

## Algorithm EE398000 hw01

106061146 陳兆廷

**1. Introduction:**

The analysis and comparisons of the four sorting algorithms: selection sort, insertion sort, bubble sort and shaker sort.

The goal of the algorithms is to sort a list of vocabulary into alphabetical order.

The input of the algorithms is a list of vocabulary, which is in a form of 2d array. The output of the algorithms is a list of sorted, alphabetically ordered vocabulary.

**2. Approach:**

## a. Analysis:

Analysis of **Selection Sort**:

- i. **Intro:** Find minimum value, put it in the front of array.
- ii. **Proof:** Each iteration in the  $i$ th iterations, it brings the smallest value among the  $n-1$  items to the front of the array. Therefore, the algorithm finds the values in ascending order, undoubtedly.

iii. **Space complexity:**

Fixed part: int: (i, j, k, n), \*char: (tmp)

Variable part: \*char (A) x n

Total:  $n (*char) + 4$

iv. **Time complexity:**

| Statement   | s/e    | Freq.    | Total                  |
|---|--------|----------|------------------------|
| 1. <b>Algorithm</b> SelectionSort(A, n)                 | 0      | 0        | 0                      |
| 2. {  | 0      | 0        | 0                      |
| 3.   for i := 1 to n do {                               | n+1    | 1        | n+1                    |
| 4.     j := i;  | 1      | n        | n                      |
| 5.     for k := i+1 to n do {                           | n+1    | n        | $n(n+1)$               |
| 6.       if (A[k] < A[j]) then j = k                    | 2 or 1 | $n(n+1)$ | $n(n)*2 \text{ or } 1$ |
| 7.     }  | 0      | 0        | 0                      |
| 8.     tmp = list[i]; list[i] = list[j]; list[j] = tmp; | 3      | n        | 3n                     |
| 9.   }  | 0      | 0        | 0                      |
| 10. }   | 0      | 0        | 0                      |
| Total   |        |          | $3n^2 + 4n + 0$        |

\*since avg of i is  $n/2$ , I calculate T(P) with  $i = n/2$ .

Best case:  $2n^2 + 4n + 2$  ( $O(n^2)$ ).

Worst Case:  $3n^2 + 4n + 2$  ( $O(n^2)$ ).

Average Case:  $O(n^2)$ .

**Therefore, the time complexity of Selection sort is  $O(n^2)$ .**

Analysis of **Insertion Sort**:

- i. **Intro:** Compare with the minimum value, if true, swap, put it in the front of array.
- ii. **Proof:** Like Selection sort, but Insertion sort gradually inserts the items from 2 to n into array in the front, in the right order.

iii. **Space complexity:**

Fixed part: int: (i, j, n), \*char: (tmp)

Variable part: \*char (A) x n

Total: n (\*char) + 3

iv. **Time complexity:**

| Statement                                     | s/e   | Freq.     | Total                |
|---|-------|-----------|----------------------|
| 1. <b>Algorithm</b> InsertionSort(A, n)       | 0     | 0         | 0                    |
| 2. {  | 0     | 0         | 0                    |
| 3.   for j := 2 to n do {                     | n-1   | 1         | n-2                  |
| 4.     tmp := A[j];                           | 1     | n-2       | n-2                  |
| 5.     i := j - 1;                            | 1     | n-2       | n-2                  |
| 6.     while ((i >= 1) and (tmp < A[i])) do { | n (*) | n-2       | n <sup>2</sup> -2n   |
| 7.       A[i + 1] := A[i]; i := i-1;          | 2     | (n-2)(c)n | (2n-4)cn             |
| 8.     }                                      | 0     | (n-2)(c)n | 0                    |
| 9.     A[i + 1] = tmp;                        | 1     | n-2       | n-2                  |
| 10. }   | 0     | 0         | 0                    |
| 11. }   | 0     | 0         | 0                    |
| <b>Total</b>                                  |       |           | (1+2c)n <sup>2</sup> |

\*Assume >=, <, and are all performed. (I recall that if i<1, computer won't bother do the rest computation.)

