CS224d
Deep NLP

Lecture 5:
Project Information
+
Neural Networks & Backprop

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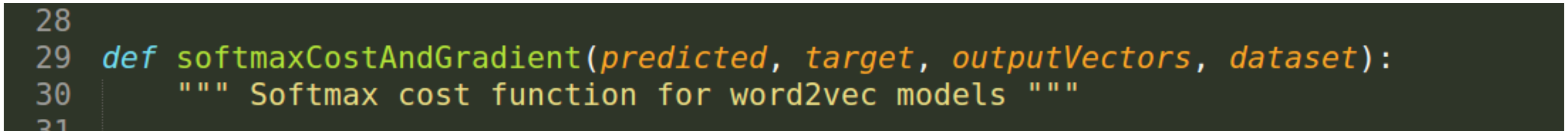
Overview Today:

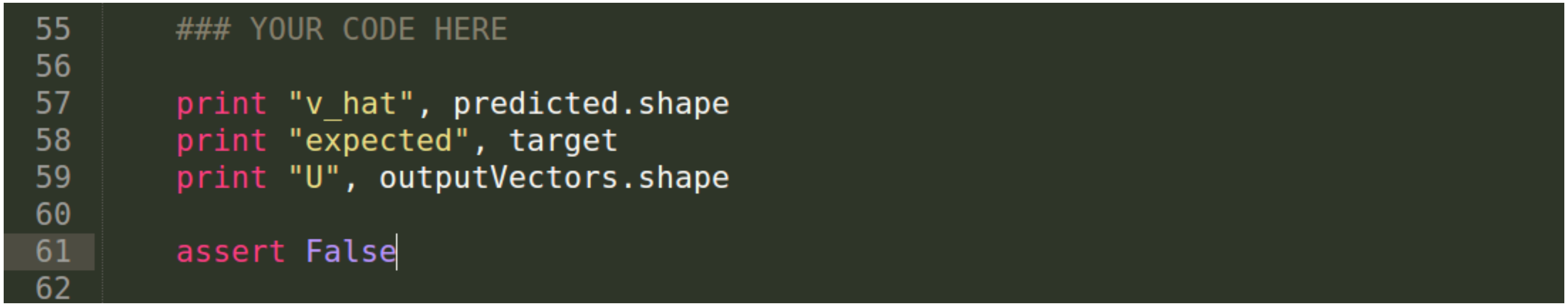
- Organizational Stuff
- Project Tips
- From one-layer to multi layer neural network!
- Max-Margin loss and **backprop!**
  (This is the hardest lecture of the quarter)

Announcement:

- 1% extra credit for Piazza participation!
- Hint for PSet1: Understand math and dimensionality, then add print statements, e.g.



```
def softmaxCostAndGradient(predicted, target, outputVectors, dataset):
    """ Softmax cost function for word2vec models """
```



```
    ### YOUR CODE HERE

    print "v_hat", predicted.shape
    print "expected", target
    print "U", outputVectors.shape

    assert False
```

- Student survey sent out last night, please give us feedback to improve the class :)

Class Project

- Most important (40%) and lasting result of the class
- PSet 3 a little easier to have more time
- Start early and clearly define your task and dataset

- Project types:
  1. Apply existing neural network model to a new task
  2. Implement a complex neural architecture
  3. Come up with a new neural network model
  4. Theory

Class Project: Apply Existing NNets to Tasks

1. Define Task:
   - Example: **Summarization**

2. Define Dataset
   1. Search for academic datasets
      - They already have baselines
      - E.g.: Document Understanding Conference (DUC)

   2. Define your own (harder, need more new baselines)
      - If you're a graduate student: connect to your research
      - Summarization, Wikipedia: Intro paragraph and rest of large article
      - Be creative: Twitter, Blogs, News

Class Project: Apply Existing NNets to Tasks

3. Define your metric
   - Search online for well established metrics on this task
   - Summarization: Rouge (Recall-Oriented Understudy for Gisting Evaluation) which defines n-gram overlap to human summaries

4. Split your dataset!
   - Train/Dev/Test
   - Academic dataset often come pre-split
   - Don’t look at the test split until ~1 week before deadline!

Class Project: Apply Existing NNets to Tasks

5. Establish a baseline
    - Implement the simplest model (often logistic regression on unigrams and bigrams) first
    - Compute metrics on train AND dev
    - Analyze errors
    - If metrics are amazing and no errors:
      done, problem was too easy, restart :)

6. Implement existing neural net model
    - Compute metric on train and dev
    - Analyze output and errors
    - Minimum bar for this class

Class Project: Apply Existing NNets to Tasks

7. Always be close to your data!
   - Visualize the dataset
   - Collect summary statistics
   - Look at errors
   - Analyze how different hyperparameters affect performance

8. Try out different model variants
   - Soon you will have more options
     - Word vector averaging model (neural bag of words)
     - Fixed window neural model
     - Recurrent neural network
     - Recursive neural network
     - Convolutional neural network

Class Project: A New Model -- Advanced Option

- Do all other steps first (Start early!)
- Gain intuition of why existing models are flawed

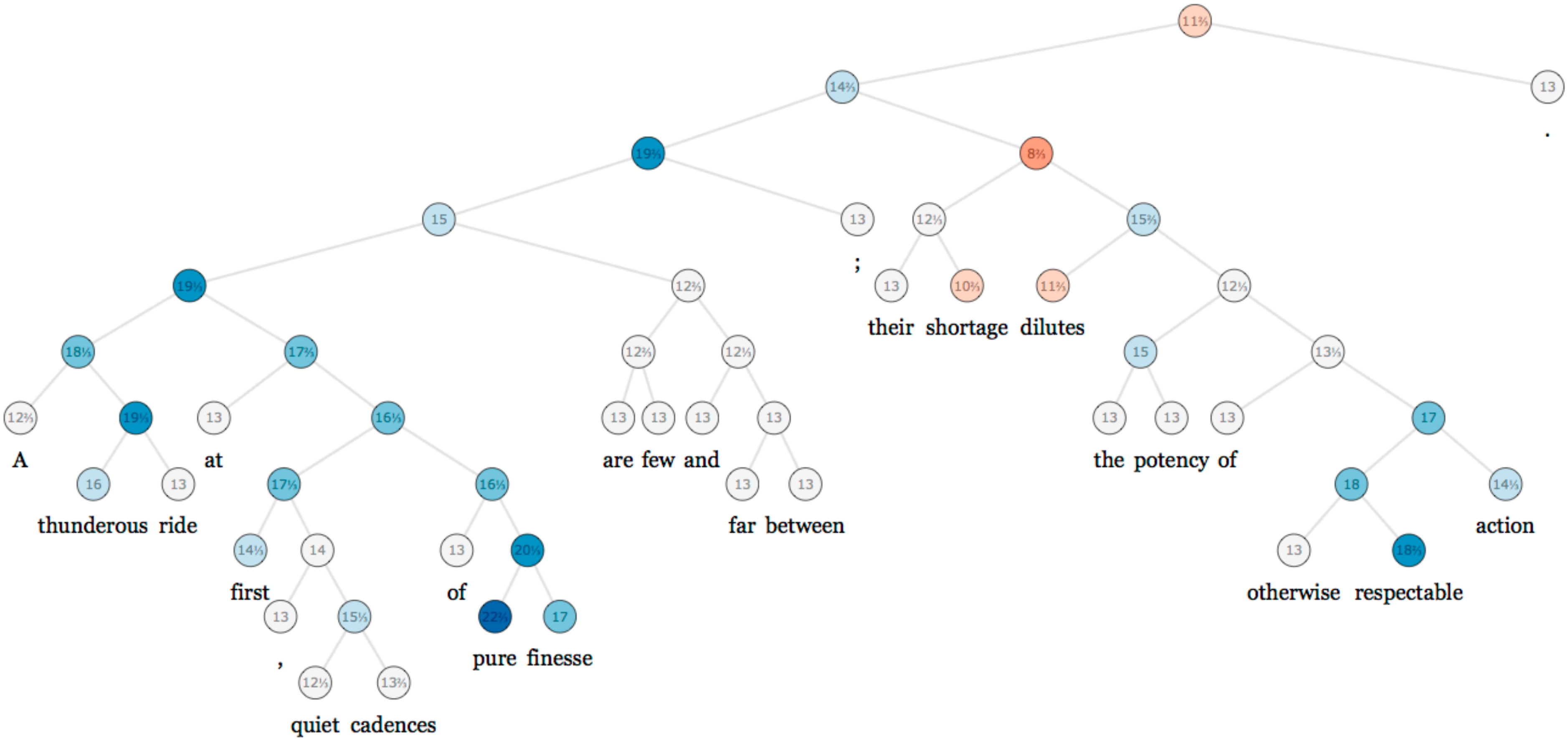
- Talk to other researchers, come to my office hours a lot
- Implement new models and iterate quickly over ideas
- Set up efficient experimental framework
- Build simpler new models first
- Example Summarization:
  - Average word vectors per paragraph, then greedy search
  - Implement language model or autoencoder (introduced later)
  - Stretch goal for potential paper: Generate summary!

