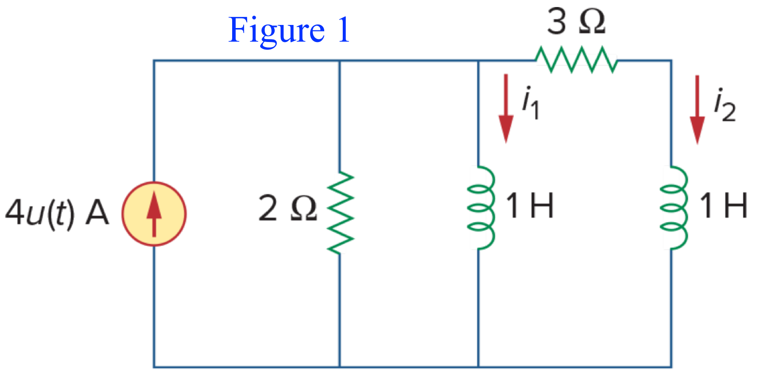
112-1
Electrical Engineering Fundamentals I

Quiz 6

Keys

1. 30% Obtain $\boldsymbol{i}_1$ and $\boldsymbol{i}_2$ for t > 0 in the circuit of Figure 1.

Figure 1



At t < 0, 4u(t) = 0, L shorted

$$i_1(0^-) = i_2(0^-) = 0 \text{ (A)}$$

At t > 0, 4u(t) = 4 (A)

Nodal analysis:

$$4 = \frac{1}{2}\times 1\times\frac{di_1}{dt} + i_1 + i_2$$

$$\Rightarrow \frac{di_1}{dt} + 2i_1 + 2i_2 = 8$$

$$\Rightarrow \frac{d^2i_1}{dt^2} + 2\frac{di_1}{dt} + 2\frac{di_2}{dt} = 0$$

$$i_2 = \frac{v_{L1} - v_{L2}}{3}$$

$$\Rightarrow 3i_2 = \frac{di_1}{dt} - \frac{di_2}{dt}$$

$$\Rightarrow \frac{di_2}{dt} = \frac{di_1}{dt} - 3i_2$$

$$= \frac{di_1}{dt} - 3\left(4 - \frac{1}{2}\frac{di_1}{dt} - i_1\right)$$

$$\frac{d^2i_1}{dt^2} + 2\frac{di_1}{dt} + 2\frac{di_2}{dt} = 0$$

$$\Rightarrow \frac{d^2i_1}{dt^2} + 2\frac{di_1}{dt} + 2\left(\frac{di_1}{dt} - 12 + \frac{3}{2}\frac{di_1}{dt} + 3i_1\right) = 0$$

$$\Rightarrow \frac{d^2i_1}{dt^2} + 7\frac{di_1}{dt} + 6i_1 = 24$$

$$\Rightarrow s^2 + 7s + 6 = 0$$

$$\Rightarrow s = -1, -6$$

$$i_1(0) = i_1(0^+) = i_1(0^-) = 0 \text{ (A)}$$

$$i_2(0) = i_2(0^+) = i_2(0^-) = 0 \text{ (A)}$$

$$\frac{di_1}{dt} + 2i_1 + 2i_2 = 8 \Rightarrow \frac{di_1}{dt} = 8 - 2i_1 + 2i_2$$