What is c?

Best case: if c = 0 or 1, it is mostly sorted, it won't do anything. (O(1))

Worst case: if c ≈ n, it is almost decreasing, it is ~ (1+2c)n<sup>2</sup>. (O(n<sup>2</sup>))

Average case: O(n<sup>2</sup>) since most of the case require sorting.

**Therefore, the time complexity of Selection sort is O(n<sup>2</sup>).**

Analysis of **Bubble Sort**:

- i. **Intro:** Swap whenever it's in the wrong order.
- ii. **Proof:** It swaps whenever two neighbors are not in the right order.  
From ith iteration in 1 to n-1 iterations, it does the aforementioned behavior from n to i+1. This way, no single thing is in the wrong order.

iii. **Space complexity:**

Fixed part: int: (i, j, n), \*char: (tmp)

Variable part: \*char (A) x n

Total: n (\*char) + 3

iv. **Time complexity:**

| Statement | s/e | Freq. | Total |
|-----------|-----|-------|-------|
|-----------|-----|-------|-------|

|  |          |             |             |
|--|----------|-------------|-------------|
| 1. <b>Algorithm</b> BubbleSort( <b>A</b> , <b>n</b> )  | 0        | 0           | 0           |
| 2. {   | 0        | 0           | 0           |
| 3. <b>for</b> <b>i</b> := 1 <b>to</b> <b>n</b> - 1 <b>do</b> {   | <b>n</b> | 1           | <b>n</b>    |
| 4. <b>for</b> <b>j</b> := <b>n</b> <b>to</b> <b>i</b> + 1 <b>step</b> -1 <b>do</b> {   | <b>n</b> | <b>n</b> -1 | $n^2-n$     |
| 5. <b>if</b> ( <b>A</b> [ <b>j</b> ] < <b>A</b> [ <b>j</b> - 1]) {   | 1        | <b>n</b> -1 | <b>n</b> -1 |
| 6. <b>tmp</b> = <b>A</b> [ <b>j</b> ]; <b>A</b> [ <b>j</b> ] = <b>A</b> [ <b>j</b> -1]; <b>A</b> [ <b>j</b> -1] = <b>tmp</b> ; | 3        | $c(n-1)$    | $3c(n-1)$   |
| 7.       }   | 0        | 0           | 0           |
| 8.     }   | 0        | 0           | 0           |
| 9. }   | 0        | 0           | 0           |
| 10. }  | 0        | 0           | 0           |
| <b>Total</b>   |          |             | $n^2+...$   |

Best case: if  $c = 0$  or  $1$ , the 2<sup>nd</sup> iteration does nothing, then it is  $O(n)$ .

Worst case: if  $c \approx n$ , it is almost decreasing, it is  $\sim n^2+... (O(n^2))$

Average case:  $O(n^2)$  since most of the case require sorting.

**Therefore, the time complexity of Bubble sort is  $O(n^2)$ .**

Analysis of **Shaker Sort**:

- i. **Intro:** Swap whenever it's in the wrong order but goes from both ends repeatedly.
- ii. **Proof:** Like Bubble sort, but it reduces the 1 to **n** cycle to half of it, and repeat the swapping from **n** to **i**+1 again, from **i** to **n**-1. Same as Bubble sort, no single thing is in the wrong order.
- iii. **Space complexity:**  
Fixed part: int: (**j**, **l**, **r**, **n**), \*char: (**tmp**)  
Variable part: \*char (**A**) x **n**  
Total:  $n (*char) + 4$