Project Ideas

- Summarization
- NER, like PSet 2 but with larger data
  Natural Language Processing (almost) from Scratch, Ronan Collobert, Jason Weston, Leon Bottou, Michael Karlen, Koray Kavukcuoglu, Pavel Kuksa, http://arxiv.org/abs/1103.0398
- Simple question answering, A Neural Network for Factoid Question Answering over Paragraphs, Mohit Iyyer, Jordan Boyd-Graber, Leonardo Claudino, Richard Socher and Hal Daumé III (**EMNLP 2014**)
- Image to text mapping or generation,
  Grounded Compositional Semantics for Finding and Describing Images with Sentences, Richard Socher, Andrej Karpathy, Quoc V. Le, Christopher D. Manning, Andrew Y. Ng. (**TACL 2014**)
  or
  Deep Visual-Semantic Alignments for Generating Image Descriptions, Andrej Karpathy, Li Fei-Fei
- Entity level sentiment
- Use DL to solve an NLP challenge on kaggle,
  Develop a scoring algorithm for student-written short-answer responses, https://www.kaggle.com/c/asap-sas

Default project: sentiment classification

- Sentiment on movie reviews: http://nlp.stanford.edu/sentiment/
- Lots of deep learning baselines and methods have been tried



A more powerful window classifier

- Revisiting
- $X_{window} = [\ x_{museums} \quad x_{in} \quad x_{Paris} \quad x_{are} \quad x_{amazing}\ ]$
- Assume we want to classify whether the center word is a location or not

A Single Layer Neural Network

- A single layer is a combination of a linear layer and a nonlinearity:

$$\begin{aligned} z &= Wx + b \\ a &= f(z) \end{aligned}$$

- The neural activations *a* can then be used to compute some function
- For instance, an unnormalized score or a softmax probability we care about:

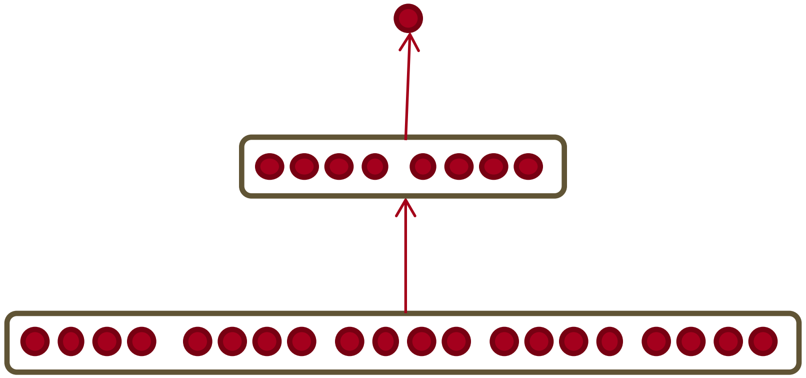
$$score(x) = U^T a \in \mathbb{R}$$

Summary: Feed-forward Computation

Computing a window's score with a 3-layer neural net: *s = score*(museums in Paris are amazing)

$$s = U^T f(Wx+b) \qquad x \in \mathbb{R}^{20\times 1}, W \in \mathbb{R}^{8\times 20}, U \in \mathbb{R}^{8\times 1}$$

$$\begin{aligned} s &= U^T a \\ a &= f(z) \\ z &= Wx+b \end{aligned}$$

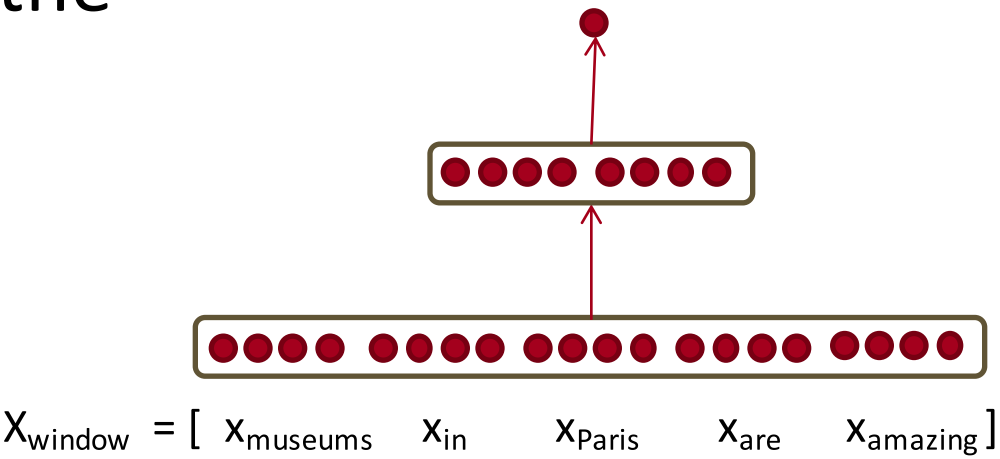


x_{window} = [x_{museums} x_{in} x_{Paris} x_{are} x_{amazing}]

Main intuition for extra layer

The layer learns non-linear interactions between the input word vectors.

Example:
only if “*museums*” is first vector should it matter that “*in*” is in the second position



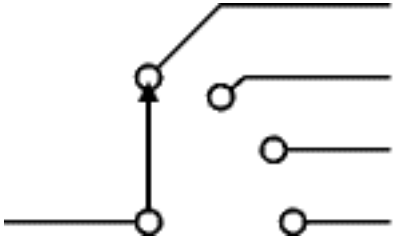
$x_{window} = [\ x_{museums} \quad x_{in} \quad x_{Paris} \quad x_{are} \quad x_{amazing}\ ]$

Summary: Feed-forward Computation

- s = score(museums in Paris are amazing)
- s_c = score(Not all museums in Paris)

- Idea for training objective: make score of true window larger and corrupt window's score lower (until they're good enough): minimize

$$\boxed{J = \max(0, 1 - s + s_c)}$$



- This is continuous, can perform SGD

Max-margin Objective function

- Objective for a single window:

$$\boxed{J = \max(0, 1 - s + s_c)}$$

- Each window with a location at its center should have a score +1 higher than any window without a location at its center
- xxx |← 1 →| ooo

- For full objective function: Sum over all training windows

Training with Backpropagation

$$\boxed{J = \max(0, 1 - s + s_c)}$$

$$s = U^T f(Wx + b)$$
$$s_c = U^T f(Wx_c + b)$$

Assuming cost *J* is > 0,

compute the derivatives of *s* and s_c wrt all the involved variables: *U, W, b, x*

$$\frac{\partial s}{\partial U} = \frac{\partial}{\partial U} U^T a \qquad \frac{\partial s}{\partial U} = a$$

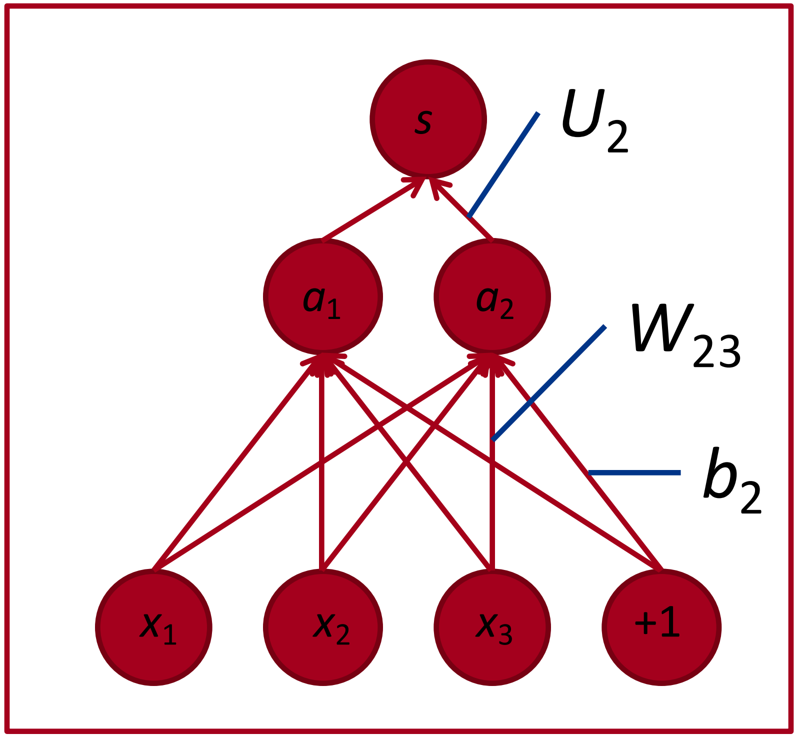
Training with Backpropagation

- Let’s consider the derivative of a single weight W_{ij}

$$\frac{\partial s}{\partial W} = \frac{\partial}{\partial W} U^T a = \frac{\partial}{\partial W} U^T f(z) = \frac{\partial}{\partial W} U^T f(Wx + b)$$

