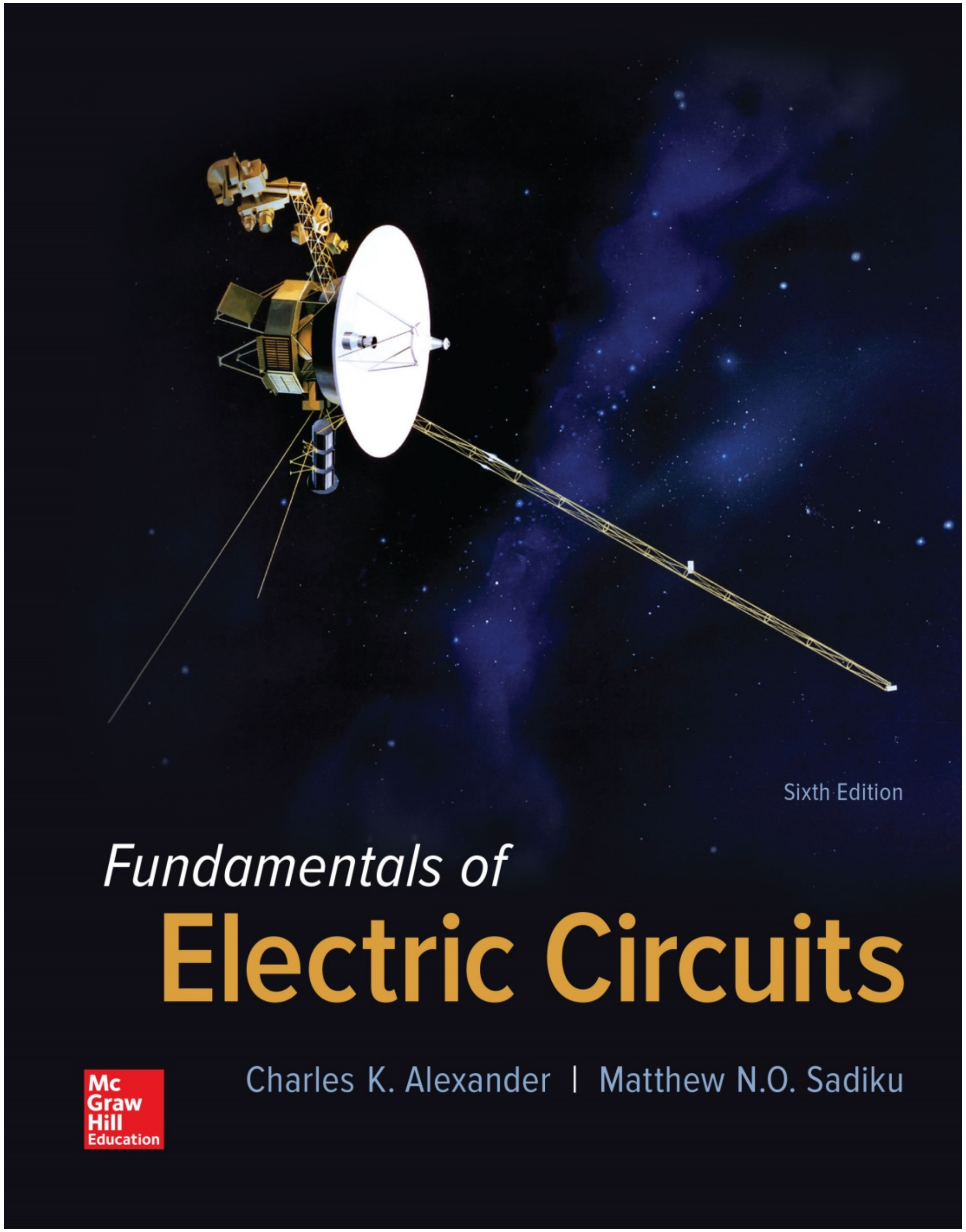
Fundamentals of
Electric Circuits
Chapter 2

Basic Laws



Overview

- This chapter will introduce Ohm's law: a central concept in electric circuits.
- Resistors will be discussed in more detail.
- Circuit topology and the voltage and current laws will be introduced.
- Finally, meters for measuring voltage, current, and resistivity will be presented.

Resistivity

- Materials tend to resist the flow of electricity through them.
- This property is called "resistance"
- The resistance of an object is a function of its
  - length, l,
  - cross sectional area, A,
  - material's resistivity:

$$R = \rho \frac{l}{A}$$

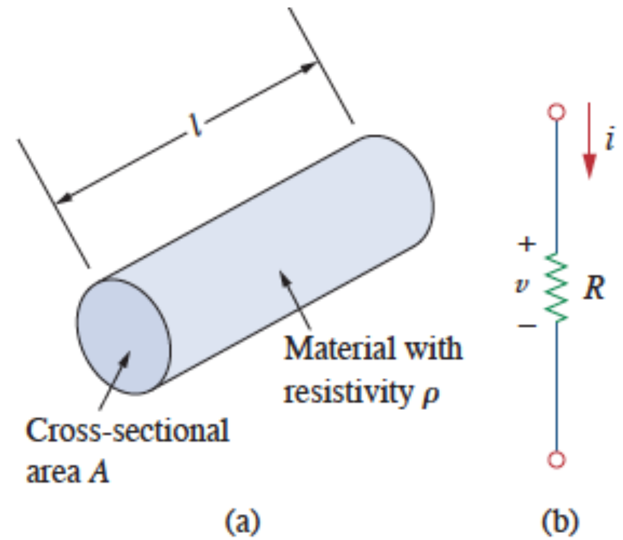


Figure 2.1
(a) Resistor, (b) Circuit symbol for resistance.

Ohm’s Law

- In a resistor, the voltage across a resistor is directly proportional to the current flowing through it.

$$V = IR$$

- The resistance of an element is measured in units of Ohms, Ω, (V/A)
- The higher the resistance, the less current will flow through for a given voltage.
- Ohm’s law requires conforming to the passive sign convention.

Resistivity of Common Materials

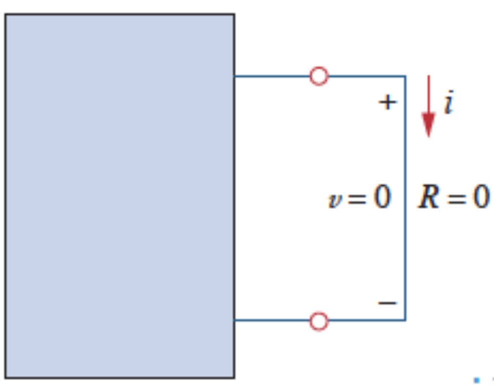
TABLE 2.1

Resistivities of common materials.

Material	Resistivity ($\Omega \cdot m$)	Usage
Silver	1.64×10^{-8}	Conductor
Copper	1.72×10^{-8}	Conductor
Aluminum	2.8×10^{-8}	Conductor
Gold	2.45×10^{-8}	Conductor
Carbon	4×10^{-5}	Semiconductor
Germanium	47×10^{-2}	Semiconductor
Silicon	6.4×10^{2}	Semiconductor
Paper	10^{10}	Insulator
Mica	5×10^{11}	Insulator
Glass	10^{12}	Insulator
Teflon	3×10^{12}	Insulator

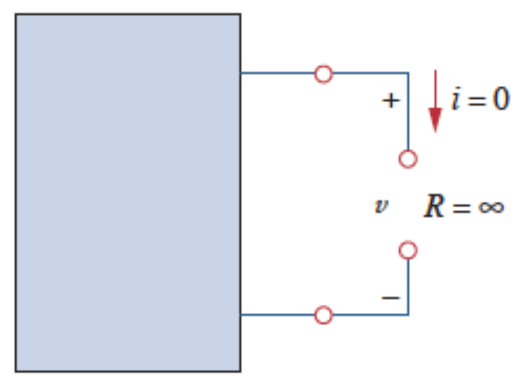
Short and Open Circuits

- A connection with almost **zero resistance** is called a **short circuit**.
- Ideally, any current may flow through the short.
- In practice this is a connecting wire.
- A connection with **infinite resistance** is called an **open circuit**.
- Here no matter the voltage, no current flows.



$$v = iR = 0$$

(a)



(b)

$$i = \lim_{R\to\infty} \frac{v}{R} = 0$$

Conductance

- A useful quantity in circuit analysis is the reciprocal of resistance R, known as conductance and denoted by G.

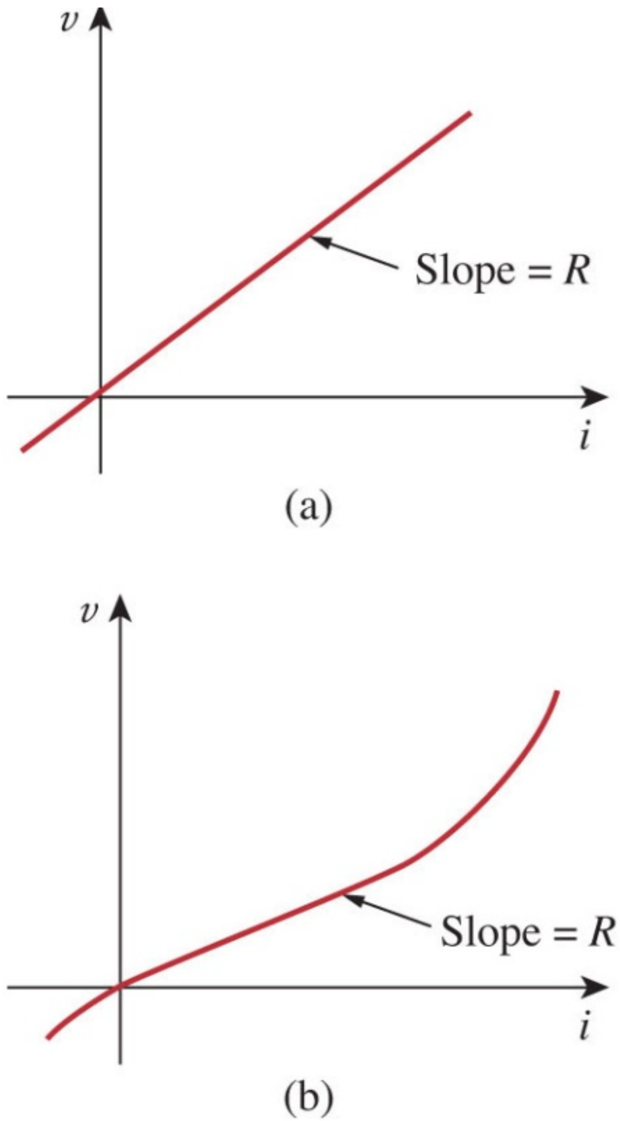
$$G = \frac{1}{R} = \frac{i}{v}$$

- The conductance is a measure of how well an element will conduct electric current.
- The unit of conductance is the mho (ohm spelled backward) or reciprocal ohm ($1/\Omega$), with symbol, ℧
- This book uses the siemens (S), the SI unit of conductance,

$$1\ \mathrm{S} = 1\ \mho = 1\ \mathrm{A/V}$$

Linearity

- Not all materials obey Ohm's Law.
- Resistors that do are called linear resistors because their current voltage relationship is always linearly proportional.
- Diodes and light bulbs are examples of non-linear elements



Power Dissipation

- Running current through a resistor **dissipates power**.

$$p = vi = i^2 R = \frac{v^2}{R}$$

- The power dissipated is a non-linear function of current or voltage
- **Power dissipated is always positive**
- A resistor can never generate power

Example 2.2

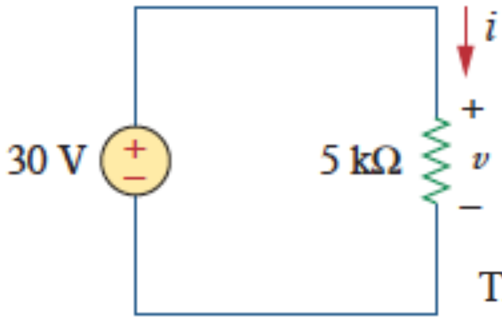


Figure 2.8

In the circuit shown in Fig. 2.8, calculate the current i, the conductance G, and the power p.

