written assignment 2

Jhao Ting Chen, Jui Li

July 2018

1 Question 1

for $i = 0$ to $mainString$'s length
    for $j = 0$ to bag's length
        if $bag[j] > mainString[i]$ and $bag[j] > large$
        then $large \leftarrow bag[j]$
    $mainString[i] \leftarrow large$
    $bag[j] \leftarrow a$
return $mainString$

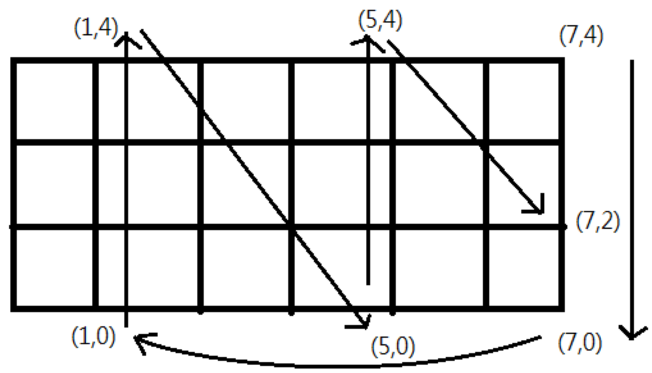
running time: $\mathcal{O}(m)$

2 Question 2 (a)

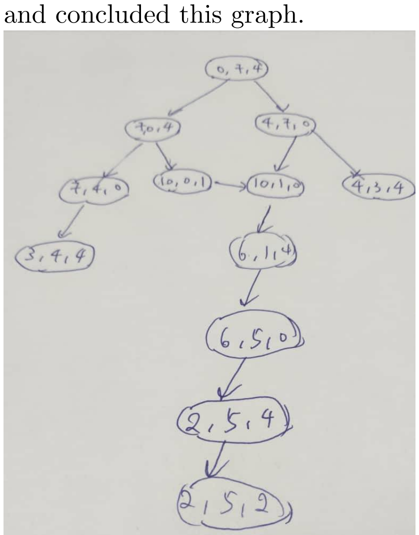
$s \to b \to c \to t$
$s \to b \to d \to t$
$s \to b \to d \to c \to t$

3 Question 2 (b) (i)

From (7,4), always walk all the way to the edge, whether it is straight or at 45 degree. Unless the 10L glass is full(ex: (7,0)$\rightarrow (1,0)$), the water left have to be more then 1L($(0,1) or (1,0)$).



For this graph, we go from the original state (0, 7, 4), and see if any glass is not full and make the other two glass fill it up. However when the state is repeated, it is not the shortest path. We deleted the loop that could happened and concluded this graph.



4 Question 2 (b) (ii)

BFS.

5 Question 2 (b) (iii)

Algorithm:

1. Check: which glass are not full.
2. make the other two glass fill it up. That makes two solution.
3. Check: if 7L or 4L glass has 2L in it. If not, repeat solutions.

Presudocode:

let Q be the sequence of the amount of water in the glasses
$Q.enqueue(s)$
mark s as visited
while Q is not empty
    $v = Q.dequeue()$
    for all neighbours w of v in graph G
        if w is not visited
            $Q.enqueue(w)$
            mark w as visited

6 Question 3 (a)

Let V be the set of vertices. We have $\Sigma deg(u) \leq M*v$ $(u \in V)$, since $deg(u) \leq M$ for all u . But$\Sigma deg(u) = 2e$. Therefore, $2e \leq M * v$ and thus $2e/v \leq M$.

7 Question 3 (b)

Let V be the set of vertices. We have $\Sigma deg(u) \geq m*v$ $(u \in V)$, since $deg(u) \geq m$ for all u . But$\Sigma deg(u) = 2e$. Therefore, $2e \geq m * v$ and thus $2e/v \geq m$.

