APP Empirical Equation_ Ring and Arch Pendulums

1. Objective:

The objectives of this experiment are to understand empirical equations by measuring the period of oscillation and the diameter of a ring pendulum, to establish a mathematical relationship between them, and to prove that an arch pendulum and a ring pendulum with an equivalent diameter have the same period of oscillation.

2. Theory:

1. Empirical equations
   1. An empirical equation is a mathematical relationship expressing relationships among physical quantities as determined by experimental data generated by controlling variables.
   2. Below, we use a ring pendulum as an example:

      In this experiment, the observed objects comprise a set of five metal rings made of the same material but with different sizes. As shown in Fig. 1, each ring has a notch that fits on the knife-edge and serves as the fulcrum as the metal ring swings (the axis must be horizontally level and perpendicular to the plane of oscillation). This device is called a ring pendulum. By fixing several variables (such as appearance, material, range of oscillation …), we can determine the relationship between the period of oscillation and the diameter of the ring pendulum. To derive mathematical relationships from experimental data, we generally assume that these relationships among the variables are simple. Based on the data in this experiment, we assume that the period of oscillation T is proportional to the nth power of the equivalent diameter d, and therefore,

      $$T = A \cdot d^{n} \qquad (1)$$

      where A is a constant of proportionality, and n is an unknown constant that may be either positive or negative. It is generally hoped that n is a small integer or a fraction containing two small integers. The logarithmic form of Eq. (1) is

      $$\log T = \log A + n \cdot \log d \qquad (2)$$

      If Eq. (1) is an accurate model of the physical system, then a corresponding straight line will be able to be drawn with $\log d$ and $\log T$ as the horizontal and vertical coordinates, respectively.

2. Principles of full logarithmic (log-log) graph paper:

Drawing the relationship between two logarithms on graph paper is inconvenient as it requires logarithmic tables or calculators. This led to the design of full logarithmic (log-log) graph paper, in which the axis graduations are based on logarithmic scales, and as a result, the graduations are not evenly spaced. To draw the graph, you need only mark the T and d values on the graph paper to obtain the log T-log d graph, as shown in Fig. 2.

As presented in Eq. (2), n is the slope of the log-log graph, which can be calculated using $n = \Delta y / \Delta x$; $\Delta x, \Delta y$ can be measured with a ruler. Furthermore, A is point at which the curve intercepts the horizontal axis where d=1 (log d=0).

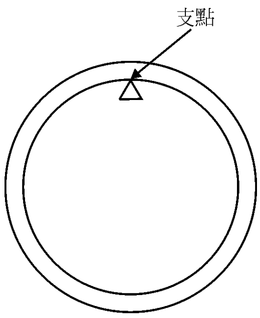

Fig. 1

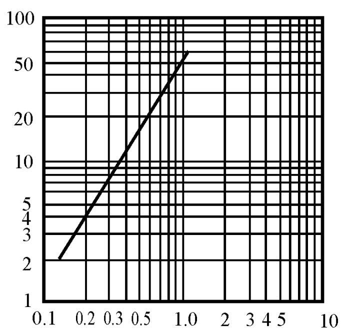

Fig.2

3. Calculation of the period of oscillation for a ring pendulum

1. As shown in Fig. 3, d_i and d_0 are the inner and outer diameters of the ring, and M is the mass of the ring. The moment of inertia of the ring about its center of mass is

$$I_{cm} = \frac{1}{2} M \left[\left(\frac{d_0}{2} \right)^2 + \left(\frac{d_i}{2} \right)^2 \right] \tag{3}$$

Let the root mean square of the inner and outer diameters be the equivalent diameter d:

$$d = \left(\frac{d_0^2 + d_i^2}{2} \right)^{1/2} \tag{4}$$

Thus $$I_{cm} = \frac{1}{4} M d^2 \tag{5}$$

已註解 **[t1]:** Thus,

The distance between the fulcrum and the center of the ring is the equivalent radius $d/2$, and therefore, based on the parallel axis theorem, the moment of inertia of the ring about the fulcrum is

$$I = I_{cm} + M\left(\frac{d}{2}\right)^2 = \frac{1}{2}Md^2 \tag{6}$$

As τ=Iα, we can derive

$$-Mg\frac{d}{2}\sin\theta = \frac{1}{2}Md^2\frac{d^2\theta}{dt^2} \tag{7}$$

When the swinging angle is small, $\sin\theta \cong \theta$, and Eq. (7) can be simplified to simple harmonic motion:

$$\frac{d^2\theta}{dt^2} + \frac{g}{d}\theta = 0 \tag{8}$$

Therefore, the period of oscillation for the ring pendulum is

$$T = 2\pi\sqrt{\frac{d}{g}} = \frac{2\pi}{\sqrt{g}}d^{1/2} \tag{9}$$

From Eq. (9), we can see that the period of oscillation for the ring pendulum is associated with the equivalent diameter and g, but is unrelated to its mass. By comparing Eqs. (9) and (1), we can derive theoretical values for A and n.

