M 362K Pre-Class Work for 3/24

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5-2

(a)

We can see that x^3 is a non-decreasing function when $0 \leq x \leq 1$. Also, $F_X(0) = 0$ and $F_X(1) = 1$. Therefore $F_X(x)$ is a non-decreasing function

$\lim_{x \to -\infty} F_X(x) = 0$

$\lim_{x \to +\infty} F_X(x) = 1$

$Pr(a < X \leq b) = F_X(b) - F_X(a)$

Therefore the properties of cumulative distribution functions hold for $F_X(x)$

(b)

Since $f(x) = F'_X(x)$, we can know that

$$f(x) = \begin{cases} 0 & \text{if } x < 0 \text{ or } x > 1 \quad (1) \\ 3x^2 & \text{if } 0 \leq x \leq 1 \quad (2) \end{cases}$$

(c)

Distribution function:

$Pr(0.1 < X \leq 0.5) = F_X(0.5) - F_X(0.1) = 0.5^3 - 0.1^3 = 0.124$

Density function:

$Pr(0.1 < X \leq 0.5) = \int_{0.1}^{0.5} f(x)dx = \int_{0.1}^{0.5} 3x^2 dx = 0.124$

(d)

$Pr(0.1 < X | X \leq 0.5) = \frac{Pr(0.1 < X \leq 0.5)}{Pr(0.5)} = \frac{F_X(0.5) - F_X(0.1)}{F_X(0.5)} = \frac{0.124}{0.5^3} = 0.992$

5-4

(a)

The cumulative probability distribution function is given as:

$$F_T(t) \begin{cases} 0 & \text{if } t < 82 \quad (3) \\ \dfrac{t-82}{8} & \text{if } 82 \leq t \leq 90 \quad (4) \\ 1 & \text{if } T > 90 \quad (5) \end{cases}$$

The probability density function $f(t)$ is therefore:

$$f(t) \begin{cases} 0 & \text{if } t < 82 \text{ and } t > 90 \quad (6) \\ \dfrac{1}{8} & \text{if } 82 \leq t \leq 90 \quad (7) \end{cases}$$

$\therefore E[T] = \int_{82}^{90} tf(t)dt = \int_{82}^{90} \frac{t}{8} dt = 86$

(b)

According to the property of cumulative probability distribution,

$Pr(t = 87) = F_T(87) - F_T(87) = 0$

(c)

$Pr(86.5 < T < 87.5) = F_X(87.5) - F_X(86.5) = \frac{1}{8}$