- This only appears inside a_i

- For example: W_{23} is only used to compute a_2



Training with Backpropagation

$$\frac{\partial s}{\partial W} = \frac{\partial}{\partial W} U^T a = \frac{\partial}{\partial W} U^T f(z) = \frac{\partial}{\partial W} U^T f(Wx + b)$$

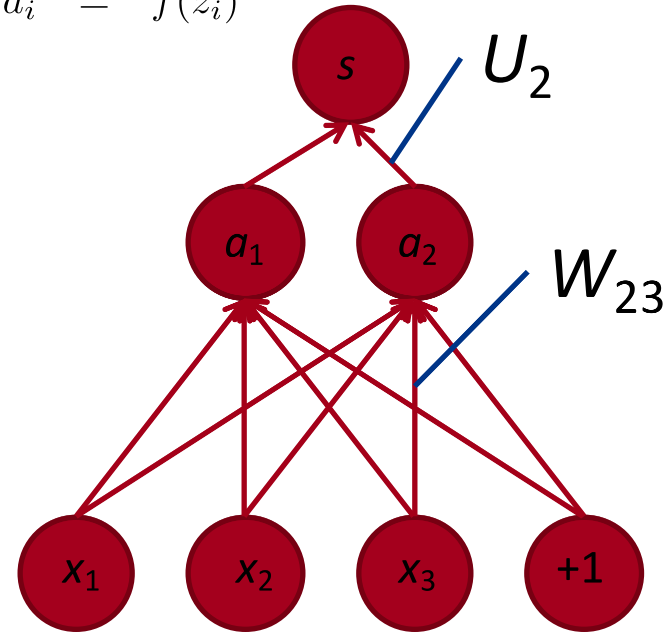
Derivative of weight W_{ij}:

$$\frac{\partial y}{\partial x} = \frac{\partial y}{\partial u} \frac{\partial u}{\partial x}$$

$$\frac{\partial}{\partial W_{ij}} U^T a \quad \rightarrow \quad \frac{\partial}{\partial W_{ij}} U_i a_i$$

$$\begin{aligned} U_i \frac{\partial}{\partial W_{ij}} a_i &= U_i \frac{\partial a_i}{\partial z_i} \frac{\partial z_i}{\partial W_{ij}} \\ &= U_i \frac{\partial f(z_i)}{\partial z_i} \frac{\partial z_i}{\partial W_{ij}} \\ &= U_i f'(z_i) \frac{\partial z_i}{\partial W_{ij}} \\ &= U_i f'(z_i) \frac{\partial W_{i\cdot} x + b_i}{\partial W_{ij}} \end{aligned}$$

$$\begin{aligned} z_i &= W_{i\cdot} x + b_i &= \sum_{j=1}^{3} W_{ij} x_j + b_i \\ a_i &= f(z_i) \end{aligned}$$



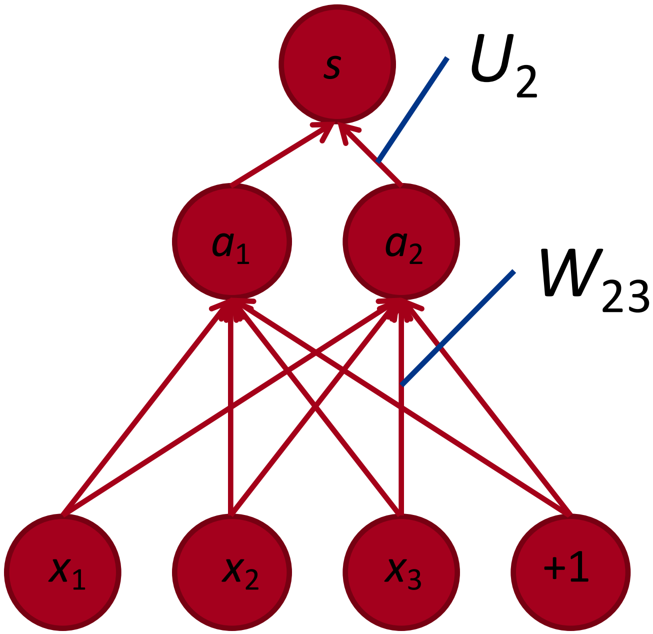
Training with Backpropagation

Derivative of single weight W_{ij} :

$$\begin{aligned} z_i &= W_{i\cdot}x + b_i &= \sum_{j=1}^{3} W_{ij}x_j + b_i \\ a_i &= f(z_i) \end{aligned}$$

$$\begin{aligned} U_i \frac{\partial}{\partial W_{ij}} a_i &= U_i f'(z_i) \frac{\partial W_{i\cdot}x + b_i}{\partial W_{ij}} \\ &= U_i f'(z_i) \frac{\partial}{\partial W_{ij}} \sum_k W_{ik}x_k \\ &= \underbrace{U_i f'(z_i)} x_j \\ &= \underbrace{\delta_i}_{\text{Local error signal}} \quad \underbrace{x_j}_{\text{Local input signal}} \end{aligned}$$

where $f'(z) = f(z)(1 - f(z))$ for logistic f



Training with Backpropagation

- From single weight W_{ij} to full W:

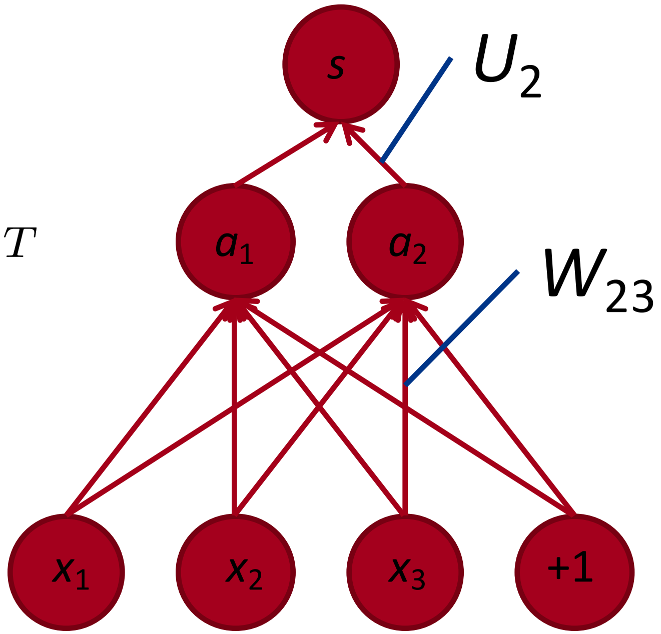
$$\frac{\partial \text{ s}}{\partial W_{ij}} = \underbrace{U_i f'(z_i)}_{} x_j$$

$$= \delta_i \quad x_j$$

$$z_i = W_{i\cdot}x + b_i = \sum_{j=1}^{3} W_{ij}x_j + b_i$$

$$a_i = f(z_i)$$

- We want all combinations of *i* = 1, 2 and *j* = 1, 2, 3 → ?
- Solution: Outer product: $\frac{\partial J}{\partial W} = \delta x^T$ where $\delta \in \mathbb{R}^{2\times 1}$ is the "responsibility" or error message coming from each activation *a*

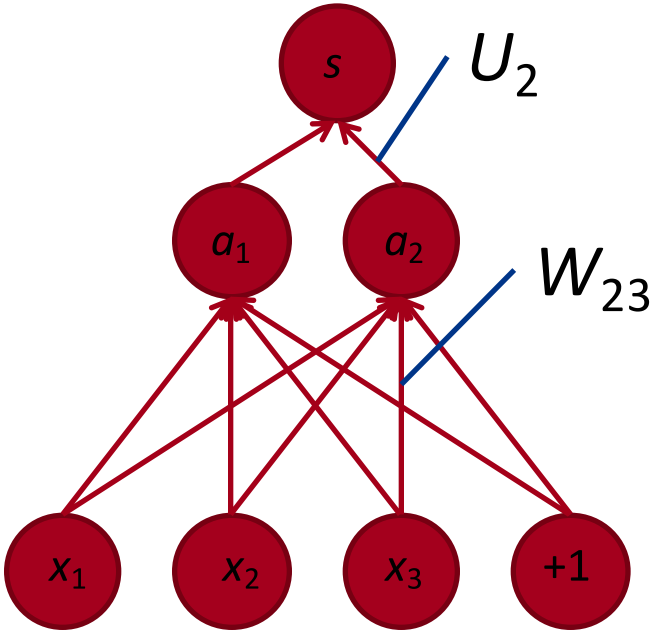


Training with Backpropagation

- For biases *b*, we get:

$$\begin{aligned} z_i &= W_{i\cdot}x + b_i &= \sum_{j=1}^{3} W_{ij}x_j + b_i \\ a_i &= f(z_i) \end{aligned}$$

$$\begin{aligned} & U_i \frac{\partial}{\partial b_i} a_i \\ = \quad & U_i f'(z_i) \frac{\partial W_{i\cdot}x + b_i}{\partial b_i} \\ = \quad & \delta_i \end{aligned}$$



Training with Backpropagation

That's almost backpropagation

It's simply taking derivatives and using the chain rule!