$$\Rightarrow \frac{di_1(0)}{dt} = 8 - 2i_1(0) + 2i_2(0) = 8\ (A/s)$$

$$i_1(\infty) = 4(\text{A}) \Rightarrow I_{1SS} = 4(A)$$

$$i_1(t) = I_{1SS} + Ae^{s_1t} + Ae^{s_2t} = 4 + Ae^{-t} + Be^{-6t}$$

$$\frac{di_1}{dt} = -Ae^{-t} - 6Be^{-6t}$$

$$i_1(0) \rightarrow 4 + A + B = 0 \Rightarrow A + B = -4$$

$$\frac{di_1(0)}{dt} \rightarrow -A - 6B = 8$$

$$\Rightarrow \begin{cases} A = -3.2 \\ B = -0.8 \end{cases}$$

$$\boxed{i_1(t) = 4 - 3.2e^{-t} - 0.8e^{-6t}\ (A)}$$

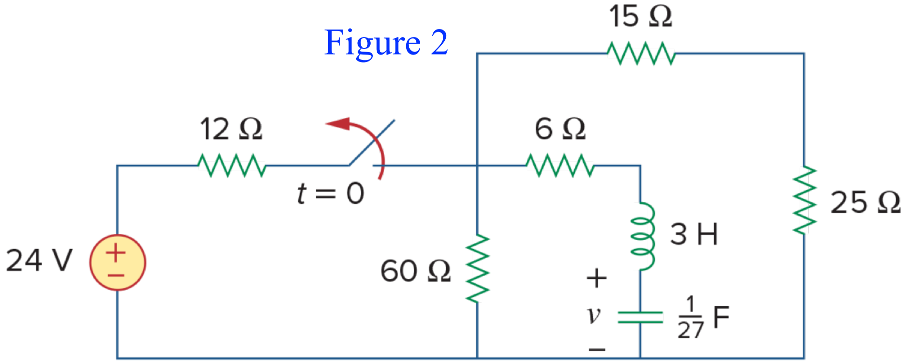
$$\frac{di_1}{dt} + 2i_1 + 2i_2 = 8 \Rightarrow i_2 = 4 - i_1 - 0.5\frac{di_1}{dt}$$

$$i_2(t) = 4 - (4 - 3.2e^{-t} - 0.8e^{-6t}) - 0.5(3.2e^{-t} + 4.8e^{-6t})$$

$$\boxed{\begin{aligned} i_2(t) &= 1.6e^{-t} - 1.6e^{-6t} \\ &= 1.6(e^{-t} - e^{-6t})(A) \end{aligned}}$$

2. 25% Find α and ω_0 and calculate $v(t)$ for $t > 0$ in the circuit of Fig. 2.

Figure 2

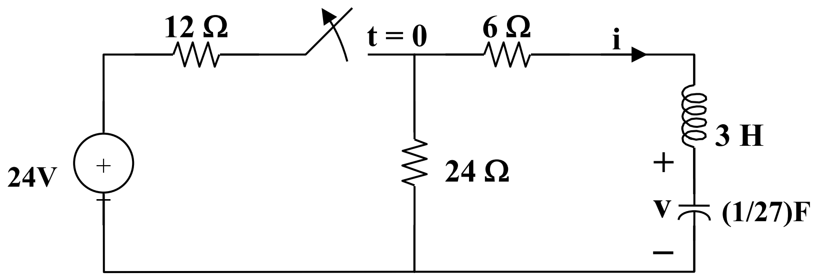


At $t < 0$, sw closed, L shorted & C opened

$$v_C(0^-) = 24\times\frac{60 /\!/ (15+25)}{12 + 60 /\!/ (15+25)} = 24\times\frac{24}{12+24} = 16\ (V)$$

$$i_L(0^-) = 0\ (A)$$

For $t > 0$, sw opened, Series RLC circuit



$$R_{eq} = 6 + 60 /\!/ (15+25) = 6 + 24 = 30\ \Omega$$

$$\alpha = \frac{R}{2L} = \frac{30}{2\times 3} = 5$$

$$\omega_0 = \frac{1}{\sqrt{3\times\frac{1}{27}}} = 3 \qquad \alpha > \omega_0$$

$\Rightarrow$ system is *overdamped*

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2} = -5 \pm \sqrt{5^2 - 3^2} = -9, -1$$

$$i(t) = A_1 e^{-9t} + A_2\, e^{-t}$$

$$\frac{di}{dt} = -9A_1 e^{-9t} - A_2\, e^{-t}$$

$$v_C(0^+) = v_C(0^-) = 16\ (V)$$

$$i(0^+) = i_L(0^+) = i_L(0^-) = 0\ (A)$$

$$v_L(0^+) = -i(0^+)\times(6+24) - v_C(0^+)$$
$$= -0\times(6+24) = -16(V)$$

$$\frac{di(0^+)}{dt} = \frac{1}{L}v_L(0^+) = \frac{-16}{3}\ (V)$$

$$i(0) = A_1 + A_2 = 0 \cdots (1)$$

$$\frac{di(0)}{dt} = -9A_1 - A_2 = \frac{-16}{3} \cdots (2)$$

$$\Rightarrow \begin{cases} A_1 = \dfrac{2}{3} \\ A_2 = \dfrac{-2}{3} \end{cases}$$

