Data Structures

CH3 Stacks & Queues

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NTHU EE

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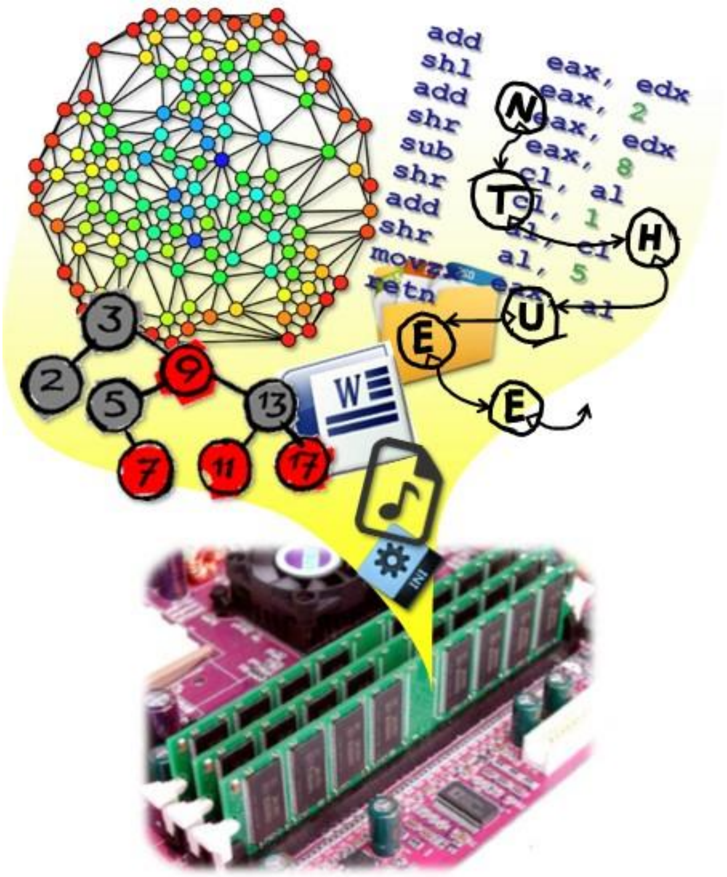

Outline

- **3.1 Templates in C++**
- 3.2 The stack ADT
- 3.3 The queue ADT
- 3.4 Subtyping and inheritance in C++
- 3.5 A mazing problem
- 3.6 Evaluation of expressions

Observations

- Many codes look the same for different types
    - Sorting functions that handle
        - 32-bit integers
        - 64-bit integers
        - float
        - …
    - Sparse matrix classes that handle
        - 32-bit integers
        - 64-bit integers
        - float
        - …

Non-Template Solutions

- Implement the same behavior over and over
  - Hard to maintain code
  - Hard to globally modify code

- Write general code for a common base type
  - Lose the benefits of compiler's type checking
  - Incurs overhead

- Use macros (#define)
  - Sacrifice readability
  - Sacrifice debuggability

Template

- Template can be instantiated to any data type
  - So called "parameterized types"

- C++ language supports
  - Template functions
  - Template classes

Template Function Example

```
void SelectionSort (int *a , const int n )
{
    for (int i = 0 ; i < n ; i++ )
    {
        int j = i;
        for ( int k = i + 1 ; k < n ; k++ )
            if ( a[k] < a[j] ) j = k;
                swap ( a[i], a[j] );
    }
}
```

```
template <class T>
void SelectionSort (T *a , const int n )
{
    for (int i = 0 ; i < n ; i++ )
    {
        int j = i;
        for ( int k = i + 1 ; k < n ; k++ )
            if ( a[k] < a[j] ) j = k;
                swap ( a[i], a[j] );
    }
}
```

- template <class T> is identical to template <typename T>
- It is a convention to use "T", but one can use any other name

Bag Class (for integers)

```
class Bag
{
public:
    Bag ( int bagCapacity = 10 );     // constructor
    ~Bag( );                          // destructor
    int Size( ) const;        // return number of elements in bag
    int Element( ) const;     // return an element that is in the bag
    void push(const int);     // add an integer into the bag
    void pop();               // delete an integer in the bag

private:
    int *array;
    int capacity;    // capacity of array
    int top;    // array position of top element
};
```

const member function

Specifies that the function does not modify the object for which it is called.

const Bag emptyBag;
emptyBag.size();   //valid
emptyBag.push(1); //error

Bag Class (for integers)

```
Bag::Bag (int bagCapacity)
:capacity ( bagCapacity )
{
    if ( capacity < 1 )
        throw "Capacity must be > 0";

    array = new int [ capacity ];
    top = -1;
}

Bag::~Bag ( )
{ delete [] array; }

inline int Bag::Size( ) const
{ return top + 1; }
```

Initialization list

initialize member variables when they are created rather than afterwards

Bag Class (for integers)

```
inline int Bag::Element ( ) const
{
    if ( IsEmpty ( ) )
        throw “Bag is empty”;

    return array [0]; // always return 0th element
}

void Bag::Push (const int x)
{
    if (capacity == top + 1) {
        ChangeSize1D (array, capacity, 2 * capacity);
        capacity *= 2;
    }
    array[++top] = x;
}
```

Template Bag

```
template<class T>
class Bag
{
public:
    Bag( int bagCapacity = 10 );     // constructor
    ~Bag( );                          // destructor
    int Size( ) const;   // return number of elements in bag
    T Element( ) const; // return an element that is in the bag
    void push(const T&); // add an integer into the bag
    void pop();
private:
    T *array;
    int capacity;        // capacity of array
    int top;             // array position of top element
};
```

Template Bag

```
template<class T>
Bag<T>::Bag(int bagCapacity) : capacity (bagCapacity)
{
    if (capacity < 1)
        throw “Capacity must be > 0”;
    array = new T [capacity];
    top = -1;
}

template <class T>
Bag<T>::~Bag( )
{delete [ ] array;}

template <class T>
inline int Bag<T>::Size( ) const
{ return top + 1; }
```

Template Bag

```
template <class T>
inline T Bag<T>::Element( ) const
{
    if ( IsEmpty() )
        throw “Bag is empty”;
    return array [0];
}

template <class T>
void Bag<T>::Push(const T& x)
{
    if (capacity == top + 1) {
        ChangeSize1D (array, capacity, 2 * capacity);
        capacity *= 2;
    }
    array [++top];
}
```

Use of the Template Bag

```
int main()
{
   Bag<int> myIntBag;
   myIntBag.push(1);
   myIntBag.push(9);
   cout << myIntBag.size << endl;
   cout << myIntBag.element();

   Bag<float>  myFloatBag;
   for(int i=0; i<10; i++)
       myFloatBag.push(1.0/i);

   Bag<Bag<int> > myManyIntBag;

   myManyIntBag.push(myIntBag);
   …
   return;
}
```

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- **3.3 The queue ADT**
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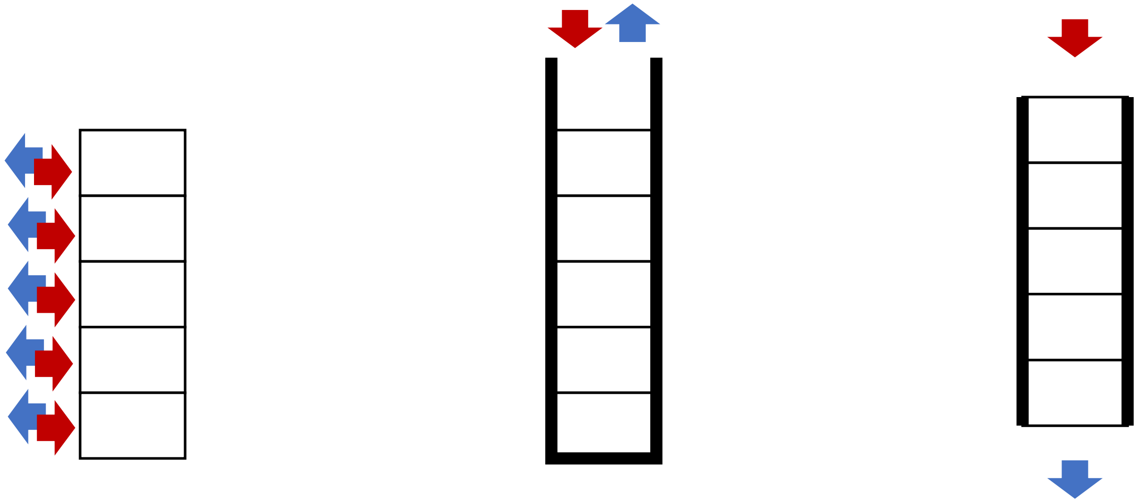
Stacks and Queues

- Two frequently used data structures
- They are special cases of the more general data structure type, lists



List
(Random add/del)

Stack
(Last In First Out)

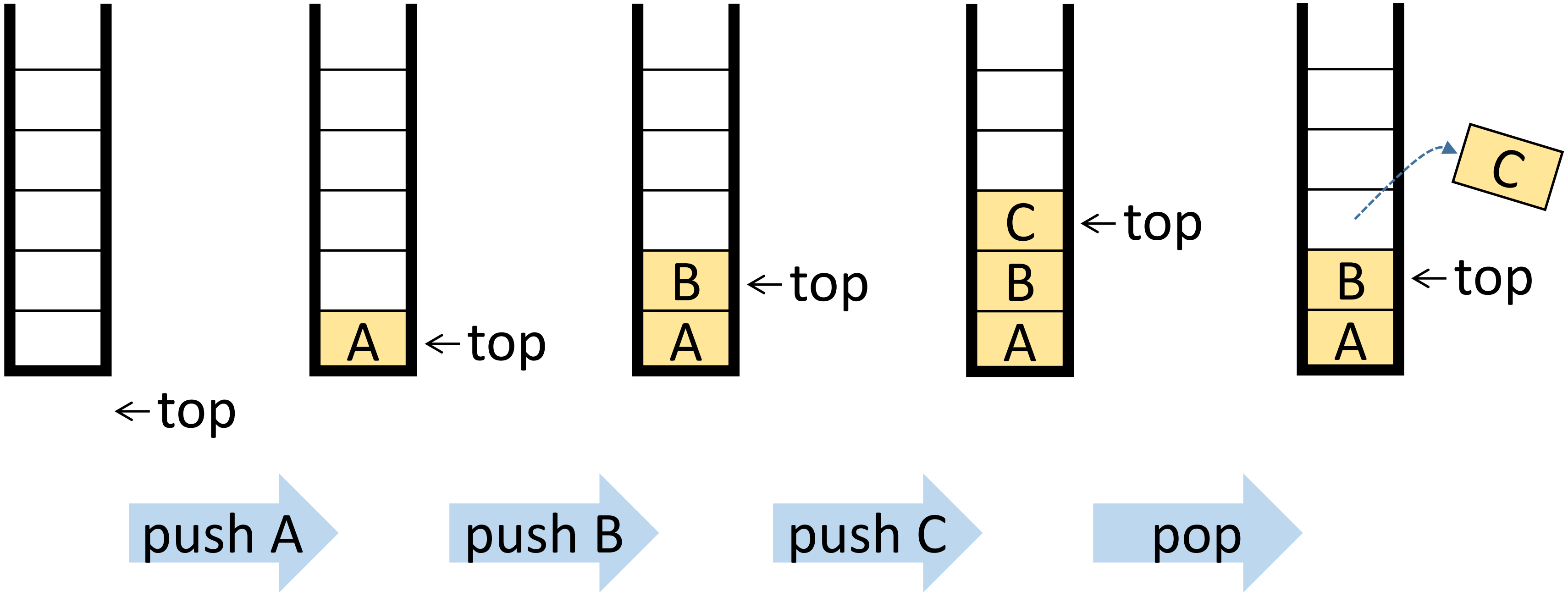
Queue
(First In First Out)

Stack and Queue ADTs

```
template < class T >
class Stack
{
public:
    Stack (int stackCapacity = 10);

    bool IsEmpty( ) const;

    void Push(const T& item);
    // add an item into the stack

    void Pop( );
    // delete an item
    …

};
```

```
template < class T >
class Queue
{
public:
    Queue (int queueCapacity = 0);

    bool IsEmpty( ) const;

    void Push(const T& item);
    // add an item into the queue

    void Pop( );
    // delete an item
    …

} ;
```

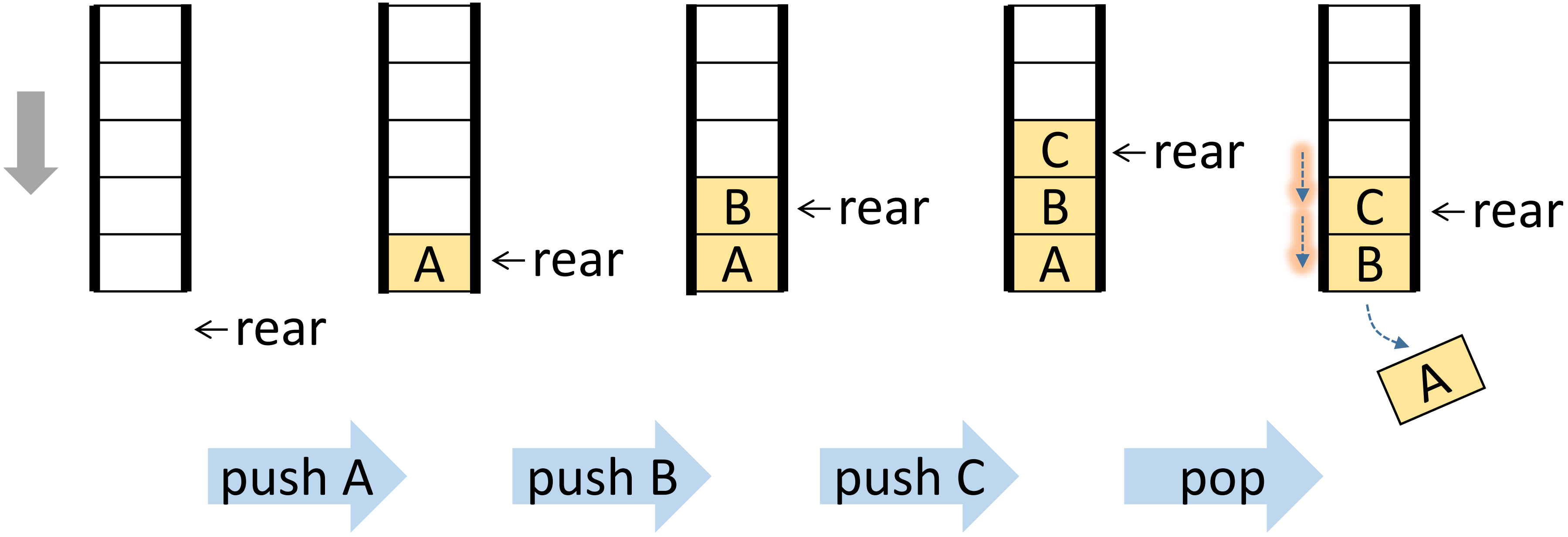
Stack



Time complexity

- push(): Θ(1)
- pop(): Θ(1)

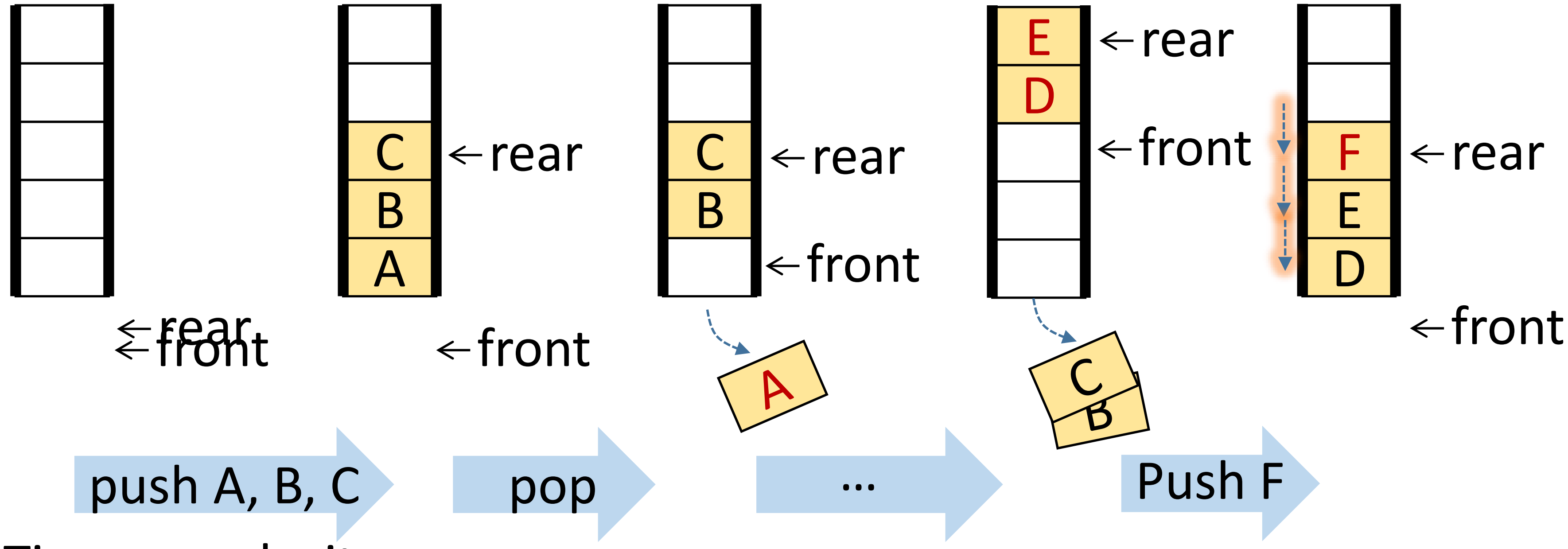
Queue (Single Pointer)



Time complexity

- push(): Θ(1)
- pop(): Θ(size)

Queue (Dual Pointers)

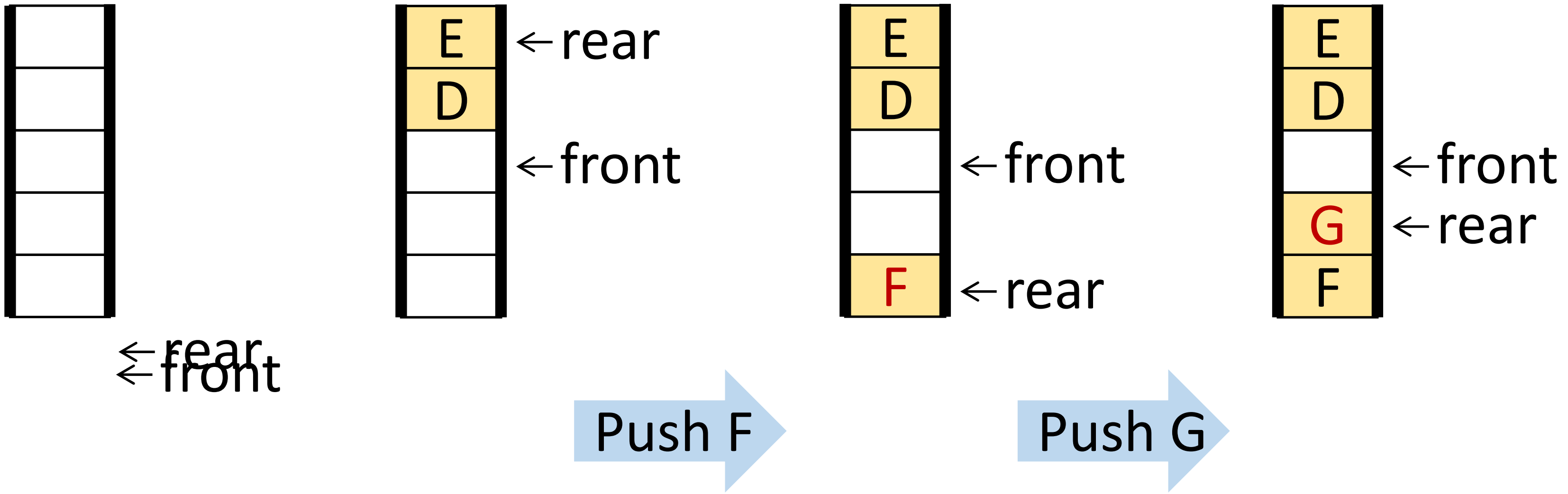


Time complexity

- push(): O(size)
  - When the rear pointer reaches the boundary and a push occurs, data need to be moved
- pop(): Θ(1)

Circular Queue

- Permit the queue to wrap around the end space



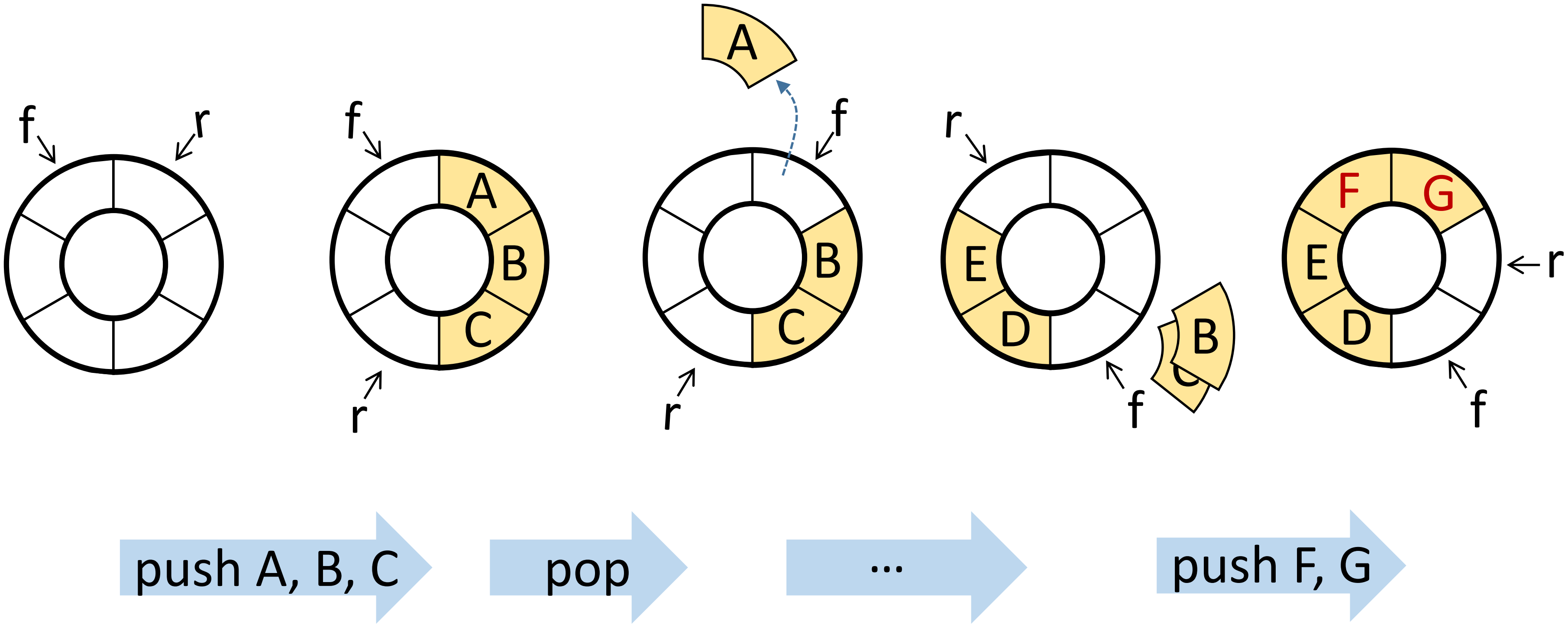
Time complexity

- push(): Θ(1)
- pop(): Θ(1)

Note that in this version of circular buffer, the position that the front pointer points to is a dead space. A slot is deliberately unused.

- Otherwise, we cannot determine whether the queue is empty or full.

Circular Queue (Circular Illustration)



push A, B, C

pop

…

push F, G

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Relationships Between Things

- We abstract things on two key dimensions
  - IS-A relationship
  - HAS-A relationship

- Real world examples
  - iPhone is a smartphone. iPhone has a battery
  - NTHU is a university. NTHU has a Math department

- ADT examples
  - Rectangle is a Polygon. Rectangle has a *height* dimension
  - Stack is a Bag. Stack has a *top* pointer
    - Stack is a specialized bag that requires elements to be deleted in the LIFO order

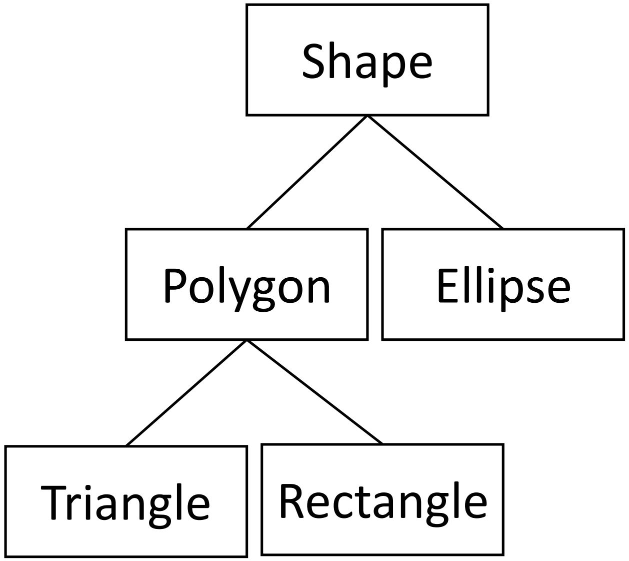
Subtype / IS-A / Subclass

- Subtype
  - Equivalent concept to the IS-A relationship
    - Rectangle is a subtype of Polygon
  - Since C++ use *classes* to denote data types, subtypes are also widely referred to as subclasses

- Subtype is conceptual relationship between ADT specifications
  - "Stack IS A Bag" is true regardless of the implementation

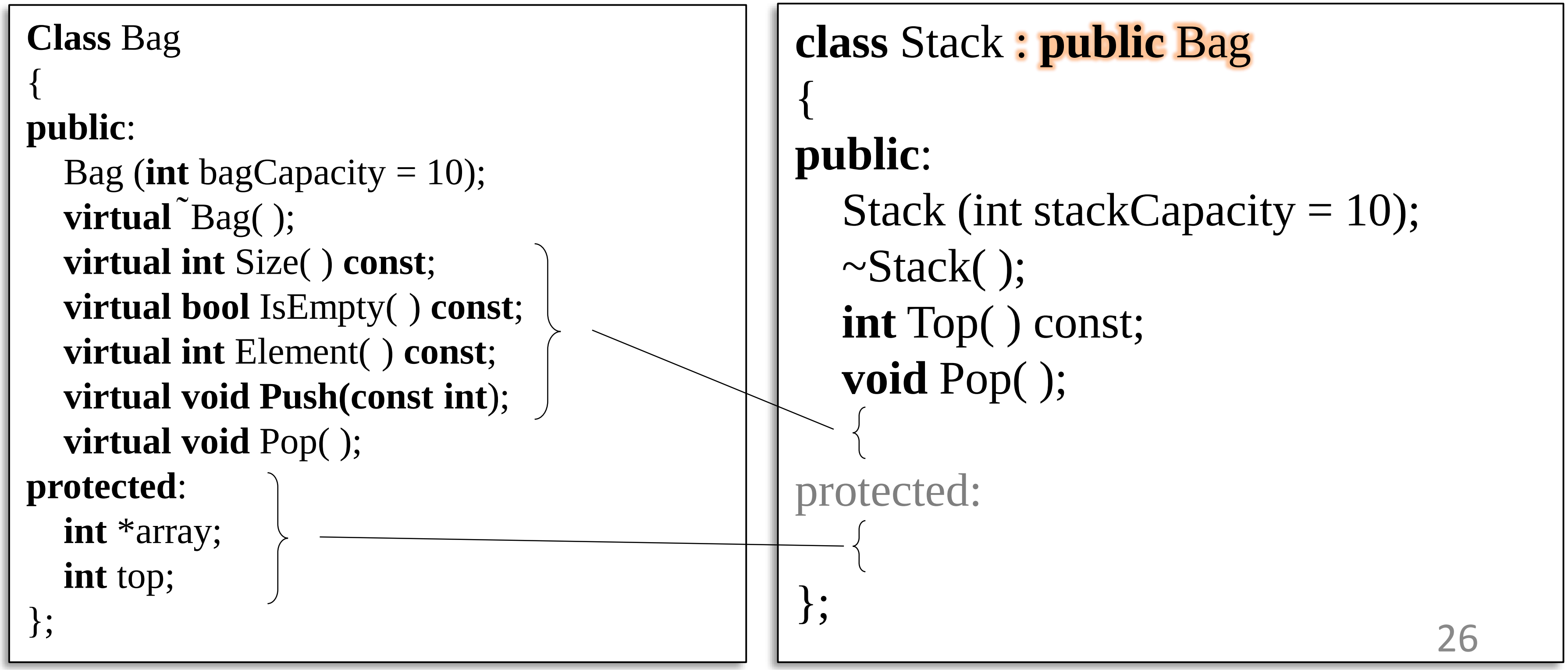
Inheritance

- Use
  - Express IS-A relationships between classes
  - Derive a new class (derived class / sub type / sub class) from an existing class (base class)
- Objective
  - Eliminate redundant implementation
    - Members (data and functions) are by default inherited from a base class to a derived class
- Different inheritance styles
  - Public inheritance
    - Access levels (public/protected/private) of the members are also inherited
  - Protected inheritance
  - Private inheritance



Effects of Inheritance

- Stack inherits from Bag
  - Stack must redefine its constructors and destructors
  - Stack can redefine its unique data and functions (pop and top)
  - Stack inherits all the other data and functions of Bag



```
Class Bag
{
public:
    Bag (int bagCapacity = 10);
    virtual ˜Bag( );
    virtual int Size( ) const;
    virtual bool IsEmpty( ) const;
    virtual int Element( ) const;
    virtual void Push(const int);
    virtual void Pop( );
protected:
    int *array;
    int top;
};
```

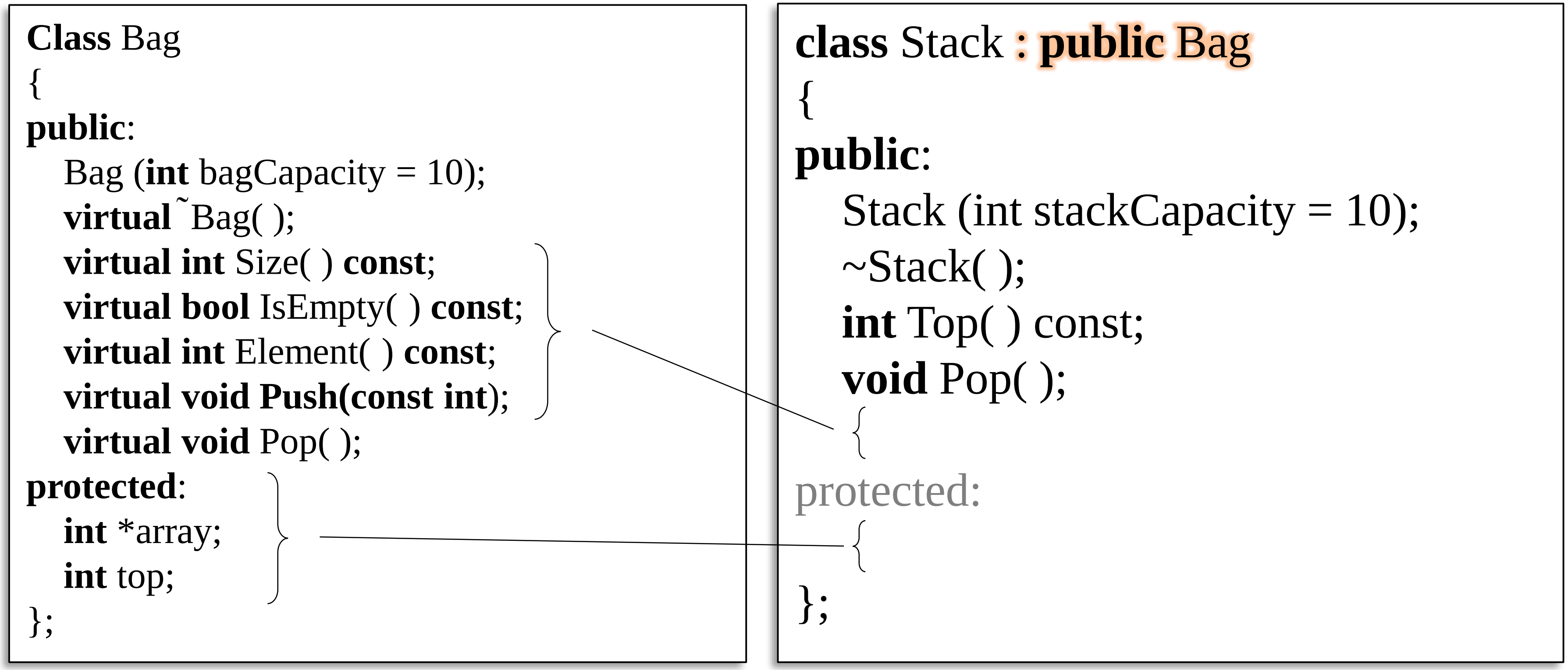
```
class Stack : public Bag
{
public:
    Stack (int stackCapacity = 10);
    ~Stack( );
    int Top( ) const;
    void Pop( );

protected:

};
```

