

PHY 362K Homework 7

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Since the particle is in infinite square well, the energy is $E_n = \frac{n^2 \pi^2 \hbar^2}{2ma^2}$

$$\psi_n(x) = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi}{a}x\right)$$

Since the transition is from state $n = 1$ to state $n = 2$, we get $\omega_{21} = \frac{E_2 - E_1}{\hbar} = \frac{3\pi^2 \hbar}{2ma^2}$

$$H'_{21}(t) = \langle 2 | H' | 1 \rangle = \int_0^a \psi_2^*(x) A x^2 e^{-\gamma t} \psi_1(x) dx = -\frac{16a^2 A e^{-t\gamma}}{9\pi^2} \quad (\text{Using Mathematica})$$

$$\therefore c_2 = -\frac{i}{\hbar} \int_0^T e^{i\omega_{21}t} H'_{21}(t) dt = \frac{32ia^4 A \left(1 - e^{-T\gamma + \frac{3i\pi^2 T \hbar}{2a^2 m}}\right) m}{9\pi^2 \hbar (2a^2 m \gamma - 3i\pi^2 \hbar)}$$

$$\therefore P_{1 \rightarrow 2} = |c_2|^2 = \frac{1024a^8 A^2 m^2}{81\pi^4 \hbar^2 (4a^4 m^2 \gamma^2 + 9\pi^4 \hbar^2)} \left(1 - e^{-T\gamma + \frac{3i\pi^2 T \hbar}{2a^2 m}}\right) \left(1 - e^{-T\gamma - \frac{3i\pi^2 T \hbar}{2a^2 m}}\right)$$

$$\text{When } T \rightarrow \infty, \left(1 - e^{-T\gamma + \frac{3i\pi^2 T \hbar}{2a^2 m}}\right) \rightarrow 0 \text{ and } \left(1 - e^{-T\gamma - \frac{3i\pi^2 T \hbar}{2a^2 m}}\right) \rightarrow 0$$

$$\text{Therefore in this case } P_{1 \rightarrow 2} = \frac{1024a^8 A^2 m^2}{81\pi^4 \hbar^2 (4a^4 m^2 \gamma^2 + 9\pi^4 \hbar^2)}$$

2

(a)

(b)

(c)

3

(a)

(b)

4

5

(a)

(b)