Backpropagation: A modular approach (Torch NN)

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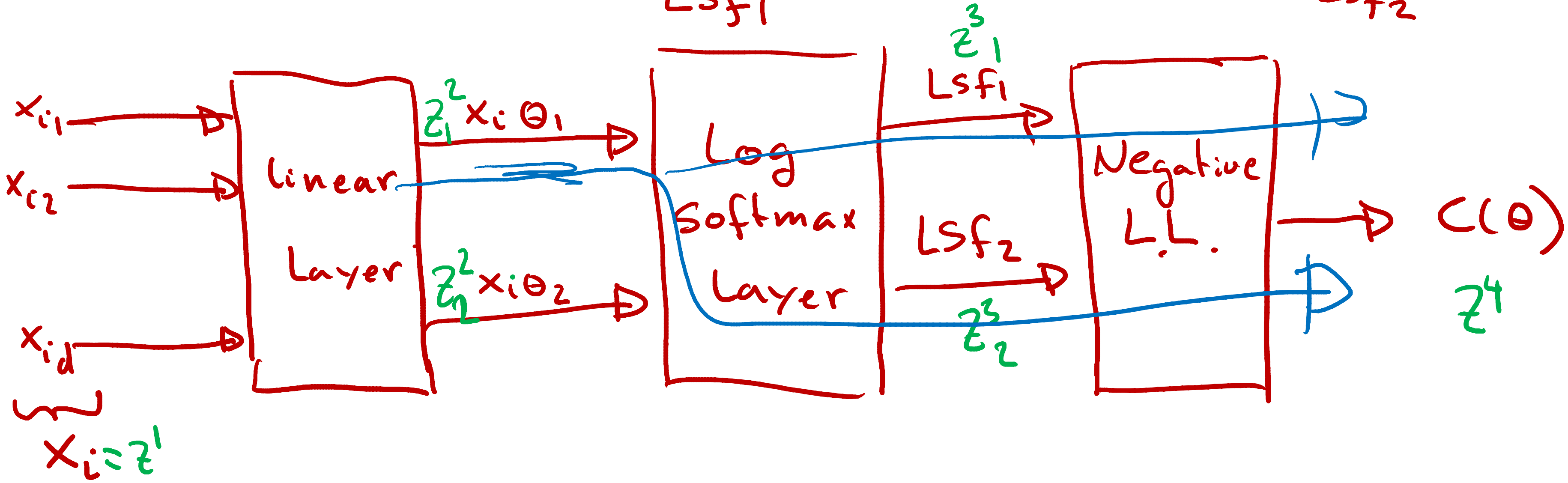
Outline of the lecture

This lecture describes modular ways of formulating and learning distributed representations of data. The objective is for you to learn:

- ❑ How to specify models such as logistic regression in layers.
- ❑ How to formulate layers and loss criterions.
- ❑ How well formulated local rules results in correct global rules.
- ❑ How back-propagation works.
- ❑ How this manifests itself in Torch.

$$C(\theta) = -\sum_{i=1}^{n} \mathbb{I}_0(y_i) \underbrace{\log\left(\frac{e^{x_i\theta_1}}{e^{x_i\theta_1}+e^{x_i\theta_2}}\right)}_{Lsf_1} + \mathbb{I}_1(y_i) \underbrace{\log\left(\frac{e^{x_i\theta_2}}{e^{x_i\theta_1}+e^{x_i\theta_2}}\right)}_{Lsf_2}$$



$$z_1^2 = x_i\theta_1 \qquad z_2^2 = x_i\theta_2$$

$$z_1^3 = \log\left(\frac{e^{z_1^2}}{e^{z_1^2}+e^{z_2^2}}\right) \qquad z_2^3 = \log\left(\frac{e^{z_2^2}}{e^{z_1^2}+e^{z_2^2}}\right)$$

$$z^4 = \sum_i \mathbb{I}_0(y_i)\, z_1^3 + \mathbb{I}_1(y_i)\, z_2^3$$

Derivative using the chain rule

$$C(\theta) = -\sum_{i=1}^{n} \mathbb{I}_0(y_i)\log\left(\frac{e^{x_i\theta_1}}{e^{x_i\theta_1}+e^{x_i\theta_2}}\right) + \mathbb{I}_1(y_i)\log\left(\frac{e^{x_i\theta_2}}{e^{x_i\theta_1}+e^{x_i\theta_2}}\right)$$

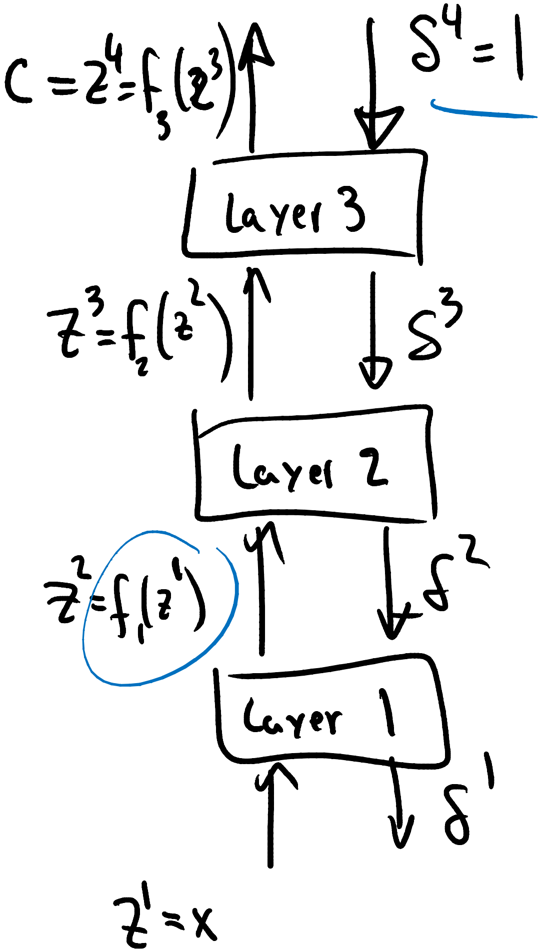
z_{12}

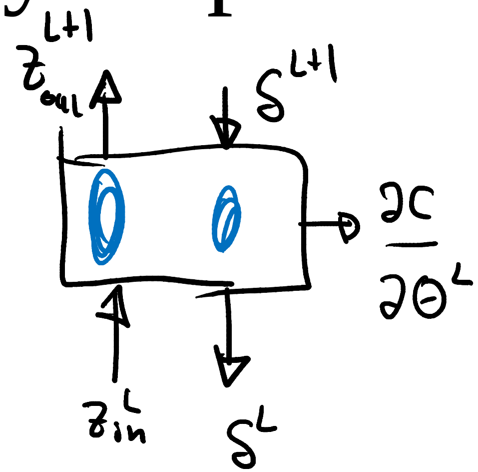
$$C(\theta) = z^4\left\{ z^3_1\left[\boxed{z^2_1(\theta_1, z^1)}, \boxed{z^2_2(\theta_2, z^1)}\right], z^3_2\left[\boxed{z^2_1(\theta_1, z^1)}, \boxed{z^2_2(\theta_2, z^1)}\right]\right\}$$

$$\frac{\partial C(\theta)}{\partial \theta_1} = \frac{\partial z^4}{z^3_1}\frac{\partial z^3_1}{\partial z^2_1}\frac{\partial z^2_1}{\partial \theta_1} + \frac{\partial z^4}{\partial z^3_1}\frac{\partial z^3_1}{\partial z^2_2}\frac{\partial z^2_2}{\partial \theta_1}$$

$$+ \frac{\partial z^4}{\partial z^3_2}\frac{\partial z^3_2}{\partial z^2_1}\frac{\partial z^2_1}{\partial \theta_1} + \frac{\partial z^4}{\partial z^3_2}\frac{\partial z^3_2}{\partial z^2_2}\frac{\partial z^2_2}{\partial \theta_1}$$