Remaining trick: we can **re-use** derivatives computed for higher layers in computing derivatives for lower layers!

Example: last derivatives of model, the word vectors in *x*

Training with Backpropagation

- Take derivative of score with respect to single element of word vector
- Now, we cannot just take into consideration one a_i because each x_j is connected to all the neurons above and hence x_j influences the overall score through all of these, hence:

$$
\begin{aligned}
\frac{\partial s}{\partial x_j} &= \sum_{i=1}^{2} \frac{\partial s}{\partial a_i} \frac{\partial a_i}{\partial x_j} \\
&= \sum_{i=1}^{2} \frac{\partial U^T a}{\partial a_i} \frac{\partial a_i}{\partial x_j} \\
&= \sum_{i=1}^{2} U_i \frac{\partial f(W_{i\cdot}x + b)}{\partial x_j} \\
&= \sum_{i=1}^{2} \underbrace{U_i f'(W_{i\cdot}x + b)} \frac{\partial W_{i\cdot}x}{\partial x_j} \\
&= \sum_{i=1}^{2} \delta_i W_{ij} \\
&= W_{\cdot j}^T \underbrace{\delta}
\end{aligned}
$$

Re-used part of previous derivative

Training with Backpropagation

- With $\frac{\partial s}{\partial x_j} = W_{\cdot j}^T \delta$,what is the full gradient? →

$$\frac{\partial s}{\partial x} = W^T \delta$$

- Observations: The error message ± that arrives at a hidden layer has the same dimensionality as that hidden layer

Putting all gradients together:

- Remember: Full objective function for each window was:

$$\boxed{J = \max(0, 1 - s + s_c)}$$

$$s = U^T f(Wx + b)$$
$$s_c = U^T f(Wx_c + b)$$

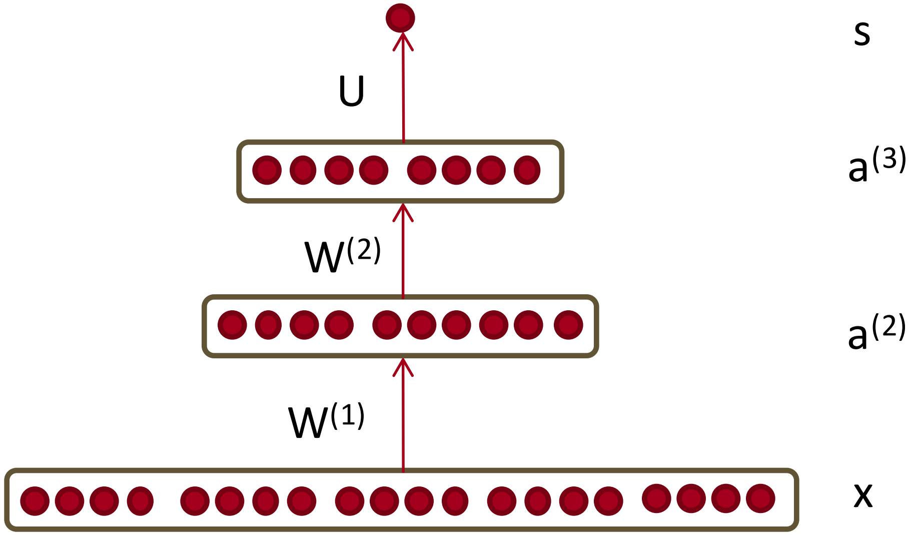
- For example: gradient for U:

$$\frac{\partial s}{\partial U} = 1\{1 - s + s_c > 0\}\left(-f(Wx + b) + f(Wx_c + b)\right)$$
$$\frac{\partial s}{\partial U} = 1\{1 - s + s_c > 0\}\left(-a + a_c\right)$$

Two layer neural nets and full backprop

- Let's look at a 2 layer neural network
- Same window definition for x
- Same scoring function
- 2 hidden layers (carefully not superscripts now!)

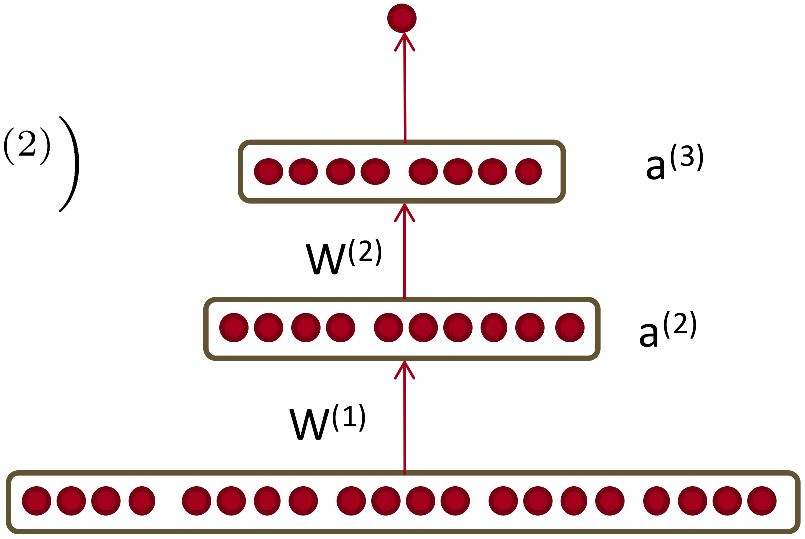
$$\begin{aligned} x &= z^{(1)} = a^{(1)} \\ z^{(2)} &= W^{(1)}x + b^{(1)} \\ a^{(2)} &= f\left(z^{(2)}\right) \\ z^{(3)} &= W^{(2)}a^{(2)} + b^{(2)} \\ a^{(3)} &= f\left(z^{(3)}\right) \\ s &= U^T a^{(3)} \end{aligned}$$



Two layer neural nets and full backprop

- Fully written out as one function:

$$\begin{aligned} s &= U^T f\left(W^{(2)} f\left(W^{(1)} x + b^{(1)}\right) + b^{(2)}\right) \\ &= U^T f\left(W^{(2)} a^{(2)} + b^{(2)}\right) \\ &= U^T a^{(3)} \end{aligned}$$



- Same derivation as before for W$^{(2)}$ (now sitting on a$^{(1)}$)

$$\begin{aligned} \frac{\partial \mathrm{s}}{\partial W_{ij}} &= \underbrace{U_i f'(z_i)}_{} x_j \\ &= \delta_i \quad x_j \end{aligned} \qquad \begin{aligned} \frac{\partial s}{\partial W_{ij}^{(2)}} &= \underbrace{U_i f'\left(z_i^{(3)}\right)}_{} a_j^{(1)} \\ &= \delta_i^{(3)} \quad a_j^{(2)} \end{aligned}$$

Two layer neural nets and full backprop

$$\begin{aligned} x &= z^{(1)} = a^{(1)} \\ z^{(2)} &= W^{(1)}x + b^{(1)} \\ a^{(2)} &= f\left(z^{(2)}\right) \\ z^{(3)} &= W^{(2)}a^{(2)} + b^{(2)} \\ a^{(3)} &= f\left(z^{(3)}\right) \\ s &= U^T a^{(3)} \end{aligned}$$

- Same derivation as before for top W(2) :

$$\frac{\partial s}{\partial W_{ij}^{(2)}} = \underbrace{U_i f'\left(z_i^{(3)}\right)} a_j^{(1)}$$

$$= \quad \delta_i^{(3)} \quad a_j^{(2)}$$

- In matrix notation: $\frac{\partial s}{\partial W^{(2)}} = \delta^{(3)} {a^{(2)}}^T$

where $\delta^{(3)} = U \circ f'\left(z^{(3)}\right)$ and ± is the element-wise product also called Hadamard product

- Last missing piece for understanding general backprop: $\frac{\partial s}{\partial W^{(1)}}$

Two layer neural nets and full backprop

- Last missing piece: $\frac{\partial s}{\partial W^{(1)}}$

- What's the bottom layer's error message $\pm^{(2)}$?

- Similar derivation to single layer model

- Main difference, we already have ${W^{(2)}}^T\delta^{(3)}$ and need to apply the chain rule again on $f'(z^{(2)})$

$$\begin{aligned}
x &= z^{(1)} = a^{(1)} \\
z^{(2)} &= W^{(1)}x + b^{(1)} \\
a^{(2)} &= f\left(z^{(2)}\right) \\
z^{(3)} &= W^{(2)}a^{(2)} + b^{(2)} \\
a^{(3)} &= f\left(z^{(3)}\right) \\
s &= U^T a^{(3)}
\end{aligned}$$

Two layer neural nets and full backprop

- Chain rule for: $s = U^T f\left(W^{(2)} f\left(W^{(1)} x + b^{(1)}\right) + b^{(2)}\right)$

- Get intuition by deriving $\frac{\partial s}{\partial W^{(1)}}$ as if it was a scalar

- Intuitively, we have to sum over all the nodes coming into layer

- Putting it all together: $\delta^{(2)} = \left({W^{(2)}}^T \delta^{(3)}\right) \circ f'\left(z^{(2)}\right)$

Two layer neural nets and full backprop

- Last missing piece: $\frac{\partial s}{\partial W^{(1)}} = \delta^{(2)} x^T$

- In general for any matrix $W^{(l)}$ at internal layer *l* and any error with regularization E_R all backprop in standard multilayer neural networks boils down to 2 equations:

$$
\begin{aligned}
x &= z^{(1)} = a^{(1)} \\
z^{(2)} &= W^{(1)} x + b^{(1)} \\
a^{(2)} &= f\left(z^{(2)}\right) \\
z^{(3)} &= W^{(2)} a^{(2)} + b^{(2)} \\
a^{(3)} &= f\left(z^{(3)}\right) \\
s &= U^T a^{(3)}
\end{aligned}
$$

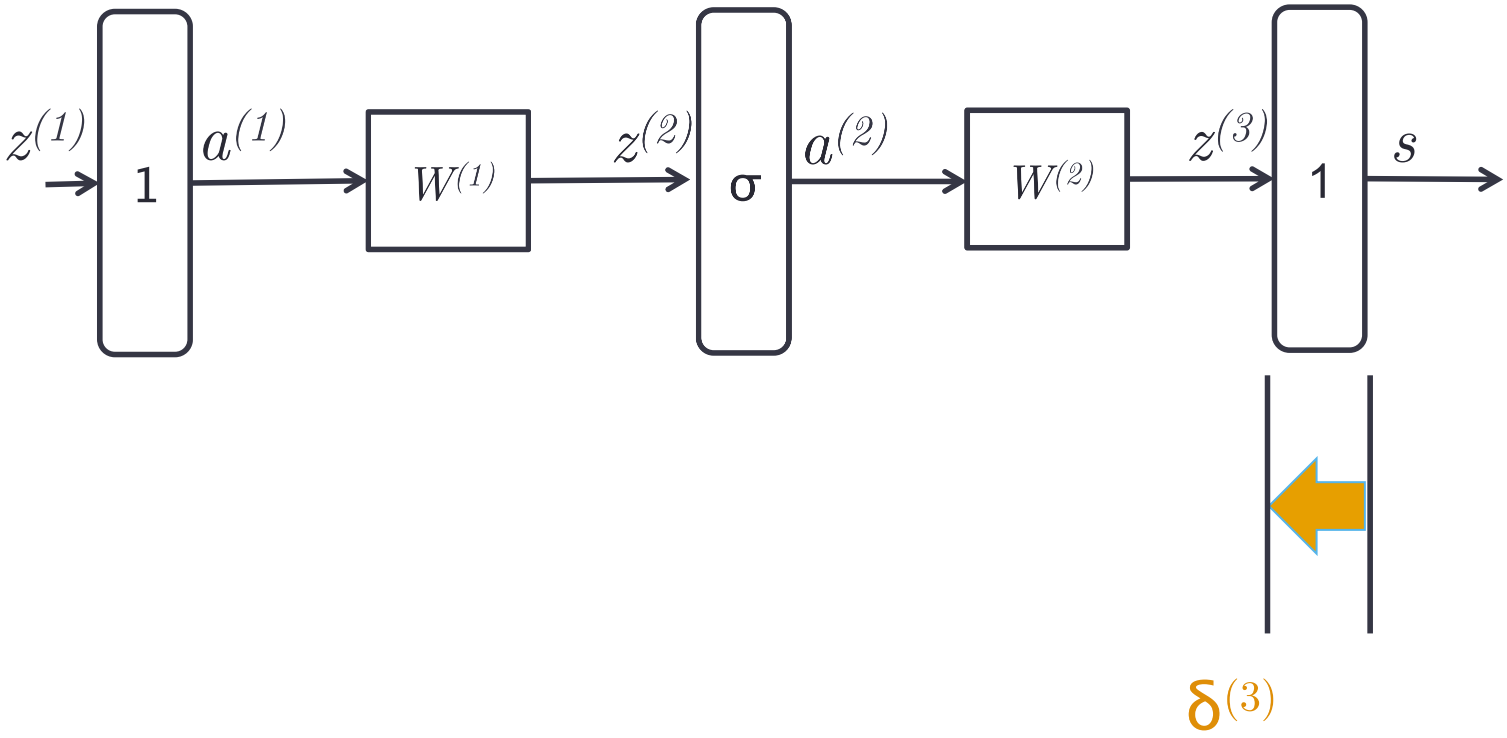
$$\boxed{\delta^{(l)} = \left((W^{(l)})^T \delta^{(l+1)}\right) \circ f'(z^{(l)}),}$$

$$\boxed{\frac{\partial}{\partial W^{(l)}} E_R = \delta^{(l+1)} (a^{(l)})^T + \lambda W^{(l)}}$$

- Top and bottom layers have simpler ±

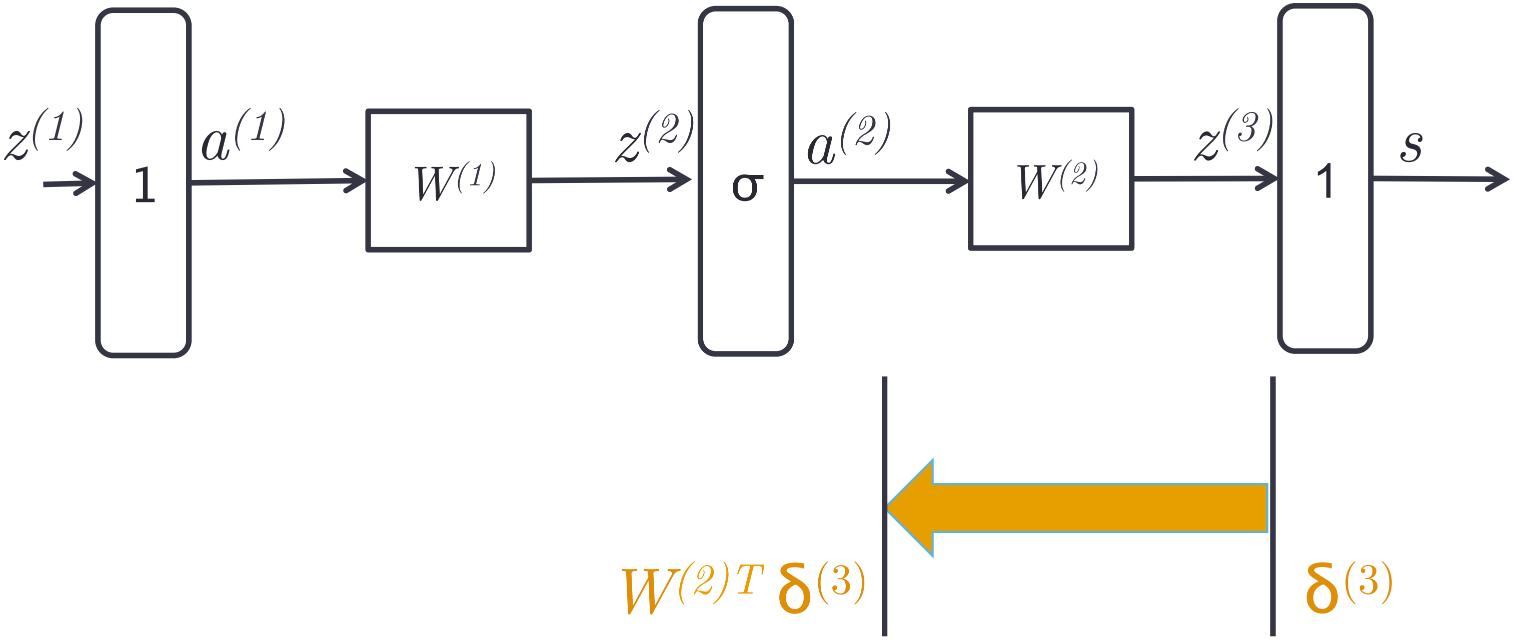
Visualization of intuition

- Let’s say we want $\frac{\partial s}{\partial W^{(1)}} = \delta^{(2)} {a^{(1)}}^T$ with previous layer and f = ¾



