Feng Chia University 111-1 Class Purdue II Multivariate Calculus Final Exam

(Time：90 minutes. Pages: Three Pages, Total 100 points)

Name：____________________ SID：____________________

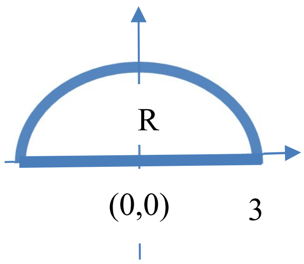
A、Computations : (Total 100%, Show all your work, NO DETAIL WORK, NO POINTS!!)

1. Find (a) div$\vec{\mathbf{F}}$, the divergence at (2,1,-1)
   (b) curl$\vec{\mathbf{F}}$, the curl at (2,1,-1) for the vector field
   $\vec{\mathbf{F}}(\mathrm{x}, \mathrm{y}, \mathrm{z}) = x^3y^2z\vec{\boldsymbol{\imath}} + x^2z\vec{\boldsymbol{\jmath}} + x^2y\vec{\boldsymbol{k}}$.

2. Evaluate $\int_C (x+2)ds$, where C is the curve represented by $\vec{\mathbf{r}}(\mathrm{t}) = \mathrm{t}\vec{\boldsymbol{\imath}} + \frac{4}{3}t^{\frac{3}{2}}\vec{\boldsymbol{\jmath}} + \frac{1}{2}t^2\vec{\boldsymbol{k}}$, for $0 \leq \mathrm{t} \leq 2$.

3. Find the work done by the force field
   $\vec{\mathbf{F}}(\mathrm{x}, \mathrm{y}, \mathrm{z}) = e^x \cos y\,\vec{\boldsymbol{\imath}} - e^x \sin y\,\vec{\boldsymbol{\jmath}} + 2\vec{\boldsymbol{k}}$
   on an object moving along a curve C from the point $(0,\frac{\pi}{2},1)$ to the point $(1, \pi, 3)$.

4. Use Green's theorem, find the work done by the force $\vec{\mathbf{F}}(x, y) = y^3\vec{\boldsymbol{\imath}} + (x^3 + 3\mathrm{x}y^2)\vec{\boldsymbol{\jmath}}$ acting on a particle travelling once around (as fig.) the upper-semicircle of radius 3 centered at (0,0).



5. Let R be the region inside the circle C_1: $x^2 + y^2 = 9$ with counterclockwise orientation and outside the ellipse C_2: $x^2 + (\frac{y}{2})^2 = 1$ with clockwise orientation. Evaluate the line integral $\int_C 2xy\,dx + (x^2 + 2x)dy$ where $C = C_1 + C_2$ is the boundary of R.	6. Find an equation of the tangent plane to the paraboloid $\vec{\mathbf{r}}(\mathrm{u},\mathrm{v}) = \mathrm{u}\vec{\boldsymbol{\imath}} + v\vec{\boldsymbol{\jmath}} + (u^2 + v^2)\vec{\boldsymbol{k}}$ at the point $(1, 2, 5)$.
7. Find the area of the surface over the given region $\vec{\mathbf{r}}(\mathrm{u},\mathrm{v}) = 4\mathrm{u}\vec{\boldsymbol{\imath}} - v\vec{\boldsymbol{\jmath}} + v\vec{\boldsymbol{k}},\ 1 \le \mathrm{u} \le 2,$ $0 \le \mathrm{v} \le 1.$	8. Evaluate the surface integral $\iint_S y^2 + 2yzdS$ where S is the first octant of the plane $2\mathrm{x} + \mathrm{y} + 2\mathrm{z} = 6$

9. Let Q be the region bounded by the unit sphere $x^2+y^2+z^2=1$. Find the outward flux $\iint_S \vec{\boldsymbol{F}}\cdot\mathbf{N}dS$ of the vector field $\vec{\boldsymbol{F}}(x,y,z)=x^3\vec{\boldsymbol{\imath}}+y^3\vec{\boldsymbol{\jmath}}+z^3\vec{\boldsymbol{k}}$ through the sphere S by the Divergence Theorem.	10. Let S be the portion of the paraboloid $z=4-x^2-y^2$ lying above the xy-plane, oriented upward. Let *C* be the boundary curve in the xy-plane, oriented counterclockwise. Evaluate $\iint_S curl\vec{\mathbf{F}}\cdot\mathbf{N}d\mathbf{S}$, by the Stokes's Theorem where $\vec{\boldsymbol{F}}(x,y,z)=2z\vec{\boldsymbol{\imath}}+x\vec{\boldsymbol{\jmath}}+y^2\vec{\boldsymbol{k}}$.