Layer specification





$$z^{L+1} = f(z^L)$$

forward pass

backward pass

$$\delta_i^L = \frac{\partial C}{\partial z_i^L} = \sum_j \frac{\partial C}{\partial z_j^{L+1}} \frac{\partial z_j^{L+1}}{\partial z_i^L} = \sum_j \delta_j^{L+1} \boxed{\frac{\partial z_j^{L+1}}{\partial z_i^L}}$$

$$\frac{\partial C}{\partial \theta^L} = \sum_j \frac{\partial C}{\partial z_j^{L+1}} \frac{\partial z_j^{L+1}}{\partial \theta^L} = \sum_j \delta_j^{L+1} \left(\frac{\partial z_j^{L+1}}{\partial \theta^L} \right)$$

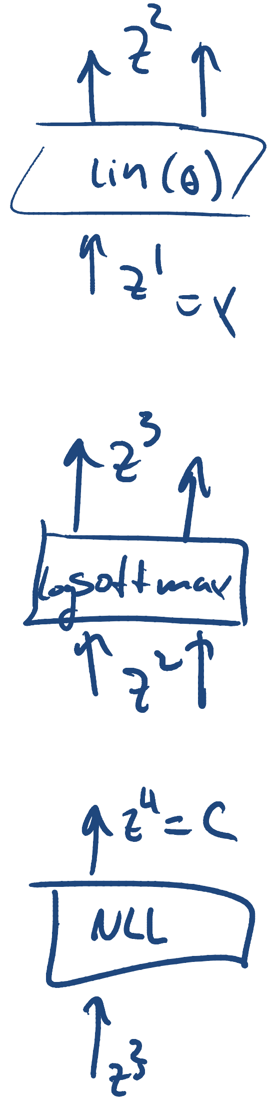
Derivative via layer-specification

$$\frac{\partial c}{\partial \theta_1} = \sum_j \underbrace{\frac{\partial c}{z_j^2}} \frac{\partial z_j^2}{\partial \theta_1}$$

$$= \sum_j \left(\sum_k \frac{\partial c}{z_k^3} \frac{\partial z_k^3}{\partial z_j^2} \right) \frac{\partial z_j^2}{\partial \theta_1}$$

$$= \sum_{j=1}^{2} \sum_{k=1}^{2} \overset{1}{\cancel{\frac{\partial c}{z^4}}} \left(\frac{\partial z^4}{\partial z_k^3} \frac{\partial z_k^3}{\partial z_j^2} \frac{\partial z_j^2}{\partial \theta_1} \right)$$

= as before.