$$\boxed{i(t) = A_1 e^{-9t} + A_2\, e^{-t} = \frac{2}{3}e^{-9t} - \frac{2}{3}e^{-t}}$$

$$\boxed{\begin{aligned} v(t) &= -v_L(t) - i(t)\times 30 \\ &= -L\frac{di(t)}{dt} - 30i(t) \\ &= -3\left(-6e^{-9t} + \frac{2}{3}e^{-t}\right) - 20e^{-9t} + 20e^{-t} \\ &= -2e^{-9t} + 18e^{-t} \end{aligned}}$$

3. 45% For each of the circuits Fig. 3.A~3.C, assume all diodes are ***ideal***.

(A) Derive v_O in terms of v_I;

(B) Sketch transfer characteristics v_O vs v_I, ;

(C) Draw output waveforms $v_O(t)$, as v_I is a 1-kHz, 5-V peak sine wave.

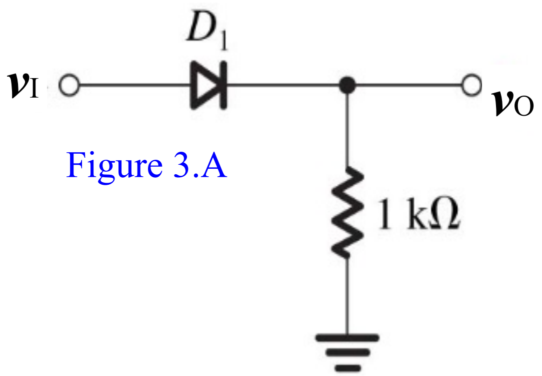


Figure 3.A

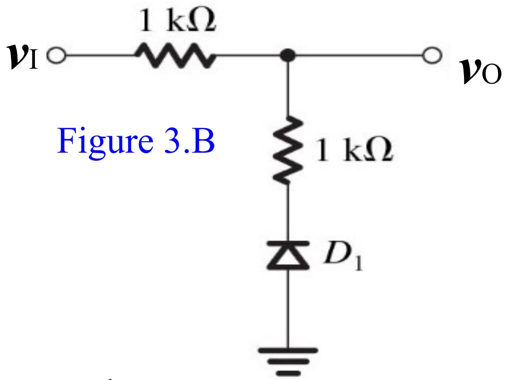


Figure 3.B

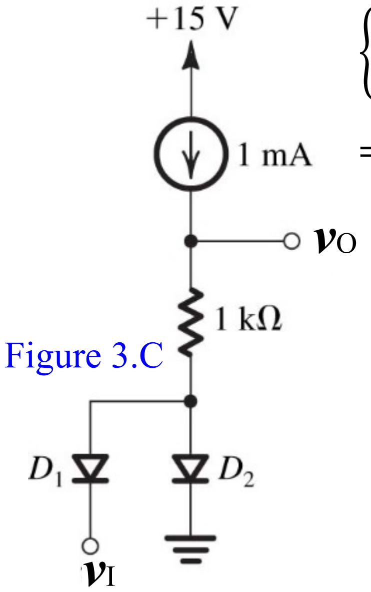
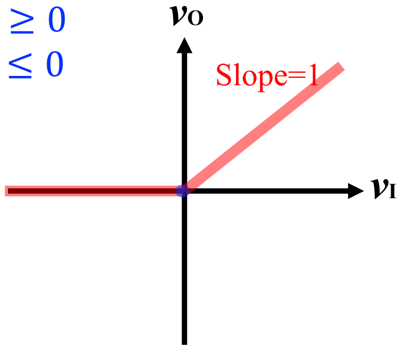
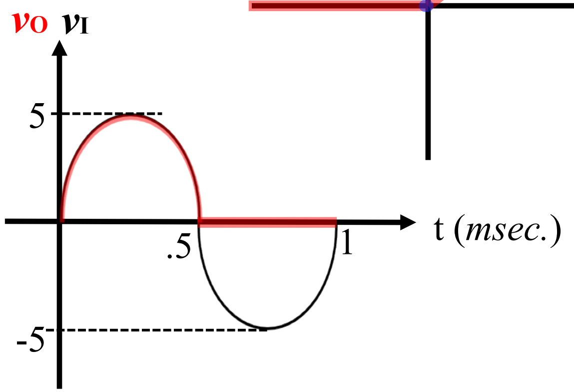