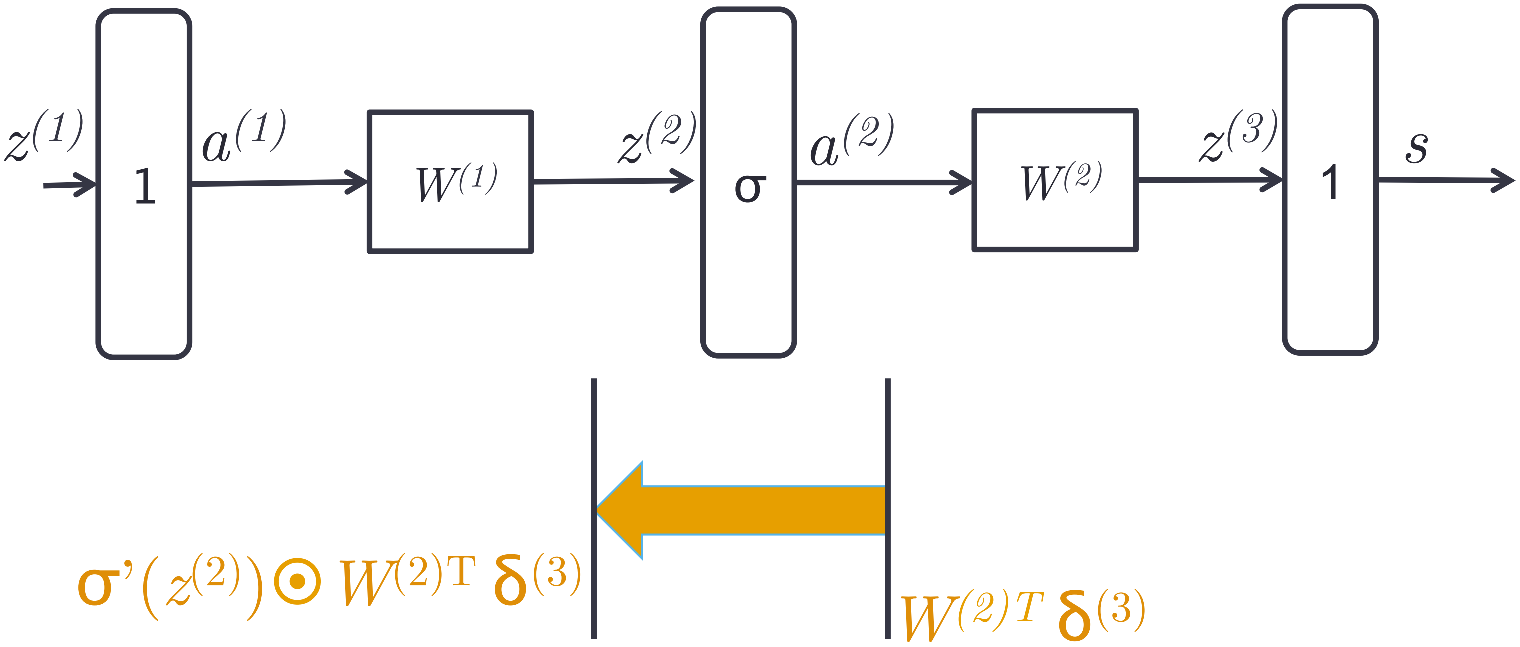
Gradient w.r.t $W^{(2)} = \delta^{(3)} a^{(2)\mathrm{T}}$

Visualization of intuition



--Reusing the $\delta^{(3)}$ for downstream updates.
--Moving error vector across affine transformation simply requires multiplication with the transpose of forward matrix
--Notice that the dimensions will line up perfectly too!

Visualization of intuition



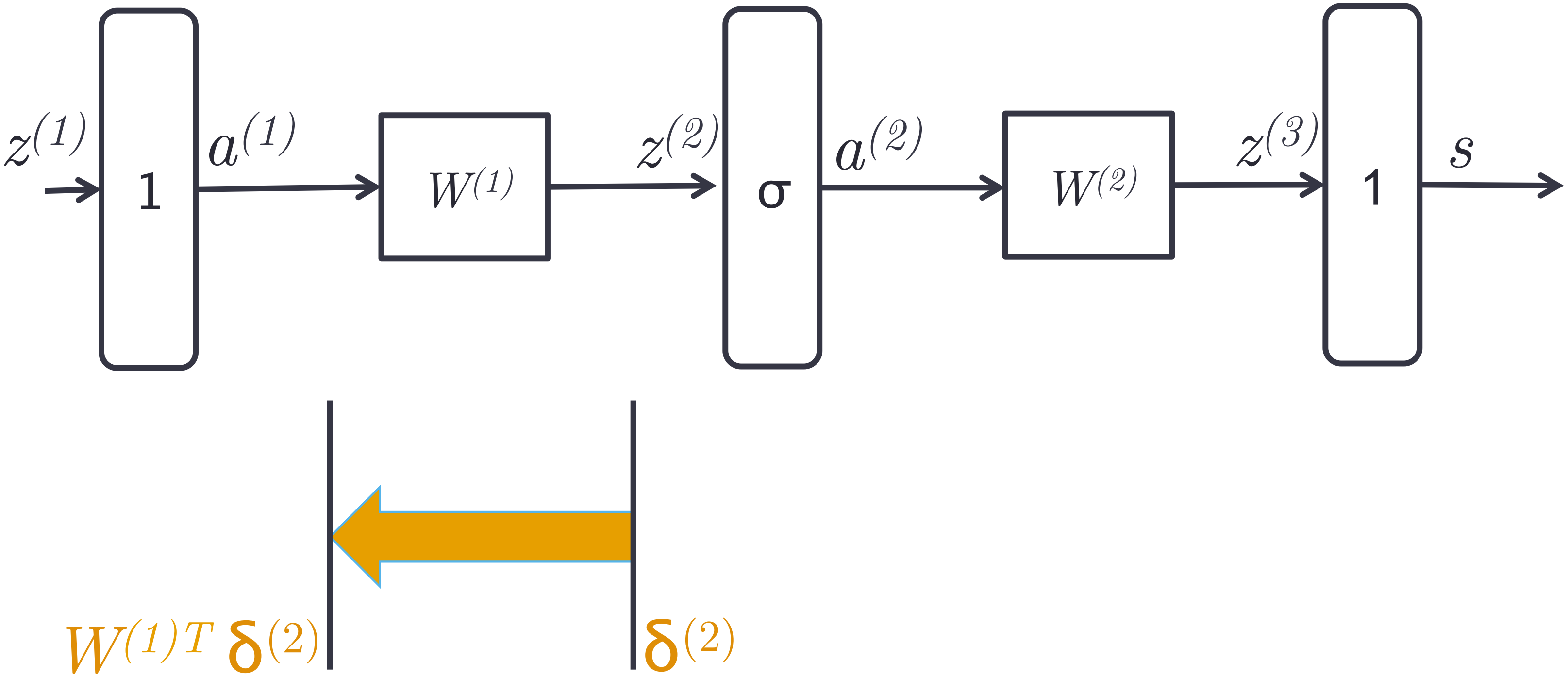
$\sigma'(z^{(2)}) \odot W^{(2)\mathrm{T}} \delta^{(3)}$

$W^{(2)T} \delta^{(3)}$

$= \delta^{(2)}$

--Moving error vector across point-wise non-linearity requires point-wise multiplication with local gradient of the non-linearity

Visualization of intuition



$$\text{Gradient w.r.t } W^{(1)} = \delta^{(2)} a^{(1)\mathrm{T}}$$

Backpropagation (Another explanation)

