EE3980 Algorithms

Hw02 Random Data Searches

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Introduction:

In this homework, I'll be analyzing, implementing and observing 3 searching algorithms: Linear search, Bidirectional search and Random-directional search. The goal of the algorithms is to find a specific data in an array. The input of them will be a string, and the output of them will be the position of the input in the array.

During the analyzing process, I'll be using table-counting method to calculate the time complexities of 3 algorithms. Furthermore, I'll find the worst-case conditions in 3 algorithms each accordingly. Before implementation on C code, I'll try to predict the result based on my analysis.

The implementation of 3 algorithms on C code mainly focus on the average time and worst-case conditions. 9 testing data are given by professor, and the length of them are 10 times power of 2s, from 1 to 9.

Lastly, the observation of them will be focusing on the time complexity of the results from implementation. Moreover, I'll compare 3 algorithms with themselves and make some rankings. Finally, I'll check the experimented results with my analyzation.

Analysis:

1. **Linear Search:**
   a. Abstract:

      Linear search is a rather direct approach of searching. It goes through all the items in the array until it finds the requested item, or else it returns "not found".

   b. Algorithm:

```
1.  // Find the location of word in array list.
2.  // Input: word, array list, int n
3.  // Output: int i such that list[i] = word.
4.  Algorithm search(word, list, n)
5.  {
6.      for i := 1 to n do {
7.          if(list[i] = word) return i;
8.      }
9.      return -1;
10. }
```

c. Proof of correctness:

We can find that there's a truth: at the start of the i-th iteration, there is no requested item, v, in the array list A[0, …, i - 1]. The search index will be n.

During the i-th iteration, there are 2 conditions: A[i] == v or A[i] != v. If A[i] == v, the requested item's position will be found, which is i. If A[i] != v, let j = i + 1, then by the aforementioned truth, during the j-th iteration, where j = i + 1, there is no requested item, v, in the list A[1, …, j - 1], or in A[1, …, i]. Therefore, the truth applies to all iterations from 1 to n.

The loop terminates in 2 ways. It either ends when v is found, or the iteration will go through n iterations same as what is mentioned in the last paragraph, and finds that in the n + 1-th iteration, A[1, …, n] does not contain v, which indicates "not found". Hence proved.

d. Time complexity:

	s/e	freq	total
1. Algorithm search(word, list, n)	0	0	0
2. {	0	0	0
3. for i := 1 to n do {	n+1	1	n+1
4. if(list[i] = word) return i;	1	n	n
5. }	0	0	0
6. return -1;	1	1	1
7. }	0	0	0
			2n+2

Worst-case:

When the specific item is at the end of the array, the steps would be 2n+2, as the table shows, and the time complexity would be **O(n)**.

Best-case:

When the specific item is at the front of the array, the steps would be 1, and the time complexity would be **O(1)**.

Average-case:

When the specific item randomly exists in the array, the steps would be 2c+2, where c is an integer between 1 to n. I choose n/2 in this case, since it's the average. Therefore, the steps would be n+2, and the time complexity would be **O(n)**.

2. Bidirectional Search:

a. Abstract:

Bidirectional Search is the transformation of Linear Search. It only runs

n/2 iterations, however in each iteration, it searches 2 items gradually from both directions.

b. Algorithm:

```
1.  // Bidirectional search to find the location of word in array list.
2.  // Input: word, array list, int n
3.  // Output: int i such that list[i] = word.
4.  Algorithm search(word, list, n)
5.  {
6.      for i := 1 to n/2 do {
7.          if(list[i] = word) return i;
8.          if(list[n – i - 1] = word) return n – i - 1;
9.      }
10.     return -1;
11. }
```

c. Proof of correctness:

We can find that there's a truth: at the start of the i-th iteration, there is no requested item, v, in the array lists A[1, …, i - 1] and A[n - i, …, n]. The search index will be n, and the iteration number will be n/2.

During the i-th iteration, there are 2 conditions: {A[i] == v or A[n – i + 1] == v}, and {A[i] != v or A[n – i + 1] != v}. If it fits the first condition, the requested item's position will be found, which is either i or n – i + 1. If it fits the second condition, then by the aforementioned truth, during the j-th iteration, where j = i + 1, there is no requested item, v, in the list A[1, …, j - 1], or in A[1, …, i]. Therefore, the truth applies to all iterations from 1 to n.

The loop terminates in 2 ways. It either ends when v is found, or the iteration will go through n/2 iterations same as what is mentioned in the last paragraph, and finds that in the n/2 + 1-th iteration, A[1, …, n] does not contain v, which indicates "not found". Hence proved.

d. Time complexity:

	s/e	freq	total
`Algorithm search(word, list, n)`	0	0	0
`{`	0	0	0
`for i := 1 to n/2 do {`	n/2+1	1	n/2+1
`if(list[i] = word) return i;`	1	n/2	n/2
`if(list[n – i - 1] = word) return n – i - 1;`	1	n/2	n/2
`}`	0	0	0

7. return -1; 8. }	1 0	1 0	1 0
			3/2n+2

Worst-case:

When the specific item is at the middle of the array, the steps would be 3/2n+2 steps, as the table shows, and the time complexity would be **O(n)**.

Best-case:

When the specific item is at the front of the array, the steps would be 1 step, and the time complexity would be **O(1)**.

Average-case:

When the specific item randomly exists in the array, the steps would be 3/2c+2, where c is an integer between 1 to n. I choose n/2 in this case, since it's the average. Therefore, the steps would be 3/4n+2, and the time complexity would be **O(n)**.

3. Random-directional Search

a. Abstract:

It is basically Linear search, but Random-directional Search changes it's search direction based on a random number between 0 and 1.

b. Algorithm:

```
1.  // Random-direction search to find the location of word in array list.
2.  // Input: word, array list, int n
3.  // Output: int i such that list[i] = word.
4.  Algorithm search(word, list, n)
5.  {
6.      choose j randomly from the set {0, 1};
7.      if (j=1) then
8.          for i := 1 to n do {
9.              if(list[i] = word) return i;
10.         }
11.     else
12.         for i := n to 1 step -1 do {
13.             if(list[i] = word) return i;
14.         }
15.     return -1;
16. }
```

c. Proof of correctness:

The proof of Random-directional Search is basically the same with Linear Search. Change the direction and apply it again.

d. Time complexity:

	s/e	freq	total
1. Algorithm search(word, list, n)	0	0	0
2. {	0	0	0
3. choose j randomly from the set {0, 1};	1	1	1
4. if (j=1) then	1	1	1
5. for i := 1 to n do {	n+1	1	n+1
6. if(list[i] = word) return i;	1	n	n
7. }	0	n	0
8. else	0	1	0
9. for i := n to 1 step -1 do {	n+1	1	n+1
10. if(list[i] = word) return i;	1	n	n
11. }	0	n	0
12. return -1;	1	1	1
13. }	0	0	0
			2n+3

Worst-case:

When the specific item is at the end of the array when j == 1, or the specific item is at the front of the array when j == 0, the steps would be 2n+3, as the table shows, and the time complexity would be **O(n)**.

Best-case:

When the specific item is at the front or the end of the array, accordingly, the steps would be 1, and the time complexity would be **O(1)**.

Average-case:

When the specific item randomly exists in the array, the steps would be 2c+3, where c is an integer between 1 to n. I choose n/2 in this case, since it's the average. Therefore, the steps would be n+3, and the time complexity would be **O(n)**.

4. Comparison:

	Linear		Bidirectional		Random-directional	
Best	1	O(1)	1	O(1)	1	O(1)
Worst	2n+2	O(n)	3/2n+2	O(n)	2n+3	O(n)
Average	n+2	O(n)	3/4n+2	O(n)	n+3	O(n)

Average Speed (fast>slow): BDSearch > LinearSearch > RDSearch.

Worst-case Speed (fast>slow): BDSearch > LinearSearch ~= RDSearch.

Implementation:

1. **Average-case scenario:**

Since the most average case of testing the algorithm is randomly pick an item and search, I pick an item in given data randomly, implement on the algorithm, and repeat the procedure 500 times. The mean of the recorded CPU time between those steps are the average CPU runtime of the algorithm.

Workflow:

```
1.   t = GetTime();                          // initialize time counter
2.   for i := 0 to 500 do {
3.       random_num = RandomNumber(N);      // generate a random number
4.       Search(random_num);          // search item accordingly
5.  }
6.   t = (GetTime() - t) / 500;                // calculate CPU time / iteration
```

Results:

N(num)	Linear(s)	Bidirectional(s)	Random-directional(s)
10	2.00272E-07	1.83582E-07	2.34127E-07
20	2.57969E-07	2.52247E-07	4.95911E-07
40	3.84331E-07	3.63827E-07	4.42028E-07
80	6.46114E-07	6.0606E-07	7.10011E-07
160	1.30606E-06	1.08576E-06	1.28174E-06
320	2.27022E-06	2.04182E-06	2.33221E-06
640	4.50611E-06	3.78609E-06	4.53568E-06
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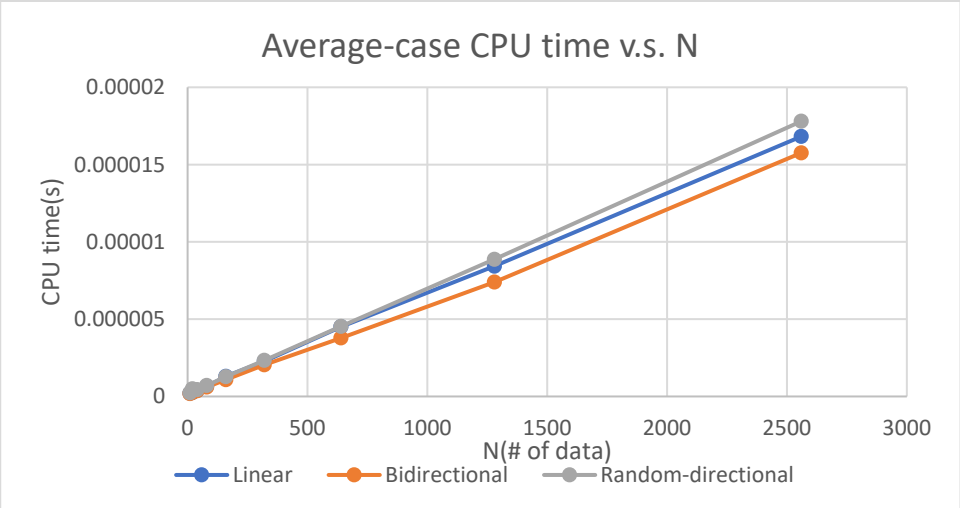
2560	1.682E-05	1.57599E-05	1.78042E-05



2. **Worst-case scenario:**

For Linear Search, the worst case will happen when the item for searching is at the end of the array. Therefore I choose to **search data[N - 1]** as the worst-case condition.

For Bidirectional Search, the worst case will happen when the item for searching is at the middle of the array. Therefore, I choose to **search data[N/2]** as the worst-case condition.

For Bidirectional Search, the worst case will happen when the item for searching is at the front or end of the array, depending on the random number algorithm generates. But, there's no way for me to set the item for searching according to the random number generated by algorithm without altering the algorithm itself. Therefore, I choose to **search data[N/2],** as the worst-case condition.

Since the worst-case scenario must compare with average case, I generate a random number in this workflow, too. This way, they are compared with each other in the same condition.

Workflow:

```
1.  t = GetTime();                          // initialize time counter
2.  for i := 0 to 500 do {
3.          random_num = RandomNumber(N);          // generate a random number
4.          Search(n);      // search for item (list[N])
5.  }
6.  t = (GetTime() - t) / R;                // calculate CPU time / iteration
```

Results:

N(num)	Linear(s)	Bidirectional(s)	Random-directional(s)
10	2.39849E-07	2.26021E-07	1.99795E-07
20	3.85761E-07	3.48091E-07	2.89917E-07
40	6.38008E-07	5.8794E-07	4.26292E-07
80	1.16205E-06	1.0643E-06	6.97613E-07
160	2.2378E-06	1.98603E-06	1.266E-06
320	4.34637E-06	3.96013E-06	2.36225E-06
640	8.57401E-06	7.61604E-06	4.54807E-06
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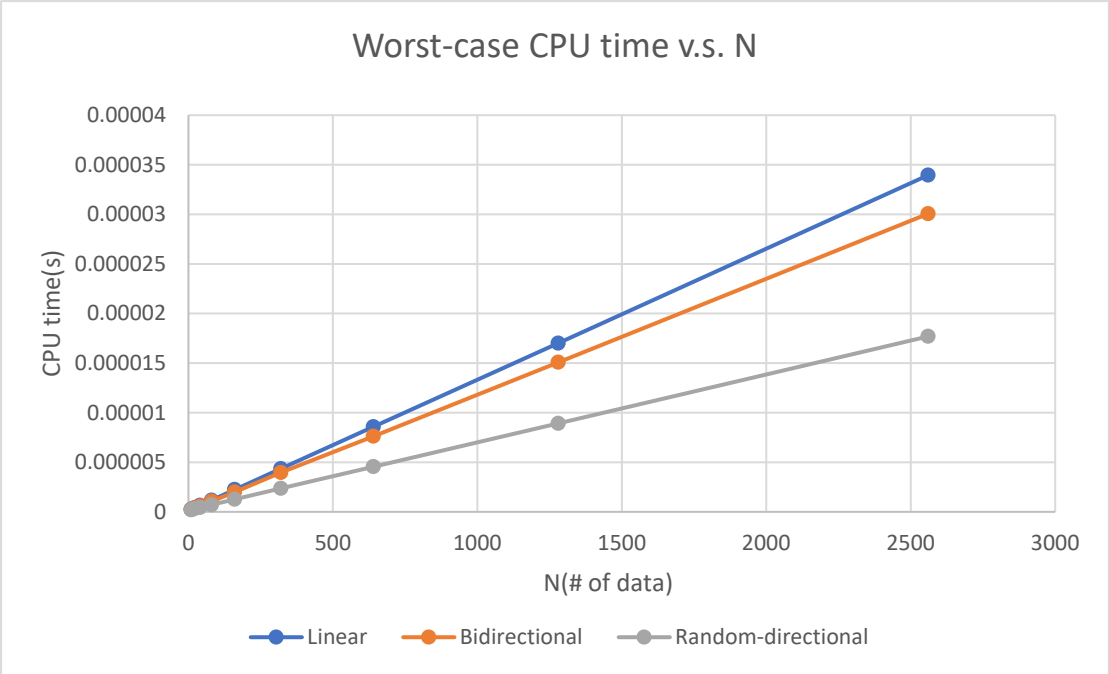
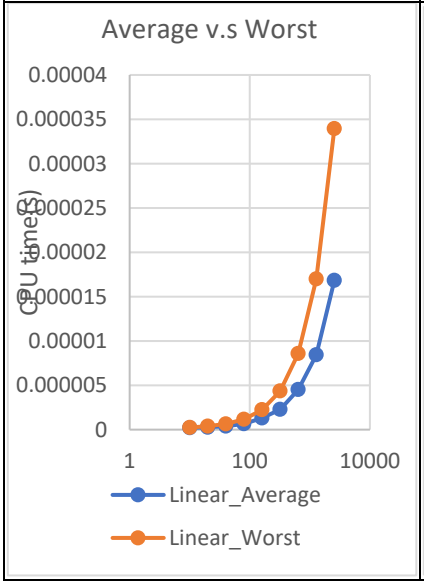
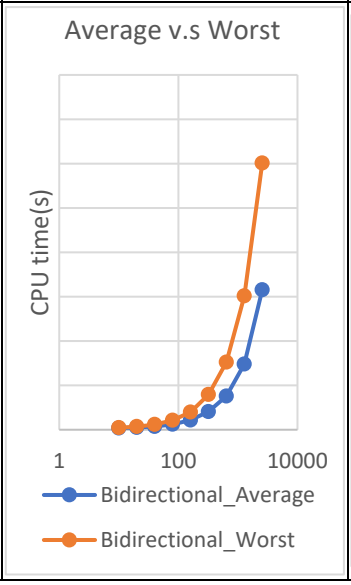
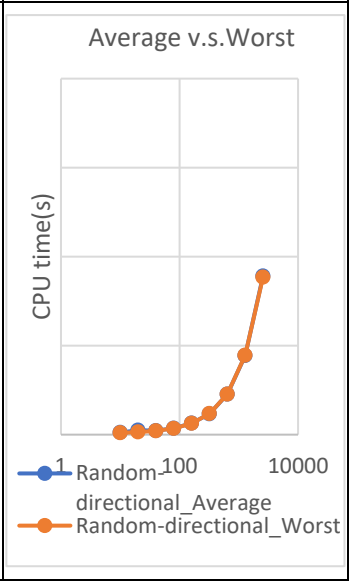
2560	3.3946E-05	3.00579E-05	1.76859E-05



Observation:

1. Average-case v.s. Worst-case comparisons:

Linear	Bidirectional	Random-directional
		

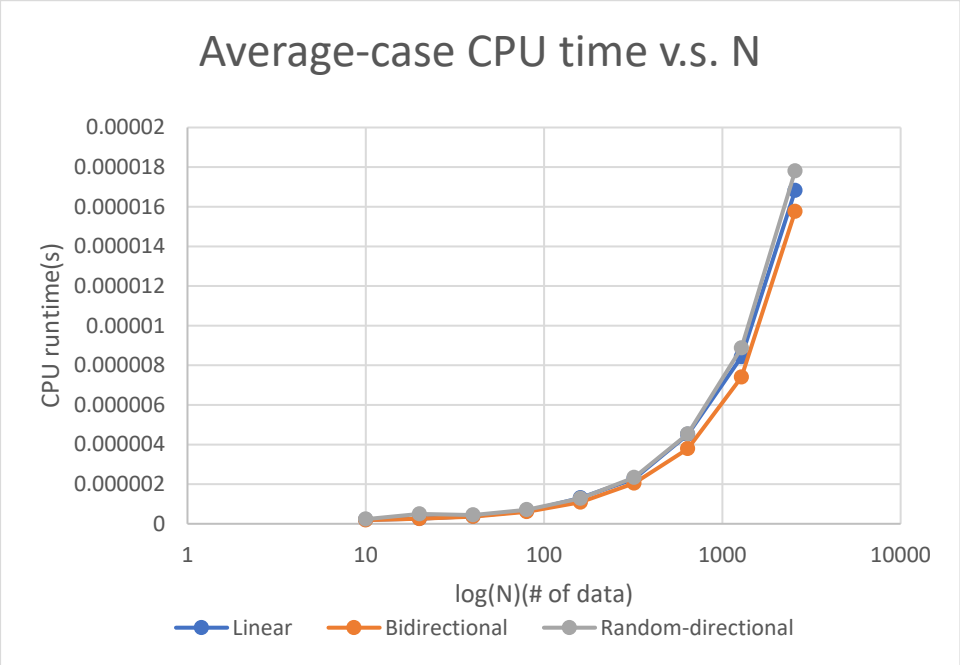
p.s. The x-axis is log(# of data), y-axis is CPU runtime. The scale of three charts are the same, y: from 0 to 0.00004, x: 1 to 10000.

For Linear Search and Bidirectional Search, **the Worst-case scenario has a longer CPU runtime.** This indicates that the experiments between average case and worst case is valid. However, the result of Random-directional Search did not seem valid since the worst-case scenario should spend longer time than average-case. The reason of this is because the worst-case of this is to pick the target in the middle, and this seem like an average case, too.

Therefore, there's no big changes between average and worst case of RDSearch.

