Middle East Technical University
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Homework 3 - Jacobians and Mobile Robots

Assigned - Dec 12, 2017, **Due - Dec 26, 2017**

1. Consider the two-DOF PR planar robot arm given in Figure 1, which consists of a prismatic joint aligned with the x-axis, followed by a rotary joint connected to a link of length d_2. Derive the forward kinematics to compute the end effector position in O and the associated 2×2 Jacobian matrix.

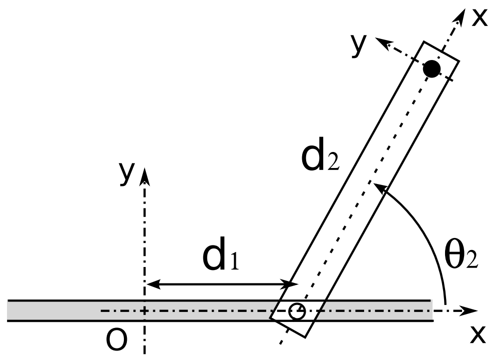

Figure 1: Two DOF planar robot arm.

2. Write a Matlab script `hw3_script1.m` that uses resolved-rate motion control to make the end effector move at a constant speed along a vector $\mathbf{v}$, starting from any given joint configuration $[d_1(0), \theta_2(0)]$, using $d_2 = 1$. Your implementation should be in the form of a function that takes $\mathbf{v}$ and the initial configuration as well the number of steps n and a step size *delta* as inputs and generates a vector of robot states as an output. Include a few example simulations and plots in your report with different choices for $\mathbf{v}$ and the starting point. Discuss what would eventually happen if the robot keeps moving when the desired direction of motion is not horizontal. Would the robot arm be able to follow the line indefinitely? Discuss your findings.

3. Write a similar Matlab script, `hw3_script2.m`, that uses resolved-rate motion control to make the same robot follow a circular path with radius r, centered at c (assuming that the entire circle stays within the reachable workspace). Your implementation should be in the form of a function that takes r and c as well the number of steps n and a step size *delta* as inputs and generates a vector of robot states as an output. Is the resolved-rate motion control appropriate for this application? Why? Discuss your findings and include a few informative examples and plots in your report.

4. Consider the mobile robot with three omni-direction wheels illustrated in Figure 2. The black point marks the “front” of the robot. The three wheels are located symmetrically, 120^o from each other and all at a distance D from the center of

the robot, and the wheels A and C are symmetric with respect to the front of the robot. The position and orientation of the robot body are denoted with $[x, y, \theta]$ in the world frame O.

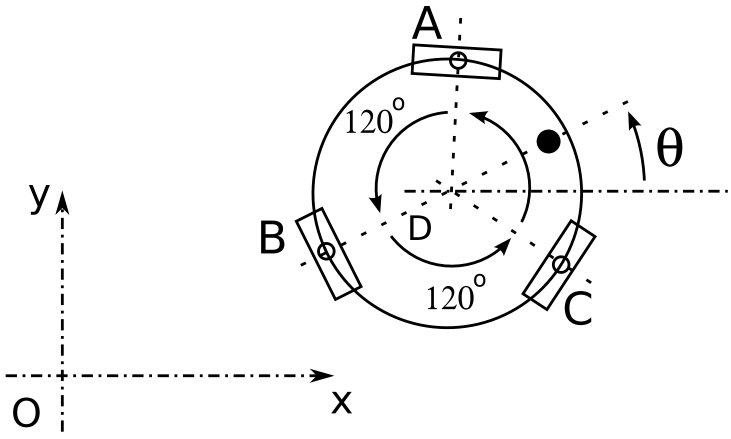

Figure 2: Planar omnidirectional mobile robot with three wheels.

Now, suppose that a body frame F is defined on the robot, located at the center point such that the x_F points towards the front of the robot and y_F towards the left side. Given a particular body pose with $[x, y, \theta]$, derive formulas for the positions of the centers of all wheels p_A, p_B and p_C in the world frame O.

5. Suppose that the robot (and hence the frame F) is moving with a translational velocity $\dot{x}$ and $\dot{y}$ in O as well as a rotational velocity $\dot{\theta}$. Using kinematic equations from earlier chapters, derive expressions for the translational velocities of the centers of all wheels, $\dot{p}_A$, $\dot{p}_B$ and $\dot{p}_C$ in the world frame O. Assuming these wheels are simple omnidirecional wheels with sideways rollers and radius R, find expressions for the rotational speeds w_A, w_B and w_C for all three wheels. Write the (linear) function from the variables $\dot{x}$, $\dot{y}$ and $\dot{\theta}$ to w_A, w_B and w_C in matrix form.

6. Write a Matlab script, `hw3_script3.m`, that implements a function taking three wheel rotational speeds w_A, w_B and w_C as a column vector, returning the associated robot velocities $\dot{x}$, $\dot{y}$ and $\dot{\theta}$ as a column vector using your derivations above.

7. Use Matlab's `ode45` function to simulate a robot with $D = 1$, $R = 0.2$ for different combinations of rotational velocities for the wheels. Include figures in your report to illustrate and discuss the resulting robot motion in each case. For example, what happens if all wheels rotate at the same speed? What happens if only one of the three wheels is rotating?

Submission

Submitted solutions must be typeset in a word processing environment such as LaTeX. Submissions are expected to be in the form of a ZIP file named `460_name_surname_hw#.zip`, including a PDF report with answers to theoretical questions with your name and student ID indicated clearly, as well as Matlab or other source files that are requested in the homework text. Late submissions will be penalized with a deduction of $8n^2$ points where n is the number of late days.

Note: You can discuss your discoveries and knowledge with your classmates but you must write your own answers and code for all questions above. If any significant similarities are found between your answers and other homeworks, you will be audited on your understanding of your own solutions.