M 362K Post-Class Homework 13

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5-9

$E[X] = \int_0^\infty x(1+x)^{-4}dx = \frac{1}{6}$

Therefore the answer is (A)

5-12

From the property of probability density function, we can know that $\int_0^1 cxdx = \frac{c}{2} = 1$

$\therefore c = 2$

$\mu_X = E[X] = \int_0^1 2x^2dx = \frac{2}{3}$

$E[X^2] = \int_0^1 2x^3dx = \frac{1}{2}$

$Var[X] = E[X^2] - (E[X])^2 = \frac{1}{2} - \frac{4}{9} = \frac{1}{18}$

$\sigma_X = \sqrt{Var[X]} = \sqrt{\frac{1}{18}} \approx 0.235702$

Let COV denotes the coefficient of variation of X

$\therefore COV = \frac{\sigma_X}{\mu_X} \approx 0.353553$

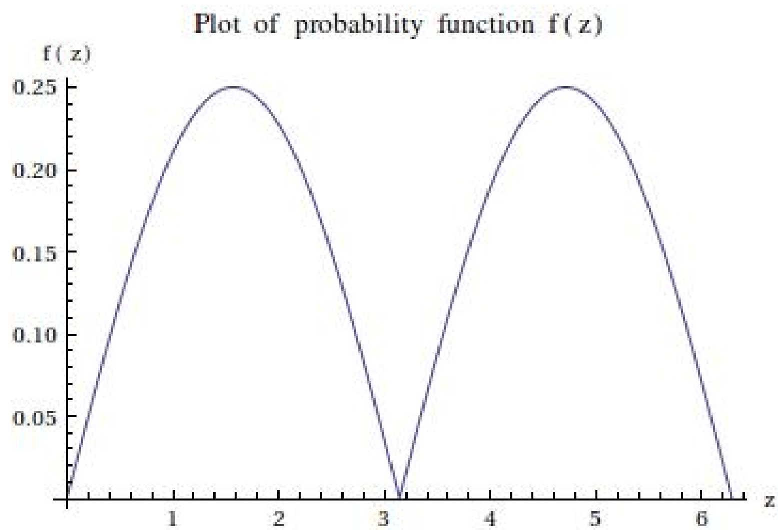

Figure 1: Plot of f(z)

5-14

(a)

$E[X] = \int_1^8 \frac{x}{7} dx = 4.5$

$E[X^2] = \int_1^8 \frac{x^2}{7} dx = \frac{73}{3}$

$\sigma_X = \sqrt{Var[X]} = \sqrt{E[X^2] - (E[X])^2} \approx 2.02073$

(b)

$E[\sqrt{X+1}] = \int_1^8 \frac{\sqrt{X+1}}{7} = \frac{18}{7} - \frac{4\sqrt{2}}{21} \approx 2.30205$

5-21

The plot of the probability density function is shown in Figure 1

$\therefore E[Z] = \int_0^{2\pi} z \frac{|\sin z|}{4} dx = \pi$

From Figure 1, we know that when $0 \le z \le pi$, $F_X(x) = -\frac{\cos z}{4} + \frac{1}{4}$

When $\pi \leq z \leq 2\pi$, $F_X(x) = \frac{1}{2} - \frac{\cos(z-\pi)}{4} + \frac{1}{4} = \frac{3}{4} + \frac{\cos(z-\pi)}{4}$

This means

$$F_X(x) \begin{cases} -\dfrac{\cos z}{4} + \dfrac{1}{4} & \text{if } 0 \leq z \leq \pi \qquad (1) \\ \dfrac{3}{4} + \dfrac{\cos(z-\pi)}{4} & \text{if } \pi \leq z \leq 2\pi \qquad (2) \end{cases}$$

When $z = \pi$, $F_X(x) = 0.5$

$\therefore z_{0.5} = \pi$

From Figure 1, the modes are $\frac{\pi}{2}$ and $\frac{3\pi}{2}$

5-25

We know that

$$f(x) \begin{cases} 0 & \text{if } x < 0 \text{ or } x > 3 \qquad (3) \\ \dfrac{1}{9}\left(4x - x^2\right) & \text{if } 0 \leq x \leq 3 \qquad (4) \end{cases}$$

Therefore $f'(x) = \frac{1}{9}(4 - 2x)$

To get $f'(x) = 0$, $x = 2$

$f(2) = \frac{4}{9} > 0$

Therefore $x_{mode} = 2$

The answer is (D)