# M 362K Post-Class Homework 11

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March 9, 2015

## 4-28

$Pr[D < 2] = Pr[D = 0] + Pr[D = 1] = 0.8 + 0.2 * 0.8 = 0.96$

$E[D] = \frac{1-0.8}{0.8} = 0.25$

$E[D^2] = Var[D] + (E[D])^2 = \frac{0.2}{0.8^2} + (0.25)^2 = 0.375$

## 4-30

Let X be the number of trials Macon made prior passing the exam

$\therefore E[X] = \frac{1-0.4}{0.4} = 1.5$

## 4-34

Let p denote the probability of success in each trial

$Pr[X = 3] = (1-p)^2 p$

$Pr[X = 6] = (1-p)^5 p$

$\because Pr[X = 3] = 5 \cdot Pr[X = 6]$

$\therefore (1-p)^2 p = 5(1-p)^5 p$

Solving the above equation, we get $p = 1 - \frac{1}{5^{\frac{1}{3}}} \approx 0.415196$

$E[X] = \frac{1-p}{p} \approx 1.4085$

## 4-41

We know that $p = \frac{1}{6}$

Let $Z = X|Y = 2$, then

$Pr[Z = 1] = Pr[X = 1|Y = 1, 3, 4, 5] = \frac{1}{5}$

$Pr[Z = 2] = 0$

$Pr[Z = 3] = \frac{4}{5} \cdot 1 \cdot \left(\frac{1}{6}\right)$

$Pr[Z = 4] = \frac{4}{5} \cdot \left(\frac{1}{6}\right) \cdot \left(\frac{5}{6}\right)$

$Pr[Z = k] = \frac{4}{5} \cdot \left(\frac{1}{6}\right) \cdot \left(\frac{5}{6}\right)^{k-1}$ for all $k > 3$

$\therefore E[X|Y = 2] = 1 * \frac{1}{5} + 2 * 0 + 3 * \frac{4}{5} * \frac{1}{6} + \ldots = \frac{1}{5} + \frac{4}{5}(E[X] + 2) = 6.6$

Therefore the answer is (D)

## 4-46

$E[X] = \frac{rq}{p} = 6$

$Var[X] = \frac{rq}{p^2} = 12$

$\therefore r = 6$, $p = 0.5$, $q = 0.5$

$Pr[X < 3] = Pr[X = 0] + Pr[X = 1] + Pr[X = 2] = 0.5^6 + {}_{6-1+1}C_1 0.5 * 0.5^6 + {}_{6-1+2}C_2 0.5^2 *$

$0.5^6 = \frac{37}{256}$

## 4-49

Let X be the number of hurricanes before the home destroyed

$Pr(X = 0) = 0.4^2 = 0.16$

$Pr(X = 1) = {}_{2-1+1}C_1 0.6 * 0.4^2 = 0.1920$

$Pr(X = 2) = {}_{2-1+2}C_2 0.6^2 * 0.4^2 = 0.1728$

We can know that $Pr(X = k)$ decreases as k increases

So the mode is $X = 1$

## 4-52

Let A be the number of administration selected and F be the number of faculty selected

$E[A] = 8 * \frac{25}{100+25} = 1.6$

Since the expected number of faculty selected is way less than 4, the selection is indeed unfair to administration

## 4-55

Let $A_1$ be the number of non-defective from source A selected, $A_2$ be the number of defective from source A selected, $B_1$ be the number of non-defective from source B selected, $B_2$ be the number of defective from source B selected

Source A has $30 * 0.2 = 6$ defective units and source B has $50 * 0.08 = 4$ defective units

$Pr(A_2 + B_2 = 2, A_1 + B_1 = 3) = Pr(A_2 = 2, A_1 + B_1 = 3) + Pr(A_2 = B_2 = 1, A_1 + B_1 = 3) + Pr(B_2 = 2, A_1 + B_1 = 3) = \frac{{}_6C_2 * {}_{70}C_3}{{}_{80}C_5} + \frac{{}_6C_1 * {}_4C_1 * {}_{70}C_3}{{}_{80}C_5} + \frac{{}_4C_2 * 70_3C}{{}_{80}C_5} \approx 0.102$

Therefore the answer is (C)