Usage Example of Derived Classes

```
Bag b(4);    // invoke Bag constructor
Stack s(7);  // invoke Stack constructor, which also invokes Bag constructor
b.Push(2017);   // use Bag::Push()
s.Push(330);     // Stack does not contains a specialized Push(), so use Bag::Push
b.Pop();       // use Bag::Pop()
s.Pop();       // Stack contains a specialized Pop() overriding Bag::Pop(), so use Stack::Pop()
```



```
Class Bag
{
public:
    Bag (int bagCapacity = 10);
    virtual ˜Bag( );
    virtual int Size( ) const;
    virtual bool IsEmpty( ) const;
    virtual int Element( ) const;
    virtual void Push(const int);
    virtual void Pop( );
protected:
    int *array;
    int top;
};
```

```
class Stack : public Bag
{
public:
    Stack (int stackCapacity = 10);
    ~Stack( );
    int Top( ) const;
    void Pop( );
    {
protected:
    {
};
```

Syntax of Implementing Derived Classes

```
Stack::Stack(int stackCapacity)
: Bag(stackCapacity)
// explicitly call to the Bag constructor that has arguments
{
     // here is code specifically for creating a stack, if any
}
int Stack::Stack( )
{      // here is code specifically for destroying a stack, if any
}
//Bag destructor is automatically called when a stack is destroyed

int Stack::Top( ) const
{
        if (IsEmpty( )) throw "Stack is empty.";
        return array[top];
}
void Stack::Pop( )
{
        if (IsEmpty( )) throw "Stack is empty. Cannot delete.";
        top--;
}
```

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Evaluation of Expressions

- Arithmetic expressions
  - X = (A / B) – C + D * E – A * C
- Boolean expressions
  - X = (A == B) || !(C>D)
- Expressions are made up of
  - Operands: A, B, C, D, E
  - Binary arithmatic operators: +, -, *, /, %
  - Unary arithmatic operators: -
  - Relational operators: <, <=, ==, !=, >=, >
  - Binary logical operators: &&, ||
  - Unary logical operators: !
  - Delimiters: (,)

Evaluation of Expressions

- Let's focus on an arithmetic expression
  - X = A / B – C + D * E – A * C

- Order of operations matters
  - Let A = 4, B = C = 2, D = E = 3
  - ((4/2)–2)+(3*3)–(4*2) = 0 + 9 – 8 = 1
  - (4/(2–2+3))*(3–4)*2 = (4/3)*(–1)*2 = -2.666…

- How can computers uniquely define the order of an expression?

Priority of Operators

- Priority is introduced to help defining the order

High

Priority	Operator
1	Unary minus (負號), !
2	*, /, %
3	+, -
4	<, <=, >=, >
5	= =, !=
6	&&
7	| |

* and / have higher priority than + and -

  - Tie break rule: left to right

- Example

Two operators compete for one operand

'/' and '*' win

  - A / B – C + D * E – A * C → (A/B) – C + (D*E) – (A*C)
  - A/B*C/D → ((A/B)*C)/D *Tie-break rule*

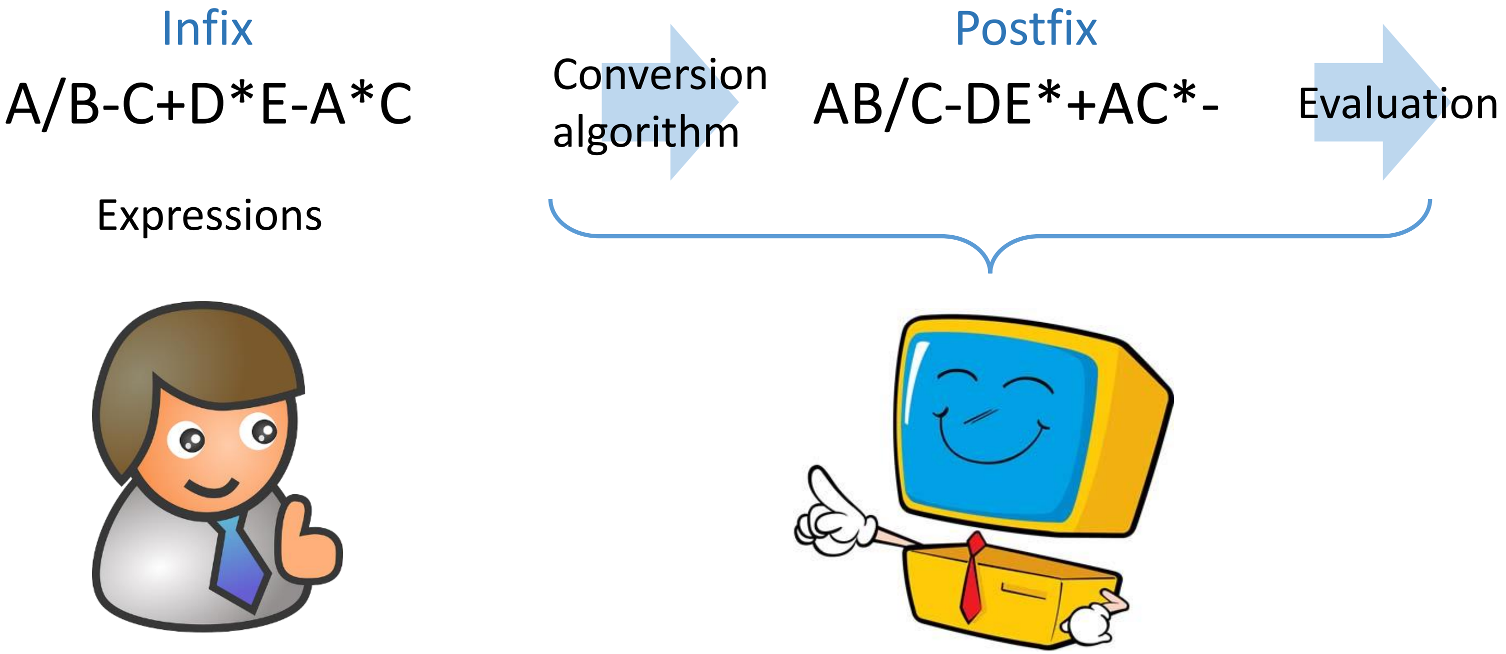
Infix and Postfix Notations

- Infix
  - Binary operators come in-between their operands
  - A*B/C
- Postfix
  - Binary operators appear after their operands
- Examples

• Infix:	A*B	A/B-C+D*E-A*C
• Postfix:	AB*	AB/C-DE*+AC*-

Two Essential Algorithms

- Combining two algorithms enables computers to handle human-written expressions
  - Infix-to-Postfix conversion
  - Postfix evaluation (just mentioned)



Infix

A/B-C+D*E-A*C

Expressions

Conversion algorithm

Postfix

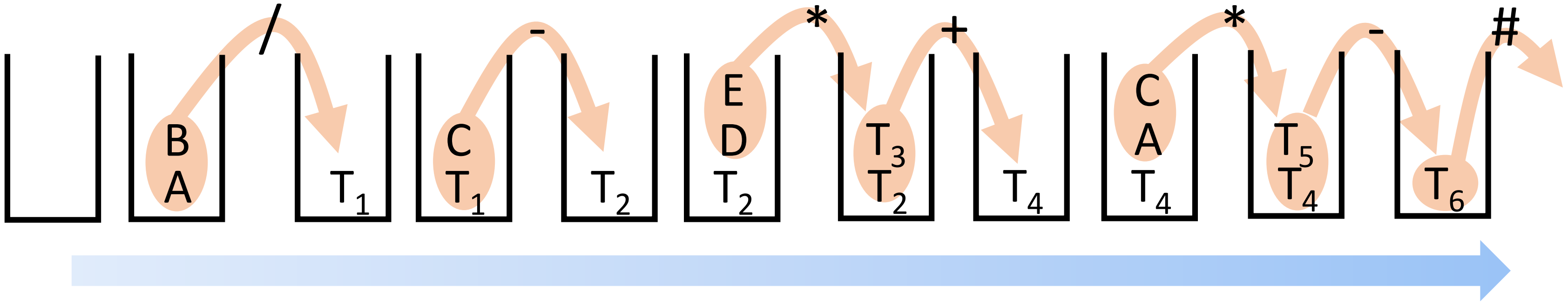
AB/C-DE*+AC*-

Evaluation

Postfix Evaluation

- Rules
  - Left to right scan
  - Push operands onto a stack
  - Evaluate operators using the required number of operands from the stack
  - Push the evaluating results onto the stack again