iv. **Time complexity:**

| Statement   | s/e         | Freq.    | Total       |
|---|-------------|----------|-------------|
| 1. <b>Algorithm</b> ShakerSort( <b>A</b> , <b>n</b> )   | 0           | 0        | 0           |
| 2. {  | 0           | 0        | 0           |
| 3. <b>l</b> := 1; <b>r</b> := <b>n</b> ;  | 2           | 1        | 2           |
| 4. <b>while</b> <b>l</b> <= <b>r</b> <b>do</b> {  | <b>n</b> +1 | 1        | <b>n</b> +1 |
| 5. <b>for</b> <b>j</b> := <b>r</b> <b>to</b> <b>l</b> + 1 <b>step</b> -1 <b>do</b> {  | <b>n</b>    | <b>n</b> | $n^2$       |
| 6. <b>if</b> ( <b>A</b> [ <b>j</b> ] < <b>A</b> [ <b>j</b> - 1]) {  | <b>c</b>    | <b>n</b> | <b>cn</b>   |
| 7. <b>tmp</b> = <b>A</b> [ <b>j</b> ]; <b>A</b> [ <b>j</b> ] = <b>A</b> [ <b>j</b> -1]; <b>A</b> [ <b>j</b> -1] = <b>tmp</b> ;  | 3           | <b>n</b> | 3 <b>n</b>  |
| 8.       }  | 0           | <b>n</b> | 0           |
| 9.     }  | 0           | <b>n</b> | 0           |
| 10. <b>l</b> = <b>l</b> + 1;  | 1           | <b>n</b> | 1 <b>n</b>  |
| 11. <b>for</b> <b>j</b> := <b>l</b> <b>to</b> <b>r</b> - 1 <b>do</b> {  | <b>n</b>    | <b>n</b> | $n^2$       |
| 12. <b>if</b> ( <b>A</b> [ <b>j</b> ] > <b>A</b> [ <b>j</b> + 1]) {   | <b>c</b>    | <b>n</b> | <b>cn</b>   |
| 13. <b>tmp</b> = <b>A</b> [ <b>j</b> ]; <b>A</b> [ <b>j</b> ] = <b>A</b> [ <b>j</b> +1]; <b>A</b> [ <b>j</b> +1] = <b>tmp</b> ; | 3           | <b>n</b> | 3 <b>n</b>  |
| 14.       }   | 0           | <b>n</b> | 0           |
| 15.    }  | 0           | <b>n</b> | 0           |
| 16. <b>r</b> := <b>r</b> - 1;   | 1           | <b>n</b> | 1 <b>n</b>  |
| 17. }   | 0           | 0        | 0           |
| 18. }   | 0           | 0        | 0           |
| <b>Total</b>  |             |          | $2n^2+...$  |

Best case: if  $c = 0$  or  $1$ ,  $2^{\text{nd}}$  iterations do nothing, then it is  $O(n)$ .

Worst case: if  $c \approx 1/2n$ , it is almost decreasing, it is  $O(n^2)$ .

Average case:  $O(n^2)$  since most of the case require sorting, but it goes 2 times slower than bubble sort.

**Therefore, the time complexity of Shaker sort is  $O(n^2)$ .**

#### Comparing Table:

|       | Selection                  | Insertion                  | Bubble                     | Shaker                     |
|-------|----------------------------|----------------------------|----------------------------|----------------------------|
| Space | $n (*\text{char}) + 4$     | $n (*\text{char}) + 3$     | $n (*\text{char}) + 3$     | $n (*\text{char}) + 4$     |
| Time  | <b><math>O(n^2)</math></b> | <b><math>O(n^2)</math></b> | <b><math>O(n^2)</math></b> | <b><math>O(n^2)</math></b> |

Prediction: Shaker > Bubble ? Insertion > Selection Prediction of what?

The reason of Insertion > Selection is because the comparing step is lesser in Insertion sort. Shaker sort reduces the swapping steps by comparing from 2 ends.

The speed of the algorithms depends on whether writing data(swapping) or going through more steps is faster. Based on my learning in Computer Architecture, writing data is way slower.

