

1. Introduction:

The analysis and comparisons of the four sorting algorithms: selection sort, insertion sort, bubble sort and shaker sort.

The goal of the algorithms is to sort a list of vocabulary into alphabetical order.

The input of the algorithms is a list of vocabulary, which is in a form of 2d array. The output of the algorithms is a list of sorted, alphabetically ordered vocabulary.

2. Approach:

a. Analysis:

Analysis of **Selection Sort**:

- i. **Intro:** Find minimum value, put it in the front of array.
- ii. **Proof:** Each iteration in the i th iterations, it brings the smallest value among the $n-1$ items to the front of the array. Therefore, the algorithm finds the values in ascending order, undoubtedly.

iii. Space complexity:

Fixed part: int: (i, j, k, n), *char: (tmp)

Variable part: *char (A) x n

Total: $n (*char) + 4$

iv. Time complexity:

Statement	s/e	Freq.	Total
1. Algorithm SelectionSort(A, n)	0	0	0
2. {	0	0	0
3. for i := 1 to n do {	n+1	1	n+1
4. j := i;	1	n	n
5. for k := i+1 to n do {	n+1	n	$n(n+1)$
6. if (A[k] < A[j]) then j = k	2 or 1	$n(n+1)$	$n(n)*2 \text{ or } 1$
7. }	0	0	0
8. tmp = list[i]; list[i] = list[j]; list[j] = tmp;	3	n	3n
9. }	0	0	0
10. }	0	0	0
Total			$3n^2 + 4n + 0$

*since avg of i is $n/2$, I calculate T(P) with $i = n/2$.

Best case: $2n^2 + 4n + 2$ ($O(n^2)$).

Worst Case: $3n^2 + 4n + 2$ ($O(n^2)$).

Average Case: $O(n^2)$.

Therefore, the time complexity of Selection sort is $O(n^2)$.

Analysis of **Insertion Sort**:

- i. **Intro:** Compare with the minimum value, if true, swap, put it in the front of array.
- ii. **Proof:** Like Selection sort, but Insertion sort gradually inserts the items from 2 to n into array in the front, in the right order.

iii. **Space complexity:**

Fixed part: int: (i, j, n), *char: (tmp)

Variable part: *char (A) x n

Total: n (*char) + 3

iv. **Time complexity:**

Statement	s/e	Freq.	Total
1. Algorithm InsertionSort(A, n)	0	0	0
2. {	0	0	0
3. for j := 2 to n do {	n-1	1	n-2
4. tmp := A[j];	1	n-2	n-2
5. i := j - 1;	1	n-2	n-2
6. while ((i >= 1) and (tmp < A[i])) do {	n (*)	n-2	n ² -2n
7. A[i + 1] := A[i]; i := i-1;	2	(n-2)(c)n	(2n-4)cn
8. }	0	(n-2)(c)n	0
9. A[i + 1] = tmp;	1	n-2	n-2
10. }	0	0	0
11. }	0	0	0
Total			(1+2c)n ²

*Assume >=, <, and are all performed. (I recall that if i<1, computer won't bother do the rest computation.)

Best case: if c = 0 or 1, it is mostly sorted, it won't do anything. (O(1))

Worst case: if c ~ n, it is almost decreasing, it is ~ (1+2c)n².(O(n²))

Average case: O(n²) since most of the case require sorting.

Therefore, the time complexity of Selection sort is O(n²).

Analysis of **Bubble Sort**:

- i. **Intro:** Swap whenever it's in the wrong order.
- ii. **Proof:** It swaps whenever two neighbors are not in the right order.
From ith iteration in 1 to n-1 iterations, it does the aforementioned behavior from n to i+1. This way, no single thing is in the wrong order.

iii. **Space complexity:**

Fixed part: int: (i, j, n), *char: (tmp)

Variable part: *char (A) x n

Total: n (*char) + 3

iv. **Time complexity:**

Statement	s/e	Freq.	Total
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1. Algorithm BubbleSort(A, n)	0	0	0
2. {	0	0	0
3. for i := 1 to n - 1 do {	n	1	n
4. for j := n to i + 1 step -1 do {	n	n-1	n^2-n
5. if (A[j] < A[j - 1]) {	1	n-1	n-1
6. tmp = A[j]; A[j] = A[j-1]; A[j-1] = tmp;	3	$c(n-1)$	$3c(n-1)$
7. }	0	0	0
8. }	0	0	0
9. }	0	0	0
10. }	0	0	0
Total			$n^2+...$

Best case: if $c = 0$ or 1 , the 2nd iteration does nothing, then it is $O(n)$.

Worst case: if $c \approx n$, it is almost decreasing, it is $\sim n^2+... (O(n^2))$

Average case: $O(n^2)$ since most of the case require sorting.

Therefore, the time complexity of Bubble sort is $O(n^2)$.

Analysis of **Shaker Sort**:

- i. **Intro:** Swap whenever it's in the wrong order but goes from both ends repeatedly.
- ii. **Proof:** Like Bubble sort, but it reduces the 1 to n cycle to half of it, and repeat the swapping from n to i+1 again, from i to n-1. Same as Bubble sort, no single thing is in the wrong order.
- iii. **Space complexity:**
Fixed part: int: (j, l, r, n), *char: (tmp)
Variable part: *char (A) x n
Total: $n (*char) + 4$

iv. **Time complexity:**

Statement	s/e	Freq.	Total
1. Algorithm ShakerSort(A, n)	0	0	0
2. {	0	0	0
3. l := 1; r := n;	2	1	2
4. while l <= r do {	n+1	1	n+1
5. for j := r to l + 1 step -1 do {	n	n	n^2
6. if (A[j] < A[j - 1]) {	c	n	cn
7. tmp = A[j]; A[j] = A[j-1]; A[j-1] = tmp;	3	n	3n
8. }	0	n	0
9. }	0	n	0
10. l = l + 1;	1	n	1n
11. for j := l to r - 1 do {	n	n	n^2
12. if (A[j] > A[j + 1]) {	c	n	cn
13. tmp = A[j]; A[j] = A[j+1]; A[j+1] = tmp;	3	n	3n
14. }	0	n	0
15. }	0	n	0
16. r := r - 1;	1	n	1n
17. }	0	0	0
18. }	0	0	0
Total			$2n^2+...$

Best case: if $c = 0$ or 1 , 2^{nd} iterations do nothing, then it is $O(n)$.

Worst case: if $c \approx 1/2n$, it is almost decreasing, it is $O(n^2)$.

Average case: $O(n^2)$ since most of the case require sorting, but it goes 2 times slower than bubble sort.

Therefore, the time complexity of Shaker sort is $O(n^2)$.

Comparing Table:

	Selection	Insertion	Bubble	Shaker
Space	$n (*\text{char}) + 4$	$n (*\text{char}) + 3$	$n (*\text{char}) + 3$	$n (*\text{char}) + 4$
Time	$O(n^2)$	$O(n^2)$	$O(n^2)$	$O(n^2)$

Prediction: Shaker > Bubble ? Insertion > Selection

The reason of Insertion > Selection is because the comparing step is lesser in Insertion sort. Shaker sort reduces the swapping steps by comparing from 2 ends.

The speed of the algorithms depends on whether writing data(swapping) or going through more steps is faster. Based on my learning in Computer Architecture, writing data is way slower.

Therefore, **my prediction is: Insertion > Selection > Shaker > Bubble.**

b. Implementation: (hw01.c on NTHUEE workstation, gcc 4.1.2)

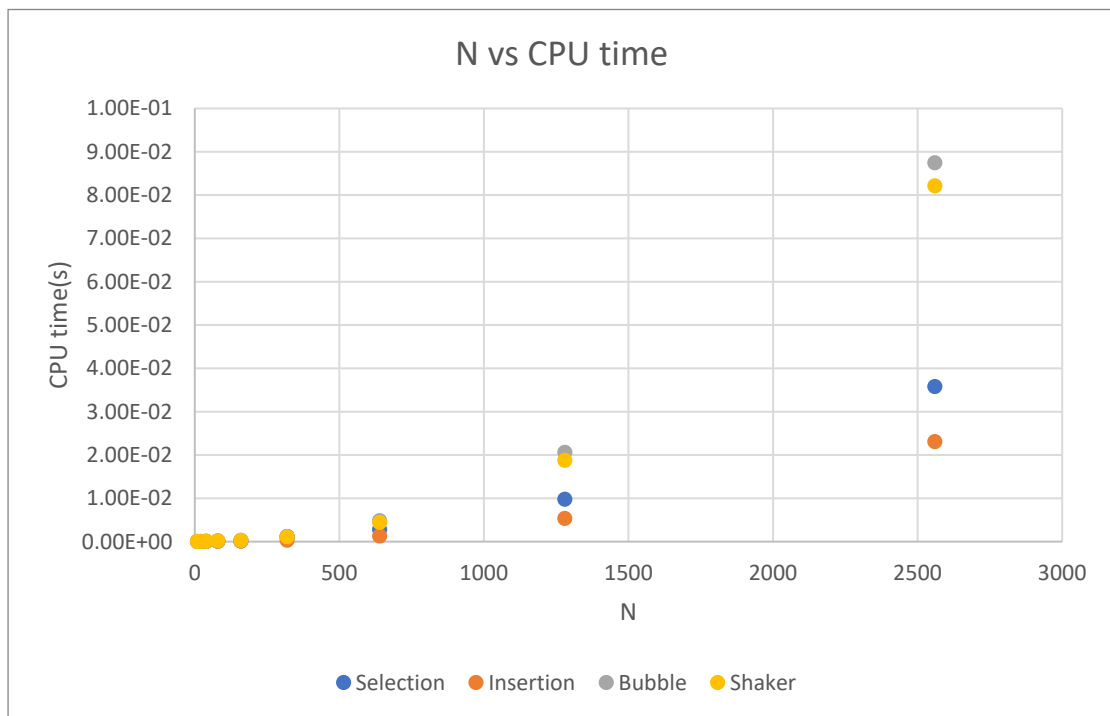
3. Result:

a. Result Table:

N	Selection	Insertion	Bubble	Shaker
10	9.90E-07	7.30E-07	1.11E-06	1.12E-06
20	6.79E-06	5.09E-06	1.00E-05	9.99E-06
40	2.33E-05	1.44E-05	3.96E-05	4.04E-05
80	8.60E-05	5.27E-05	1.64E-04	1.64E-04
160	1.33E-04	7.21E-05	2.77E-04	2.65E-04
320	1.14E-03	3.03E-04	1.13E-03	1.07E-03
640	2.76E-03	1.24E-03	4.81E-03	4.39E-03
1280	9.75E-03	5.32E-03	2.06E-02	1.87E-02
2560	3.58E-02	2.31E-02	8.74E-02	8.21E-02

b. Observation:

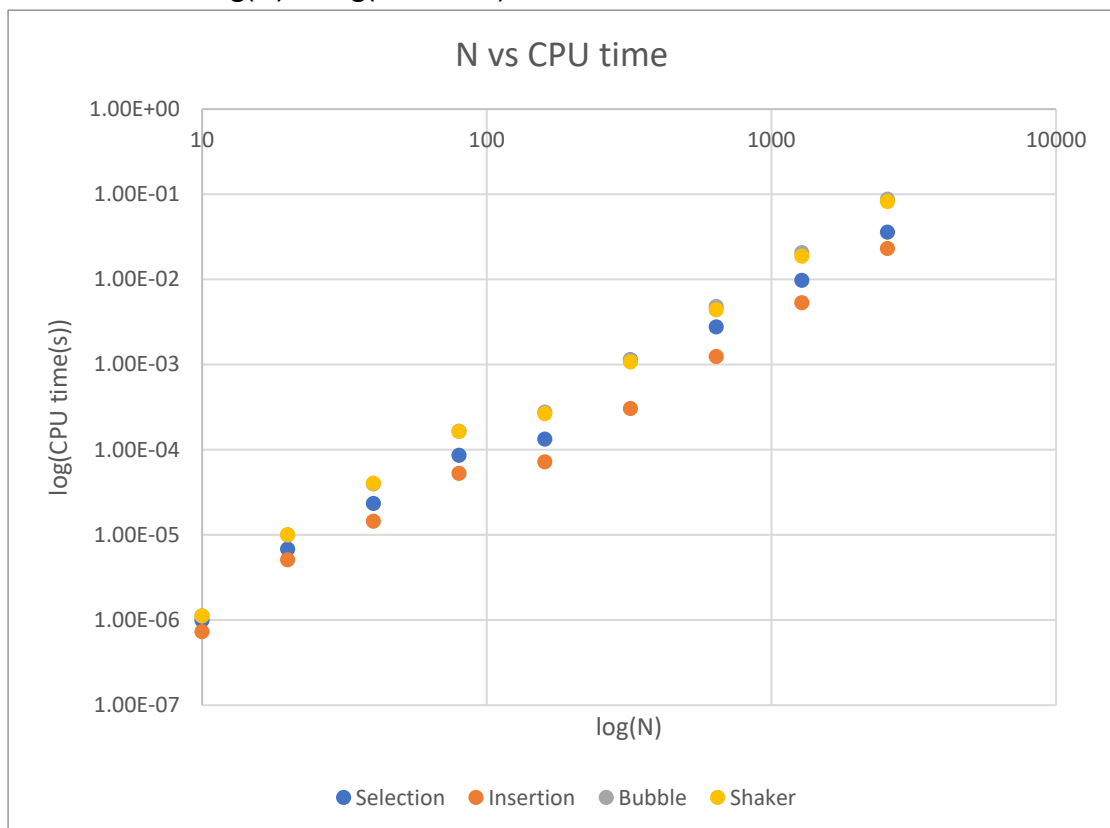
- N vs CPU time chart:



Speed: Insertion > Selection > Shaker > Bubble.

It seems that the result meets by prediction. Furthermore, the time complexities of the four algorithms are $O(n^2)$, same as those that I've calculated.

- Log(N) vs log(CPU time) chart:



When y-axis is logged, the graph looks linear. It indicates that they are $O(n^2)$.

Logged x-axis is just for good spacing between different N s.

*The $N=10$ & $N=40$ case's bubble sort is faster than shaker sort. They might just be the special case when N is small.

c. Conclusion:

- i. Experimented Speed: **Insertion > Selection > Shaker > Bubble.**
- ii. Time Complexity: **Insertion = Selection = Shaker = Bubble = $O(n^2)$.**
- iii. Best Case: **Insertion > Shaker = Bubble > Selection.**
- iv. Space Complexity: **Insertion = Selection = Shaker = Bubble = $\theta(1)$**
- v. **Swapping (data writing) is much slower than comparing.**