- AB/C-DE*+AC*-# *(# denotes the end of an expression)*



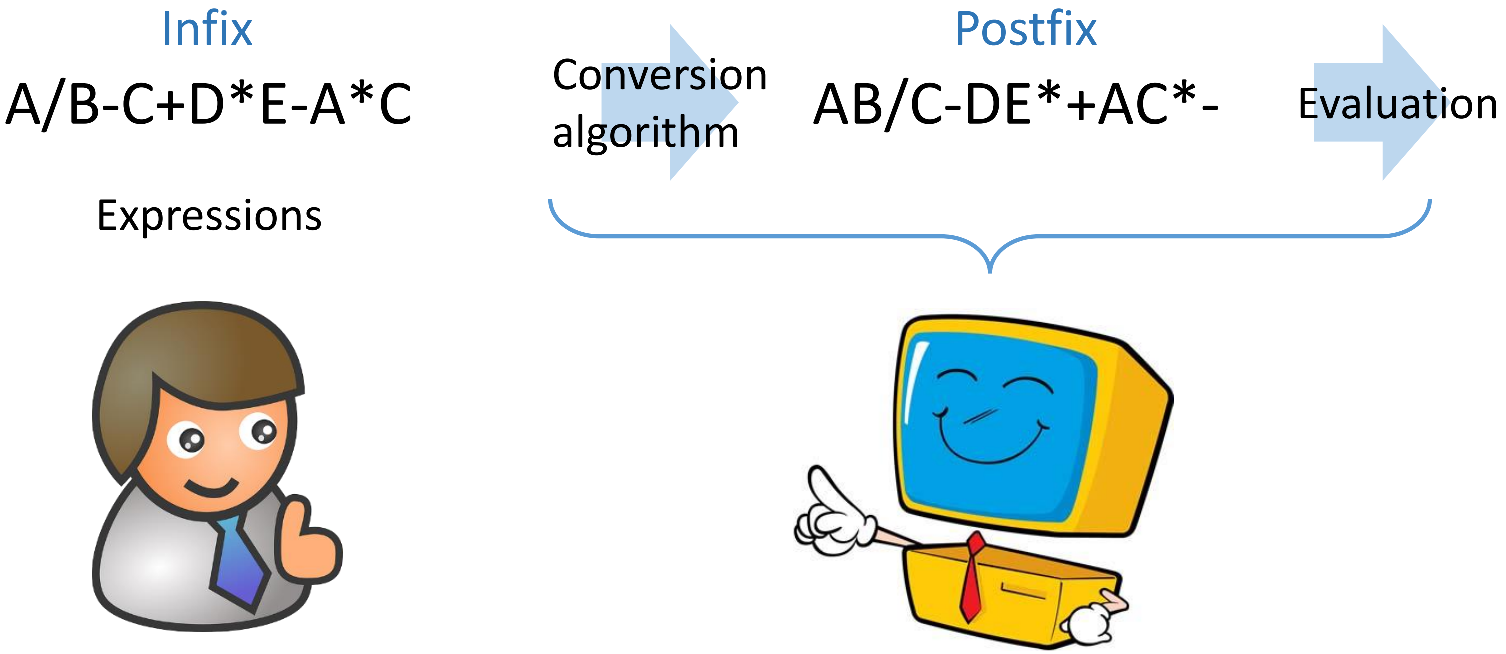
Advantages of Postfix Notation

- Evaluation is simpler than infix notation
  - The need for parenthesis is gone
  - The need for operator priority is gone

```
void Eval(Expression e)
{
    Stack<Token> stack;    // initialize a stack
    for (Token x = NextToken(e); x!= end of expression; x=NextToken(e))
    {
        if (x is an operand) {
            stak.Push(x)
        } else {// x is an operator
            pop from the stack the correct number of operands for the operator;
            perform the operation x and store the result (if any) onto the stack;
    }
}
```

Two Essential Algorithms

- Combining two algorithms enables computers to handle human-written expressions
  - Infix-to-Postfix conversion
  - Postfix evaluation (just mentioned)



Infix

A/B-C+D*E-A*C

Expressions

Conversion algorithm

Postfix

AB/C-DE*+AC*-

Evaluation

Infix to Postfix Conversion

- Observations
  - Number of operands and operators do not change
  - Order of operands (A, B, C…) do not change

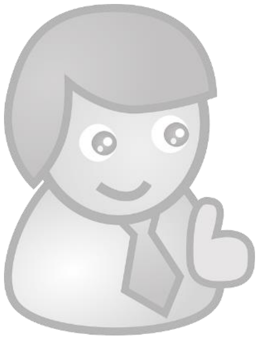
Infix

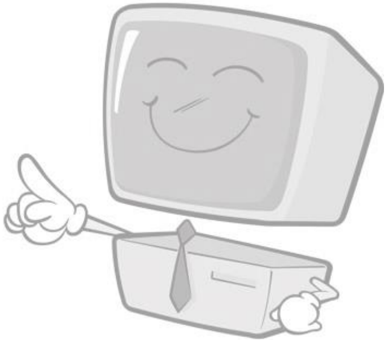
A/B-C+D*E-A*C

Conversion algorithm

Postfix

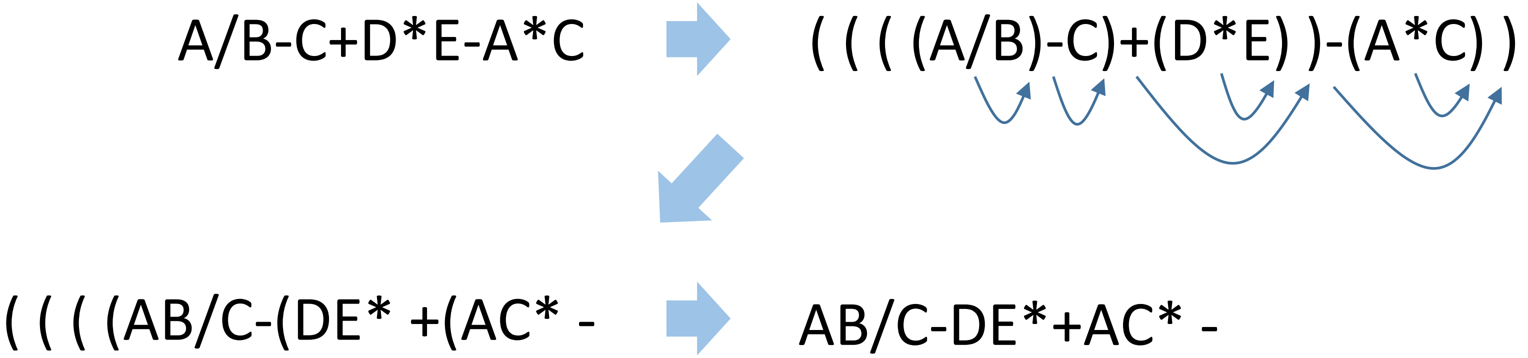
AB/C-DE*+AC*-





Infix to Postfix Conversion

- Method 1
  - Fully parenthesize the expression (based on the operator priorities)
  - Move all operators so that they replace their corresponding right parentheses
  - Delete all parentheses



A/B-C+D*E-A*C

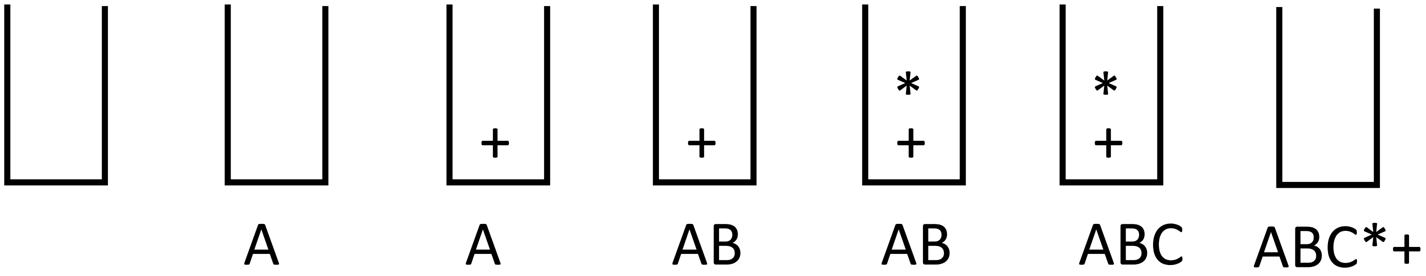
((((A/B)-C)+(D*E))-(A*C))