8 Question 3 (c)

Consider a maximum path $P = \{v_1, v_2, ..., v_l\}$ in G. Suppose that there exists a vertex $w \in V_G\{v_1, ..., v_l\}$ s.t. v_1 is adjacent to w, Then the new path $P' = \{w, v_1, v_2, ..., v_l\}$ is longer then P, contradiction to assumption.
Therefore, v_1 can only be adjacent to $\{v_2, ..., v_1\}$, which means that $deg(v_1) \leq l - 1$, and $deg(v_1)$ has degree at least m, which imply that there exists a simple path (P) of length at least m.

9 Question 3 (d)

Assume that G has no cycle, and consider the longest path in G is P. Let v be the final vertex in P since v has degree > 2, it must have two edges e_1 and e_2 . If e_1 is the last edge of P. Then e_2 cannot be incident on any other vertex of

P since that would create a cycle . So e_2 and its other endpoint are not part of P, and can make P a longer path, which contradicts our assumption. Hence G must contain a cycle and must be connected.

10 Question 3 (e)

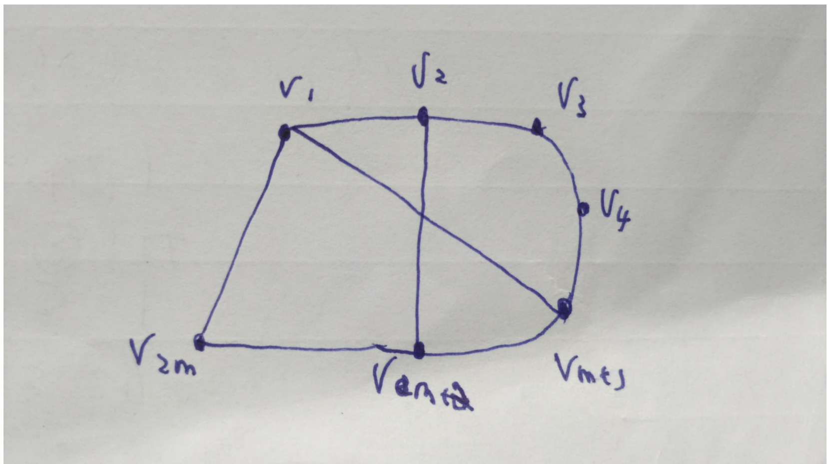
Suppose there is an odd vertex A we can take a component (which is a maximal connected subgraph) that contains A. Thus we can assume the graph is connected to begin with. In any simple graph the number of odd vertices is even. Thus if A is a odd vertex, there must be at least one other odd vertex call it B. Because the graph is connected therefore there is a path from A to B.

11 Question 4 (a)

$(i-j-1) = k*2m \Rightarrow i-j = 1-k*2m$, then $k=1 \Rightarrow i-j = 1-2m$
$(i-j-2m-1) = k*2m \Rightarrow i-j = k*2m+2m-1$, $k=-1 \Rightarrow i-j=-1$
$(i-j-m) = k*2m \Rightarrow i-j = k*2m+m$, $k=-1 \Rightarrow i-j=-m$
because $i=1 \Rightarrow 1-j = 1-2m$, $1-j=-1$, $1-j=-m$
$\Rightarrow -j = 2m$, $-j=2$, $-j=-m-1$
therefore, when $j=2m$, $m+1$, 2, distance = 1.

if($j == 2m \| j == m+1 \| j == 2$), distance = 1
else if ($j \leq (m+3)/2$), distance = $j-2+1 = j-1$
else if ($j < m+1$), distance = $m+1-j+1 = m-j+2$
else if ($j \leq (3m+1)/2$), distance = $j-m-1+1 = j-m$
else distance = $2m-j+1$

12 Question 4 (b)



when $j' - j = -1$, exists an edge between v_j, $v_{j'}$
when $j' - j = 2m$, exists an edge between v_j, $v_{j'}$
when $j' - j = -m$, exists an edge between v_j, $v_{j'}$
then remove edge between v_2 and v_3, v_2 and v_{m+2}, v_1 and v_2
then graph G' is disconnected
therefore G is not 4-edge-connected