Figure 3.C

$$\begin{cases} \text{As } v_I \ge 0 \to D_1\ on \to v_O = v_I \\ \text{As } v_I \le 0 \to D_1\ off \to v_O = 0 \end{cases}$$

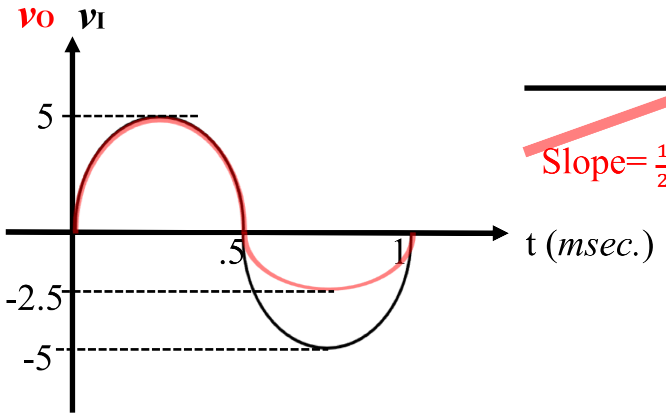
$$\Rightarrow v_O = \begin{cases} v_I & \text{as } v_I \ge 0 \\ 0 & \text{as } v_I \le 0 \end{cases}$$

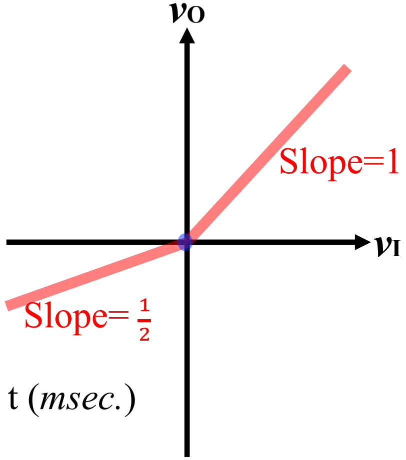




$$\begin{cases} \text{As } v_I \ge 0 \to D_1\ off \to v_O = v_I \\ \text{As } v_I \le 0 \to D_1\ on \to v_O = v_I \times \frac{1}{1+1} = \frac{1}{2}\cdot v_I \end{cases}$$

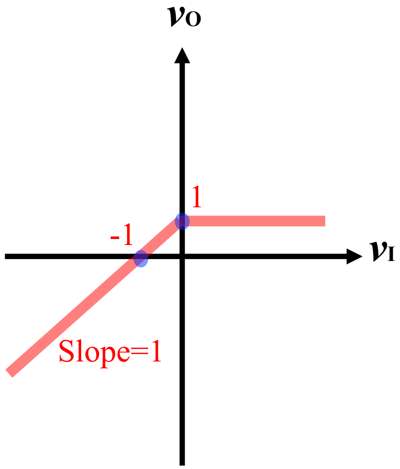
$$\Rightarrow v_O = \begin{cases} v_I & \text{as } v_I \ge 0 \\ \frac{1}{2}\cdot v_I & \text{as } v_I \le 0 \end{cases}$$

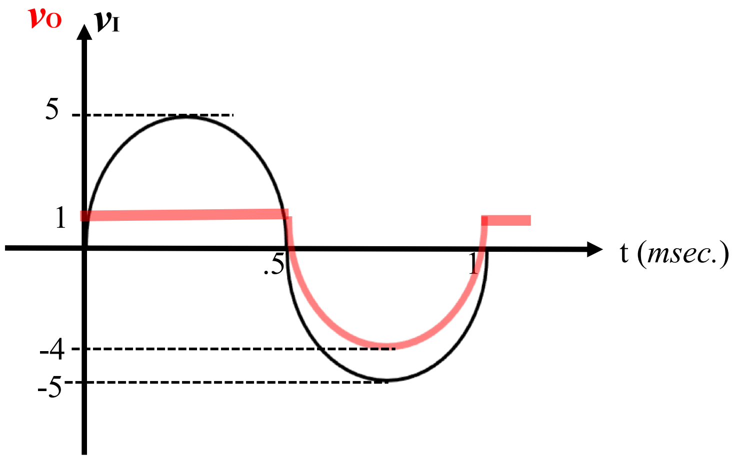




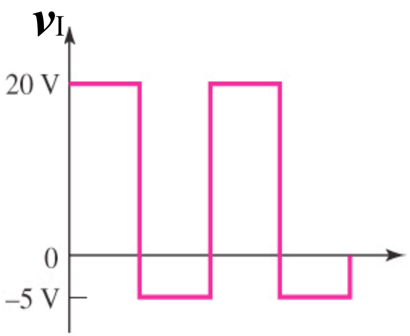
$$\begin{cases} \text{As } v_I \ge 0 \to D_1\ off, D_2\ on \\ \text{As } v_I \le 0 \to D_1\ on, D_2\ off \end{cases}$$

$$\Rightarrow v_O = \begin{cases} 1(mA)\times 1(k\Omega) = 1(V) & \text{as } v_I \ge 0 \\ 1\times 1 + v_I = 1 + v_I & \text{as } v_I \le 0 \end{cases}$$





4. 20% Assume the diodes have a constant voltage drop V_D=0.7 (V) while conducting (Constant voltage drop model). For Figure 4.A~D, with the shown input v_I, sketch the output waveform v_o.



