PHY 362K Midterm 2 Review Note

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1 Normal Zeeman Effect

Bohr magneton: $\mu_B = \frac{e\hbar}{2m_e}$

The levels with $m_l = 0, +1, -1$ have energies $E_P^{(0)}$, $E_P^{(0)} + \mu_B B$, $E_P^{(0)} - \mu_B B$

The Bohr transition frequencies of these levels to a lower s level are just ω_0, $\omega_0 + \frac{\mu_B B}{\hbar}$, $\omega_0 - \frac{\mu_B B}{\hbar}$ respectively

The normal Zeeman effect does occur in real atoms when all electron spins are paired, such that the total spin of all electrons is zero

Fine structure constant: $\alpha = \frac{e^2}{4\pi\epsilon_0\hbar c} \approx \frac{1}{137.04}$

Physics spectroscopic notation: $n^{2s+1}l_j$

2 Stark Effect in Hydrogen

$\vec{\mathcal{E}} = \mathcal{E}z$

$H' = e\mathcal{E}z$

$E_1^{(1)} = \langle\psi_{100}^{(0)}|H'|\psi_{100}^{(0)}\rangle = e\mathcal{E}\langle\psi_{100}^{(0)}|z|\psi_{100}^{(0)}\rangle = 0$

$|\psi_{100}^{(1)}\rangle = \sum\limits_{(nlm)\neq 100} \frac{\langle\psi_{nlm}^{(0)}|H'|\psi_{100}^{(0)}\rangle}{E_1^{(0)} - E_n^{(0)}}$

$\psi^{(0)}_{nlm}|z|\psi^{(0)}_{n'l'm'}\rangle = 0$ unless

(1) $m = m'$ and

(2) $l = l' + 1$ and $l = l' - 1$

Energy Shift:

$E_1^{(2)} = \frac{e\mathcal{E}^2}{E_1^{(0)}} \sum_{n=2}^{\infty} \frac{|\psi^{(0)}_{n10}|z|\psi^{(0)}_{100}\rangle|^2}{1-\frac{1}{n^2}} = -2 * 4\pi\epsilon_0 a_0 \mathcal{E}^2 \sum_{n=2}^{\infty} \frac{|\psi^{(0)}_{n10}|z|\psi^{(0)}_{100}\rangle|^2}{1-\frac{1}{n^2}}$

3 Electron Spin

$[S_x, S_y] = i\hbar S_z$, $[S_y, S_z] = i\hbar S_x$, $[S_z, S_x] = i\hbar S_y$

$S^2|s\ m\rangle = \hbar^2 s(s+1)|s\ m\rangle$

$S_z|s\ m\rangle = \hbar m|s\ m\rangle$

3.1 Raising and Lowering Operators

$L_\pm|l\ m_l\rangle = \hbar\sqrt{l(l+1) - m_l(m_l \pm 1)}|l\ (m_l \pm 1)\rangle$

$S_\pm|s\ m_s\rangle = \hbar\sqrt{s(s+1) - m_s(m_s \pm 1)}|s\ (m_s \pm 1)\rangle$

$\because \vec{j} = \vec{l} + \vec{s}$

$\therefore J_\pm = L_\pm + S_\pm$

$J_\pm|j\ m_j\rangle = \hbar\sqrt{j(j+1) - m_j(m_j \pm 1)}|s\ (m_j \pm 1)\rangle$

4 State of Hydrogen Atom including Spin

$\{|nlm_lm_s\rangle\}$ are also eigenvectors of H_z

$H_z|nlm_lm_s\rangle = \mu_B B(m_l + 2m_s)|nlm_lm_s\rangle$

$\{|nljm_j\rangle\}$ are also eigenvectors of H_{fs}

$H_{fs}|nljm_j\rangle = -|E_n^{(0)}| \left(\frac{\alpha}{n}\right)^2 \left[\frac{n}{j+\frac{1}{2}} - \frac{3}{4n^4}\right]$

Neither set are eigenvectors of both H_{fs} and H_z

Need to use degenerate perturbation theory with $H' = H_{fs} + H_z$ for accurate description

5 Relativistic Effects of the Hydrogen Atom

The energy of a free particle is $E = \sqrt{(m_e c^2)^2 + (pc)^2}$. Therefore, $i\hbar\frac{\partial\psi}{\partial t} = \sqrt{m_e^2 c^4 - \hbar^2 c^2 \vec{\nabla}^2}\psi$

Alternatively, we should write $H^2 = m_e^2 c^4 + p^2 c^2$, then we write $-\hbar^2 \frac{\partial^2\psi}{\partial t^2} = (m_e^2 c^4 - \hbar^2 c^2 \vec{\nabla}^2)\psi$

The atom is placed in uniform applied fields $\vec{\mathcal{E}} = \mathcal{E}\hat{z}$ and $\vec{B} = B\hat{z}$. Then the Hamiltonian is

$H = H_0 - \vec{\mu}\cdot\vec{B} + \frac{e^2 B^2}{8m_e}(x^2 + y^2) + e\mathcal{E}z + H_{fs}$

$H_0 = \frac{\vec{p}^2}{2m_e} - \frac{e^2}{4\pi\epsilon_0 r}$

$\vec{\mu} = \vec{\mu}_l + \vec{\mu}_s = -\mu_B\left(\frac{\vec{l} + g_e\vec{s}}{\hbar}\right)$

