國立清華大學電機系 106 學年度下學期 (2017 Spring)

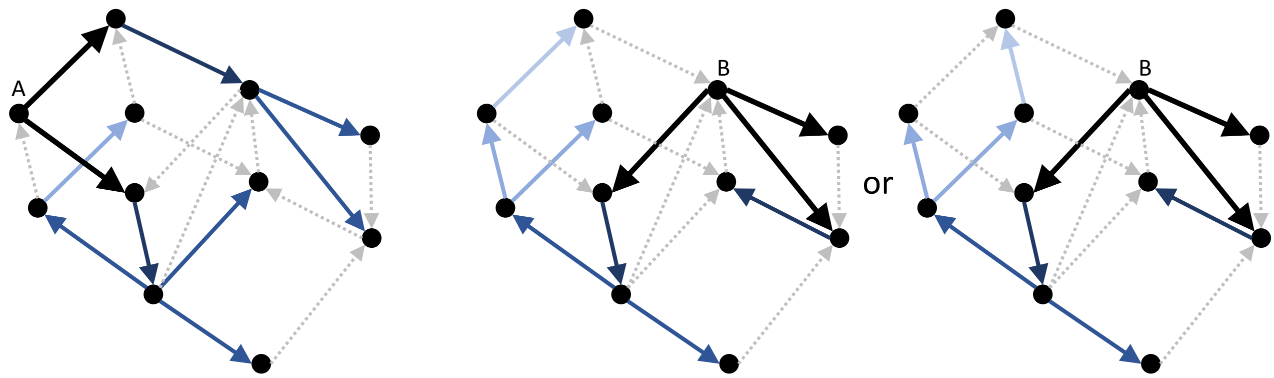
NTHU EE 10520EE241000 Data Structures Final Exam

3:30pm-5:20pm (110 minutes), June. 12, 2017

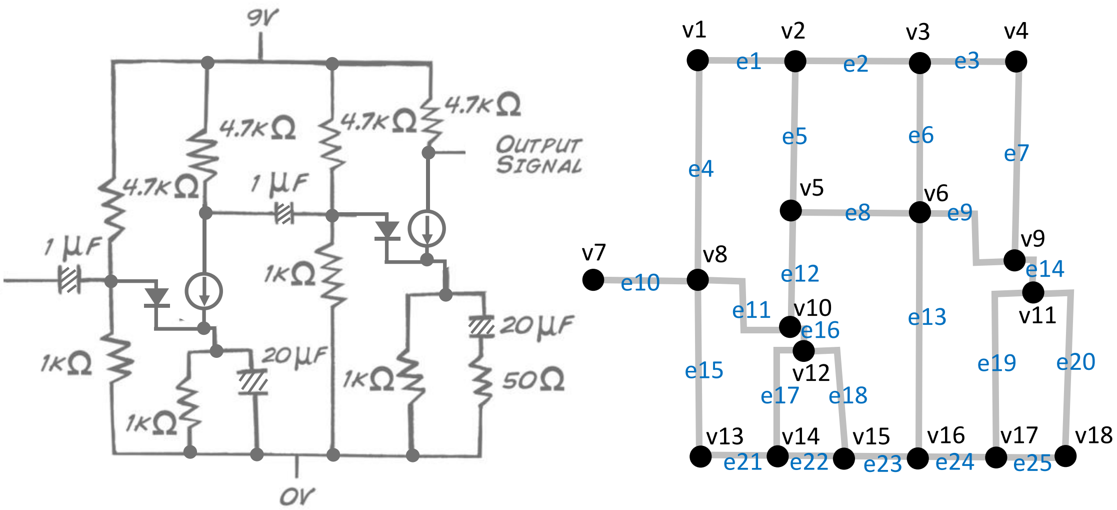
#: ________ Student ID: __________________ Name: ________________________

- ✧ Read each question carefully before you start answering it. **看清題目再作答**。
- ✧ You can use both ballpoint pens and pencils.
- ✧ If there are more than one answers for a problem, just answer one of them. If there is any question on the problems, ask or use reasonable assumptions to solve the problems.
- ✧ There are 12 problems, each being 10 points. You can obtain up to 120 points.

1. Please use solid lines to display the breadth-first search (**BFS**) trees starting from vertices A and B, respectively.



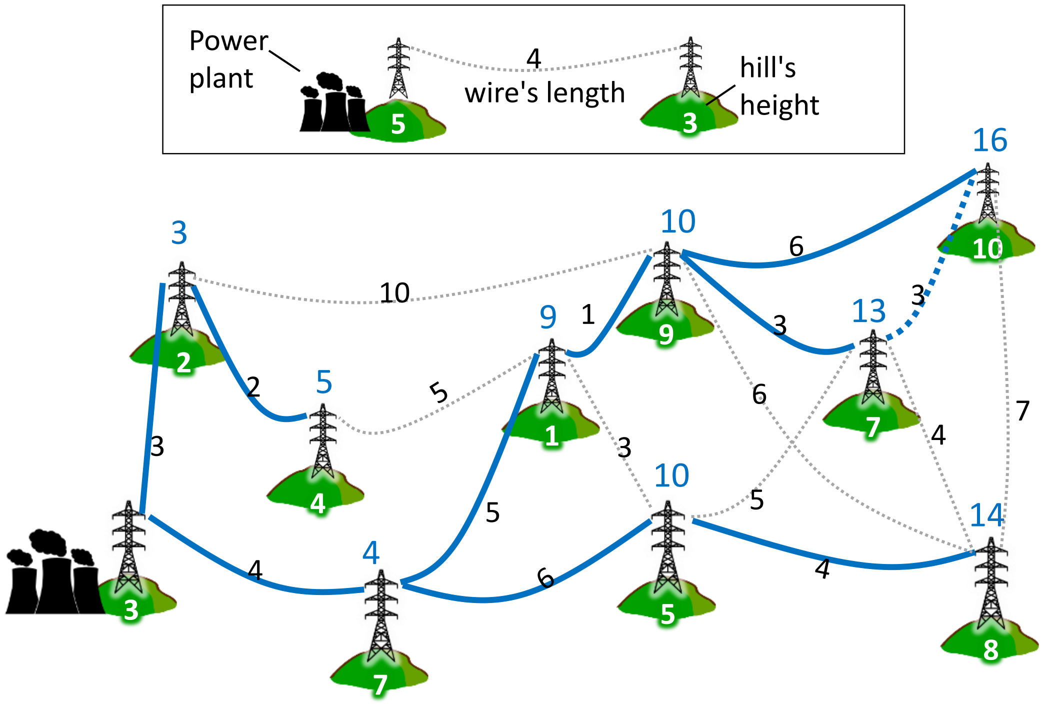
2. How many **independent cycles** are there in the following circuit? Hint: view the circuit as a graph.



|V| = 18, |E| = 25 → |E| - |V| + 1 = 8

3. The Power Company wants to build a power distribution tree spanning several hills.

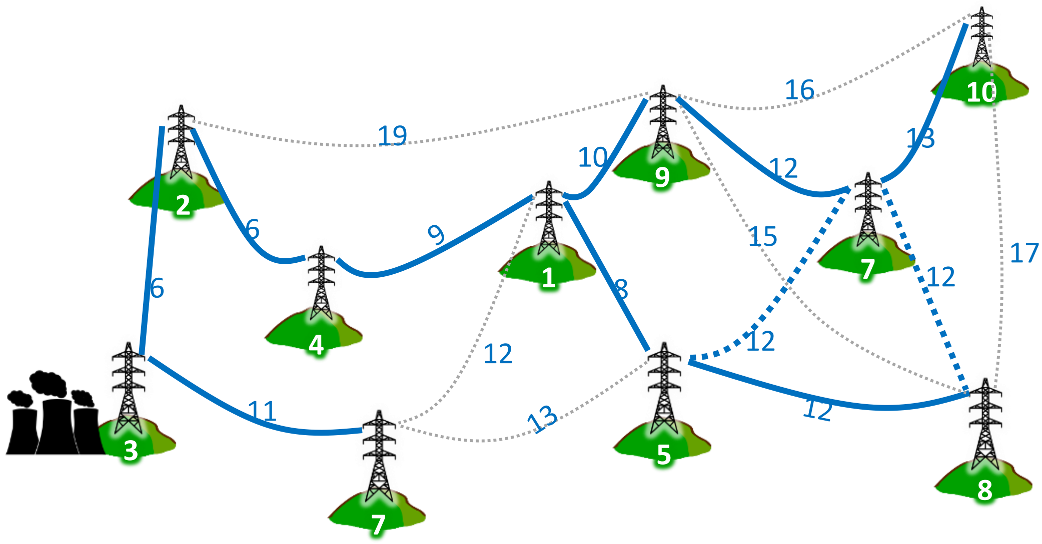
(1) If we want every path from the power plant to every hill to be as short as possible, please **use thick solid lines to depict** what the company should do.



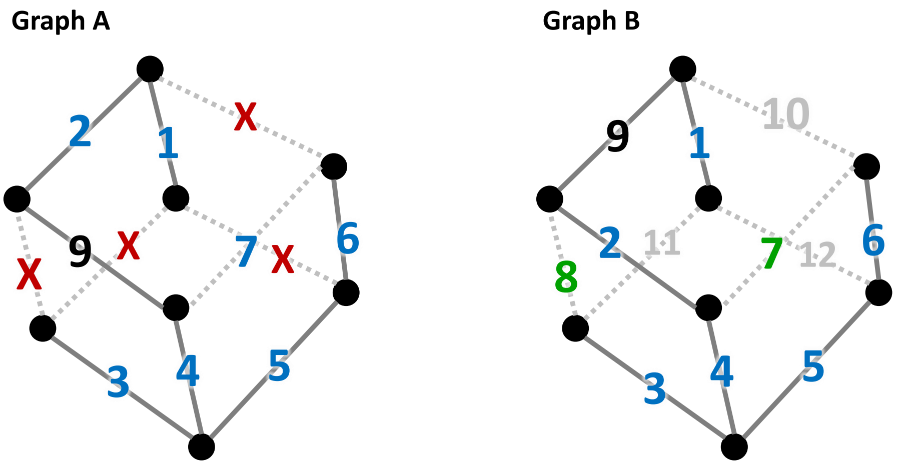
(2) Considering the following cost functions:

> - Cost of building a tower = hill's height
> - Cost of building a link = wire's length + maximum hill's height of the two ends.

If we only want to **minimize the overall cost** of building the power distribution tree, please **use thick solid lines to depict** what the company should do.



4. Please assign 1, 2, 3, …, 12 to the 12 edges of each of the following two graphs (9 is already assigned) so that the minimum-cost span trees (MSTs) of the graphs match what are indicated by the solid lines. Please give a brief explanation if such an assignment does not exist.



The cost of any of X edges in Graph A cannot ≤ 8. Otherwise, one of the X edges would be part of the MST according to the Kruskal's algorithm

→ An assignment does not exist for Graph A.

5. Consider a Bloom filter with a 10-bit vector and the following three hash functions:

h1(x) = x % 10

h2(x) = x^2 % 10

h3(x) = x^3 % 10

(1) Please show the bit vector status after three keys, 12, 16, and 17, are inserted.

0	1	2	3	4	5	6	7	8	9
0	0	1	1	1	0	1	1	1	1

(2) Given the above bit vector status, how many false-positive keys are there in the range of **0, 1, 2, …, 99**?

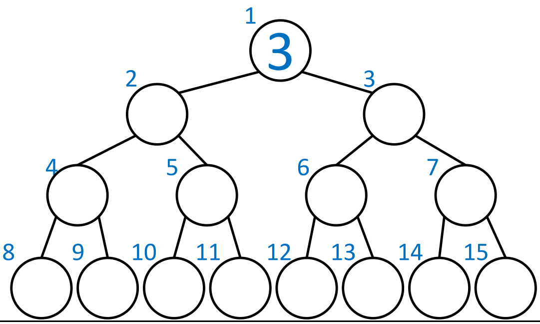
(10x+2), (10x+6), (10x+7)   x = 0, 2 ~ 9   → 3*9 = 27 個

(10x+3), (10x+4), (10x+8)   x = 0 ~ 9   → 3*10 = 30 個

Ans: 57

6. Please show the tree and associated heap statuses after one progressively inserts 3, 9, 5, 1, 2, 7 into an empty binary search tree.

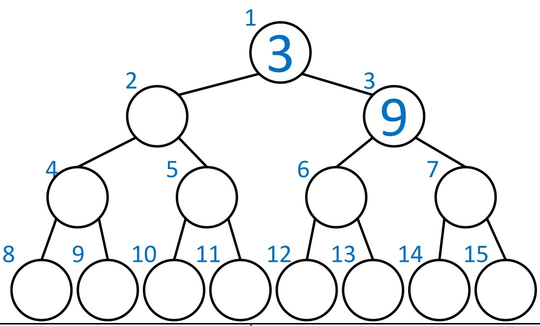
3



0	1	2	3	4	5	6	7
	3						

8	9	10	11	12	13	14	15

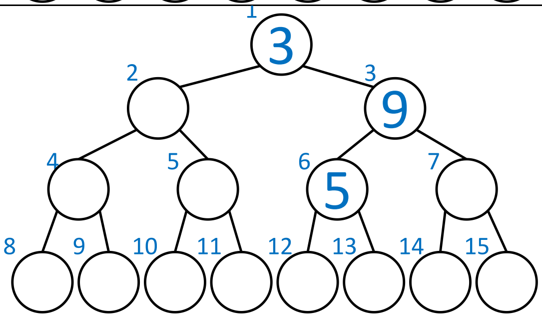
9



0	1	2	3	4	5	6	7
	3		9				

8	9	10	11	12	13	14	15

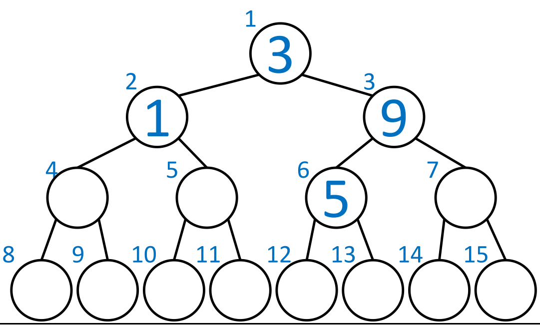
5



0	1	2	3	4	5	6	7
	3		9			5	

8	9	10	11	12	13	14	15

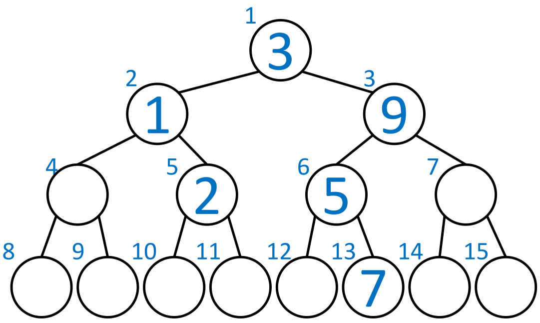
1



0	1	2	3	4	5	6	7
	3	1	9			5	

8	9	10	11	12	13	14	15

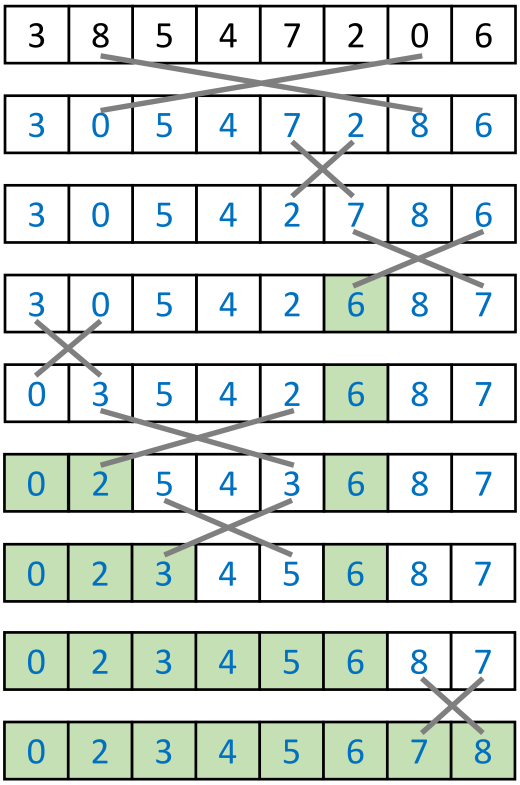
2, 7



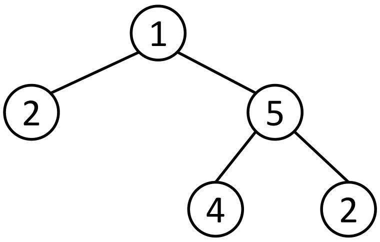
0	1	2	3	4	5	6	7
	3	1	9		2	5	

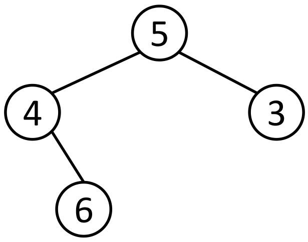
8	9	10	11	12	13	14	15
					7		

7. Please show the process of **Quick Sort** (in an ascending order, 由小到大) that always selects the **right-most** entry as the pivot. Each step only swaps two items.



8. Please design a **hash function** that can map **any binary tree of integers** to an integer between 0 and 15. Please also show the hash values for the following two trees.





Example 1:

Hash(any binary tree) = (Sum of all integers in the tree) % 16

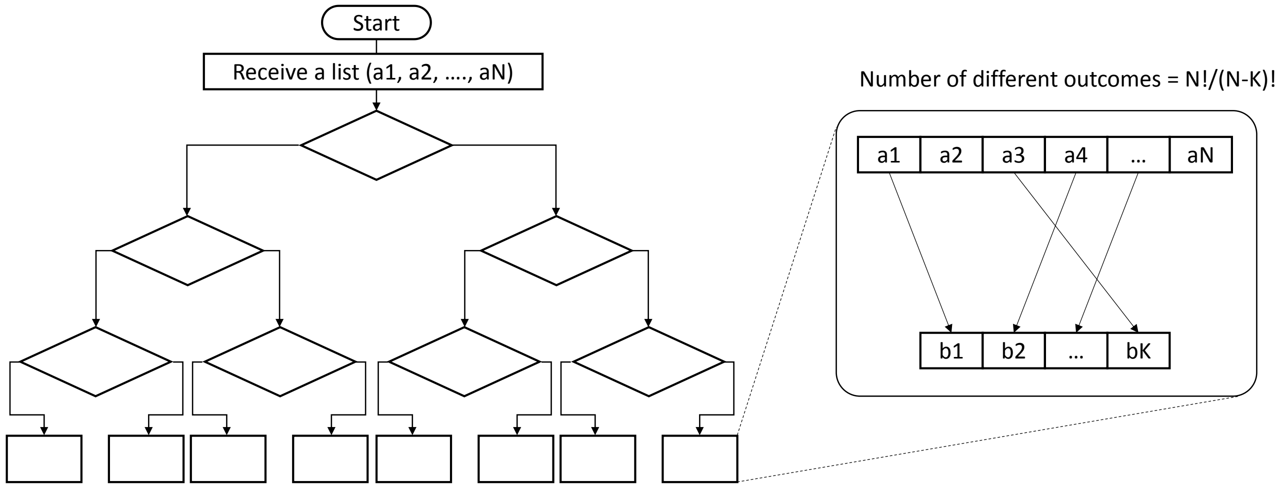
The hash values of the two trees are 14 and 2 respectively.

Example 2:

Hash(any binary tree) = (The root integer) % 16

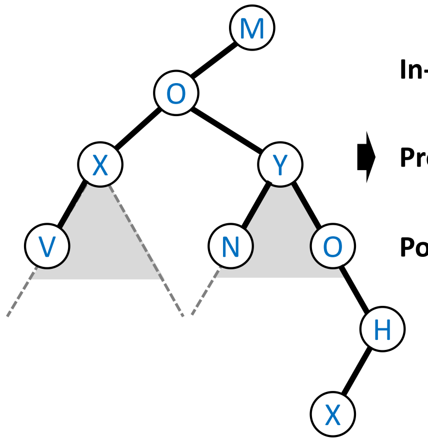
The hash values of the two trees are 1 and 5 respectively.

9. Please analyze the **worst-case number of comparisons** required for **Partial Sort** that receives an N-entry list and produces an ordered list consisted of the K largest entries among the N entries.



The longest path is at least **$\log_2(N!/(N-K)!)$**

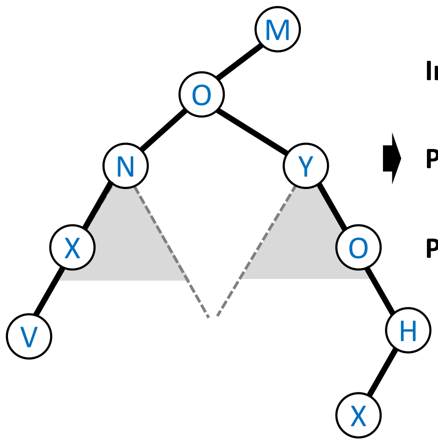
10. Please **plot the tree** and complete its **tree traversals**. Hint: 1) As you can see in the traversals, there are duplicated entries in the tree, such as X and O. 2) Some trial and error may be necessary.



In-order: V X O N Y O X H M

Pre-Order: M O X V Y N O H X

Post-order: V X N X H O Y O M



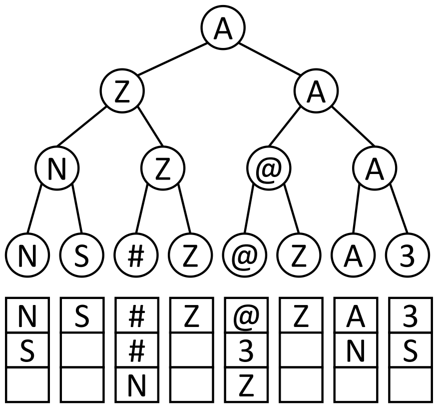
In-order: V X N O Y O X H M

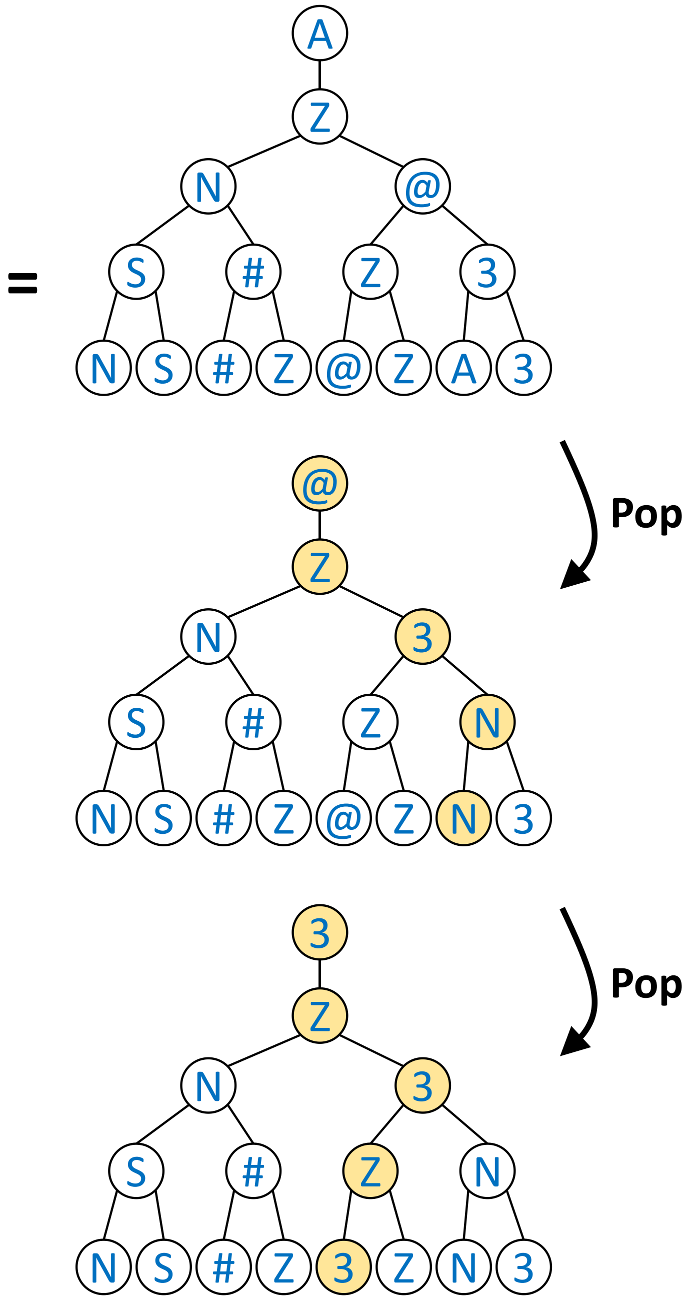
Pre-Order: M O N X V Y O H X

Post-order: V X N X H O Y O M

11. Given a winner tree, please show the associated loser trees.

Winner Tree **Loser Tree**





A > @ > 3 > Z > # > N > S

12. Please write a recursive function, TreeToChain(), to read all the entries in a **binary search tree** and return a pointer pointing to an **ascending-order, singly-linked chain**. Please show your algorithm concretely enough. You can use the LastNode() function.

```
struct TreeNode{
   int data;
   TreeNode * left, * right;
};
struct ChainNode{
   int data;
   ChainNode * next;
}
// return a pointer pointing to the last node of the input chain.
ChainNode * LastNode(ChainNode * head);


// return a pointer pointing to the first node of the generated chain.
ChainNode * TreeToChain(TreeNode * root)
{
   ChainNode * ret, * last;

   if(root->left != 0) {
     ret = TreeToChain(root->left);
     last = LastNode(ret);
   }else{
     ret = 0;
   }

   ChainNode * tmp = new ChainNode;
   tmp->data = root->data;
   tmp->next = 0;

   if(ret == 0) ret = tmp;
   else  last->next = tmp;

   if(root->right != 0)
     tmp->next = TreeToChain(root->right);
   }

   return ret;

}
```