The voltage across the resistor is the same as the source voltage (30 V) because the resistor and the voltage source are connected to the same pair of terminals. Hence, the current is

$$i = \frac{v}{R} = \frac{30}{5 \times 10^3} = 6 \text{ mA}$$

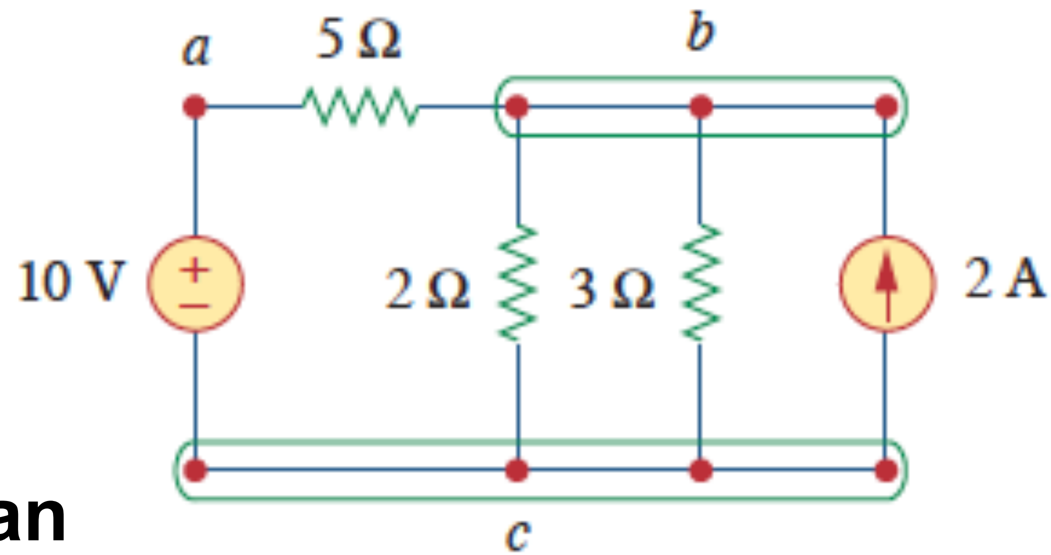
$$G = \frac{1}{R} = \frac{1}{5 \times 10^3} = 0.2 \text{ mS}$$

$$p = vi = 30(6 \times 10^{-3}) = 180 \text{ mW}$$

$$p = i^2R = (6 \times 10^{-3})^2 5 \times 10^3 = 180 \text{ mW}$$

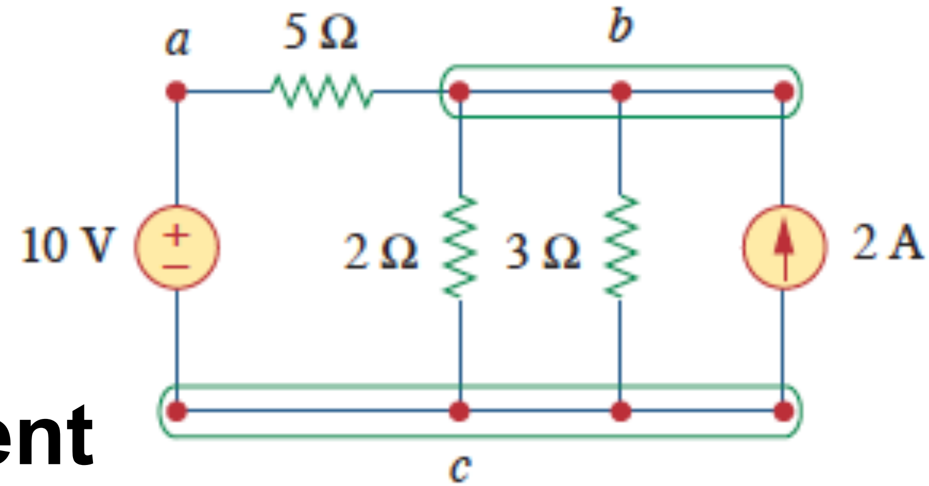
$$p = v^2G = (30)^2 0.2 \times 10^{-3} = 180 \text{ mW}$$

Nodes, Branches and Loops



- Circuit elements can be interconnected in multiple ways.
- To understand this, we need to be familiar with some network topology concepts.
- A branch represents a single element such as a voltage source or a resistor.
- A node is the point of connection between two or more branches.
- A loop is any closed path in a circuit.

Network Topology



- A loop is independent if it contains at least one branch not shared by any other independent loops.
- Two or more elements are in series if they share a single node and thus carry the same current
- Two or more elements are in parallel if they are connected to the same two nodes and thus have the same voltage.

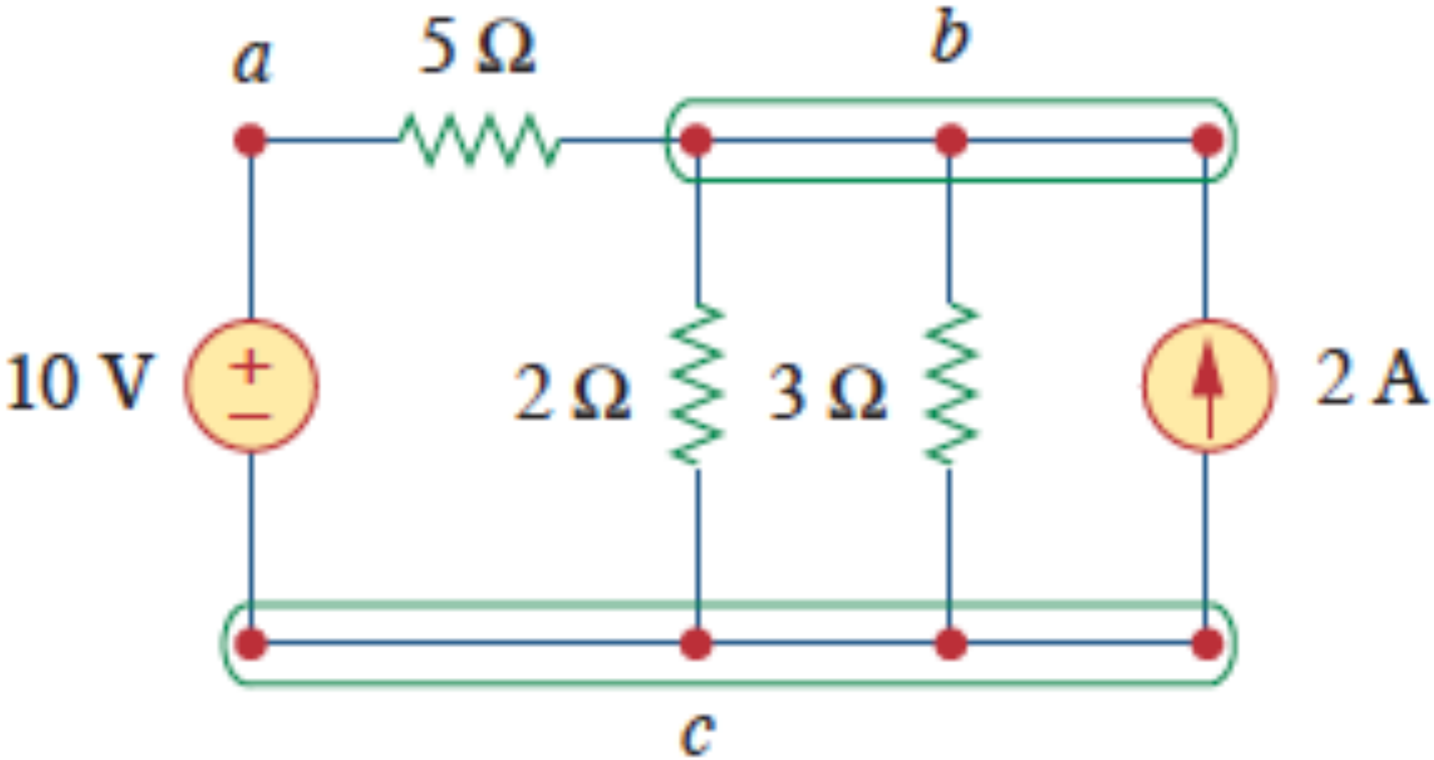


Figure 2.10
Nodes, branches, and loops.

Example 2.4

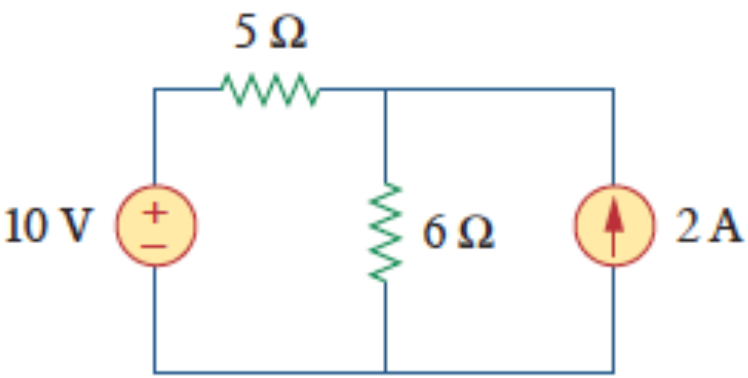


Figure 2.12
For Example 2.4.

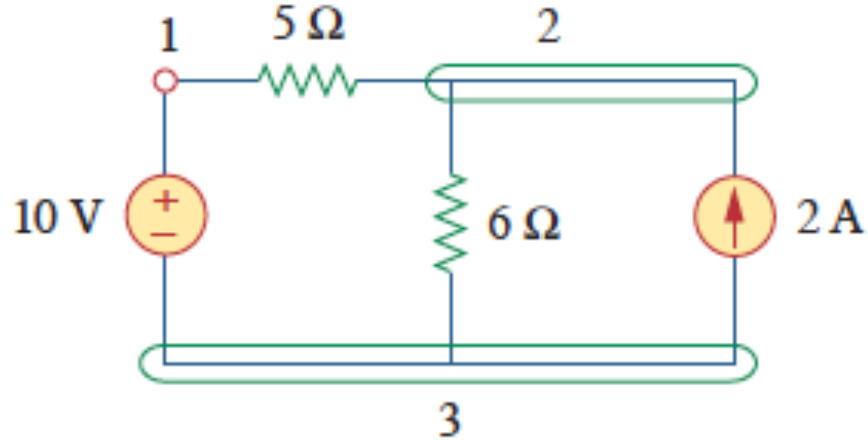


Figure 2.13
The three nodes in the circuit of Fig. 2.12.

Practice Problem 2.4

Answer: Five branches and three nodes are identified in Fig. 2.15. The 1-Ω and 2-Ω resistors are in parallel. The 4-Ω resistor and 10-V source are also in parallel.

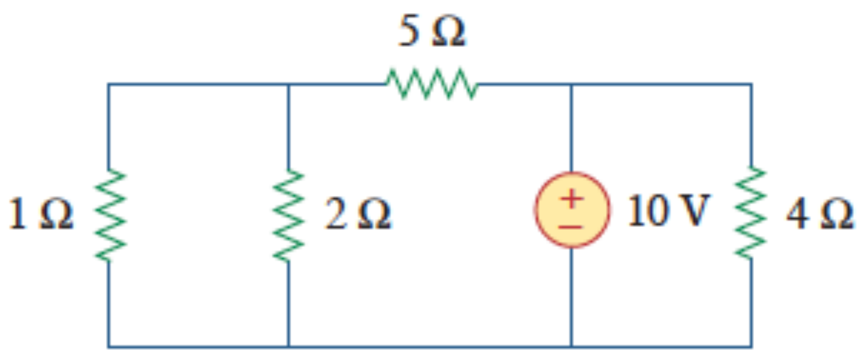


Figure 2.14
For Practice Prob. 2.4.

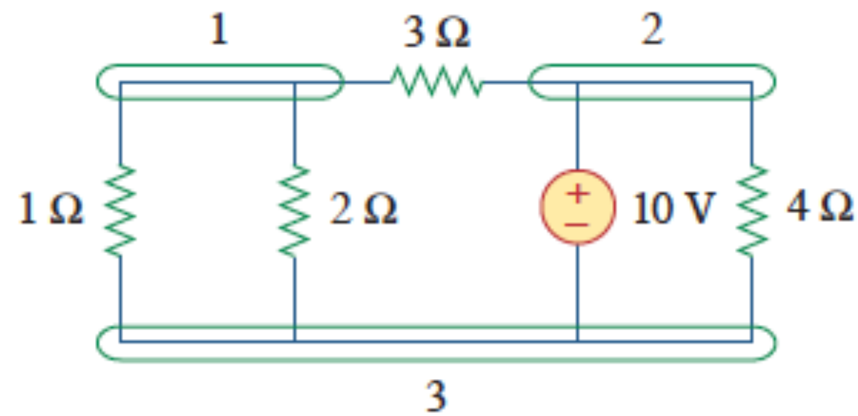


Figure 2.15

Kirchhoff's Laws

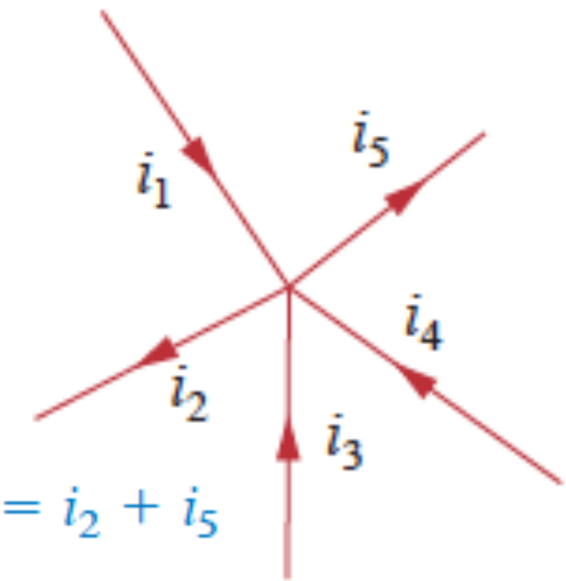
- Ohm's law is not sufficient for circuit analysis
- Kirchhoff's laws complete the needed tools
- There are two laws:
  - Kirchhoff's Current Law, KCL
  - Kirchhoff's Voltage Law, KVL

