M 362K Post-Class Homework 1

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1-7

For each of the digit positions, there are 10 possible outcomes

For the first and third alpha positions, each has 26-3=23 possible outcomes since I,O and Q are excluded

For the second alpha position, there are 26 outcomes

Let #plates denotes the number of possible licence plates, then #plates = $10^3 * 23^2 * 26 = 1.375 * 10^7$

1-9

From the question we can know that the first digit has 10 - 2 = 8 outcomes and the fourth digit has 10 - 1 = 9 outcomes

Let #phone denotes the possible number of ten-digit phone numbers. Therefore,

$$#phone = 8 * 9 * 10^{10-2} = 7.2 * 10^9$$

1-12

According to the question, the number of possible outcomes for the trifeeta with 8 horses is ${}_{8}P_{1} * {}_{7}P_{1} * {}_{6}P_{1}$

Therefore, $Pr(winning\ ticket) = \frac{1}{8P_1*7P_1*6P_1} = \frac{1}{8*7*6} = \frac{1}{336}$

1-17

According to the question, there are 11 possible floors and 7 people. The total number of possible outcomes is 11^7

If no two will get of at the same floor, this is equivalent to the permutation of 11 floors chosen 7, which is $_{11}P_7$

:. $Pr(no\ two\ get\ of\ at\ the\ same\ floor) = \frac{11}{11^7} = \frac{11!}{(11-7)!11^7} \approx 0.085$

1-24

(a)

There are 13 toppings, 2 sizes and 2 types of crust

Let #2-top-pizza be the number of different two-topping pizzas

$$\therefore #2 - top - pizza = {}_{13}C_2 * {}_{2}C_1 * {}_{2}C_1 = 312$$

(b)

There are 6 meat toppings, 7 vegetable toppings, 2 sizes and 2 types of crust

Let #2 - top - mv - pizza be the number of different two-topping pizzas with exactly one meat and exactly one vegetable topping

$$\therefore #2 - top - mv - pizza = {}_{6}C_{1} * {}_{7}C_{1} * {}_{2}C_{1} * {}_{2}C_{1} = 168$$

(c)

There are 7 vegetable toppings, 2 sizes and 2 types of crust

Let #4 - top - v - pizza be the number of different four-topping vegetarian pizzas

$$\therefore #4 - top - v - pizza = {}_{7}C_{4} * {}_{2}C_{1} * {}_{2}C_{1} = 140$$

1-32

The number of possible words with 4 letters is indeed the permutation of the 4 letters. However, since the order of letter O does not matter, the number of possible words is therefore $\frac{4!}{2} = 12$

1-36

(a)

This is equivalent to dividing 25 girls into 3 groups of 5,5,15 girls respectively

Let #partition denotes the number of such partition

$$\therefore \#partition = \begin{pmatrix} 25 \\ 5 & 5 & 15 \end{pmatrix} = \frac{25!}{5!*5!*15!} = 823727520$$

(b)

This question is also equivalent to 25 girls into 3 groups of 5,5,15 girls respectively (2 teams of 5 people and 1 group of 15 not playing)

Let #partition denotes the number of such partition

$$\therefore \#partition = \begin{pmatrix} 25 \\ 5 & 5 & 15 \end{pmatrix} = \frac{25!}{5!*5!*15!} = 823727520$$