$g_e \approx 2$

6 Fine Structure of the Hydrogen Atom

$H_{fs} = H_{kin} + H_{so} + H_D$

kinetic term: $H_{kin} = -\frac{\vec{p}^4}{8m_e^3 c^2}$

spin-orbit term: $H_{so} = \left(\frac{e^2}{4\pi\epsilon_0}\right)\left(\frac{1}{2m_e^2 c^2}\right)\frac{\vec{l}\cdot\vec{s}}{r^3}$

Darwin term: $H_D = \frac{\hbar^2}{8m_e^2 c^2} 4\pi \left(\frac{e^2}{4\pi\epsilon_0}\right)\delta^3(\vec{r})$

$\langle H_{kin}\rangle = E_n^{(0)}\frac{\alpha^2}{n^2}$

$\langle H_{so}\rangle = \left(\frac{e^2}{4\pi\epsilon_0}\right)\left(\frac{1}{2m_e^2 c^2}\right)\langle\vec{l}\cdot\vec{s}\rangle\langle\frac{1}{r^3}\rangle$

$\langle H_D\rangle = \frac{\alpha^2 m_e c^2}{2n^2}\frac{\alpha^2}{n}\delta_{l0} = -E_n^{(0)}\frac{\alpha^2}{n}\delta_{l0}$

$\langle n,l,j,m_j|H_{fs}|n,l,j,m_j\rangle = \left(-\frac{1}{2n^2}\alpha^2 m_e c^2\right)\left(\frac{\alpha^2}{n^2}\right)\left(\frac{n}{j+\frac{1}{2}} - \frac{3}{4}\right)$

$E_n^{(0)} = \left(-\frac{1}{2n^2}\alpha^2 m_e c^2\right)$

7 Abnormal Zeeman Effect

7.1 High-Field Limit

$|\langle H_z\rangle| >> |\langle H_{fs}\rangle|$

$H = H_0 + H_z + H_{fs}$

The first two terms are large while the last term is small

$E_{m_l m_s} \approx E_n^{(0)} + \mu_B(m_l + 2m_s) + \langle nlm_lm_s|H_{fs}|nlm_lm_s\rangle$

7.2 Low-Field Limit

$|\langle H_z\rangle| << |\langle H_{fs}\rangle|$

$H = H_0 + H_{fs} + H_z$

The first two terms are large while the last term is small

$E_{jm_j} \approx E_n^{(0)} - \frac{\alpha^2|E_n^{(0)}|}{n^4}\left[\frac{n}{j+\frac{1}{2}} - \frac{3}{4n^4}\right] + \frac{\mu_B B}{\hbar}\langle nljm_j|l_z + 2s_z|nljm_j\rangle$

$\langle H_z\rangle = g_j\mu_B B m_j$

Lande g-factor: $g_j = \left[1 + \frac{j(j+1)+s(s+1)-l(l+1)}{2j(j+1)}\right]$

7.3 Good Quantum Numbers

correspond to observables that approximately commute with H eigenvectors of H will be labelled by the good quantum numbers

8 Basic Properties of Nuclei

$m_p = 1.673 * 10^{-27} kg$

$m_n = 1.675 * 10^{-27} kg$

$d_{nucl} \approx 2.5 A^{\frac{1}{3}} fm$

$1fm = 10^{-15} m$

I is integer if #protrons+#electrons=even

I is half integer if #protrons+#electrons=odd

9 Hyperfine Structure

The nuclear spin eigenstate is given by $|I, M_i\rangle$, where $M_I = -I, \ldots, I$. The eigenvalue equations are

$\vec{I}^2 |I, M_I\rangle = \hbar^2 I(I+1)|I, M_I\rangle$

$I_z |I, M_I\rangle = \hbar M_I |I, M_I\rangle$

The nuclear magnetic dipole moment is $\vec{\mu_I} = g_I \mu_n \frac{\vec{I}}{\hbar}$

$\mu_n = \frac{e\hbar}{2m_p} = 5.051 * 10^{-27} J/T$ is the nuclear magneton

$H_{hf} = -\frac{\mu_0}{4\pi} \left\{ \frac{2\mu_B}{\hbar r^3} \vec{l} \cdot \vec{\mu_I} - \frac{1}{r^3} \left[\vec{\mu_e} \cdot \vec{\mu_I} - 3(\vec{\mu_I} \cdot \hat{r})(\vec{\mu_e} \cdot \hat{r}) \right] + \frac{8\pi}{3} \vec{\mu_e} \cdot \vec{\mu_I} \delta^3(\vec{r}) \right\}$

When the electron is in ground state, it is reduced to Fermi Hamiltonian

$H_{Fermi} = -\frac{8\pi}{3} \left(\frac{\mu_0}{4\pi} \right) \vec{\mu_e} \cdot \vec{\mu_I} \delta^3(\vec{r}) = \frac{8\pi}{3} \left(\frac{\mu_0}{4\pi} \right) g_e \mu_B g_I \mu_n \frac{\vec{I} \cdot \vec{s}}{\hbar^2} \delta^3(\vec{r}) = A \frac{\vec{I} \cdot \vec{s}}{\hbar^2}$

$A = \frac{8\pi}{3} \left(\frac{\mu_0}{4\pi} \right) g_e \mu_B g_I \mu_n |\psi(0)|^2$