Therefore, **my prediction is: Insertion > Selection > Shaker > Bubble.**

b. Implementation: (hw01.c on NTHUEE workstation, gcc 4.1.2)

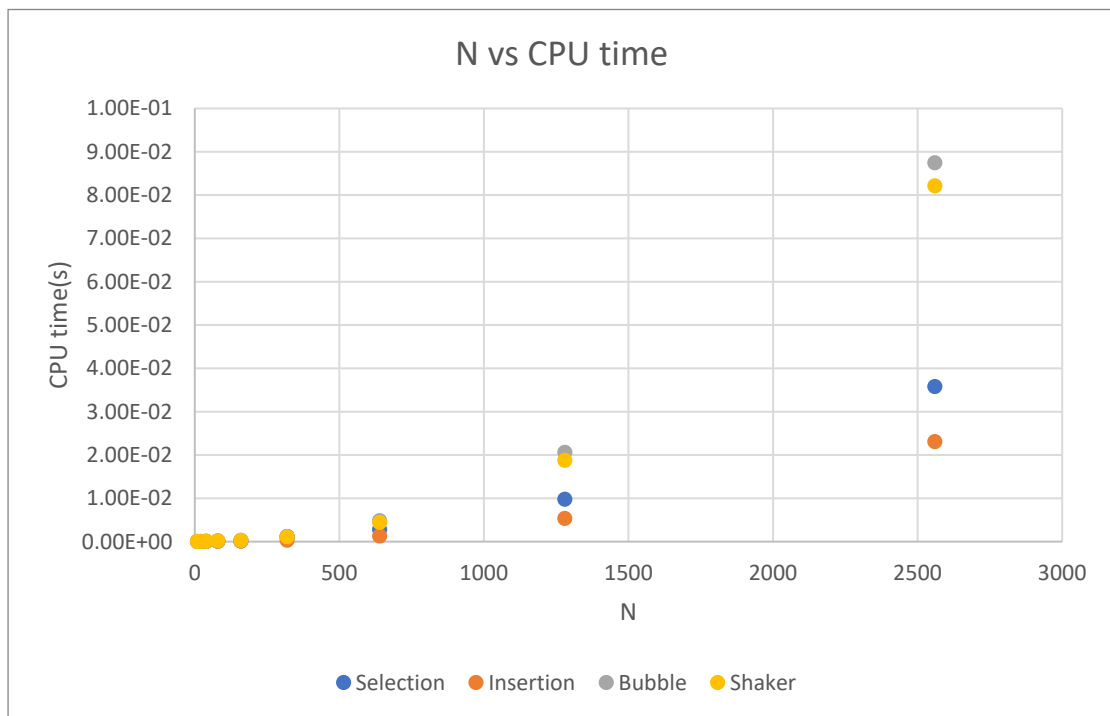
#### 3. Result:

a. Result Table: CPU time? Unit?

| N    | Selection | Insertion | Bubble   | Shaker   |
|------|-----------|-----------|----------|----------|
| 10   | 9.90E-07  | 7.30E-07  | 1.11E-06 | 1.12E-06 |
| 20   | 6.79E-06  | 5.09E-06  | 1.00E-05 | 9.99E-06 |
| 40   | 2.33E-05  | 1.44E-05  | 3.96E-05 | 4.04E-05 |
| 80   | 8.60E-05  | 5.27E-05  | 1.64E-04 | 1.64E-04 |
| 160  | 1.33E-04  | 7.21E-05  | 2.77E-04 | 2.65E-04 |
| 320  | 1.14E-03  | 3.03E-04  | 1.13E-03 | 1.07E-03 |
| 640  | 2.76E-03  | 1.24E-03  | 4.81E-03 | 4.39E-03 |
| 1280 | 9.75E-03  | 5.32E-03  | 2.06E-02 | 1.87E-02 |
| 2560 | 3.58E-02  | 2.31E-02  | 8.74E-02 | 8.21E-02 |

b. Observation:

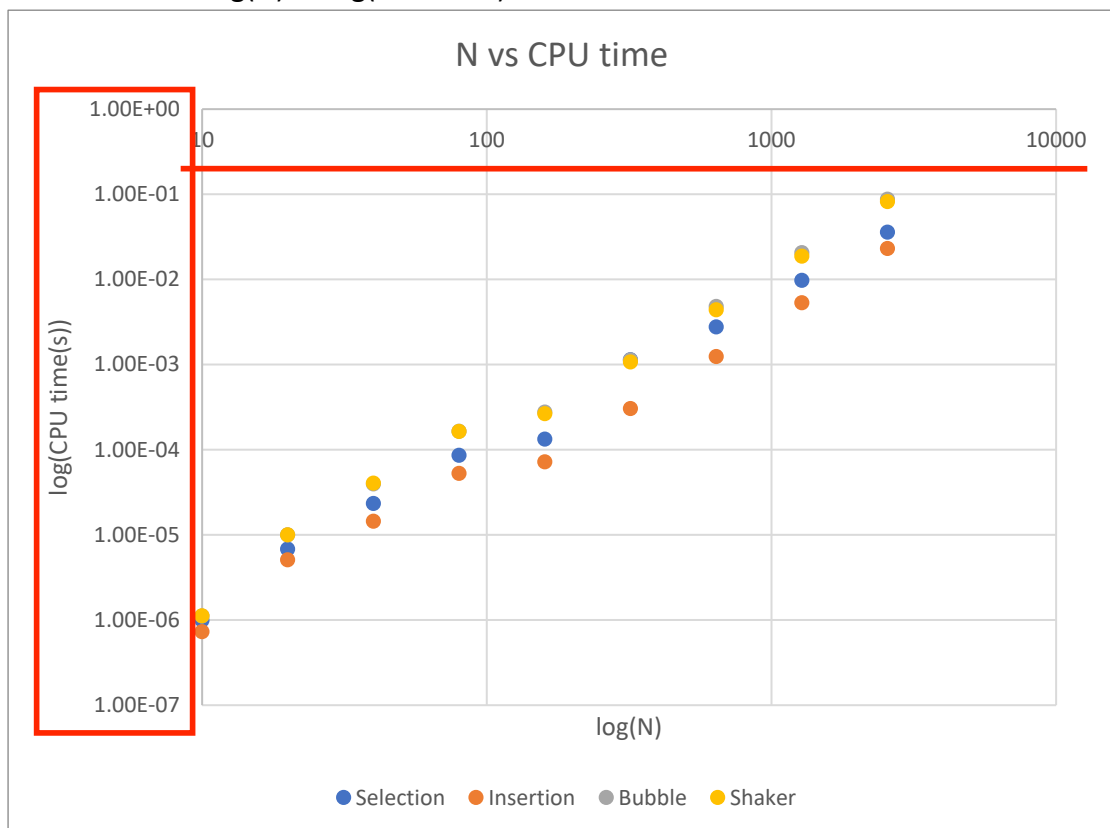
- N vs CPU time chart:



**Speed: Insertion > Selection > Shaker > Bubble.**

It seems that the result meets by prediction. Furthermore, the time complexities of the four algorithms are  $O(n^2)$ , same as those that I've calculated.

- Log(N) vs log(CPU time) chart:



How?  
When y-axis is logged, the graph looks linear. It indicates that they are  $O(n^2)$ .

Logged x-axis is just for good spacing between different Ns.

\*The N=10 & N=40 case's bubble sort is faster than shaker sort. They might just be the special case when N is small.

c. Conclusion:

- i. Experimented Speed: **Insertion > Selection > Shaker > Bubble.**
- ii. Time Complexity: **Insertion = Selection = Shaker = Bubble =  $O(n^2)$ .**
- iii. Best Case: **Insertion > Shaker = Bubble > Selection.**
- iv. Space Complexity: Insertion = Selection = Shaker = Bubble =  $\theta(1)$  Is this correct?
- v. **Swapping (data writing) is much slower than comparing.**

