CENG 460 - Introduction to Robotics
Spring 2017

Middle East Technical University
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Homework 1 - Pose Representations and Motion

Assigned - Oct 24, 2017, **Due - Nov 1, 2017**

1. Prove that a circle of radius r centered at (c_x, c_y) remains a circle with the same radius once transformed through homogeneous transformation T corresponding to a rotation R and a translation t. Note that points (x, y) on the original circle satisfy $(x - c_x)^2 + (y - c_y)^2 - r^2 = 0$. Where is the center of the new circle located? Hint: Try to express the equation of the circle above in matrix form using the vector $[x, y, 1]^T$.

2. Let $R \in SO(3)$ be a rotation matrix generated by rotating about a unit vector $\hat{w}$ by θ radians. That is, R satisfies $R = e^{[\hat{w}]_\times \theta}$.

   (a) Show that the eigenvalues of $[\hat{w}]_\times$ are 0, i, and $-i$, where $i = \sqrt{-1}$. What are the corresponding eigenvectors?

   (b) Show that the eigenvalues of R are 1, $e^{i\theta}$, and $e^{-i\theta}$. What is the eigenvector whose eigenvalue is 1?

   (c) Let $R = [r_1\ r_2\ r_3]$ be a rotation matrix. Show that $\det(R) = r_1^T(r_2 \times r_3)$

3. Compute the homogeneous transformation representing a translation of 3 units along the x-axis followed by a rotation of $\pi/2$ about the current z-axis followed by a translation of 1 unit along the fixed y-axis. Plot each intermediate frame using the Matlab toolbox and include it as a figure. What are the coordinates of the origin O_1 with respect to the original frame after each step? How did you compute these positions?

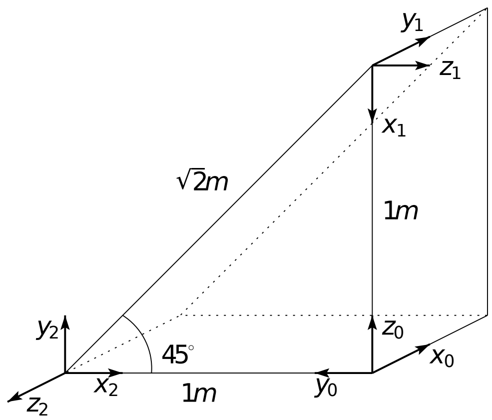

Figure 1: Illustration of coordinate frames (all right handed) for question 4.

4. Consider the coordinate frames in Figure 1. Find the homogeneous transformations T_1^0, T_2^0, T_2^1, representing transformations among the three frames shown. Verify your

results by visualizing the frames using the Matlab toolbox and include a single figure in your report with all three frames shown. Show that $T_2^0 = H_1^0 H_2^1$.

5. Write a Matlab script to animate a cube with an edge size of 1 initially centered at the origin, continuously rotated around a twist axis $[-1, 0, 1]^T$, passing through the point $[1, 0, 0]$. Your animation should be in an infinite loop, such that the cube comes back to its original position and goes through the same trajectory again and again. Explain your reasoning and derivations in the report, and submit your Matlab script $q5.m$ in your submission. Please include your name and student ID as a comment in the Matlab source file.

6. What unit quaternions correspond to rotations around the x, y and z axes? Prove, using the definition of quaternion multiplication, that the x, y and z axes indeed respectively remain fixed for these transformations.

Submission

Submitted solutions must be typeset in a word processing environment such as LaTeX. Submissions are expected to be in the form of a ZIP file named `460_name_surname_hw#.zip`, including a PDF report with answers to theoretical questions with your name and student ID indicated clearly, as well as Matlab or other source files that are requested in the homework text. Late submissions will be penalized with a deduction of $8n^2$ points where n is the number of late days.

Note: You can discuss your discoveries and knowledge with your classmates but you must write your own answers and code for all questions above. If any significant similarities are found between your answers and other homeworks, you will be audited on your understanding of your own solutions.