M 362K Post-Class Homework 9

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3-39

Let L be the random variable that represents the amount of loss

The probability distribution given the loss is greater than 0 is shown below:

l	500	1,000	10,000	50,000	100,000
$Pr(L = l \mid L > 0)$	0.6	0.3	0.08	0.01	0.01

$\therefore E[L|L > 0] = 500 * 0.6 + 1000 * 0.3 + 10000 * 0.08 + 50000 * 0.01 + 100000 * 0.01 = 2900$

The expected amount of loss is 2900

3-40

(a)

Let $x = Pr(D = 5000)$ and $y = Pr(D = 10000)$

Then $x + y = 1 - 0.82 - 0.1 = 0.08$ and $5000x + 10000y = 600 - 1000 * 0.1$

By solving the above two equations, we get $x = 0.06$ and $y = 0.02$

$\therefore Pr(D = 5000) = 0.06$ and $Pr(D = 10000) = 0.02$

(b)

$E[D^2] = 0 * 0.82 + 1000^2 * 0.1 + 5000^2 * 0.06 + 10000^2 * 0.02 = 360000$

$Var[D] = E[D^2] - (E[D])^2 = 0$

$\sigma_D = \sqrt{Var[D]} = 0$

(c)

The probability distribution given that $D > 0$ is shown below:

d	1000	5000	10000
$Pr(D = d \vert D > 0)$	$\frac{1}{9}$	$\frac{6}{9}$	$\frac{2}{9}$

$\therefore E[D|D > 0] = 1000 * \frac{1}{9} + 5000 * \frac{6}{9} + 10000 * \frac{2}{9} = \frac{17000}{3} \approx 5666.67$

(d)

$E[D^2|D > 0] = 1000^2 * \frac{1}{9} + 5000^2 * \frac{6}{9} + 10000^2 * \frac{2}{9} = 39000000$

$Var[D] = E[D^2|D > 0] - (E[D|D > 0])^2 = 39000000 - \left(\frac{17000}{3}\right)^2 = \frac{62000000}{9} \approx 6.88889 * 10^6$

3-41

(a)

The probability distribution of X and Y are shown below:

x	0	1
$Pr(X = x)$	0.5	0.5

y	0	1	2
$Pr(Y = y)$	0.25	0.5	0.25

$E[X] = 0 * 0.5 + 1 * 0.5 = 0.5$

$E[X^2] = 0 * 0.5 + 1^2 * 0.5 = 0.5$

$Var[X] = E[X^2] - (E[X])^2 = 0.25$

(b)

$E[Y] = 0 * 0.25 + 1 * 0.5 + 2 * 0.25 = 1$

$E[Y^2] = 0^2 * 0.25 + 1^2 * 0.5 + 2^2 * 0.25 = 1.5$

$Var[Y] = E[Y^2] - (E[Y])^2 = 0.5$

(c)

The probability distribution of $X + Y$ is shown below

$X + Y$	0	1	2	3
$Pr(X + Y)$	0.25	0.25	0.25	0.25

$E[X + Y] = 0 * 0.25 + 1 * 0.25 + 2 * 0.25 + 3 * 0.25 = 1.5$

$E[(X + Y)^2] = 0^2 * 0.25 + 1^2 * 0.25 + 2^2 * 0.25 + 3^2 * 0.25 = 3.5$

$Var[X + Y] = E[(X + Y)^2] - (E[X + Y])^2 = 1.25$

(d)

$Var[X] + Var[Y] = 0.25 + 0.5 = 0.75 \neq 1.25 = Var[X + Y]$

3-43

(a)

The probability distribution of X is shown below:

x	0	1	2
$Pr(X = x)$	0.25	0.5	0.25

$E[X] = 0 * 0.25 + 1 * 0.5 + 2 * 0.25 = 1$

$E[X^2] = 0^2 * 0.25 + 1^2 * 0.5 + 2^2 * 0.25 = 1.5$

$Var[X] = E[X^2] - (E[X])^2 = 0.5$

(b)