## hw01.c

```
1 // EE3980 HW01 Quadratic Sorts
2 // 106061146, Jhao-Ting, Chen
3 // 2020/03/11
4
5 #include <stdio.h>
6 #include <stdlib.h>
7 #include <string.h>
8 #include <sys/time.h>
9
10 int N; // input size
11 char **data; // input data
12 char **A; // array to be sorted
13 int R = 500; // number of repetitions
14
15 void readInput(void); // read all inputs
16 void printArray(char **A); // print the content of A
17 void copyArray(char **data, char **A); // copy data to array A
18 double GetTime(void); // get local time in seconds
19 void SelectionSort(char **list, int n); // in-place selection sort
20 void InsertionSort(char **list, int n); // in-place insertion sort
21 void BubbleSort(char **list, int n); // in-place bubble sort
22 void ShakerSort(char **list, int n); // in-place shaker sort
23
24 int main(void)
25 {
26     int i; // loop index
27     double t; // for CPU time tracking
28
29     readInput(); // read input data
30
31     t = GetTime(); // initialize time counter
32     for (i = 0; i < R; i++) {
33         copyArray(data, A); // initialize array for sorting
34         SelectionSort(A, N); // execute selection sort
35         // InsertionSort(A, N); // execute insertion sort
36         // BubbleSort(A, N); // execute bubble sort
37         // ShakerSort(A, N); // execute shaker sort
38         if (i == 0) printArray(A); // print sorted results
39     }
40     t = (GetTime() - t) / R; // calculate CPU time / iteration
41     printf("Selection sort:\n N = %d\n CPU time = %e\n", N, t); // result
42     // printf("Insertion sort:\n N = %d\n CPU time = %e\n", N, t); // result
43     // printf("Bubble sort:\n N = %d\n CPU time = %e\n", N, t); // result
```

```

    // printf("Bubble sort:\n N = %d\n CPU time = %e\n", N, t); // result
44 // printf("Shaker sort:\n N = %d\n CPU time = %e\n", N, t); // result
    // printf("Shaker sort:\n N = %d\n CPU time = %e\n", N, t); // result
45
46     return 0;
47 }
48
49 void readInput(void)                // read all inputs
50 {
51     scanf("%d", &N);                // read N for # of words
52     int i;                          // for looping
    Do not mix declarations with statements
53
54     data = (char **)calloc(N, sizeof(char *)); // initialize data size (N)
55     A = (char **)calloc(N, sizeof(char *));   // initialize A size (N)
56
57     for (i = 0; i < N; i++){
58         for (i = 0; i < N; i++) {
59             data[i] = (char *)calloc(50, sizeof(char)); // set each row 50 chars
60             A[i] = (char *)calloc(50, sizeof(char));    // for data & A
61             scanf("%s", data[i]);                      // store each word in data
62         }
63     }
64 void printArray(char **A)           // print the content of array A
65 {
66     int i;                          // for looping
67     for (i = 0; i < N; i++){
68         for (i = 0; i < N; i++) {
69             printf("%s\n", A[i]);    // print each string
70         }
71     }
72
73 void copyArray(char **data, char **A) // copy data to array A
74 {
75     int i;                          // for looping
76
77     for (i = 0; i < N; i++){
78         for (i = 0; i < N; i++) {
79             A[i] = data[i];          // copy from data to A
80         }
81     }
82 double GetTime(void)                // demonstration code from 1.1.3
83 {
84     struct timeval tv;
85
86     gettimeofday(&tv, NULL);
87     return tv.tv_sec + 1e-6 * tv.tv_usec;