KCL

- **Kirchhoff's current law** is based on conservation of charge
- It states that the **algebraic sum of currents entering a node** (or a closed boundary) **is zero**.
- It can be expressed as:

$$i_1 + (-i_2) + i_3 + i_4 + (-i_5) = 0$$

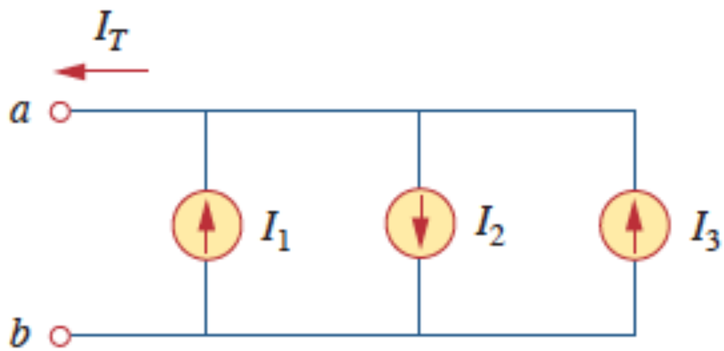
$$\sum_{n=1}^{N} i_n = 0$$

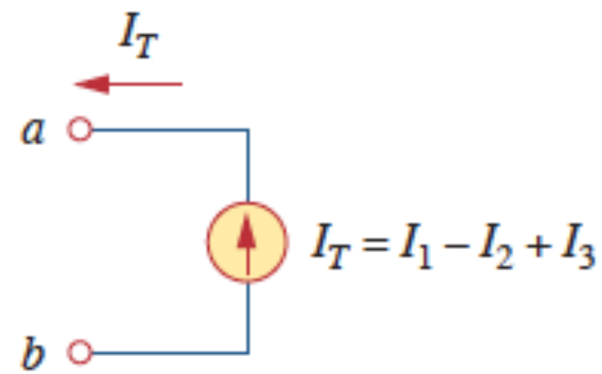


$$i_1 + i_3 + i_4 = i_2 + i_5$$

KCL

The sum of the currents entering a node is equal to the sum of the currents leaving the node.





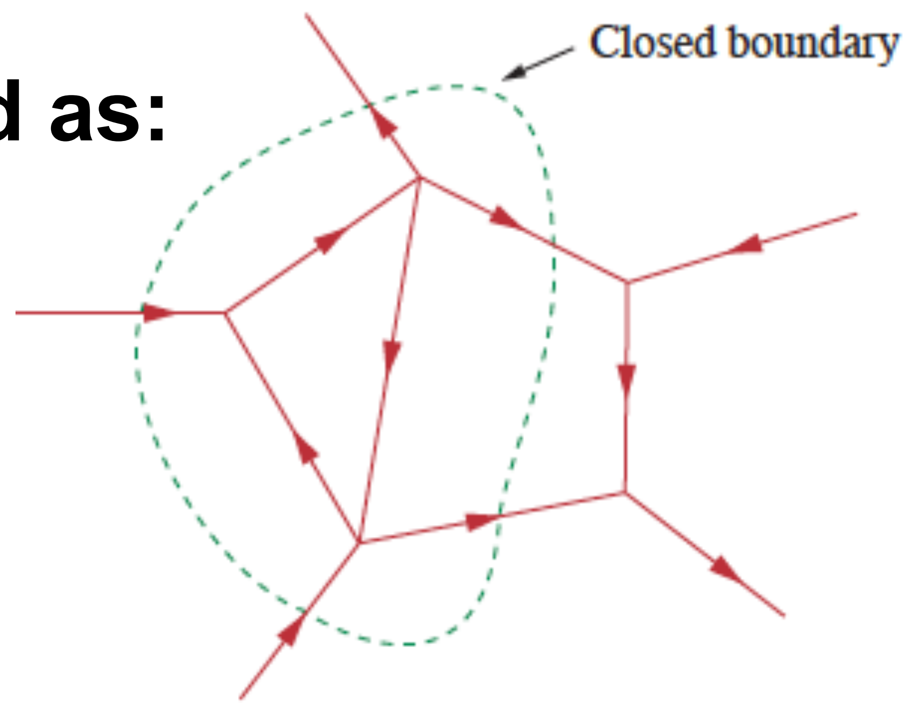
$$I_T + I_2 = I_1 + I_3$$

$$I_T = I_1 - I_2 + I_3$$

KVL

- **Kirchhoff's voltage law** is based on conservation of energy
- It states that the **algebraic sum of voltages around a closed path (or loop) is zero**.
- It can be expressed as:

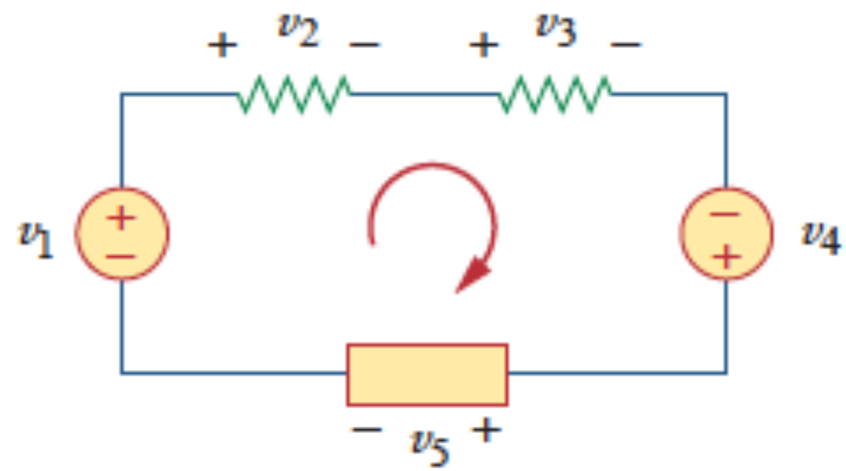
$$\sum_{m=1}^{M} v_m = 0$$



KVL

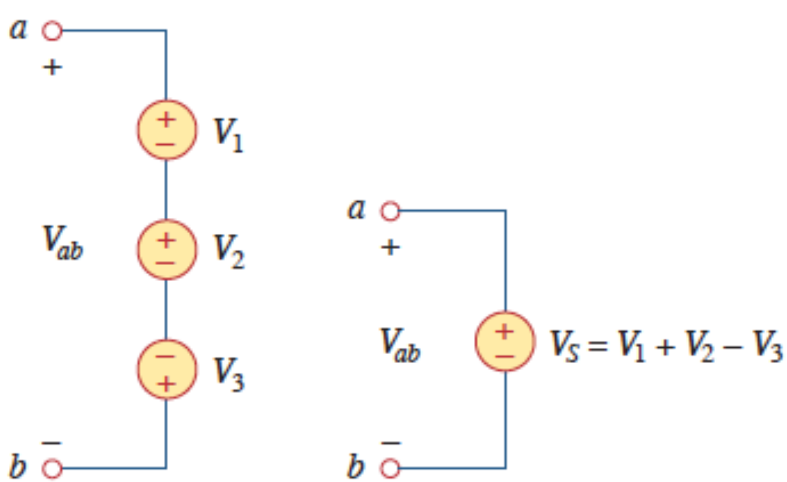
algebraic sum of voltages around a closed path (or loop) is zero

Sum of voltage drops = Sum of voltage rises



$$-v_1 + v_2 + v_3 - v_4 + v_5 = 0$$

$$v_2 + v_3 + v_5 = v_1 + v_4$$



(a) (b)

$$V_{ab} = V_1 + V_2 - V_3$$

Example 2.5

Find voltages $\boldsymbol{v}_1$ and $\boldsymbol{v}_2$.

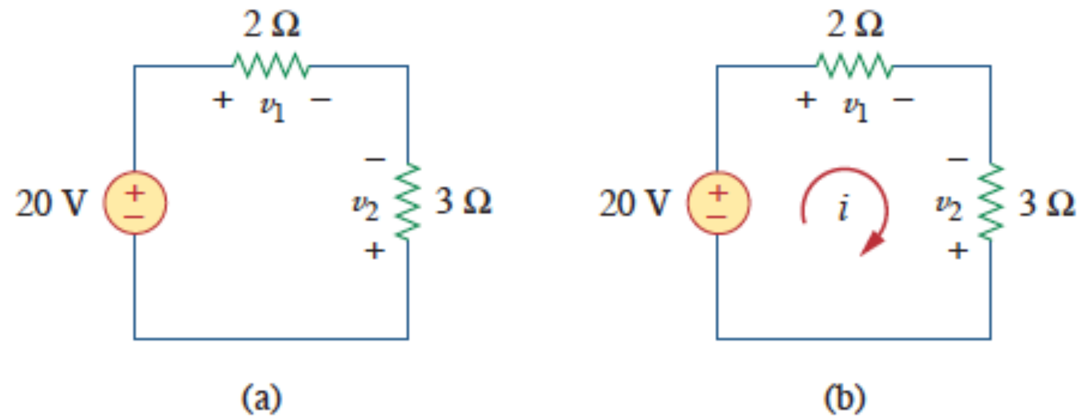


Figure 2.21

To find v_1 and v_2, we apply Ohm's law and Kirchhoff's voltage law. Assume that current i flows through the loop as shown in Fig. 2.21(b).

$$v_1 = 2i, \qquad v_2 = -3i$$

Applying KVL around the loop gives

$$-20 + v_1 - v_2 = 0$$

$$-20 + 2i + 3i = 0 \qquad \text{or} \qquad 5i = 20$$

$$\Rightarrow \qquad i = 4\text{ A}$$

$$v_1 = 8\text{ V}, \qquad v_2 = -12\text{ V}$$

Determine v_o and i in the circuit shown in Fig. 2.23(a).

Example 2.6

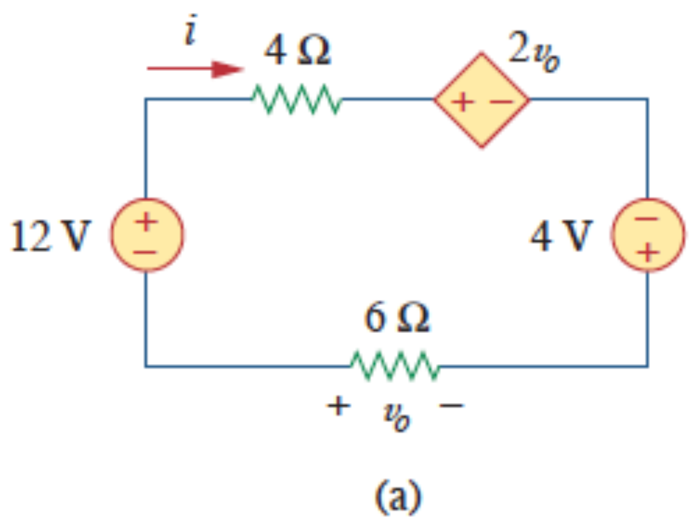


Figure 2.23

We apply KVL around the loop $\quad -12 + 4i + 2v_o - 4 + 6i = 0$

$$v_o = -6i$$

$$-16 + 10i - 12i = 0 \quad \Rightarrow \quad i = -8\ \text{A}$$

$$v_o = 48\ \text{V}.$$

Example 2.8 Find currents and voltages in the circuit shown in Fig. 2.27(a).