I did one more experience, I gave RDSearch list[0] or list[n-1] to search, depending on the direction it generated. This way, I can **simulate the true worst-case**, which would happen if it's an unlucky day. The spent time of this **worst-case scenario is much longer than the original result**.

2. Average-case comparisons:



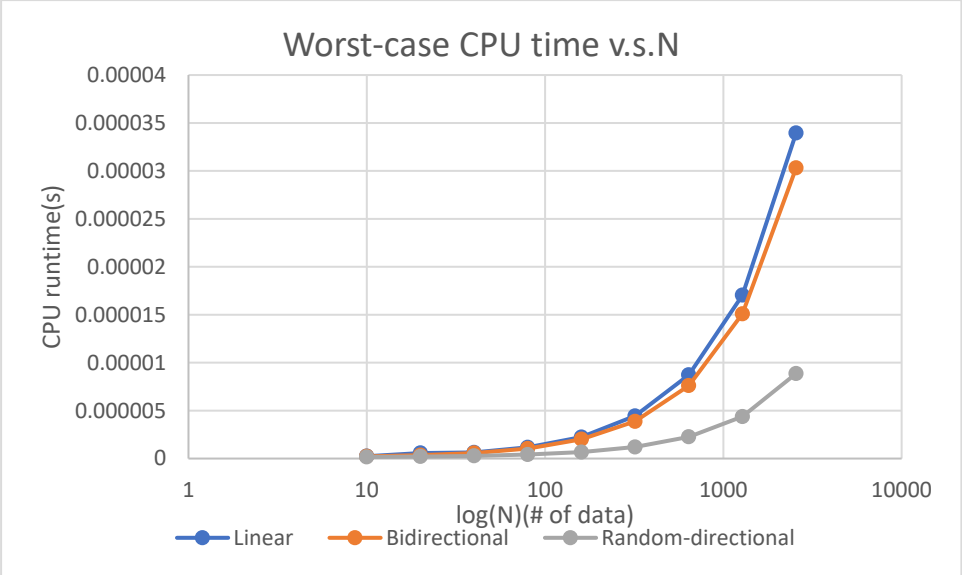
According to the graph in **Implementation**, **all 3 algorithms' time complexities are clearly O(n), since they are linear.**

Furthermore, according to this graph, we can conclude the speeds of the 3 algorithms, is **BDSearch > Linear Search > RDSearch (> means faster than)**. This result exactly meets my prediction. However, based on the results and the analysis, there's only a slightly difference between the speeds of the three algorithms.

The reason why Bidirectional search is the fastest, I think, is because: **having half of the iterations but contains double amount of statements (BDSearch) could run faster than full iterations with single statement (Linear Search)**. Bidirectional search runs through only n/2 iterations while Linear search runs though n iterations.

The reason why Random-directional search is the slowest, I think, is because it contains a random number generator and some if/else in it. Therefore, it is the slowest in the average case.

3. Worse-case comparisons:



According to the graph in **Implementation**, the **worst-case scenario fits our calculation of time complexity, which are all O(n)**.

The **worst case of Random-directional Search is much faster**. This is because we cannot make sure that which direction it will go, so we can only assign a target that is in the middle of the array for the algorithm to find. It is no different than the average-case.

I've done another worst-case scenario for Random-directional search, which is choosing the worst case depending on the random-generated direction. The result fits the observation in Average-case part and my calculation.

Conclusion:

1. Time complexities of the 3 algorithms:

	Linear	Bidirectional	Random-directional
Best	O(1)	O(1)	O(1)
Worst	O(n)	O(n)	O(n)
Average	O(n)	O(n)	O(n)

And they are verified by implementation on gcc on EE Workstation.

2. Actual runtime comparison (Average):
   BDSearch > Linear Search > RDSearch (> means runs faster)
3. Actual runtime comparison (Worst):
   RDSearch > BDSearch > Linear Search (> means runs faster)
4. Half iteration but contains double statement (BDSearch) could run faster than whole iteration with single statement (Linear Search).
5. The worst-case of Random-directional Search is not accurate.