M 362K Post-Class Homework 12

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5-1

(a)

The Plot is shown in Figure 1

(a) From the plot, we can know that $F(x)$ is a non-decreasing function

(b) $\lim_{x \to -\infty} F(x) = 0$

(c) $\lim_{x \to \infty} F(x) = 1$

Therefore $F(x)$ has the properties of a cumulative distribution function

(b)

We know that $f(x) = F'(x)$

Therefore,

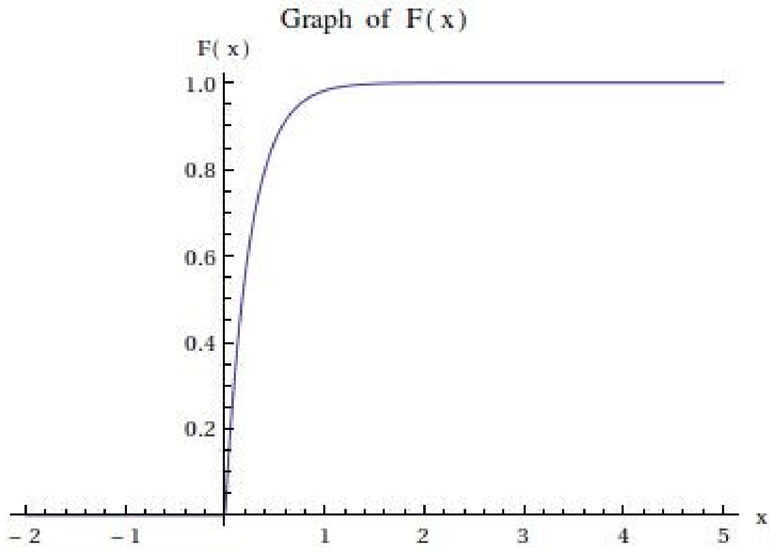

Figure 1: Plot of $F(x)$

$$f(x) \begin{cases} 0 & \text{if } x < 0 \quad (1) \\ 4e^{-4x} & \text{if } x > 0 \quad (2) \\ undefined & \text{if } x = 0 \quad (3) \end{cases}$$

(c)

Distribution function:

$Pr(1 < x \leq 2) = F(2) - F(1) = 1 - e^{-8} - 1 + e^{-4} \approx 0.0179802$

Density function:

$Pr(1 < x \leq 2) = \int_1^2 f(x)dx = \int_1^2 4e^{-4x}dx = 0.0179802$

(d)

$Pr[X > 2|X > 1] = \frac{Pr[X>2 \cap X>1]}{Pr[X>1]} = \frac{Pr[X>2]}{Pr[X>1]} = \frac{1-F(x=2)}{1-F(x=1)} = \frac{e^{-8}}{e^{-4}} = 0.0183156$

5-3

(a)

In order to have a valid density funtion, $\int_0^\infty f(y) = 1$ must be true

$\int_0^\infty ke^{-3y} = \frac{k}{3} = 1$

$\therefore k = 3$

(b)

From the density function we can know that the CDF is

$F(y) = \int_0^y ke^{-3y}dy = -\frac{1}{3}ke^{-3y} + \frac{k}{3}$

$Therefore F(\infty) = \frac{k}{3} = 1$

Therefore $k = 3$

(c)

From part (b) we know that the CDF is:

$$F(y) = \begin{cases} -e^{-3y} + 1 & \text{if } y \geq 0 \quad (4) \\ 0 & \text{if } y < 0 \quad (5) \end{cases}$$

CDF:

$Pr(2 < y \leq 3) = F(3) - F(2) = -e^{-9} + e^{-6} = 0.00235534$

Density function:

$Pr(2 < y \leq 3) = \int_2^3 3e^{-3y}dy = 0.00235534$

(d)

$Pr(Y > -3) = F(\infty) - F(-3) = 1$

5-5

$Pr(x < 2|x \geq 1.5) = \frac{Pr(1.5 \leq x < 2)}{Pr(x \geq 1.5)} = \frac{\int_{1.5}^{2} 3x^{-4} dx}{1 - \int_{1}^{1.5} 3x^{-4} dx} = 0.578125$

Therefore the answer is (A)

5-7

$Pr(x < 5) = \int_0^5 (10 + x)^{-2} dx = \frac{1}{30} = 0.0333$

Therefore the answer is (A)