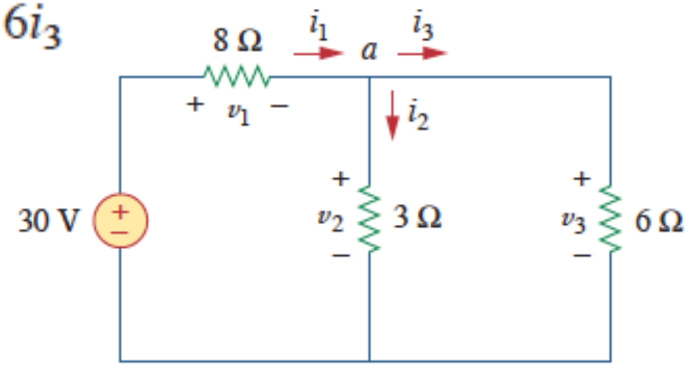
By Ohm's law, $v_1 = 8i_1, \quad v_2 = 3i_2, \quad v_3 = 6i_3$

At node a, KCL gives $i_1 - i_2 - i_3 = 0$

Applying KVL to loop 1 $\quad -30 + v_1 + v_2 = 0$

$$\Rightarrow \quad -30 + 8i_1 + 3i_2 = 0$$

$$\Rightarrow \quad i_1 = \frac{(30 - 3i_2)}{8}$$



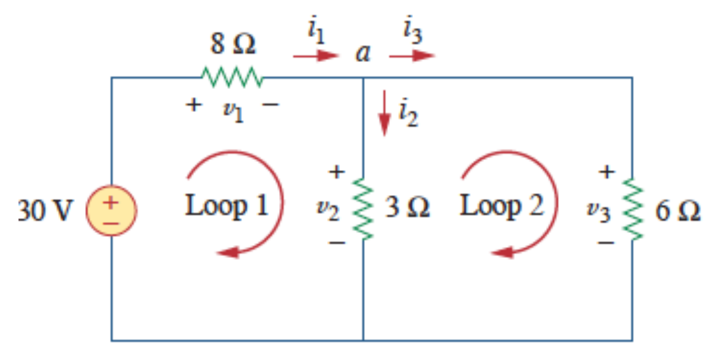
(a)

Figure 2.27

Applying KVL to loop 2,

$$-v_2 + v_3 = 0 \quad \Rightarrow \quad v_3 = v_2$$

$$6i_3 = 3i_2 \quad \Rightarrow \quad i_3 = \frac{i_2}{2}$$



(b)

$$i_1 - i_2 - i_3 = 0$$

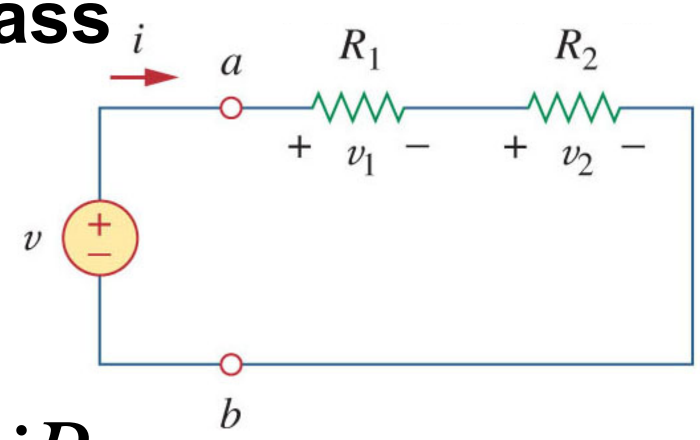
$$\Rightarrow \quad \frac{30 - 3i_2}{8} - i_2 - \frac{i_2}{2} = 0$$

$$\Rightarrow \quad i_2 = 2 \text{ A}.$$

$$\Rightarrow \quad i_1 = 3 \text{ A}, \quad i_3 = 1 \text{ A}, \quad v_1 = 24 \text{ V}, \quad v_2 = 6 \text{ V}, \quad v_3 = 6 \text{ V}$$

Series Resistors

- Two resistors are considered in series if the same current pass through them
- Take the circuit shown:



- Applying Ohm's law to both resistors

$$v_1 = iR_1 \qquad v_2 = iR_2$$

- If we apply KVL to the loop we have:

$$-v + v_1 + v_2 = 0$$

Series Resistors II

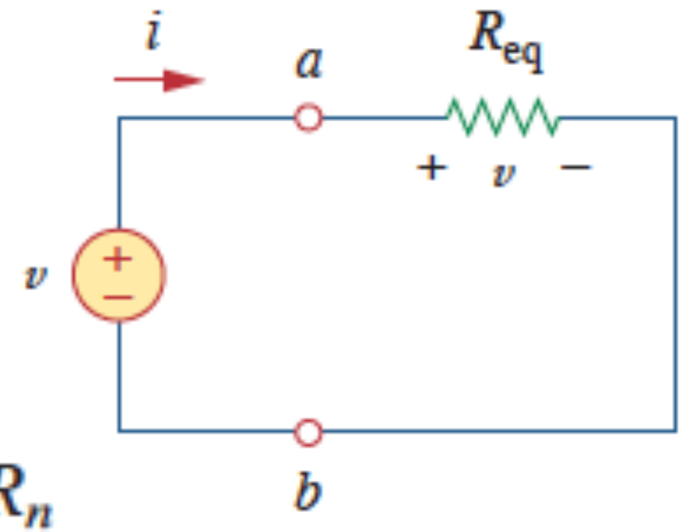
- **Combining the two equations:**

$$v = v_1 + v_2 = i\left(R_1 + R_2\right)$$

- **From this we can see there is an equivalent resistance of the two resistors:**

$$R_{eq} = R_1 + R_2$$

- **For N resistors in series:**



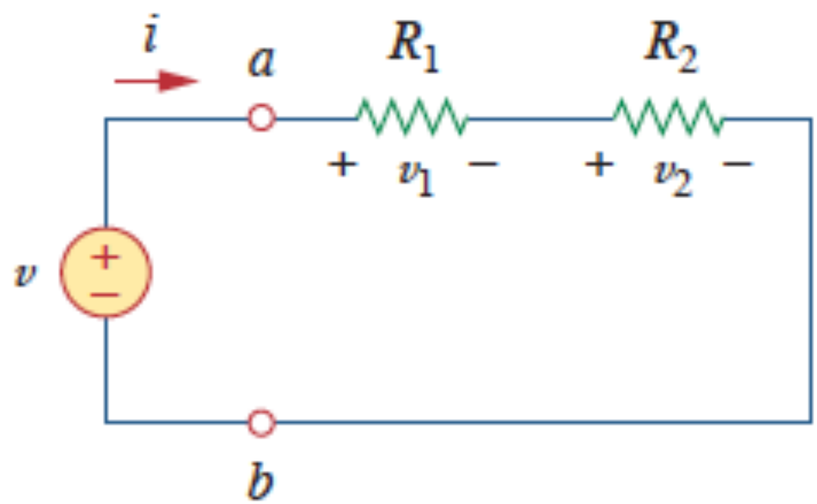
$$R_{eq} = R_1 + R_2 + \cdots + R_N = \sum_{n=1}^{N} R_n$$

Voltage Division

- The voltage drop across any one resistor can be known.
- The current through all the resistors is the same, so using Ohm's law:

$$v = v_1 + v_2 = i\left(R_1 + R_2\right)$$

$$v_1 = \frac{R_1}{R_1 + R_2}v \qquad v_2 = \frac{R_2}{R_1 + R_2}v$$



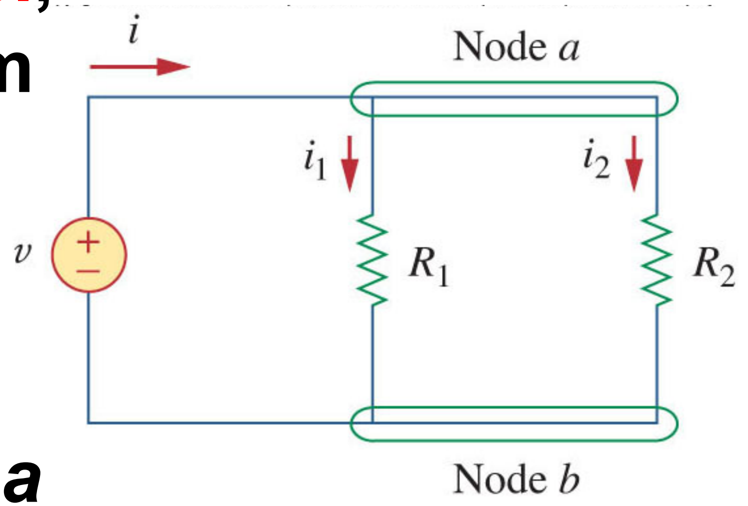
- This is the principle of voltage division

Parallel Resistors

- When **resistors** are in **parallel**, the voltage drop across them is the same

$$v = i_1 R_1 = i_2 R_2$$

$$i_1 = \frac{v}{R_1}, \qquad i_2 = \frac{v}{R_2}$$



- By KCL, the current at node *a* is

$$i = i_1 + i_2$$

$$i = \frac{v}{R_1} + \frac{v}{R_2} = v\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = \frac{v}{R_{eq}}$$

- The equivalent resistance is:

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N}$$

The **equivalent resistance** of two parallel resistors is equal to the product of their resistances divided by their sum.

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} \qquad \frac{1}{R_{eq}} = \frac{R_1 + R_2}{R_1 R_2}$$

with N resistors in parallel.

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N}$$

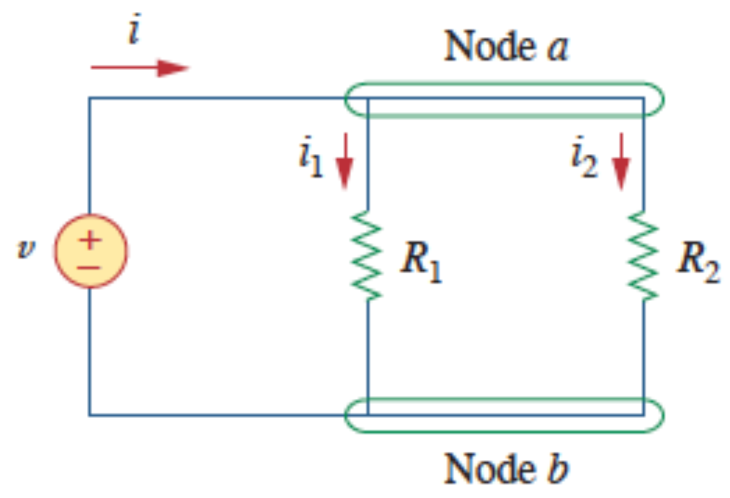
$$G_{eq} = G_1 + G_2 + G_3 + \cdots + G_N$$

Note that R_{eq} is always smaller than the resistance of the smallest resistor in the parallel combination. If $R_1 = R_2 = \cdots = R_N = R$, then

$$R_{eq} = \frac{R}{N} \tag{2.39}$$

Current Division

- Given the current entering the node, the voltage drop across the equivalent resistance will be the same as that for the individual resistors:



$$v = iR_{eq} = \frac{iR_1R_2}{R_1 + R_2}$$

- This can be used in combination with Ohm's law to get the current through each resistor:

$$i_1 = \frac{iR_2}{R_1 + R_2} \qquad i_2 = \frac{iR_1}{R_1 + R_2}$$

Current Division

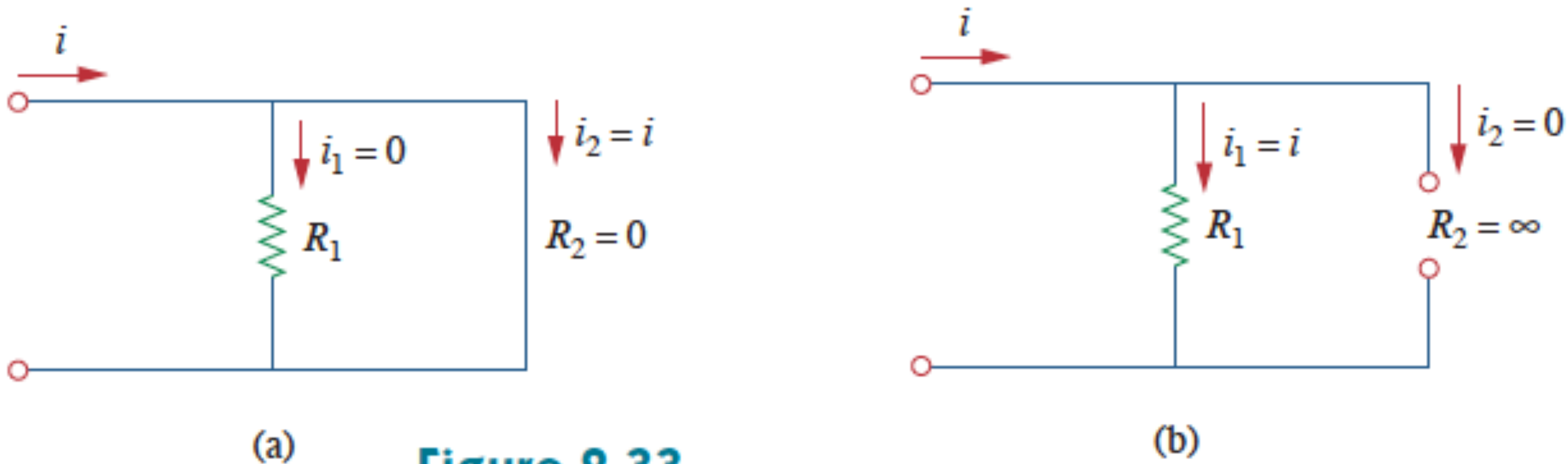


Figure 2.33
(a) A shorted circuit, (b) an open circuit.

$$i_1 = \frac{R_2\, i}{R_1 + R_2}, \qquad i_2 = \frac{R_1\, i}{R_1 + R_2}$$

Practice Problem 2.9

By combining the resistors in Fig. 2.36, find R_{eq}.

Answer: $6\ \Omega$.

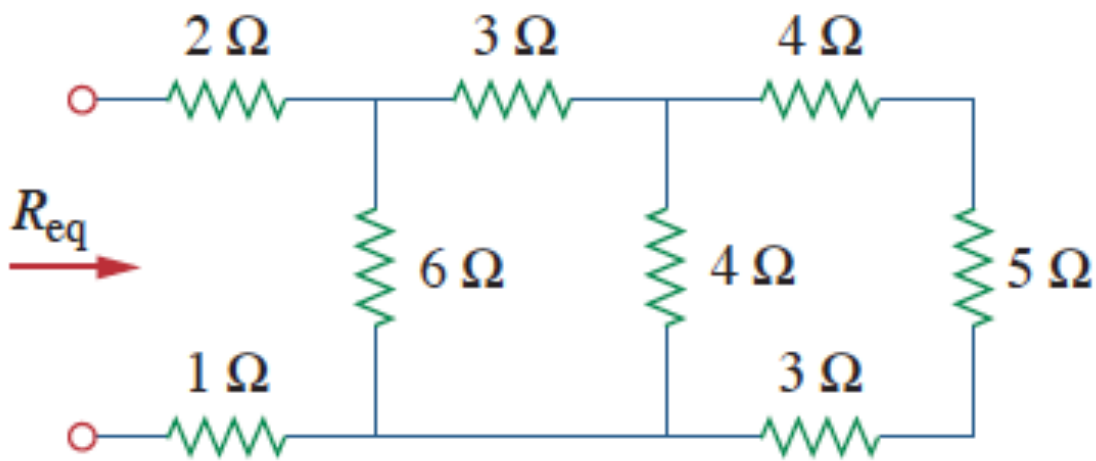


Figure 2.36

Example 2.10

Calculate the equivalent resistance R_{ab}

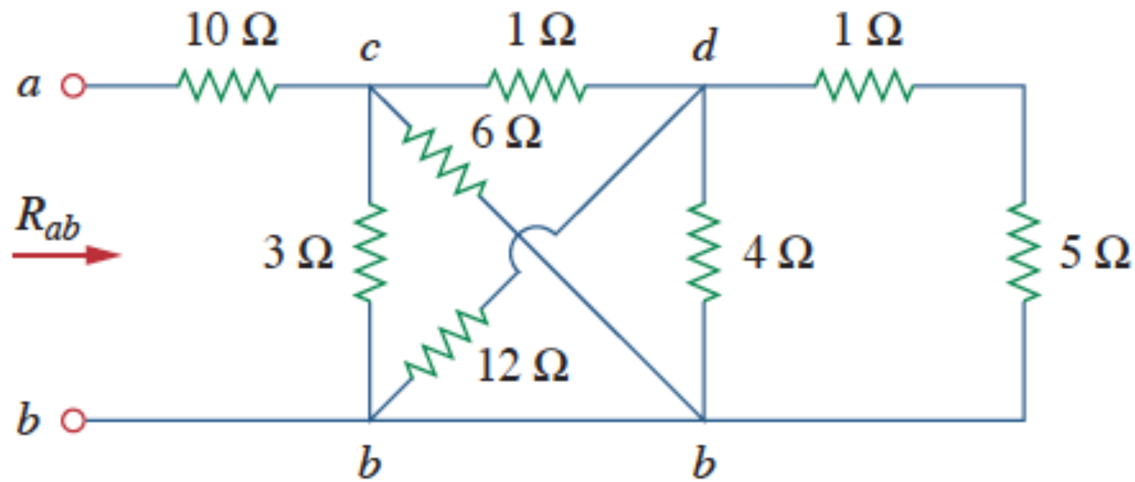
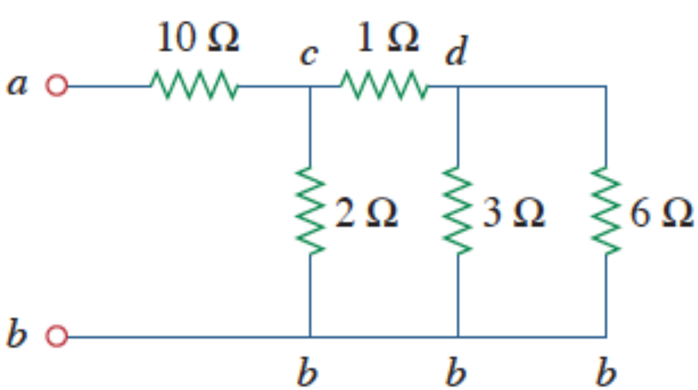


Figure 2.37

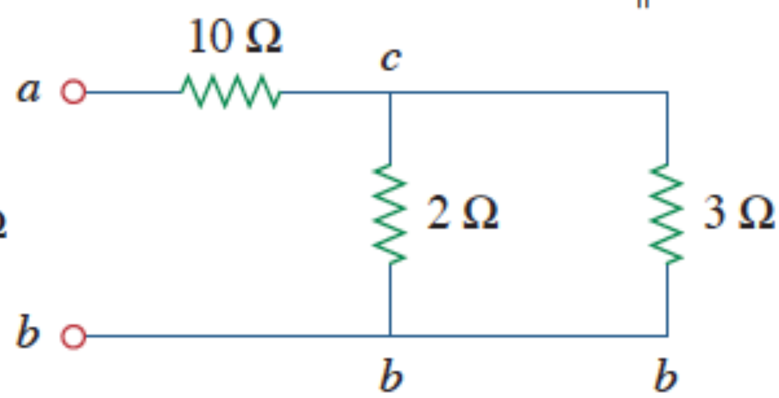
$$3\,\Omega \parallel 6\,\Omega = \frac{3 \times 6}{3 + 6} = 2\,\Omega$$

$$12\,\Omega \parallel 4\,\Omega = \frac{12 \times 4}{12 + 4} = 3\,\Omega$$

$$2\,\Omega \parallel 3\,\Omega = \frac{2 \times 3}{2 + 3} = 1.2\,\Omega$$



(a)

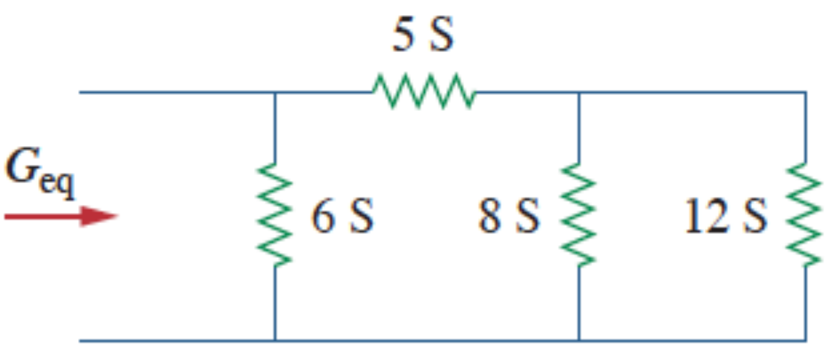


(b)

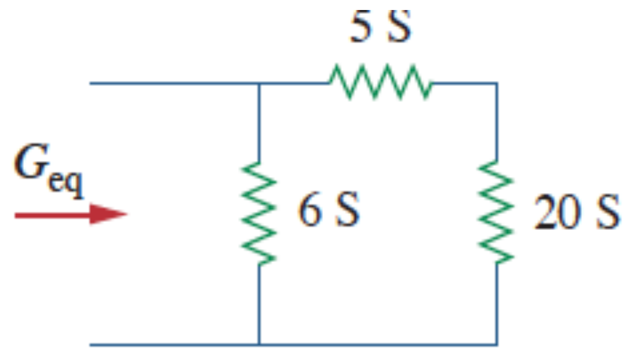
$$R_{ab} = 10 + 1.2 = 11.2\,\Omega$$

Example 2.11

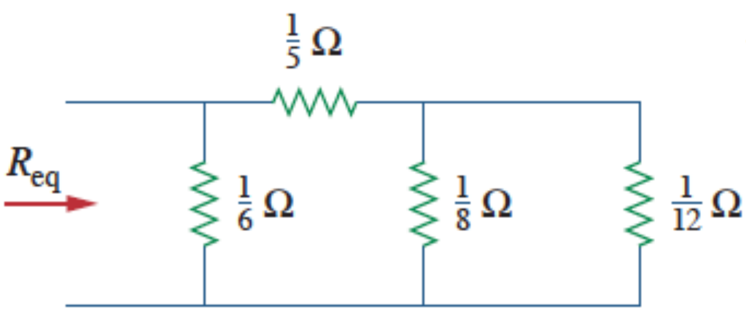
Find the equivalent conductance G_{eq} for the circuit



(a)



(b)



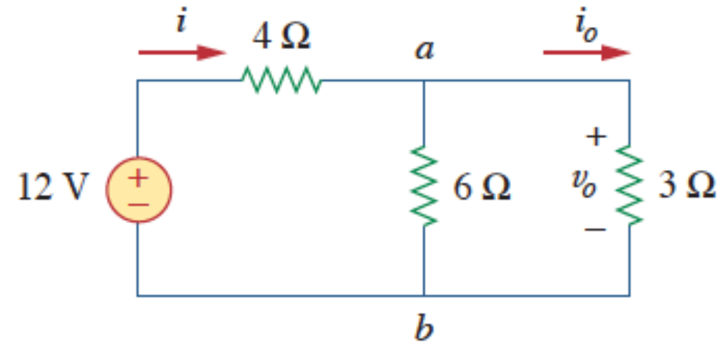
(c)

$$R_{eq} = \frac{1}{6} \left\| \left(\frac{1}{5} + \frac{1}{8} \left\| \frac{1}{12} \right. \right) = \frac{1}{6} \right\| \left(\frac{1}{5} + \frac{1}{20} \right) = \frac{1}{6} \left\| \frac{1}{4} \right.$$

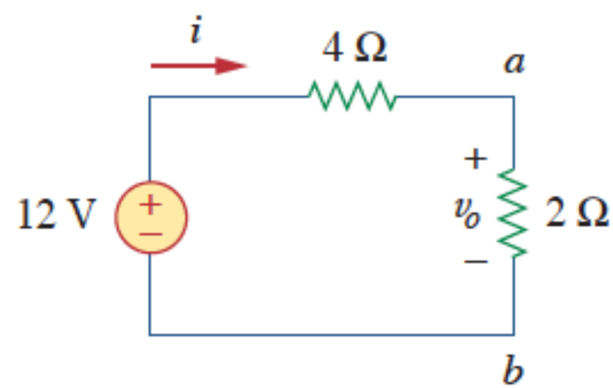
$$= \frac{\frac{1}{6} \times \frac{1}{4}}{\frac{1}{6} + \frac{1}{4}} = \frac{1}{10}\,\Omega$$

Example 2.12

Find i_o and v_o in the circuit shown in Fig. 2.42(a). Calculate the power dissipated in the 3-Ω resistor.



(a)



(b)

The 6-Ω and 3-Ω resistors are in parallel,

$$6\,\Omega \parallel 3\,\Omega = \frac{6 \times 3}{6+3} = 2\,\Omega$$

$$v_o = \frac{2}{2+4}(12\text{ V}) = 4\text{ V}$$

$$i_o = \frac{4}{3}\text{ A}$$

or,

$$i = \frac{12}{4+2} = 2\text{ A}$$

$$i_o = \frac{6}{6+3} i = \frac{2}{3}(2\text{ A}) = \frac{4}{3}\text{ A}$$

The power dissipated in the 3-Ω resistor is

$$p_o = v_o i_o = 4\left(\frac{4}{3}\right) = 5.333\text{ W}$$

Wye-Delta Transformations

- There are cases where resistors are **neither parallel nor series**
- Consider the **bridge circuit** shown here
- This circuit can be simplified to a **three-terminal equivalent**

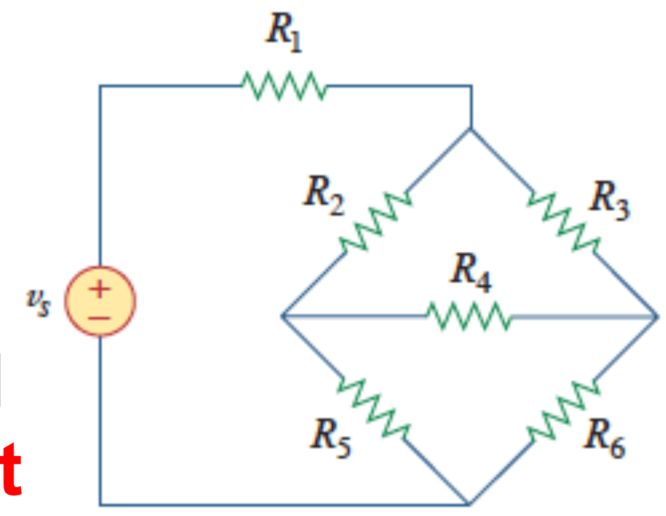


Figure 2.46
The bridge network.