((((AB/C-(DE* +(AC* -

AB/C-DE*+AC* -

Infix to Postfix Conversion

- Stack-based algorithm
  - Create a stack
  - Scan the input infix expression left to right
  - Bypass each incoming operand to the output
  - For each incoming operator
    - First, continuously pop from the stack an operator (the top) if the top has equal or lower priority than the incoming operator
    - Then, push the incoming operator onto the stack
  - Pop all operators upon the end of an expression
- Example: A + B * C



Parentheses Handling

- We want the stack algorithm to handle parentheses similarly to handling operators
- Specialized rules for left parenthesis
  - Incoming left parenthesis has the highest priority (i.e., always gets pushed onto the stack)
    - In-coming priority (ICP) = 0
  - Only gets popped from the stack upon a matched right parenthesis
  - Otherwise, behaves as one with the lowest priority
    - In-stack priority (ISP) = 8

Priority	Operator
0	In-coming (
1	Unary minus (負號), !
2	*, /, %
3	+, -
4	<, <=, >=, >
5	= =, !=
6	&&
7	||
8	In-stack (

Example

- A*(B+C)/D

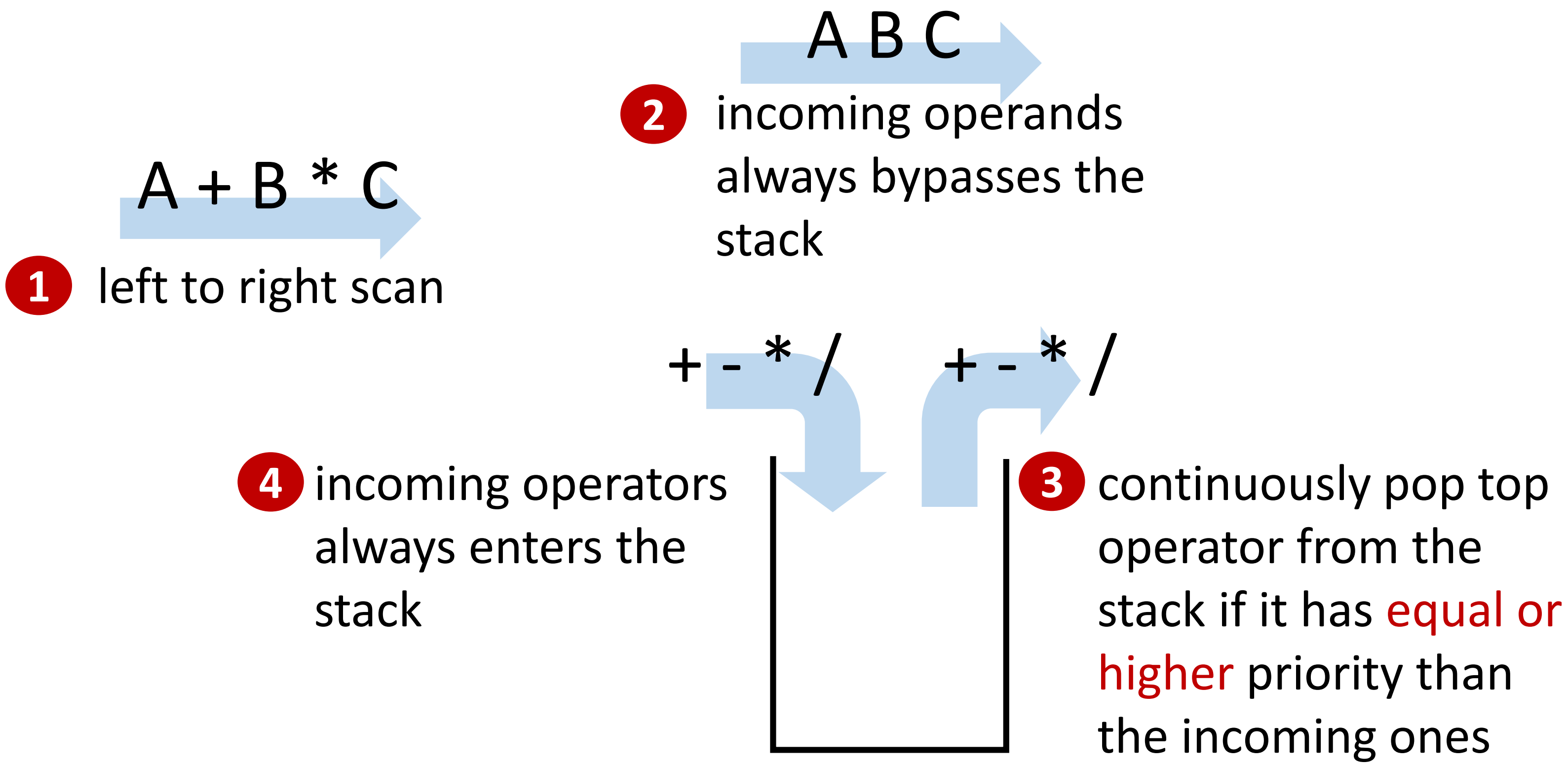
Incoming token	Stack	Output	Note
Empty	Empty	Empty	
A			
*			
(			
B			
+			
C			
)			
/			
D			
Done			

Example

- A*(B+C)/D

Incoming token	Stack	Output	Note
Empty	Empty	Empty	
A	Empty	A	Bypass operands
*	*		
(	*(		ICP('(') higher than ISP('*')
B	*(	AB	Bypass operands
+	*(+		ICP('+') higher than ISP('(')
C	*(+	ABC	Bypass operands
)	*	ABC+	Pop until a left parenthesis
/	/	ABC+*	ICP('/') == ISP('*')
D	/	ABC+*D	Bypass operands
Done	Empty	ABC+*D/	Pop all operators

Recap Infix to Postfix Conversion



A + B * C

1 left to right scan

A B C

2 incoming operands always bypasses the stack

+ - * /

+ - * /

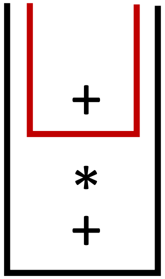
3 continuously pop top operator from the stack if it has equal or higher priority than the incoming ones

4 incoming operators always enters the stack

Recap Parenthesis Handling

- Incoming left parenthesis has the highest priority
  - It always enters the stack without popping any stacked operator
- In-stack left parenthesis has the lowest priority
  - It never gets popped from the stack until the right parenthesis appears
- Different perspective [1]
  - Left parenthesis creates an isolated, nested stack
  - Right parenthesis cleans up a nested stack

A+B*(C+D)



[1] Contributed by Mr. 陳德暉 (101061132) on April 2, 2015

Infix to Postfix Algorithm

```
void Postfix(Expression e)
{
    Stack<Token>stack; // initialize the stack
    stack.Push('#');
    for (Token x = NextToken(e); x != '#'; x = NextToken(e))
        if (x is an operand) cout << x;
        else if (x == ')' ) { // pop until a left parenthesis
            for (;stack.Top( ) != '('; stack.Pop( ))
                cout << stack.Top( );
            stack.Pop( ); // remove the left parenthesis
        } else { // x is a operator
            for (; isp(stack.Top( )) <= icp(x); stack.Pop( ))
                cout << stack.Top( );
                                            // higher or equal priority
            stack.Push(x);
        }
    // end of expression; empty the stack
    for ( ; !stack.IsEmpty( ); cout << stack.Top( ), stack.Pop( ));
    cout << endl;
}
```

Limitations of the Current Algorithm

- Characters to tokens conversion (parser)
  - Energy = Mass * LightSpeed * LightSpeed
  - Area = 3.14*radius1*radius2
- Grammar
  - X = A - B + -A
    computers need rules to differentiate the two minus symbols; Otherwise, the aforementioned postfix algorithm cannot work correctly.
- More techniques are available in a compiler course