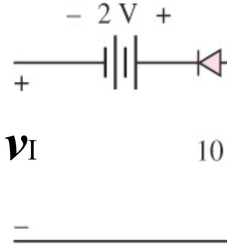


Figure 4.A

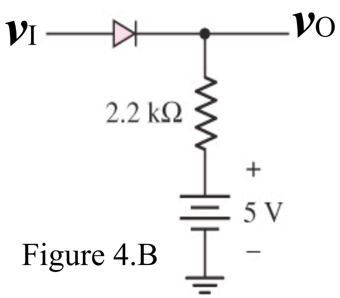


Figure 4.B

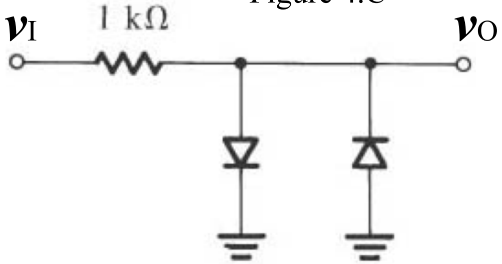


Figure 4.C

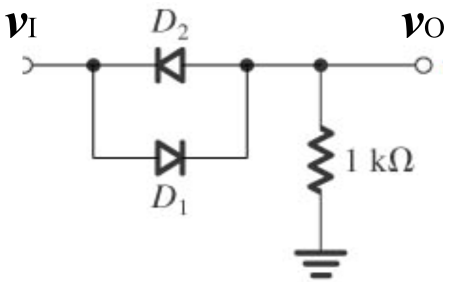
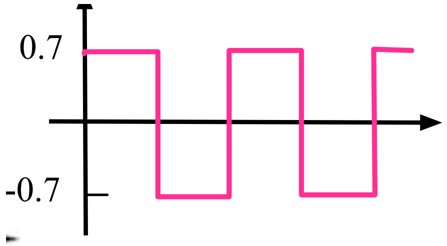
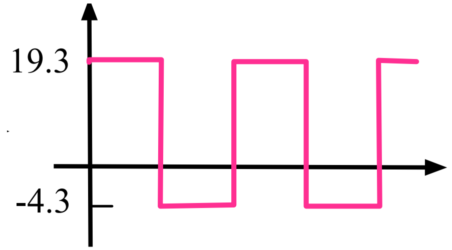
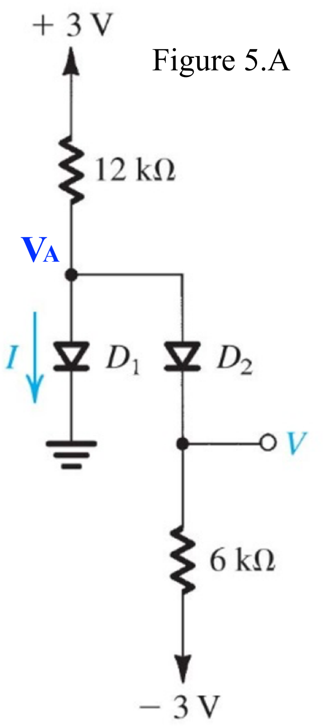


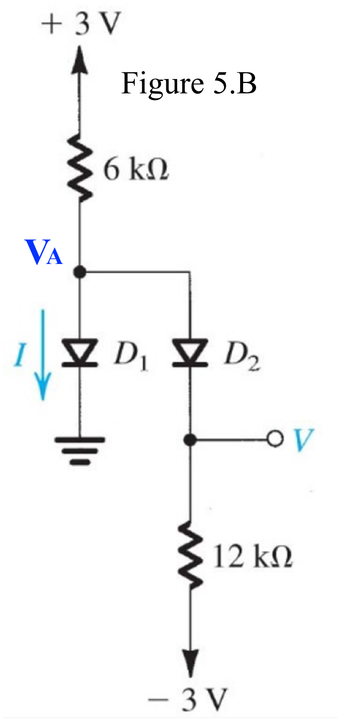
Figure 4.D





5. 20% In Figure 5, assuming that the diodes are ideal, find V and I for the circuit Figure 5.A and 5.B.





(a) Assume $\begin{cases} D_1 \\ D_2 \end{cases}$ are both on

$\Rightarrow V_A = 0 \text{ and } V = V_A = 0$

$\Rightarrow I_{12k} = \frac{3-V_A}{12k} = \frac{3-0}{12k} = 0.25(\text{mA})$

$\Rightarrow I_{D2} = I_{6k} = \frac{V-(-3)}{6k} = \frac{0+3}{6k} = 0.5(\text{mA})$

$\Rightarrow I = I_{12k} - I_{6k} = 0.25 - 0.5 = -0.25(\text{mA})$

$\Rightarrow$ Impossible

$\Rightarrow$ assumption on D_1 is incorrect

$\Rightarrow \begin{cases} D_1 \text{ is off} \\ D_2 \text{ is on} \end{cases} \Rightarrow I = 0$

$\Rightarrow I_{12k} = I_{D2} = I_{6k} = \frac{3-(-3)}{12k+6k} = \frac{1}{3}(\text{mA}) = 0.3333(\text{mA})$

$\Rightarrow V = I_{6k} \times 6(k) + (-3) = \frac{1}{3}(\text{m}) \times 6(k) - 3 = -1(V)$

$\Rightarrow \begin{cases} I = 0 \\ V = -1(V) \end{cases}$

(b) Assume $\begin{cases} D_1 \\ D_2 \end{cases}$ are both on

$\Rightarrow V_A = 0 \text{ and } V = V_A = 0$

$\Rightarrow I_{6k} = \frac{3-V_A}{6k} = \frac{3-0}{6k} = 0.5(\text{mA})$

$\Rightarrow I_{D2} = I_{12k} = \frac{V-(-3)}{12k} = \frac{0+3}{12k} = 0.25(\text{mA})$ assumption is correct!

$\Rightarrow I = I_{6k} - I_{12k} = 0.5 - 0.25 = 0.25(\text{mA})$

$\Rightarrow \begin{cases} I = 0.25(mA) \\ V = 0(V) \end{cases}$