Wye-Delta Transformations II

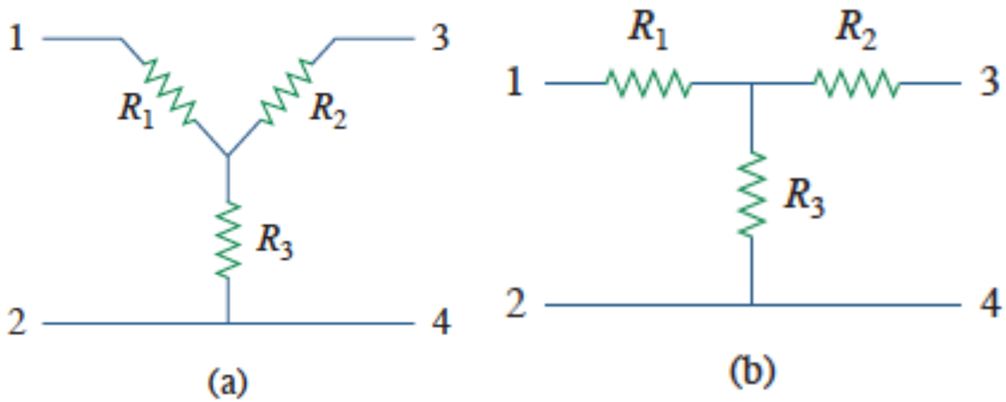


Figure 2.47
Two forms of the same network: (a) Y, (b) T.

- Two topologies can be interchanged:
  - Wye (Y) or tee (T) networks
  - Delta (Δ) or pi (Π) networks
  - Transforming between these two topologies often makes the solution of a circuit easier

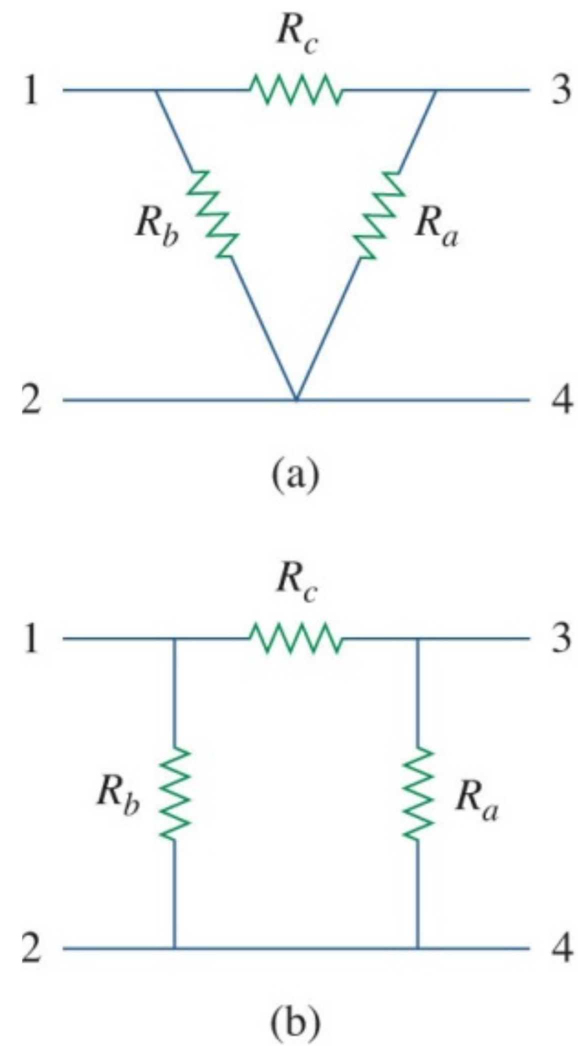


Figure 2.48
Two forms of the same network: (a) Δ, (b) Π.

Wye-Delta Transformations III

- The superimposed Y and Δ circuits shown here will used for reference
- The Δ consists of the outer resistors, labeled a, b, and c
- The Y network are the inside resistors, labeled 1, 2, and 3

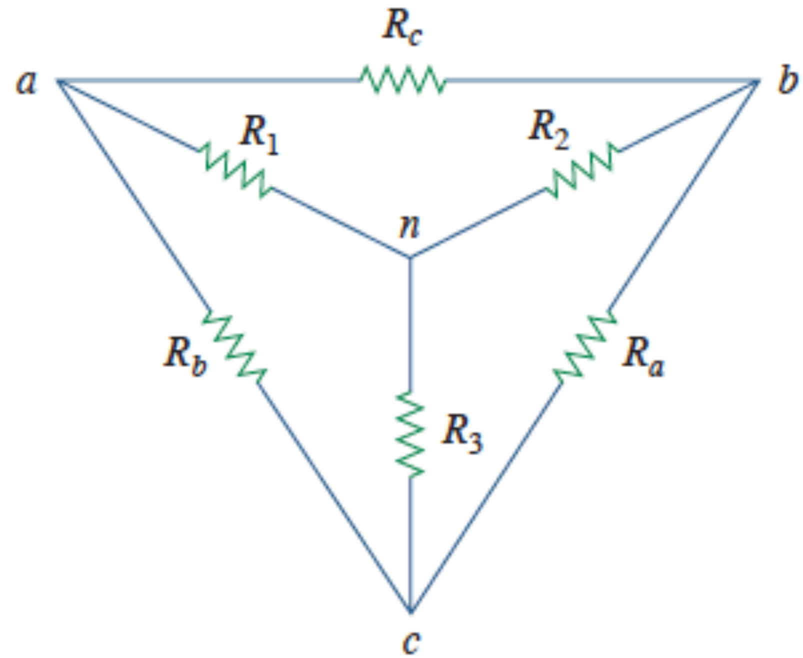


Figure 2.49
Superposition of Y and Δ networks as an aid in transforming one to the other.

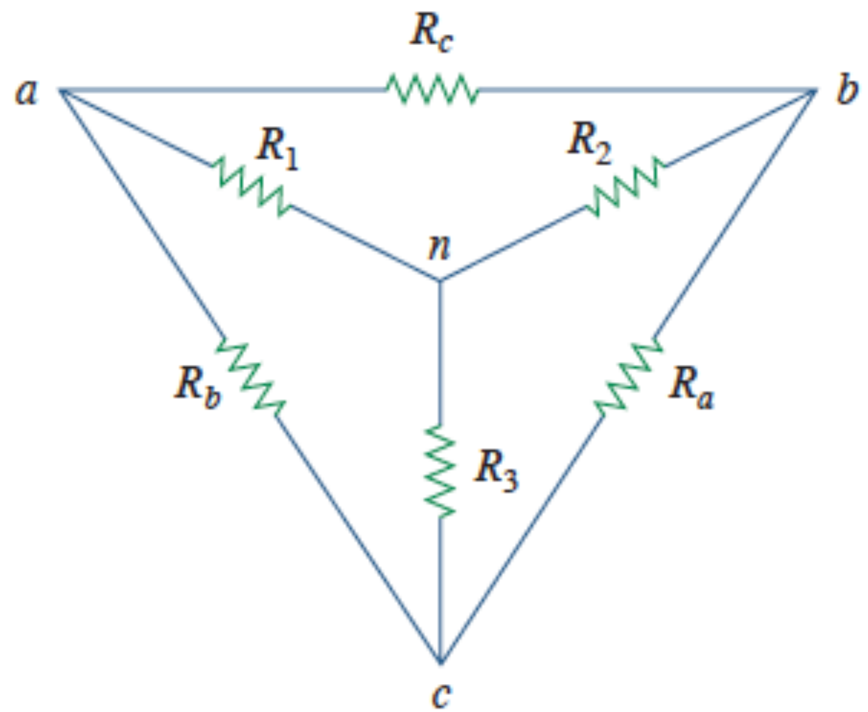
Delta to Wye $\Delta \rightarrow Y$

- **The conversion formula for a Δ to Y transformation are:**

$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c}$$

$$R_2 = \frac{R_c R_a}{R_a + R_b + R_c}$$

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$$



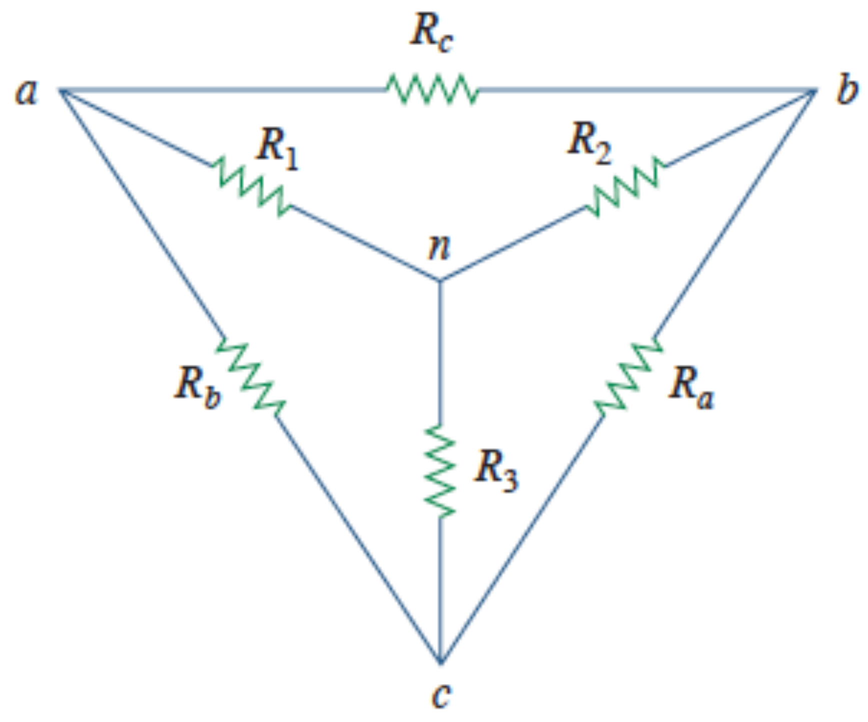
Wye to Delta $\quad Y \rightarrow \Delta$

- **The conversion formula for a Y to Δ transformation are:**

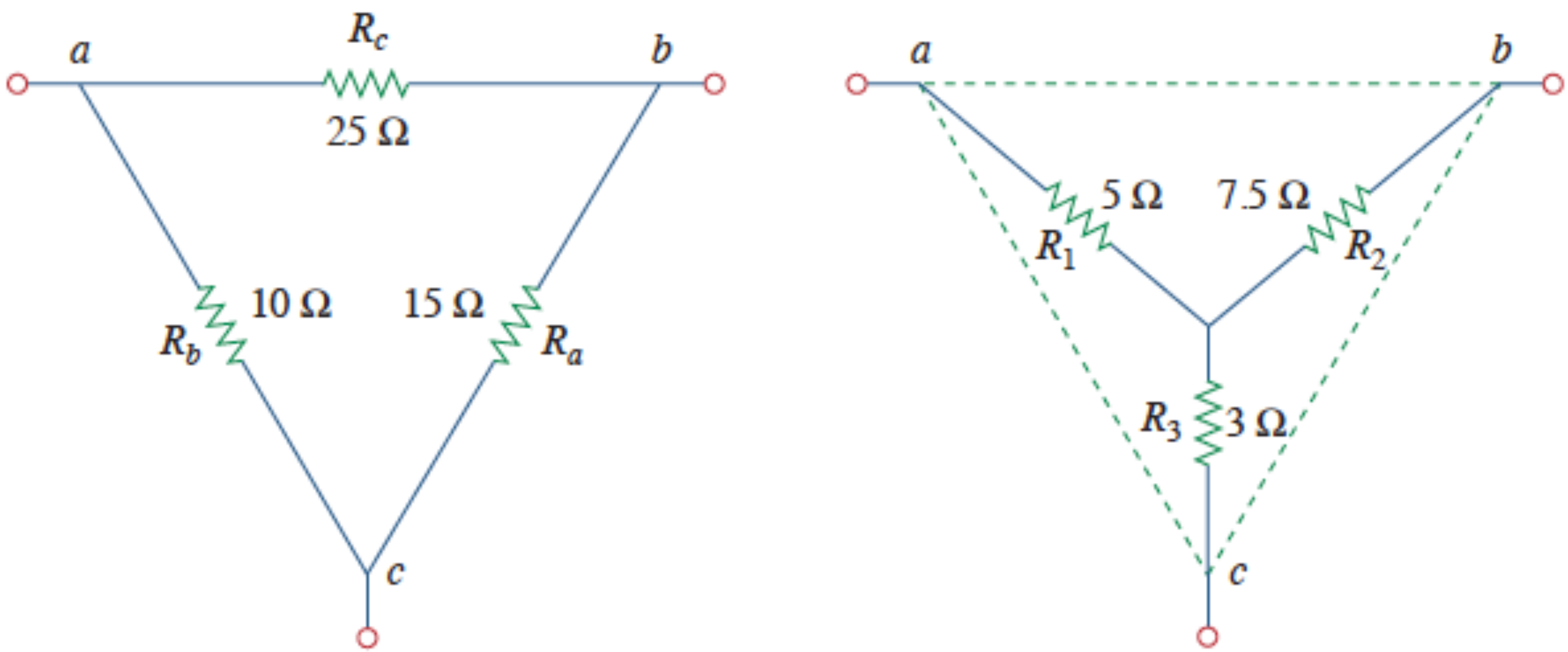
$$R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1}$$

$$R_b = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2}$$

$$R_c = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3}$$



Example 2.14 Convert the Δ network to an equivalent Y network.