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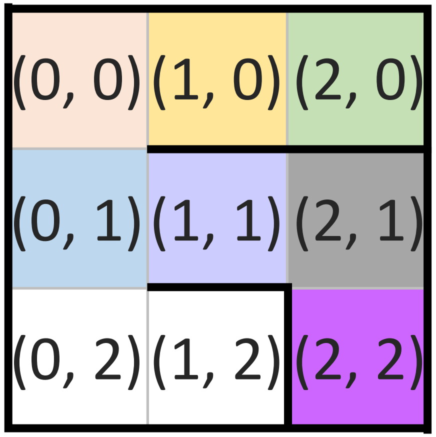
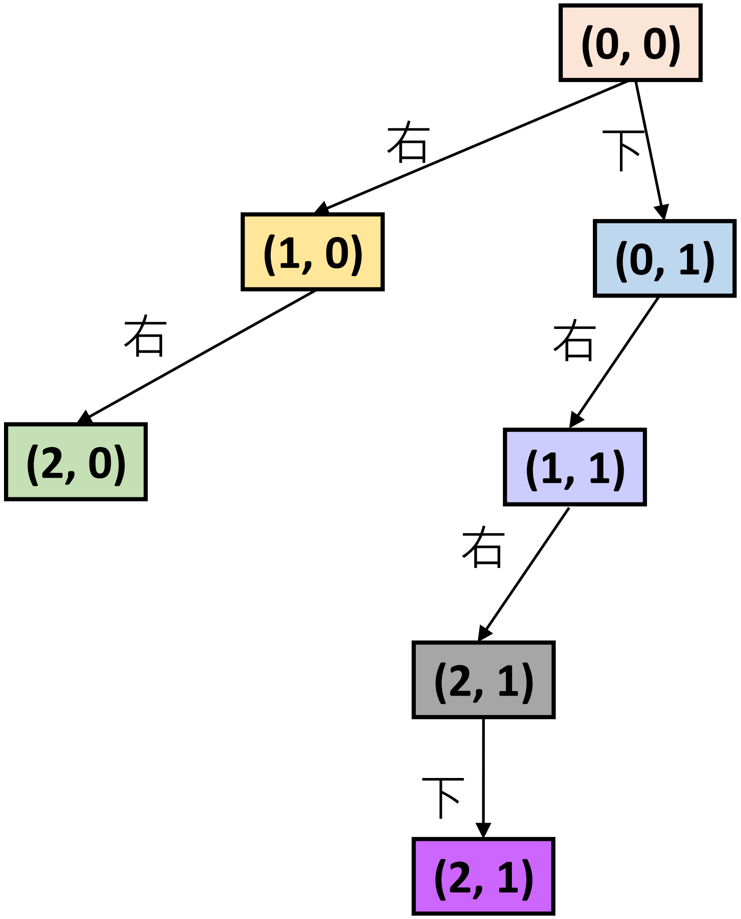
How a Computer Traverses a Maze



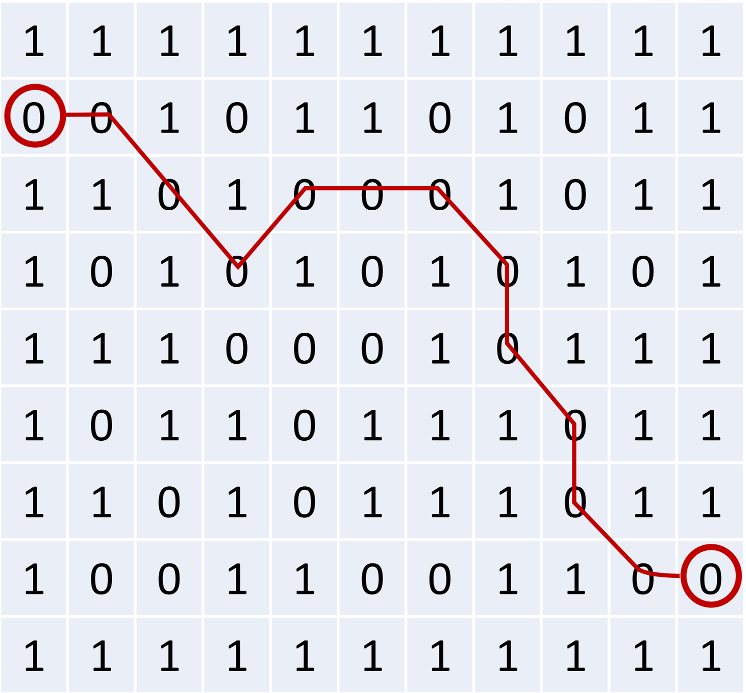
(0, 0)	(1, 0)	(2, 0)
(0, 1)	(1, 1)	(2, 1)
(0, 2)	(1, 2)	(2, 2)

右 > 下 > 上 > 左

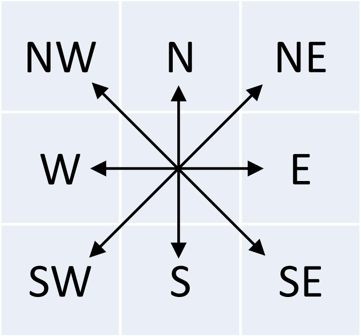


A Mazing Problem

- 2D array maze representation
  - '1' implies a blocked direction
  - '0' means otherwise
  - Borders are surrounded by '1'
- Allowable moves
  - Non-blocked squares of the eight neighboring squares
- How can a program get through the maze?



1	1	1	1	1	1	1	1	1	1	1
0	0	1	0	1	1	0	1	0	1	1
1	1	0	1	0	0	0	1	0	1	1
1	0	1	0	1	0	1	0	1	0	1
1	1	1	0	0	0	1	0	1	1	1
1	0	1	1	0	1	1	1	0	1	1
1	1	0	1	0	1	1	1	0	1	1
1	0	0	1	1	0	0	1	1	0	0
1	1	1	1	1	1	1	1	1	1	1



Algorithm (Pseudo Code)

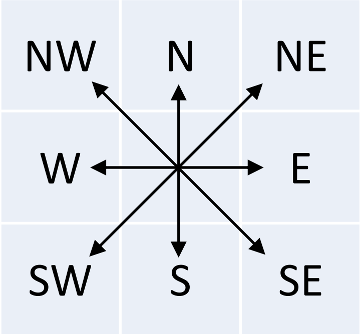
```
struct Offsets
{
    int di, dj;
}

enum directions {N, NE, E,
SE, S, SW, W, NW};

Offsets move[8];

struct Items
{
    int x, y, dir;
}
```

q	move[q].di	move[q].dj
N	−1	0
NE	−1	1
E	0	1
SE	1	1
S	1	0
SW	1	−1
W	0	−1
NW	−1	−1



Algorithm ()

```
initialize a stack // remember the point to retract
add the starting point, (0, 1, E), to the stack

while (the stack is not empty) { // there are still unexplored points
  (i, j, dir) = the top of the stack;
  remove the top of the stack;
  while(there are more move from (i, j)){
    (g, h) = nextPoint((i, j), dir);
    if ((g == m) && (h == p)) return success;

    if ((!maze [g][h]) && (!mark [g][h])) {
        dir = Next(dir);
        add (i, j, dir) to the stack;  // prepare for a dead end

        (i, j, dir) = (g, h, N); // move to (g, h), start from dir N
        mark[i][j] = 1;
    }
  }
}
cout << "No path in maze." << endl;
```

- Each position can be visited at most once.
- At most eight valid moves from each position

→ O(size of the array) time

Algorithm (Pseudo Code)

```
void driver()
{
   if (findPath(0, 1))
       cout << “Success" << endl;
   else
       cout << "No path in maze." << endl;
   return;
}
bool findPath(int i, int j) // find a path starting from (i, j)
{
    for (all eight directions) {  // explore all directions
        (g, h) = (i, j) + direction;

        if ((g == m) && (h == p)) return true;

        if ((!maze [g][h]) && (!mark [g][h])) {
            findPath(g, h); // keep finding a path…
        }
    }
    return false;
}
```

Stack Provided by C++ Library

```
#include <iostream>
#include <stack>
using namespace std;
int main()
{
    stack<int> s;
    for(int i=0; i < 5; i++){
        s.push(i);
    }
    while(!s.empty())
    {
        cout << s.size() << " ";
        cout << s.top() << endl;
        s.pop();
    }
}
```

output

```
5 4
4 3
3 2
2 1
1 0
```

Reference of STL's Stack
http://en.cppreference.com/w/cpp/container/stack