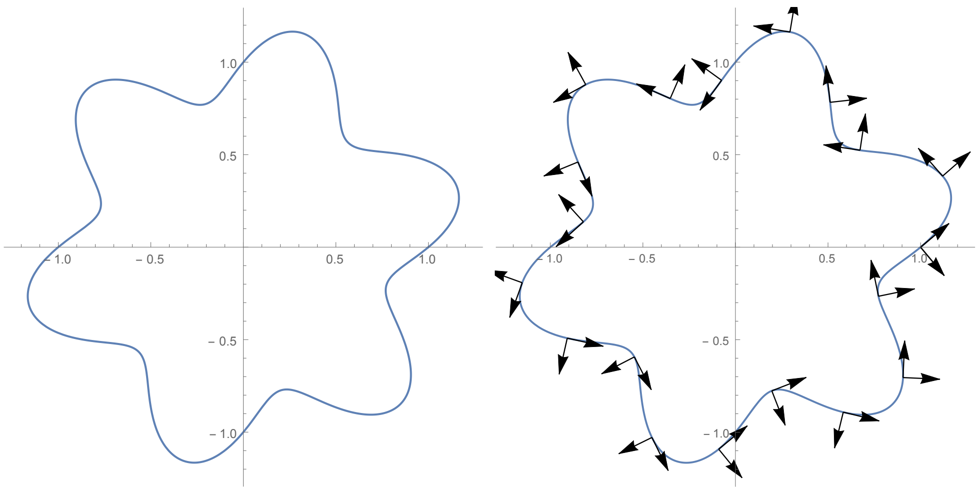

Figure 2: Left: Desired end effector path, Right: Desired end effector frame poses.

(Hint: You will first need to find an expression for the vector normal to the curve at any point. Remember that the tangent to a curve can be found by taking the derivative of its defining equation with respect to the parameter γ. If you align the y axis of E with this tangent, the x axis will be normal to the curve. Be careful about the direction though). The right plot in Figure 2 illustrates the desired frame poses.

4. Write a Matlab script (`hw2_script1.m`) that uses Corke's Matlab robotics toolbox to animate the end effector pose equation you have found in the previous question as γ goes from 0 to 1 in 100 steps. Note that your work in this question will help you debug your derivations for the previous question.

5. Consider the robot structure shown in Figure 1 with $a_1 = 0.75m$, $a_2 = 0.5m$ and $a_3 = 0.2m$. Write a Matlab script (`hw2_script2.m`) to first use Corke's Matlab toolbox to construct a serial link manipulator model with these parameters, then animate the robot following the desired path from the previous question, using the inverse kinematic equations you have derived in the second question above. You should only use Corke's toolbox to visualize the robot and not for any of the interpolation and inverse kinematics functions. Note that your work in this question will help you debug your inverse kinematics derivations.

6. You might have noticed that the end effector position proceeds at different speeds at different parts of the curve. This is due to the fact that the speed of progress along the curve (which is related to the magnitude of the tangent vector to the curve) is not constant. Find a "timing function" $s : [0, t_f] \to [0, 1]$ that takes a time instant $t \in [0, t_f]$ as input, and yields the corresponding parameter $\gamma \in [0, 1]$ that ensures that the end effector moves at constant velocity. In other words, the timing function s should be chosen such that

$$\left\| \frac{d}{dt} \left(\mathbf{c}(s(t)) \right) \right\|_2 = 1 \tag{2}$$

where $\|\mathbf{v}\|_2$ denotes the magnitude (norm) of a vector $\mathbf{v}$. Create a new Matlab script (`hw2_script3.m`) by modifying your second script from above that uses this scaling function to animate the robot arm to follow the desired trajectory of poses with a constant velocity.

7. If $a_2 = 0.5m$ and $a_3 = 0.2m$, what is the minimum value for a_1 such that the resulting robot can still follow the path in Figure 2? How did you find this minimum? Discuss your findings.

Submission

Submitted solutions must be typeset in a word processing environment such as LaTeX. Submissions are expected to be in the form of a ZIP file named `460_name_surname_hw#.zip`, including a PDF report with answers to theoretical questions with your name and student ID indicated clearly, as well as Matlab or other source files that are requested in the homework text. Late submissions will be penalized with a deduction of $8n^2$ points where n is the number of late days.

Note: You can discuss your discoveries and knowledge with your classmates but you must write your own answers and code for all questions above. If any significant similarities are found between your answers and other homeworks, you will be audited on your understanding of your own solutions.