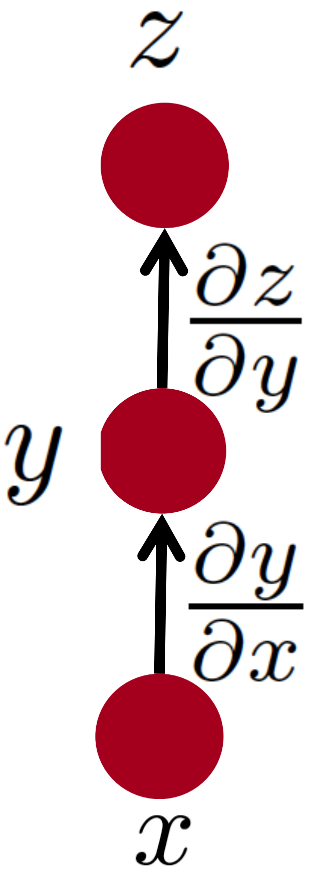
- Compute gradient of example-wise loss wrt parameters

- Simply applying the derivative chain rule wisely

$$z = f(y) \quad y = g(x) \quad \frac{\partial z}{\partial x} = \frac{\partial z}{\partial y}\frac{\partial y}{\partial x}$$

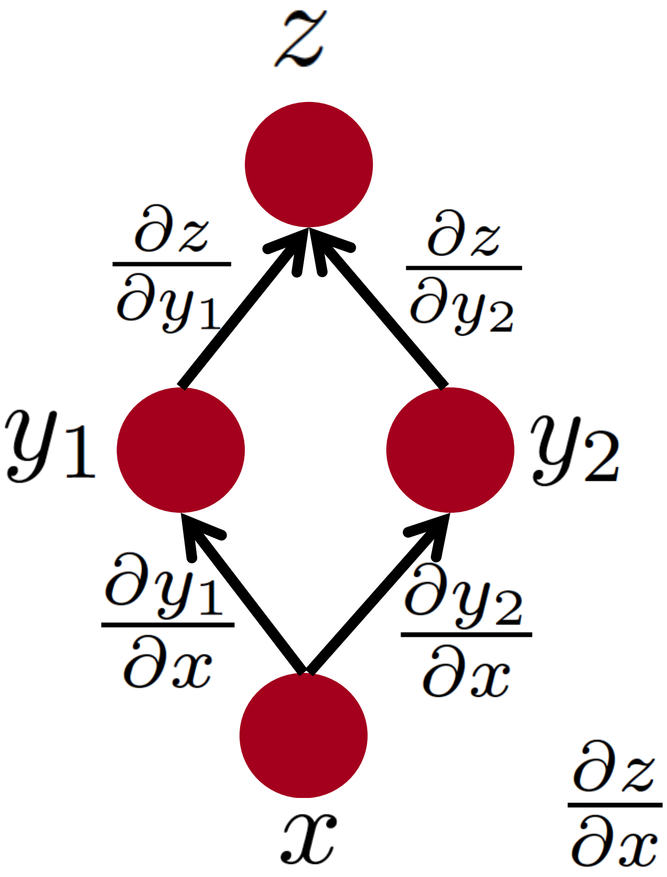
- If computing the loss(example, parameters) is *O*(*n*) computation, then so is computing the gradient

Simple Chain Rule


z
$\frac{\partial z}{\partial y}$
y
$\frac{\partial y}{\partial x}$
x

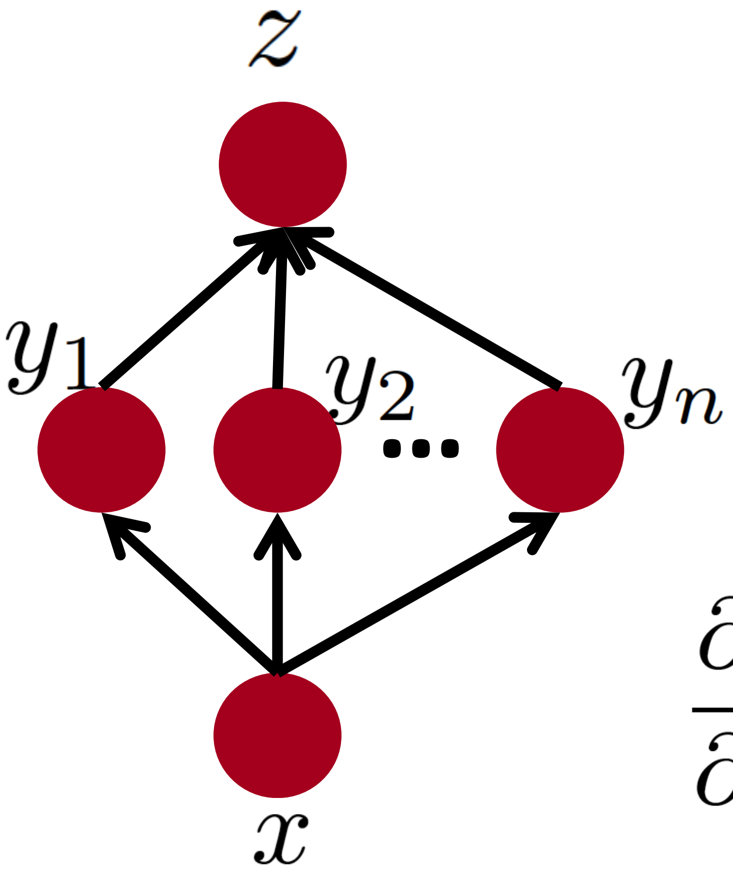
$$\frac{\partial z}{\partial x} = \frac{\partial z}{\partial y}\frac{\partial y}{\partial x}$$

Multiple Paths Chain Rule


z
$\frac{\partial z}{\partial y_1}$
$\frac{\partial z}{\partial y_2}$
y_1
y_2
$\frac{\partial y_1}{\partial x}$
$\frac{\partial y_2}{\partial x}$
x

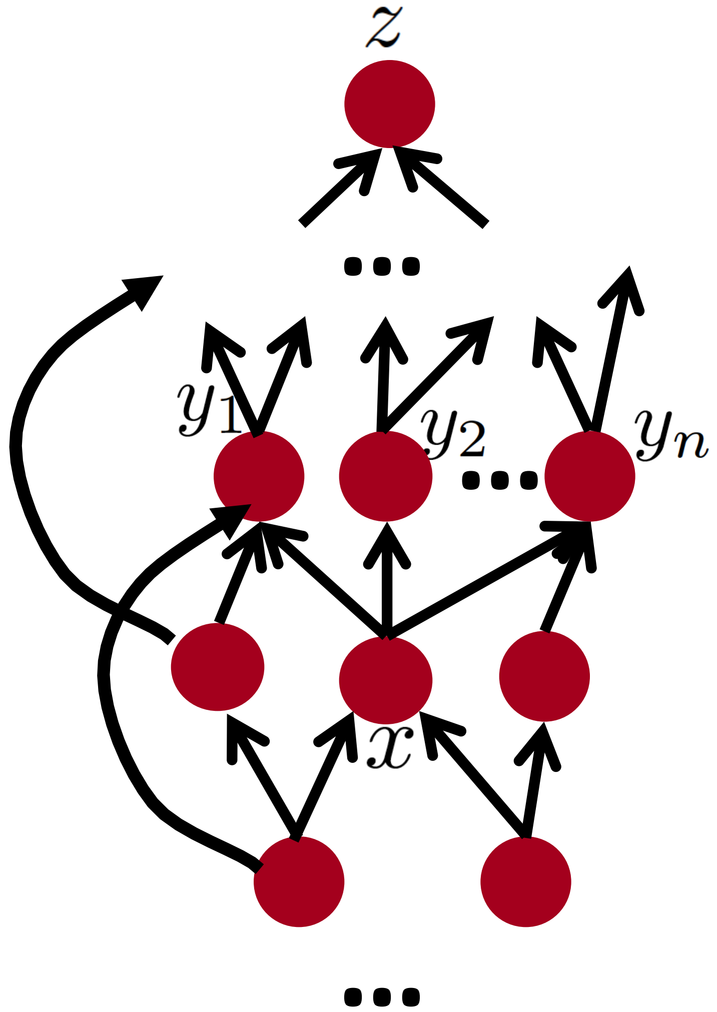
$$\frac{\partial z}{\partial x} = \frac{\partial z}{\partial y_1}\frac{\partial y_1}{\partial x} + \frac{\partial z}{\partial y_2}\frac{\partial y_2}{\partial x}$$

Multiple Paths Chain Rule - General



$$\frac{\partial z}{\partial x} = \sum_{i=1}^{n} \frac{\partial z}{\partial y_i} \frac{\partial y_i}{\partial x}$$

