M 362K Post-Class Homework 4

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2-27

(a)

We know that $Pr(car) = \frac{1}{3}$ and $Pr(goat) = \frac{2}{3}$

If you choose to keep the original door, that means you have to select the correct door at the beginning

$\therefore Pr(car|without\ changing\ doors) = Pr(car) = \frac{1}{3}$

(b)

If I select the door with goat in the first place, then I will win a car for sure if I change the door after the revelation. Otherwise I will be certain that I will win a goat

$\therefore Pr(car|changing\ doors) = \frac{2}{3} * 1 + \frac{1}{3} * 0 = \frac{2}{3}$

2-29

From the question, we know that $Pr(smoker|circulation) = 2x$, $Pr(smoker|non-circulation) = x$, $Pr(circulation) = 0.25$

$\therefore Pr(circulation|smoker) = \frac{Pr(smoker|circulation)*Pr(cirulation)}{Pr(smoker|circulation)*Pr(cirulation)+Pr(smoker|non-circulation)*Pr(non-curculation)} = \frac{2x*0.25}{2x*0.25+0.75x} = \frac{2}{5}$

Therefore the answer is (C)

2-41

	A	A'	
B	a*b	(1-a)*b	b
B'	a*(1-b)	(1-a)*(1-b)	1-b
	a	1-a	1

(a)

According to the Venn box diagram shown above, $Pr(A \cap B') = a*(1-b) = Pr(A)*Pr(B')$

Therefore, A and B' are independent

(b)

$Pr(A' \cap B') = (1-a)*(1-b) = Pr(A')*Pr(B')$

Therefore, A' and B' are independent

(c)

$Pr(A' \cap B) = (1-a) * b = Pr(A') * Pr(B)$

Therefore, A' and B are independent

2-44

$Pr(A \cap B') = Pr(A) * Pr(B') = Pr(A) * (1 - Pr(B)) = Pr(A) - Pr(A) * Pr(B)$

Similarly, we get $Pr(A' \cap B) = Pr(B) - Pr(A) * Pr(B)$

We set $Pr(A \cap B) = Pr(A) * Pr(B) = x$

$\therefore (0.2 + x) * (0.3 + x) = x$

$x = 0.2$ or $x = 0.3$

$P(A) = 0.2 + 0.2 = 0.4$ or $0.2 + 0.3 = 0.5$

$P(B) = 0.3 + 0.2 = 0.5$ or $0.3 + 0.3 = 0.6$

$\therefore Pr(A \cup B) = Pr(A) + Pr(B) - Pr(A) * Pr(B) = 0.4 + 0.5 - 0.2 = 0.7$ or $0.5 + 0.6 - 0.3 = 0.8$

2-52

$Pr(first - face) = \frac{12}{52}$ and $Pr(second - face | first - face) = \frac{11}{51}$

$\therefore Pr(first - face \cap second - face) = Pr(first - face) * Pr(second - face | first - face) =$

$\frac{12}{52} * \frac{11}{51} = \frac{11}{221} \approx 0.05$

2-55

Let the number of blue balls be x

$Pr(same\ color) = Pr(blue \cap blue) + Pr(red \cap red) = \frac{4}{10} * \frac{16}{16+x} + \frac{6}{10} * \frac{x}{16+x} = 0.44$

$\therefore x = 4$

The number of balls in the second urn is 20. The answer is (B)