```



```

88 }
89
90 void SelectionSort(char **list, int n) // in-place selection sort
91 {
92     int i, j, k; // for looping
93     char *tmp = (char *)calloc(50, sizeof(char)); // initialize tmp size
94     for (i = 0; i < N; i++){ // for every A[i]
95         for (i = 0; i < N; i++) { // for every A[i]
96             j = i; // Initialize j to i
97             for (k = i + 1; k < N; k++){ // search for smallest in A[i+1 : n]
98                 for (k = i + 1; k < N; k++) { // search for smallest in A[i+1 : n]
99                     if (strcmp(list[k], list[j]) < 0){ // found, store in j
100                         if (strcmp(list[k], list[j]) < 0) { // found, store in j
101                             j = k;
102                         }
103                     }
104                 }
105             }
106             tmp = list[i]; // swap A[i] & A[j]
107             list[i] = list[j];
108             list[j] = tmp;
109         }
110     }
111 }
112
113 void InsertionSort(char **list, int n) // in-place insertion sort
114 {
115     int i, j;
116     char *tmp = (char *)calloc(50, sizeof(char)); // initialize tmp size
117     for (j = 1; j < N; j++){ // Assume A[0: j-1] already sorted
118         for (j = 1; j < N; j++) { // Assume A[0: j-1] already sorted
119             tmp = list[j]; // Move A[j] to its proper place
120             i = j - 1; // Init i to be j-1
121             while ((i >= 0) && (strcmp(tmp, list[i]) < 0)){ // Find i for A[i] <= A[j]
122                 while ((i >= 0) && (strcmp(tmp, list[i]) < 0)) { // Find i for A[i] <= A[j]
123                     list[i + 1] = list[i]; // Move A[i] up by one position
124                     i = i - 1;
125                 }
126             }
127             A[i + 1] = tmp; // Move A[j] to A[i+1]
128         }
129     }
130 }
131
132 void BubbleSort(char **list, int n) // in-place bubble sort
133 {
134     int i, j;
135     char *tmp = (char *)calloc(50, sizeof(char)); // Initialize tmp size
136     for (i = 0; i < N - 1; i++){ // Find the smallest item for A[i]
137         for (i = 0; i < N - 1; i++) { // Find the smallest item for A[i]
138             for (j = N - 1; j > i; j--){
139                 for (j = N - 1; j > i; j--) {
140                     if (strcmp(list[j], list[j-1]) < 0){ // swap A[j] & A[j-1]
141                         if (strcmp(list[j], list[j - 1]) < 0) { // swap A[j] & A[j-1]

```

```

129         tmp = list[j];
130         list[j] = list[j-1];
131         list[j] = list[j - 1];
132         list[j-1] = tmp;
133         list[j - 1] = tmp;
134     }
135 }
136
137 void ShakerSort(char **list, int n) // in-place shaker sort
138 {
139     int l = 0, r = N-1, j;
140     char *tmp = (char *)calloc(50, sizeof(char)); // Initialize tmp size
141     while (l <= r) {
142         for (j = r; j >= l+1; j--){ // Element exchange from r down to l
143             for (j = r; j >= l + 1; j--) { // Element exchange from r down to l
144                 if (strcmp(list[j], list[j-1]) < 0){ // swap A[j] & A[j-1]
145                     if (strcmp(list[j], list[j - 1]) < 0) { // swap A[j] & A[j-1]
146                         tmp = list[j];
147                         list[j] = list[j-1];
148                         list[j] = list[j - 1];
149                         list[j-1] = tmp;
150                         list[j - 1] = tmp;
151                     }
152                 }
153             }
154             l++;
155         }
156         for (j = l; j <= r-1; j++){ // Element exchange from l to r
157             for (j = l; j <= r - 1; j++) { // Element exchange from l to r
158                 if (strcmp(list[j], list[j+1]) > 0){ // swap A[j] and A[j+1]
159                     if (strcmp(list[j], list[j + 1]) > 0) { // swap A[j] and A[j+1]
160                         tmp = list[j];
161                         list[j] = list[j+1];
162                         list[j] = list[j + 1];
163                         list[j+1] = tmp;
164                         list[j + 1] = tmp;
165                     }
166                 }
167             }
168             r--;
169         }
170     }
171 }

```

[Program Format] can be improved.

[Introduction] of the problem can be more clear.

[CPU time] measurement method should be described clearly.

[Space] complexity  $O(1)$ ?

[Figures] can be improved.

[Observation] can be strengthened.

[Report] uses 12 pt fonts, single column format and double line-space.

[Writing] needs to be logical and consistent.

Score: 74