# M 362K Pre-Class Work for 3/26

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## 5-8

### (a)

$\mu_X = E[X] = \int_0^4 x \frac{x^3}{64} dx = 3.2$

### (b)

$E[X^2] = \int_0^4 x^2 \frac{x^3}{64} dx = 10.6667$

### (c)

$F_X(x) = \int \frac{x^3}{64} dx = \frac{x^4}{256} + C$, where C is a constant

$\because F_X(0) = 0$ and $F_X(4) = 1$

$\therefore C = 0$

$F_X(x) = \frac{x^4}{256}$

### (d)

$Pr(2 < x \leq 3) = F_X(3) - F_X(2) = 0.253906$

## 5-11

$Var[X] = E[X^2] - (E[X])^2 = 10.6667 - 3.2^2 = 0.426667$

## 5-17

From the question we know that $f'(x) = 6(1-x) - 6x$

When $f'(x) = 0$, $x = \frac{1}{2}$

At the endpoints $f(x) = 0$

Therefore $x_{mode} = 0.5$

## 5-19

### (a)

According to the properties of probability density functions, $\int_0^1 cx(1-x^2)dx = 1$

$\int_0^1 cx(1-x^2)dx = \frac{c}{4} = 1$

$\therefore c = 4$

### (b)

From part (a) we can know that

$$f(x) = \begin{cases} 4x(1-x^2) & \text{for } 0 \leq x \leq 1 \qquad (1) \\ 0 & \text{otherwise} \qquad (2) \end{cases}$$

$F(x) = \int f(x)dx = 2x^2 - x^4 + C$, where C is a constant

$\because F(0) = 0$ and $F(1) = 1$

$\therefore C = 0$

$$F(x) = \begin{cases} 0 & \text{if } x < 0 & (3) \\ 2x^2 - x^4 & \text{if } 0 \leq x \leq 1 & (4) \\ 1 & \text{if } x > 1 & (5) \end{cases}$$

## (c)

$\mu_X = E[X] = \int_0^1 xf(x)dx = \int_0^1 4x^2(1-x^2)dx = \frac{8}{15}$

## (d)

$f'(x) = -8x^2 + 4(1-x^2)$

When $f'(x) = 0$, we have $x = \frac{1}{\sqrt{3}}$

When at end points, $f(x) = 0$

Therefore the mode is $x_{mode} = \frac{1}{\sqrt{3}}$

## (e)

The median occurs when $F(x) = 0.5$

Therefore $x_{median} = 0.541196$

## (f)

The value $x_{.21}$ occurs when $F(x) = 0.21$

$\therefore x_{.21} = 0.333437$