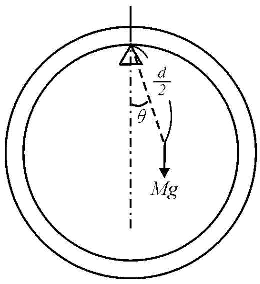

Fig. 3

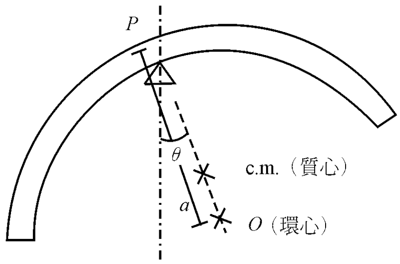

Fig.4

4. Calculation of the period of oscillation for an arch pendulum

Suppose we take any given portion of a ring, which is an arch, and swing it horizontally about its center of symmetry, as shown in Fig. 4. Because τ=*I*α,

We have $$-Mg\,h\sin\theta = I_p \frac{d^2\theta}{dt^2} \tag{10}$$

where M: the mass of the arch

h: the distance between the fulcrum and the center of mass

I_P :the moment of inertia of the arch about the fulcrum

r_{cm} :the distance between the center of mass and the center of the original ring

a: the equivalent radius of the arch

When θ is small, $\sin\theta \cong \theta$, which substituted into Eq. (10) gives

$$\frac{d^2\theta}{dt^2} = -\left(\frac{Mg\,h}{I_P}\right)\theta \tag{11}$$

Thus, the period of oscillation is (12)

where $$h = a - r_{cm} \tag{13}$$

Based on the parallel axis theorem, we can derive

$$I_P = I_{cm} + M(a - r_{cm})^2 \tag{14}$$

and $$I_0 = I_{cm} + Mr_{cm}^2 \tag{15}$$

I_{cm} :the moment of inertia of the arch about its center of mass

I_0 : the moment of inertia of the arch about the center of the original ring

$$I_0 = Ma^2 \tag{16}$$

Eqs. (14), (15), and (16) give

$$\begin{aligned} I_P &= (I_0 - Mr_{cm}^2) + M(a - r_{cm})^2 \\ &= M(2a^2 - 2ar_{cm}) \end{aligned} \tag{17}$$

Substituting Eqs. (13) and (17) into Eq. (12) give

$$T = 2\pi\sqrt{\frac{2a}{g}} = 2\pi\sqrt{\frac{d}{g}} \tag{18}$$

By comparing Eqs. (18) and (9), we can prove that an arch pendulum and a ring pendulum with the same equivalent diameter have the same period of oscillation.

3. Apparatus:

Smartphone, magnet, stand, vernier calipper, ring pendulums 3, 4, and 5, and arch pendulums3, 4, 5 and 7

4. Procedure:

1. Ring pendulum
   1. Use the vernier caliper to measure the inner diameter (d_1) and outer diameter (d_0) of each metal ring five times. Measurements should be taken at different angles. Then, calculate the equivalent diameter *d* of each ring:

$$d = \left(\frac{d_i^2 + d_0^2}{2} \right)^{1/2}$$

   2. Open the APP and use the magnet to move over the smartphone to find a point where the magnetic field reading was the highest. This position is where the magnetic sensor is located.
   3. Setup a stand, a smartphone and a ring with a magnet attached to it as shown in Fig.5.

Fig.5

   4. Let it swing in the vertical plane with a maximum amplitude of less than 5°. use a smartphone to detect the magnetic flux density. We can get the chart of magnetic flux density versus time from APP and use it to calculate the period T, repeat the measurement and average the results to calculate the mean period T .
   5. Repeat Step 4 with the other rings and record their periods of oscillation.
   6. Use Excel to plot the log *T*-log *d* graph and determine the values of *A* and *n* in Eq. (1).
   7. Use Eqs. (1) and (9) to calculate the theoretical values of *A* and *n* and determine the experimental percentage error.

2. Arch pendulum
   1. Repeat Steps 3 to 7 for the ring pendulums using arch pendulums 3, 4, and 5. Calculate the

values of A and n in Eq. (1).

2. Compare the periods of oscillation of the arch pendulums with those of the ring pendulums with the same equivalent diameter.
3. Measure the periods of oscillation of arch pendulums 7 and compare them with both the period of oscillation of arch pendulum 5, which has the same equivalent diameter, and the theoretical value.

5. Questions:

1. Based on the results of the experiment, what is the equivalent diameter of a ring pendulum with a period of oscillation of 1 second?
2. What happens if the swinging amplitude is very large?
3. Do ring pendulums and arch pendulums with the same equivalent diameter have the same period of oscillation?
4. The period of oscillation of a simple pendulum is $T = 2\pi\sqrt{\frac{l}{g}}$, where l is the length of the pendulum. If we rewrite this equation into the form of Eq. (1) and compare it with the experimental results of a ring pendulum with an equivalent diameter d, what must the ratio between l and d be for the simple pendulum to have the same period of oscillation as the ring pendulum?
5. Suppose there is a ring of mass M, where d_i and d_0 are the inner and outer diameters, as shown in the figure below, and the equivalent diameter is $d = \left(\frac{d_0^2 + d_i^2}{2}\right)^{1/2}$. Prove that the moment of inertia about the center of mass $I_{cm} = \frac{1}{4}Md^2$ is equal to that about the fulcrum I_P $= \frac{1}{2}Md^2$.