Chain Rule in Flow Graph



Flow graph: any directed acyclic graph

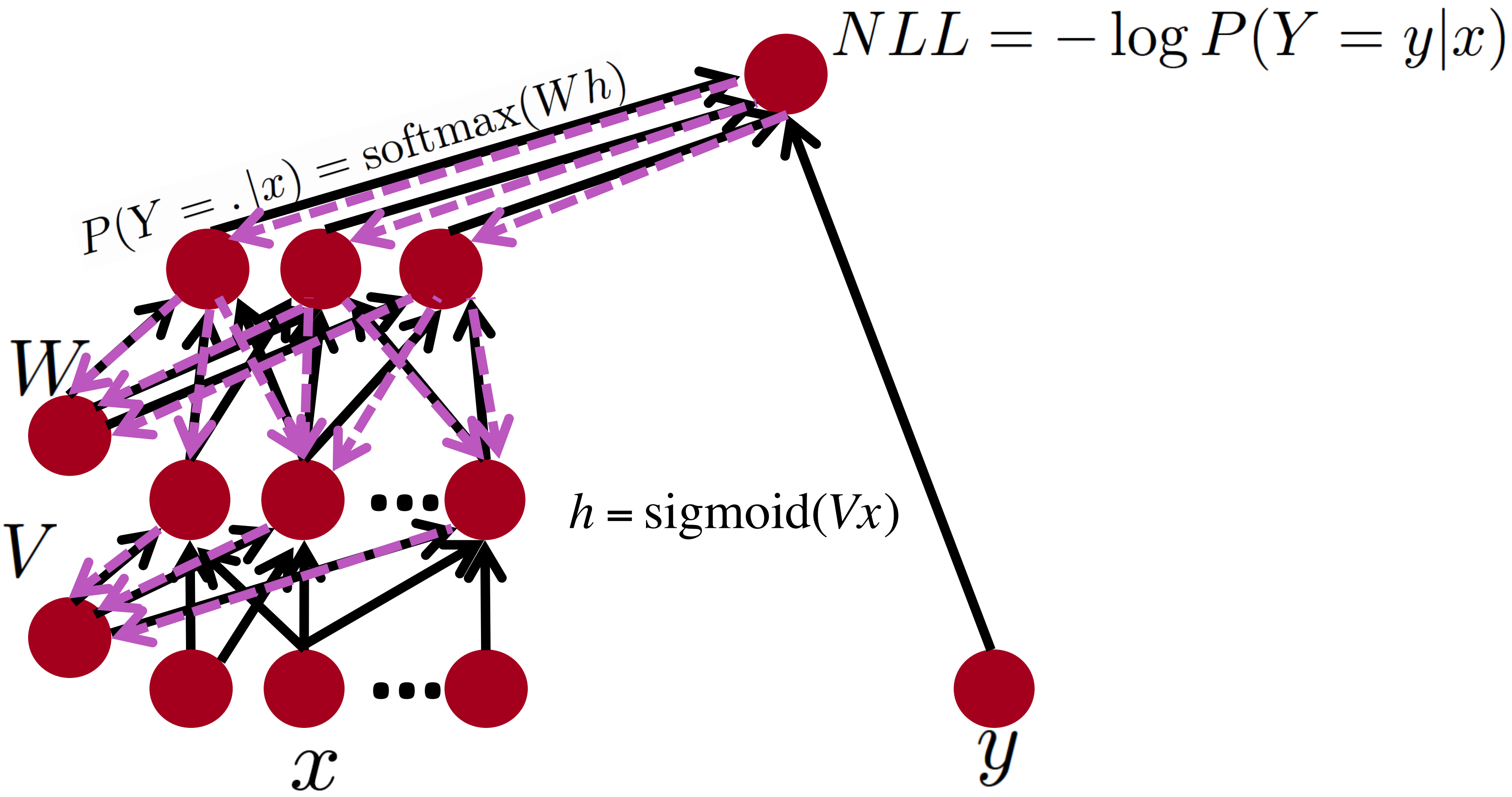
node = computation result

arc = computation dependency

$\{y_1,\ y_2,\ \ldots\ y_n\}$ = successors of x

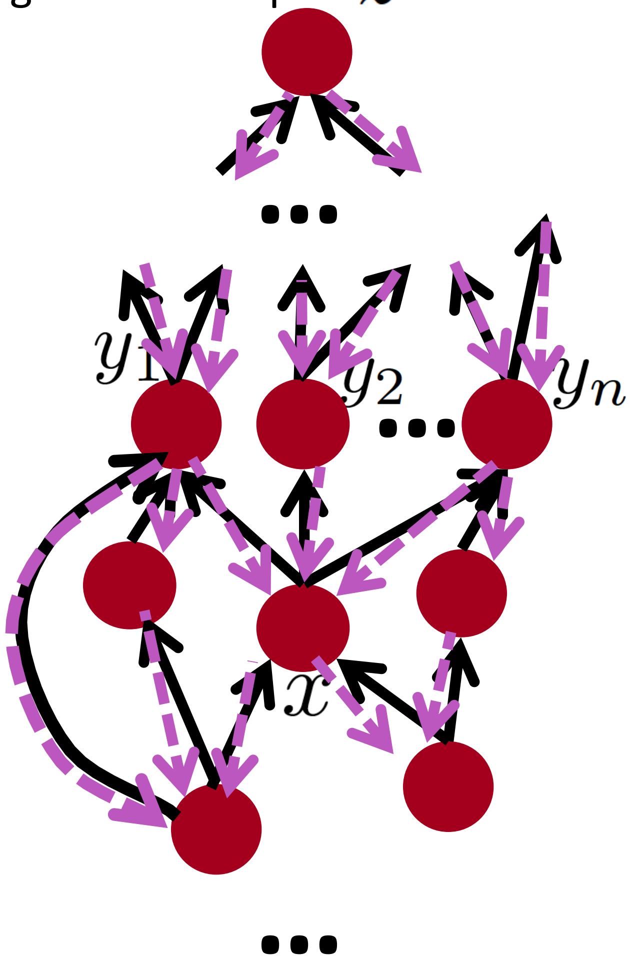
$$\frac{\partial z}{\partial x} = \sum_{i=1}^{n} \frac{\partial z}{\partial y_i} \frac{\partial y_i}{\partial x}$$

Back-Prop in Multi-Layer Net



Back-Prop in General Flow Graph

Single scalar output z



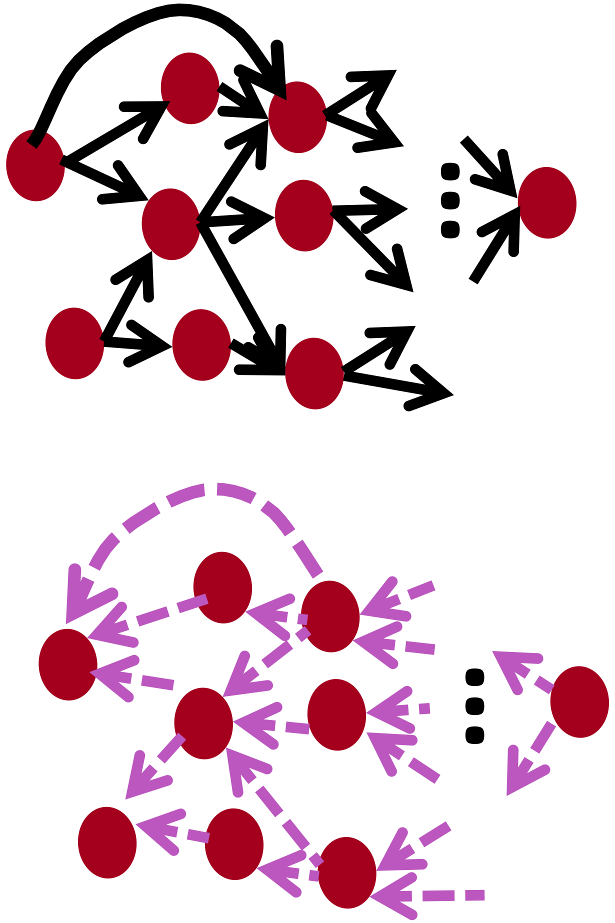
1. Fprop: visit nodes in topo-sort order
   - Compute value of node given predecessors
2. Bprop:
   - initialize output gradient = 1
   - visit nodes in reverse order:

     Compute gradient wrt each node using gradient wrt successors

$\{y_1, y_2, \dots y_n\}$ = successors of x

$$\frac{\partial z}{\partial x} = \sum_{i=1}^{n} \frac{\partial z}{\partial y_i} \frac{\partial y_i}{\partial x}$$

Automatic Differentiation



- The gradient computation can be **automatically inferred** from the symbolic expression of the fprop.
- Each node type needs to know how to compute its output and how to compute the gradient wrt its inputs given the gradient wrt its output.
- Easy and fast prototyping

Summary

- Congrats!
- You survived the hardest part of this class.
- Everything else from now on is just more matrix multiplications and backprop :)

- Next up:
  - Recurrent Neural Networks