Back-propagation algorithm

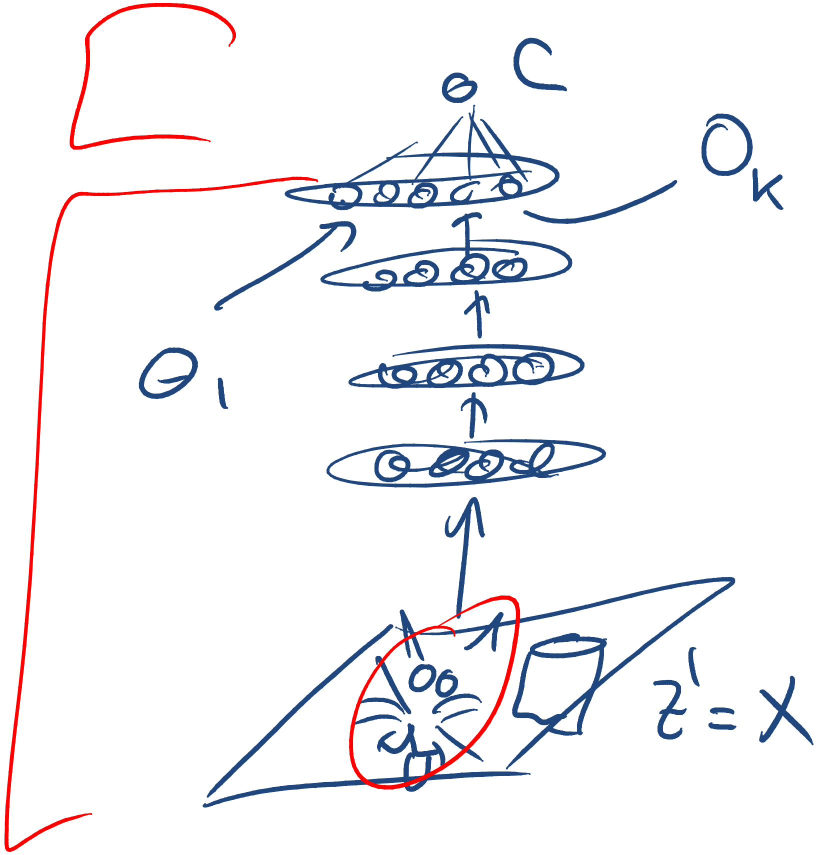
$$\log\left(\frac{e^{x_i\theta_1}}{\Sigma}\right)$$

$$z^1 = x_i \longrightarrow \underbrace{z^2(x_i)}_{x_i\theta_1} \longrightarrow z^3(x_i) \longrightarrow z^4(x_i) = C$$

$$\delta^1 \longleftarrow \delta^2 \longleftarrow \delta^3 \longleftarrow \delta^4 \leftarrow 1$$

Derivatives wrt to the input

imagenet



$$\frac{\partial C}{\partial x}$$

output

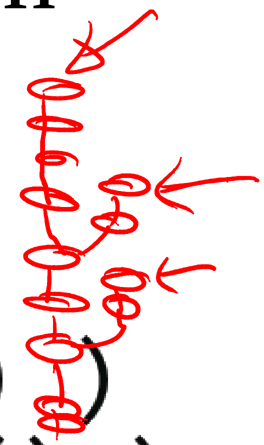
$$\left| \frac{\partial O}{\partial x_i} \right|$$

Karen Simonyan

Misha Denil

Logit Regression Model in Torch

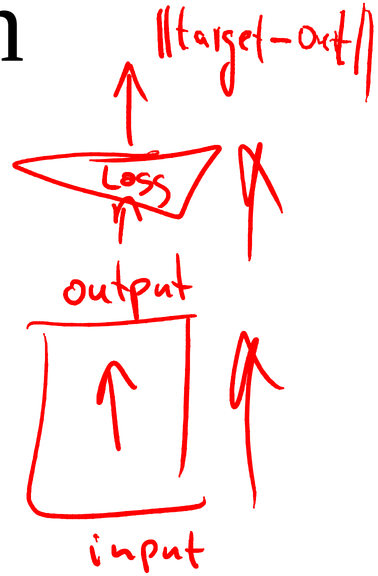
```
model = nn.Sequential()
model:add( nn.Linear(2,2) )
model:add( nn.LogSoftMax() )
```



Loss criterion in Torch

```
criterion = nn.ClassNLLCriterion()
```

Derivatives closure in Torch




```
-- params/gradients
x, dl_dx = model:getParameters()

local loss_x = criterion:forward(model:forward(inputs), target)
model:backward(inputs, criterion:backward(model.output, target))
```

Optimization in Torch

```
_,fs = optim.sgd(feval,x,sgd_params)
```

```
-- Functions in optim all return two things:
--   + the new x, found by the optimization method (here SGD)
--   + the value of the loss functions at all points that were used by
--     the algorithm. SGD only estimates the function once, so
--     that list just contains one value.
```

Next lecture

In the next lecture, we consider a generalization of logistic regression, with many logistic units, called multi-layer perceptron (MLP) or feed-forward neural network.