The probability distribution of Y is shown below:

y	0	1	2
$Pr(Y = y)$	$\frac{4}{9}$	$\frac{4}{9}$	$\frac{1}{9}$

$E[Y] = 0 * \frac{4}{9} + 1 * \frac{4}{9} + 2 * \frac{1}{9} = \frac{2}{3}$

$E[Y^2] = 0 * \frac{4}{9} + 1^2 * \frac{4}{9} + 2^2 * \frac{1}{9} = \frac{8}{9}$

$Var[Y] = E[Y^2] - (E[Y])^2 = \frac{8}{9} - \left(\frac{2}{3}\right)^2 = \frac{4}{9}$

(c)

The probability distribution of X+Y is shown below:

X+Y	0	1	2	3	4
$Pr(X + Y)$	$\frac{4}{36}$	$\frac{12}{36}$	$\frac{13}{36}$	$\frac{6}{36}$	$\frac{1}{36}$

$E[X + Y] = 0 * \frac{4}{36} + 1 * \frac{12}{36} + 2 * \frac{13}{36} + 3 * \frac{6}{36} + 4 * \frac{1}{36} = \frac{5}{3}$

$E[(X + Y)^2] = 0^2 * \frac{4}{36} + 1^2 * \frac{12}{36} + 2^2 * \frac{13}{36} + 3^2 * \frac{6}{36} + 4^2 * \frac{1}{36} = \frac{67}{18}$

$Var[X+Y] = E[(X+Y)^2] - (E[X+Y])^2 = \frac{67}{18} - \left(\frac{5}{3}\right)^2 = \frac{17}{18}$

(d)

$Var[X] + Var[Y] = 0.5 + \frac{4}{9} = \frac{17}{18} = Var[X+Y]$

It is equal because the outcomes of X and Y are independent

3-44

(a)

Let B represents a boy is born and G represents a girl is borned

Therefore the outcome is shown below:

Outcome	X_1	X_2	X_3	$X_1 + X_2 + X_3$	Probability
BBB	0	0	0	0	0.125
BBG	0	0	1	1	0.125
BGB	0	1	0	1	0.125
GBB	1	0	0	1	0.125
GGB	1	1	0	2	0.125
GBG	1	0	1	2	0.125
BGG	0	1	1	2	0.125
GGG	1	1	1	3	0.125

The probability distribution of X is shown below:

X	0	1	2	3
$Pr(X)$	0.125	0.375	0.375	0.125

Therefore $X = X_1 + X_2 + X_3$

(b)

$E[X_i] = 0 * 0.5 + 1 * 0.5 = 0.5$

$E[X_i^2] = 0^2 * 0.5 + 1^2 * 0.5 = 0.5$

$Var[X_i] = E[X_i^2] - (E[X_i])^2 = 0.25$

(c)

$E[X] = E[X_1] + E[X_2] + E[X_3] = 1.5$

$Var[X] = Var[X_1] + Var[X_2] + Var[X_3] = 0.75$

The results are consistent

Sample Exam 3

(a)

Let X be the value of the datapoint

$mean = E[X] = \frac{1}{24} * (7 + 8 + 4 + 6 + 9 + 10 + 2 + 7 + 8 + 9 + 3 + 6 + 9 + 8 + 4 + 9 + 6 + 5 +$

$9 + 8 + 7 + 9 + 0 + 3) = 6.5$

$median = \frac{7+7}{2} = 7$

$mode = 9$

$midrange = \frac{0+10}{2} = 5$

(b)

$min = 0$ and $max = 10$

(c)

Let i be the rank of the 80th percentile number

$i = 0.8 * (24 + 1) = 20$

Therefore the 80th percentile number is 9

(d)

$E[X^2] = \frac{1}{24} * (7^2 + 8^2 + 4^2 + 6^2 + 9^2 + 10^2 + 2^2 + 7^2 + 8^2 + 9^2 + 3^2 + 6^2 + 9^2 + 8^2 + 4^2 + 9^2 +$

$6^2 + 5^2 + 9^2 + 8^2 + 7^2 + 9^2 + 0^2 + 3^2) = 49$

$Var = E[X^2] - (E[X])^2 = \frac{162}{23}$

Let σ denotes the standard deviation

$\sigma = \sqrt{Var} = 9\sqrt{\frac{2}{23}} \approx 2.65396$

(e)

Let COV denotes coefficient of variance

$COV = \frac{100*\sigma}{mean}\% \approx 40.8301\%$

(f)

Let z denote the z-score

$z = \frac{11-mean}{\sigma} \approx 1.69558$