(a)

(b)

$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c} = \frac{10 \times 25}{15 + 10 + 25} = \frac{250}{50} = 5\ \Omega$$

$$R_2 = \frac{R_c R_a}{R_a + R_b + R_c} = \frac{25 \times 15}{50} = 7.5\ \Omega$$

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c} = \frac{15 \times 10}{50} = 3\ \Omega$$

Example 2.15

Y → Δ

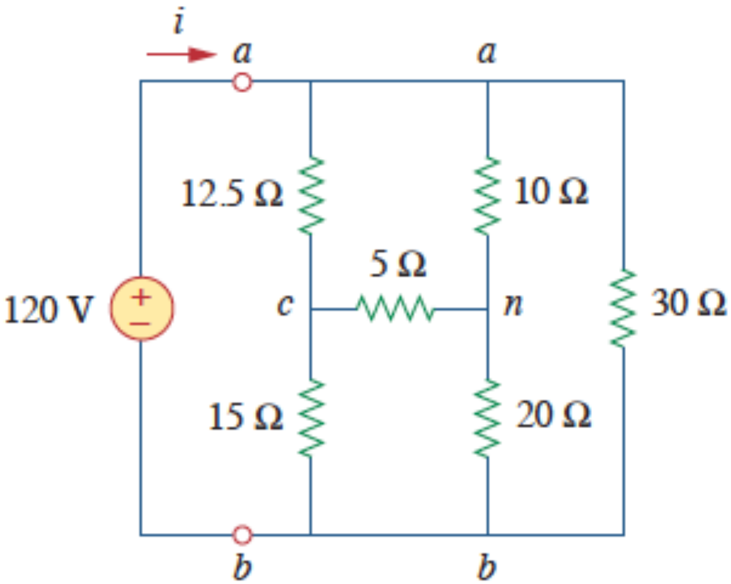
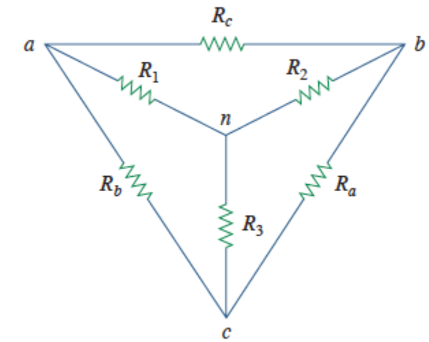


Figure 2.52

$$R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1}$$

$$R_b = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2}$$

$$R_c = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3}$$



$$R_1 = 10\ \Omega, \qquad R_2 = 20\ \Omega, \qquad R_3 = 5\ \Omega$$

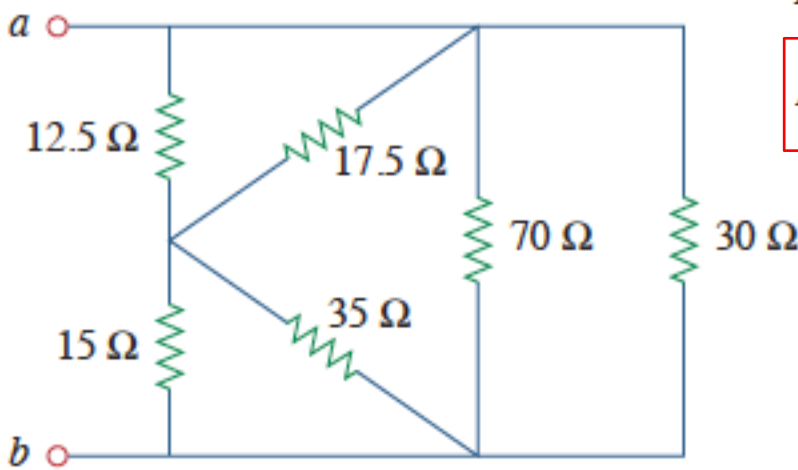
$$R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1} = \frac{10 \times 20 + 20 \times 5 + 5 \times 10}{10}$$

$$= \frac{350}{10} = 35\ \Omega$$

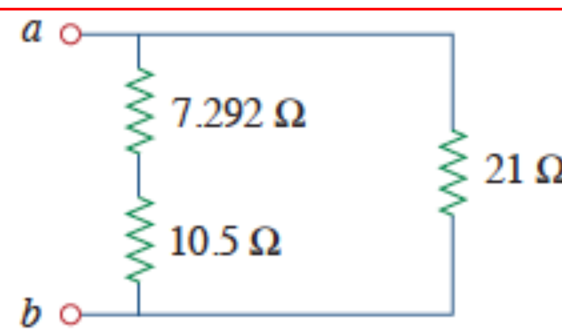
$$R_b = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2} = \frac{350}{20} = 17.5\ \Omega$$

$$R_c = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3} = \frac{350}{5} = 70\ \Omega$$

$$\boxed{R_{ab} = (7.292 + 10.5) \parallel 21 = \frac{17.792 \times 21}{17.792 + 21} = \mathbf{9.632\ \Omega}}$$



(a)



(b)

Example 2.15

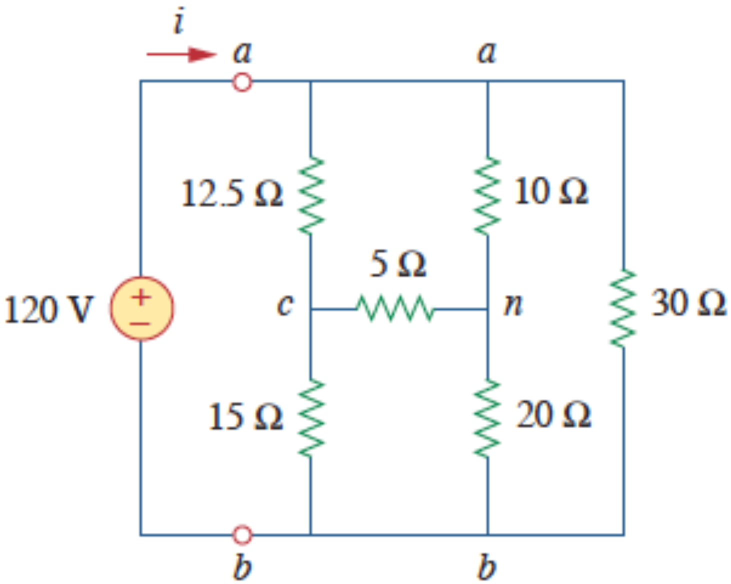
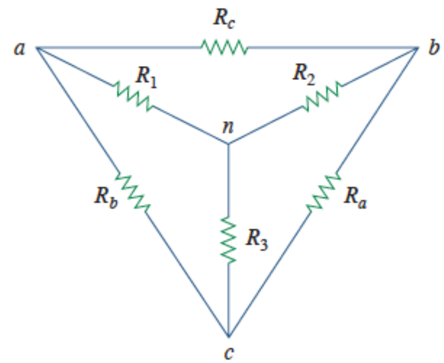


Figure 2.52

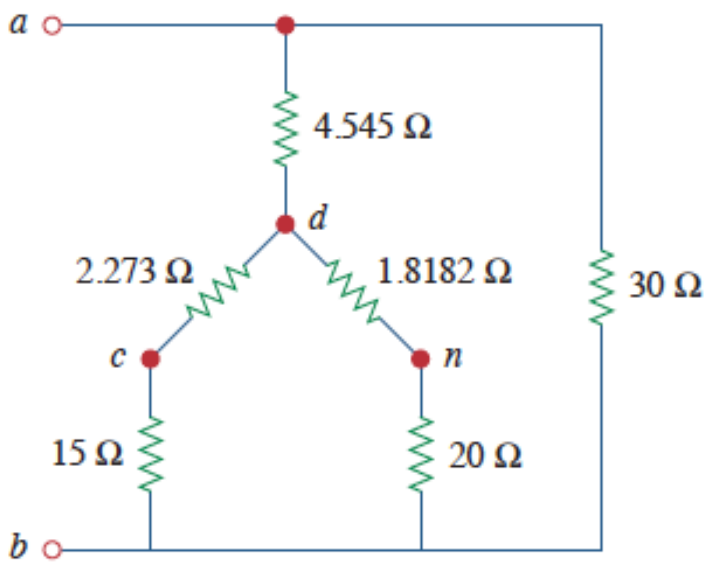
Δ → Y



$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c}$$

$$R_2 = \frac{R_c R_a}{R_a + R_b + R_c}$$

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$$



(c)

$$R_{ad} = \frac{R_c R_n}{R_a + R_c + R_n} = \frac{10 \times 12.5}{5 + 10 + 12.5} = 4.545\ \Omega$$

$$R_{cd} = \frac{R_a R_n}{27.5} = \frac{5 \times 12.5}{27.5} = 2.273\ \Omega$$

$$R_{nd} = \frac{R_a R_c}{27.5} = \frac{5 \times 10}{27.5} = 1.8182\ \Omega$$

$$R_{db} = \frac{(2.273 + 15)(1.8182 + 20)}{2.273 + 15 + 1.8182 + 20} = \frac{376.9}{39.09} = 9.642\ \Omega$$

$$\boxed{R_{ab} = \frac{(9.642 + 4.545)30}{9.642 + 4.545 + 30} = \frac{425.6}{44.19} = \mathbf{9.631\ \Omega}}$$

Practice Problem 2.15

For the bridge network, find R_{ab} and i.

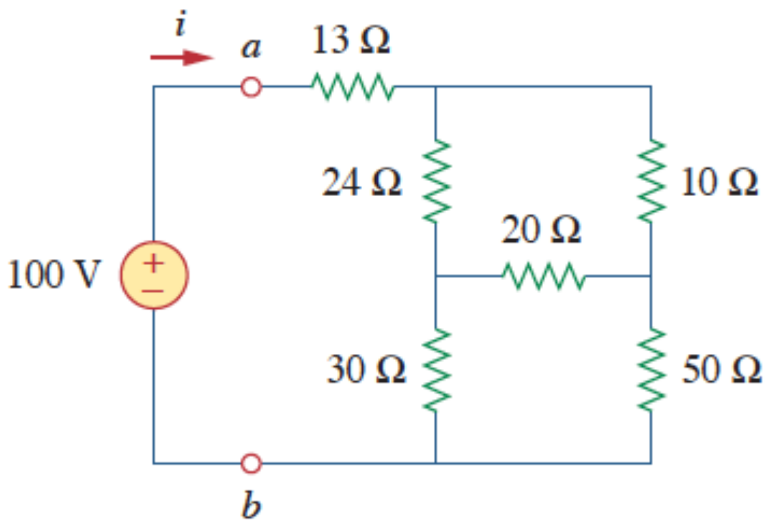
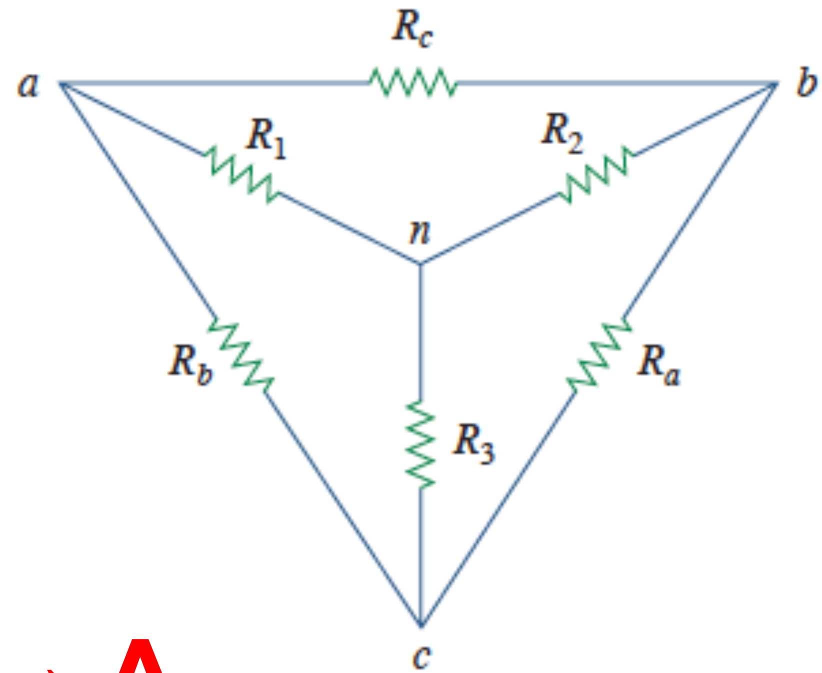


