M 362K Pre-Class Work for 2/19

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3-38

Let S be the random variable that denotes the sum of a pair of fair dice

The probability distribution is shown below

S	5	6	7	8	9	10
$Pr(S\|at\ least\ one\ 4)$	$\frac{2}{11}$	$\frac{2}{11}$	$\frac{2}{11}$	$\frac{1}{11}$	$\frac{2}{11}$	$\frac{2}{11}$

$E[S|at\ least\ one\ 4] = 5 * \frac{2}{11} + 6 * \frac{2}{11} + 7 * \frac{2}{11} + 8 * \frac{1}{11} + 9 * \frac{2}{11} + 10 * \frac{2}{11} = \frac{82}{11} \approx 7.45$

3-42

(a)

X and Y are independent. This is because the outcome of the first dice does not affect the outcome of the second dice at all

(b)

$E[X] = E[y] = \frac{1+2+3+4+5+6}{6} = 3.5$

$E[X^2] = E[Y^2] = \frac{1^2+2^2+3^2+4^2+5^2+6^2}{6} = \frac{91}{6}$

$Var[X] = Var[Y] = E[X^2] - E[X]^2 = 2.91667$

$Var[X+Y] = Var[X] + Var[Y] = 5.833$

3-45

(a)

From the table given in the question, we can know

$Pr(H = 73 \cap S = 12) = 1 - (0.25 + 0.2 + 0.15 + 0.05 + 0.12) = 0.23$

$E[S] = 68 * (0.25 + 0.05) + 70 * (0.2 + 0.12) + 73 * (0.15 * 0.23) = 45.3185$

$E[H] = 8.5 * (0.25 + 0.2 + 0.15) + 12 * (0.05 + 0.12 + 0.23) = 9.9$

(b)

$Pr(S = 8.5|H = 73) = \frac{0.15}{0.15+0.23} = 0.394837$

$Pr(S = 12|H = 73) = 1 - Pr(S = 8.5|H = 73) = 0.605263$

$E[S|H = 73] = 8.5 * 0.394837 + 12 * 0.605263 = 10.6193$

(c)

Let μ denotes mean and σ denotes the standard deviation, COV denotes coefficient of variation

$Pr(S = 8.5|H = 68) = \frac{0.25}{0.25+0.05} = 0.8333$

$Pr(S = 12|H = 68) = 1 - Pr(S = 8.5|H = 68) = 0.1667$

$\mu = E[S|H = 68] = 8.5 * 0.8333 + 12 * 0.1667 = 9.08345$

$E[S^2|H = 68] = 8.5^2 * 0.8333 + 12^2 * 0.1667 = 84.2107$

$\sigma = \sqrt{E[S^2|H = 68] - \mu^2} = 1.30448$

$COV = 100 * \frac{\sigma}{\mu}\% = 14.361\%$