

Econometrics III - Problem Set 1

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Question 1

Question 3

Let $\{X_i\}_{i \in \mathbb{N}}$ be a sequence of random variables with mean μ . Suppose that $(X_t, X_s) = 0 \forall t \neq s$ and $(X_t) \leq Kt^{1/2}$ for some constant $K > 0$. Show that $\bar{X}_t \xrightarrow{\mathbb{P}} \mu$.

Solution: Consider any $\epsilon > 0$. Using Markov inequality we get

$$\begin{aligned} P(|\bar{X}_t - \mu| > \epsilon) &\leq \frac{\mathbb{E}(|\bar{X}_t - \mu|^2)}{\epsilon^2} = \frac{V(\bar{X}_t)}{\epsilon^2} \\ &\leq \frac{\sum_{t=1}^T V(X_t)}{T^2 \epsilon^2} = \frac{K \sum_{t=1}^T t^{1/2}}{T^2 \epsilon^2} \\ &\leq \frac{KT^{3/2}}{T^2 \epsilon^2} \xrightarrow{t \rightarrow \infty} 0 \end{aligned}$$

Then $\bar{X}_t - \mu \xrightarrow{\mathbb{P}} 0$ which is equivalent to what we wanted to show.

Question 5

Suppose that $\sup_{t \in \mathcal{T}} \mathbb{E}(|X_t|^{1+\delta}) < \infty$ for some $\delta > 0$. Then $\{X_t : t \in \mathcal{T}\}$ is uniformly integrable. (Hint: Markov's inequality)

Solution:

Question 11

Interpret and prove that $e^{o_p(1)} - 1 = o_p(1)$ and $(O_p(1))^{\sqrt{2}} = O_p(1)$.

Solution: