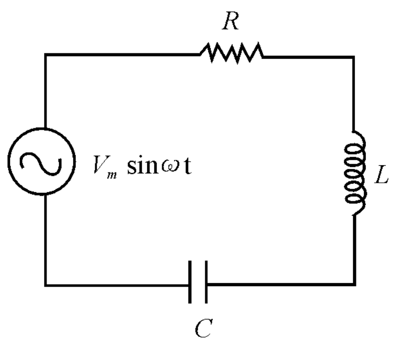
Lab 35 RLC series resonance

1.Objectives：

Understanding the resonance of RLC AC circuit and characteric of its frequency respond.

2. Introduction：

In a sinusoidal AC circuit, if power of capactive reactance Q_c equals to power of inductive reactance Q_L, the resonace effect happens. The former discharges and then the latter charges or the latter discharges and the former (capacitor) charges, due to the phase difference of two is 180°. At this moment, energies of two supply each other, self-supplied, and resonance with natural frequency happens. To introduce this effect, we use RLC circuit to explain the situation, as plot 1 shown. Accroding to Kirchhoff's voltage law, we have



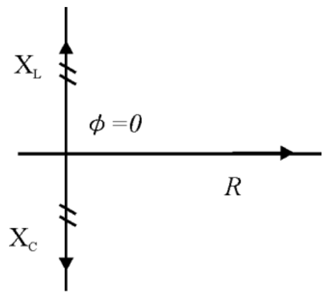
Plot 1

$$L\frac{di(t)}{dt}+Ri(t)+\frac{1}{C}Q=V_m\sin\omega t \qquad (1)$$

Solve for $$i(t)=\frac{V_m}{\sqrt{R^2+(X_L-X_c)^2}}\sin(\omega t-\phi)$$

$$\phi=\tan^{-1}\frac{X_L-X_c}{R} \qquad (2)$$

After analying this result, we find that when $X_L=X_c$, ie. when $\omega L=1/\omega C$, impedance Z＝R and phase difference is zero. So current and voltage are in phase, $I_m=V_m/R$, called series resonance. That's the reason we call angular frequency $\omega_0=1/\sqrt{LC}$ as resonance angular frequency. And because $\omega=2\pi f$, we have $f_0=1/2\pi\sqrt{LC}$ which is resonance frequency. When resonance happens, impedance has minimum value and effective current reaches its maximum value. Also the voltage acrros resistor is the maximum, and voltages of each component are



Plot 2

$$V_R = RI_m = V_m \tag{3}$$

$$V_L = X_L I_m = \left(\frac{X_L}{R}\right) V_m \tag{4}$$

$$V_C = X_C I_m = \left(\frac{X_c}{R}\right) V_m \tag{5}$$

See plot 2, $X_L + X_c = 0$. If resonace happens, power source of *RLC* circuit is $V(t) = V_m \sin \omega_0 t$ and current and voltage are in phase. So we have

$$i(t) = \frac{V_m}{R} \sin \omega_0 t \tag{6}$$

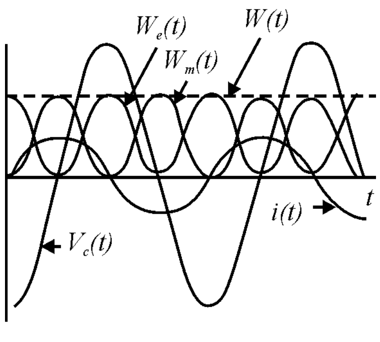
Because $$i(t) = \frac{d}{dt}\left[CV_c(t)\right] ,$$

transient voltage of capacitor is $$V_c(t) = \frac{-V}{RC\omega_0} \cos \omega_0 t\ . \tag{7}$$

Energy transient stored in circuit, sum of transient magnetic and electric energy, is

$$\begin{aligned} W(t) &= W_m(t) + W_e(t) \\ &= \frac{1}{2} L i^2(t) + \frac{1}{2} C V_C^2(t) \\ &= \frac{LV_m^2}{2R^2} \sin^2 \omega_0 t + \frac{V_m^2}{2\omega_0^2 CR^2} \cos^2 \omega_0 t \\ &= \frac{LV_m^2}{2R^2} = \frac{V_m^2}{2\omega_0^2 CR^2} \end{aligned} \tag{8}$$

$LV_m^2 / 2R^2$ is maximum magnetic energy which could store in circuit, and $V_m^2 / 2\omega_0^2 CR^2$ is maximum value of possible electric energy. From equation (8), energy stored in transient circuit is constant. From equations (6) and (7), phase difference of current and electric potential difference of capacitor is just $\pi / 2$ during resonance. That means when electric potential difference of capacitor is maximum, current drops to zero, and when there is no phase electric potential difference of capacitor, current reaches its maximum value. As electric field stores maximum electric energy, magnetic field has zero magnetic energy. By contrast, when magnetic



Plot 3

field stores maximum amount of magnetic energy, electric energy of electric field is zero. Sum of both energies of any moment is constant, as plot 3 shown. We often introduce quality factor Q to resonance problems, which describes the efficiency of device for storing energy. Factor Q is defined as

$$Q=\omega\left[\frac{\text{maximum of energy stored}}{\text{average of dissipated power}}\right]=\omega\cdot\frac{W_{\max}}{P_{a\upsilon}} \tag{9}$$

For RLC circuit, if $V(t)=V_m \sin \omega t$, $i(t)=I_m \sin(\omega t-\phi)$,

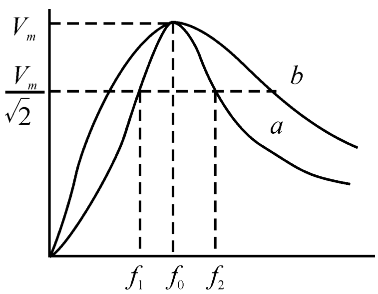
due to $P_{a\upsilon}=\frac{1}{T}\int_0^T P(t)dt$ and $\cos\phi=\frac{R}{Z}$,

after integration, we have $P_{a\upsilon}=\frac{V_m^2}{2R}=\frac{1}{2}I_m^2 R$

when resonance happens. Substitue it into equation (9), we obtain value of Q when resonance happens ($X_L=X_C$), ie.

$$Q=\omega_0\frac{W_{\max}}{P_{av}}=\frac{I_m^2 X_L/2}{I_m^2 R/2}=\frac{X_C}{R}=\frac{X_L}{R}=\frac{\omega_0 L}{R}=\frac{1}{\omega_0 CR}=\frac{1}{R}\sqrt{\frac{L}{C}} \tag{10}$$

According to equation (10), value of Q is voltage gain of circuit under resonance. Making plot of V_R to f or I to f, we get frequency response curve. Assume $V_m/\sqrt{2}$ corresponds to two frequency f_1 and f_2, and $|f_2-f_1|$ is called frequency bandwidth Δf (as plot 4). We often determine value of Q with ratio of resonance frequency f_0 and Δf :



Plot 4

because $I_m=\frac{V_m}{Z_{\min}}=\frac{V_m}{R}$ (m means maximum value)

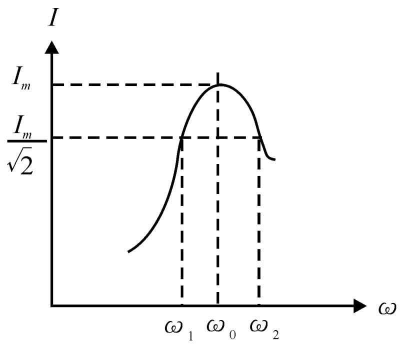
At ω_1, $\frac{I_m}{\sqrt{2}}=\frac{V_m}{\sqrt{2}R}=\frac{V_m}{\sqrt{R^2+(X_C-X_L)^2}}$ (as plot 5)

As $\sqrt{2}R=\sqrt{R^2+(X_C-X_L)^2}$,

$R^2=(X_C-X_L)^2$

When $\omega_0>\omega\equiv\omega_1$, $R=X_c-X_L$,

solve equation $R=\frac{1}{\omega_1 C}-\omega_1 L$



Plot 5

we have $$\omega_1 = \frac{-R+\sqrt{R^2+4L/C}}{2L} = 2\pi f_1 \;. \tag{11}$$

At ω_2, $\omega_0 < \omega \equiv \omega_2$, $R = X_L - X_c$,

Solve euqation $$R = \omega_2 L - \frac{1}{\omega_2 C} ,$$

we have $$\omega_2 = \frac{R+\sqrt{R^2+4L/C}}{2L} = 2\pi f_2 \,. \tag{12}$$

From (11) and (12), $$\Delta f = | f_2 - f_1 | = \frac{R}{2\pi L} \tag{13}$$

and $$f_0 = \frac{1}{2\pi\sqrt{LC}} \,. \tag{14}$$

From (13) and (14), $$\frac{f_0}{\Delta f} = \frac{(1/2\pi)\cdot 1/\sqrt{LC}}{R/2\pi L} = \frac{1}{R}\sqrt{\frac{L}{C}} \tag{15}$$

From (10) and (15), $$Q = \frac{f_0}{\Delta f} \,. \tag{16}$$

Base on the calculation above, we know the larger of value of *L*, the higher of value of *Q* and bigger of *R* or *C*. In plot 5, if we increase *L* and *C*, but other devices remain unchanged, f_0 seems doesn't change, as well as $V_{\max}$ and $I_{\max}$. But in reality, the situation of respond as frequency changes of two circuits is quite different. We can find that from curve *b*, when tuning resonance point f_0 to f_1 or f_2, voltage of circuit doesn't change much, but it drops dramatically for *a* (ie. impedance increase suddenly). On the contrary, if *f* approches f_0 from f_1 or f_2, voltage on circuit *b* (small *L*, big *C*) doesn't increase much, but it on circuit *a* will increase several times (you can feel it suddenly "get through electrically" or "triggered".). This is like tuned circuit of radio. For this case, circuit *a* is better, because we only want to hear one broadcast of station on time, but circuit *b* will broadcast more than one station at the same time. So we conclude that curve *a* has higher sensibility and better selective ability which means higher value of *Q*.

Principle of resonance circuit has applied on oscillating circuit or tuned (sychronized) circuit of emitter or receiver of radio. For electricity, it often used as aggressive way to tune (improve) power factor.

Thanks for the discovery and application of resonance, we have communiation and electrical industry nowdays, like radio communication device, radar, radio, tv…etc. As for electricity, in order to lower the power loss of current in complicate circuits, resonance circuit becomes the most important tool. Even manufacures of air conditioner use one of these methods recently to save consumptive current and reduce loss of power for improving E.E.R. value

3. Materials：

Function signal generator (see appendix 8), oscilloscope (see appendix 11), resistor 100Ω, capacitor 0.047 μF , inductor 10 mH, 4 cables and one test cable of oscilloscope.

4. Procedure：

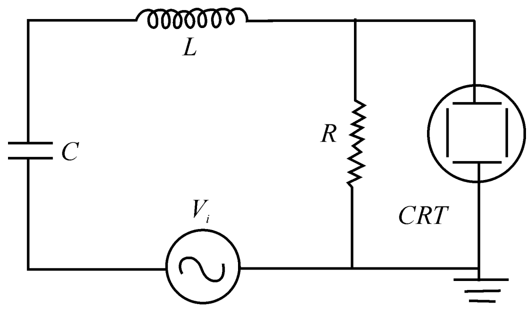
(1) Follow basic instruction and calibrate steps of oscilloscope (see step 1 and 2 of lab 33) to adjust oscilloscope to proper position.

(2) After setting, no need to tune all knobs, keys or settings on oscilloscope during whole experiment procedure.

(3) Connect signal generator with test cable, and switch oscilloscope to *AC* setting. Adjust amplitude knob to have sinusoidal wave $V_m = 1.5$ *V* on screen.

(4) Connect cables as plot 6 shown, $C = 0.047\ \mu F$, L = 10 mH and $R = 100\Omega$. Connect cable across resistor to measure peak voltage value of resistor.

(5) Tune frequency of signal generator to have maximum value of sinusoidal voltage on oscilloscope. Record frequency on signal generator as f_0, and V_R of oscilloscope as $V_{\max}$.



Plot 6

(6) Frequency starts from f_0, decreses very 500 Hz, record value of V_R. Take 10 measurements.

(7) Frequency starts from f_0, increses very 500 Hz, record value of V_R. Take 10 measurements.

(8) Draw V_R vs f frequency respond diagram.

(9) In frequency respond diagram, find f_1 and f_2 , and substitute it into equation (16) to determine measurement of *Q*. Compare it with theoretical value of *Q* from equation (15).

5. Quesitons：

(1) Why do we disscuss value of *Q* in oscillating circuit?

(2) In resonance frequency, measure electric potential across resistor and of whole system. Are these two electric potentials the same? Explain.

(3) Is there resonance frequency in *RLC* series circuit? Briefly describe it. If there is, what is it?

(4) If you can measure values of V_L and V_c when resonance happens? Try to measure them. Calculate the theoretical value of them and compare them.

(5) Determine the value of frequency bandwidth Δf .