Figure 2.54



Δ → Y

$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c}$$

$$R_2 = \frac{R_c R_a}{R_a + R_b + R_c}$$

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$$

Y → Δ

$$R_a = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_1}$$

$$R_b = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_2}$$

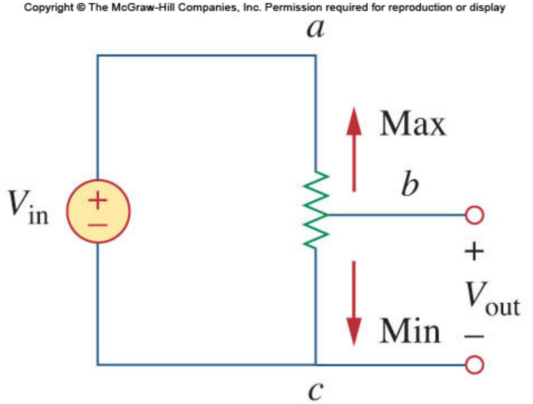
$$R_c = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_3}$$

Design of DC Meters

- Resistors by their nature control current.
- This property may be used directly to control voltages, as in the potentiometer
- The voltage output is:

$$V_{out} = V_{bc} = \frac{R_{bc}}{R_{ac}} V_{in}$$

- Resistors can also be used to make meters for measuring voltage and resistance

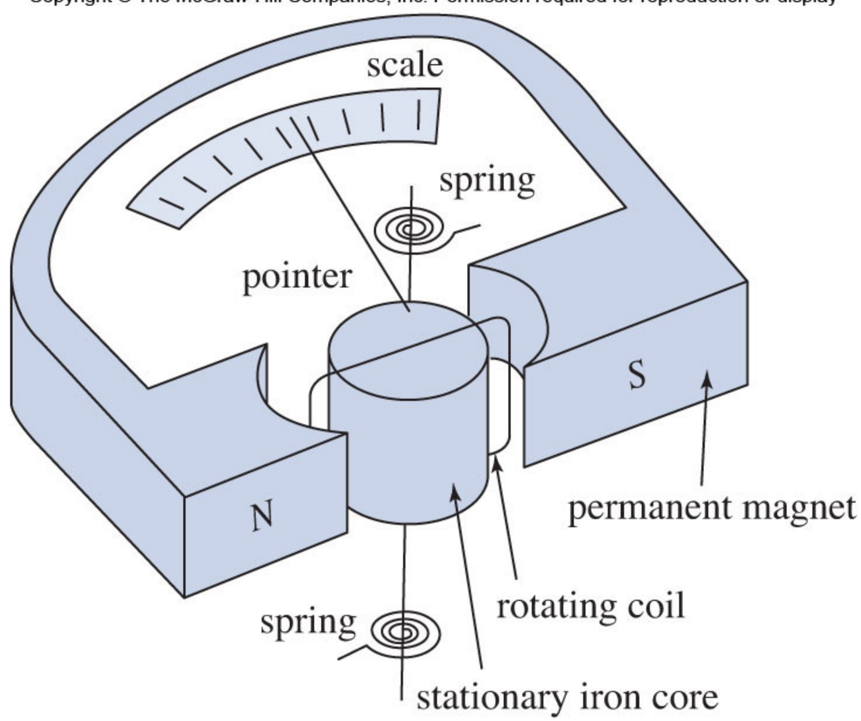


D’Arsonval Meter Movement

- Here we will look at DC analog meters
- The operation of a digital meter is beyond the scope of this chapter
- These are the meters where a needle deflection is used to read the measured value
- All of these meters rely on the D’Arsenol meter movement:
  - This has a pivoting iron core coil
  - Current through this causes a deflection

D’Arsonval Meter Movement

- **Below is an example of a D’Arsonval Meter Movement**



Ammeter

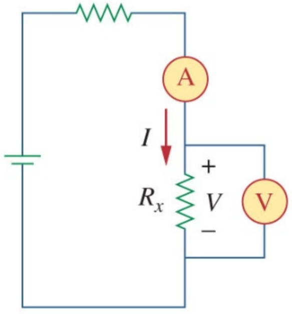
- It should be clear that the basic meter movement directly measured current.
- The needle deflection is proportional to the current up to the rated maximum value
- The coil also has an internal resistance
- In order to measure a greater current, a resistor (shunt) may be added in parallel to the meter.
- The new max value for the meter is:

Voltmeter

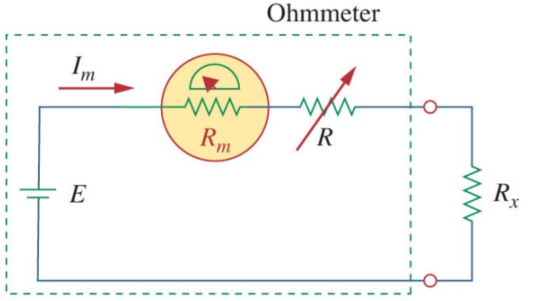
- **Ohm's law can be used to convert the meter movement into a voltmeter**
- **By adding a resistor in series with the movement, the sum of the meter's internal resistance and the external resistor are combined.**
- **A voltage applied across this pair will result in a specific current, which can be measured**
- **The full scale voltage measured is:**

Ohmmeter

- We know that resistance is related the voltage and current passing through a circuit element.
- The meter movement is already capable of measuring current
- What is needed is to add a voltage source
- By KVL:



(a)



(b)

$$R_x = \frac{E}{I_m} - (R + R_m)$$

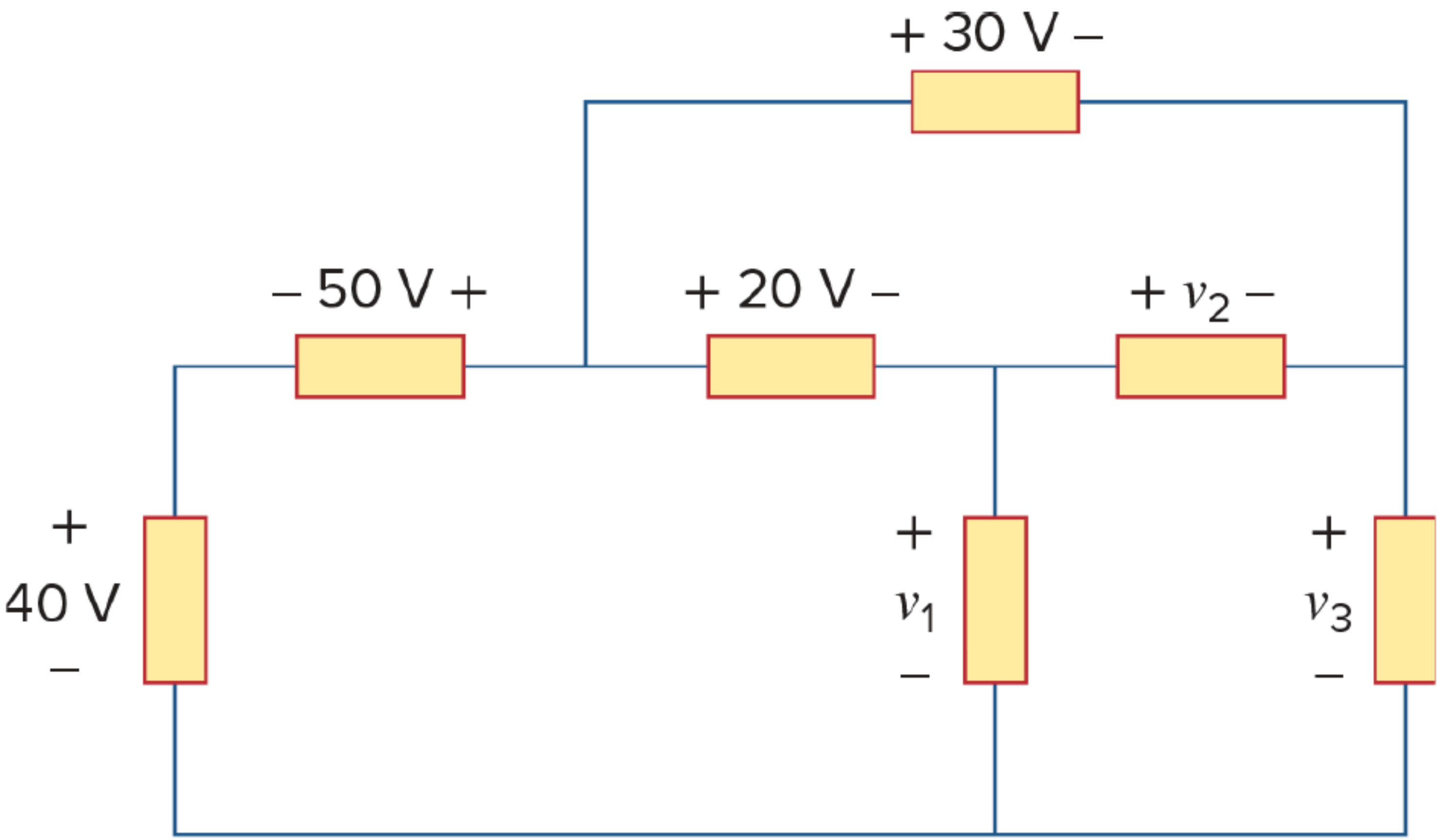
Ohmmeter II

- **The internal resistor is chosen such that when the external resistor is zero, the meter is at full deflection**
- **This yields the following relationship between measured current and resistance**

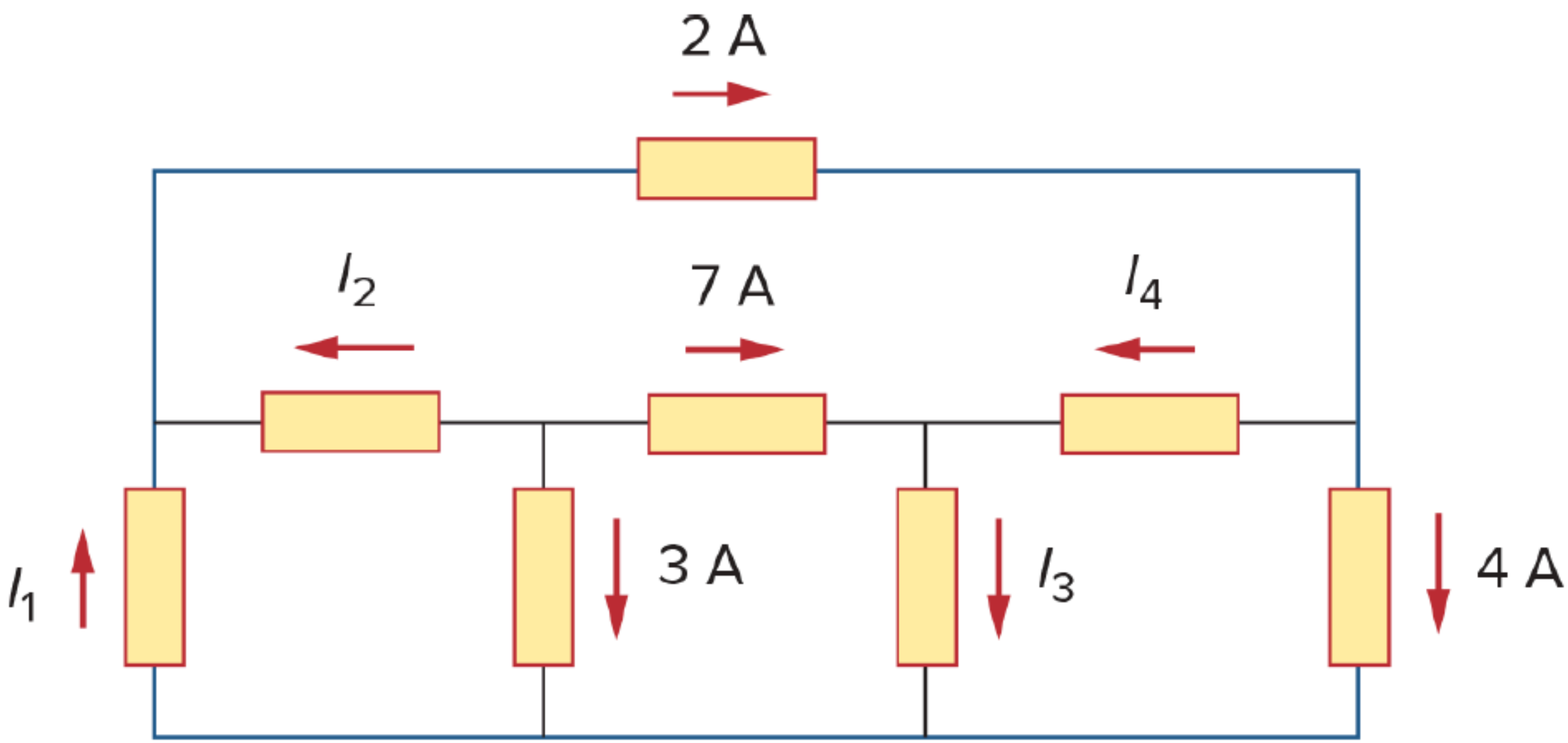
$$R_x = \left(\frac{I_{fs}}{I_m} - 1 \right)(R + R_m)$$

- **A consequence to measuring the current is that the readout of the meter will be the inverse of the resistance.**

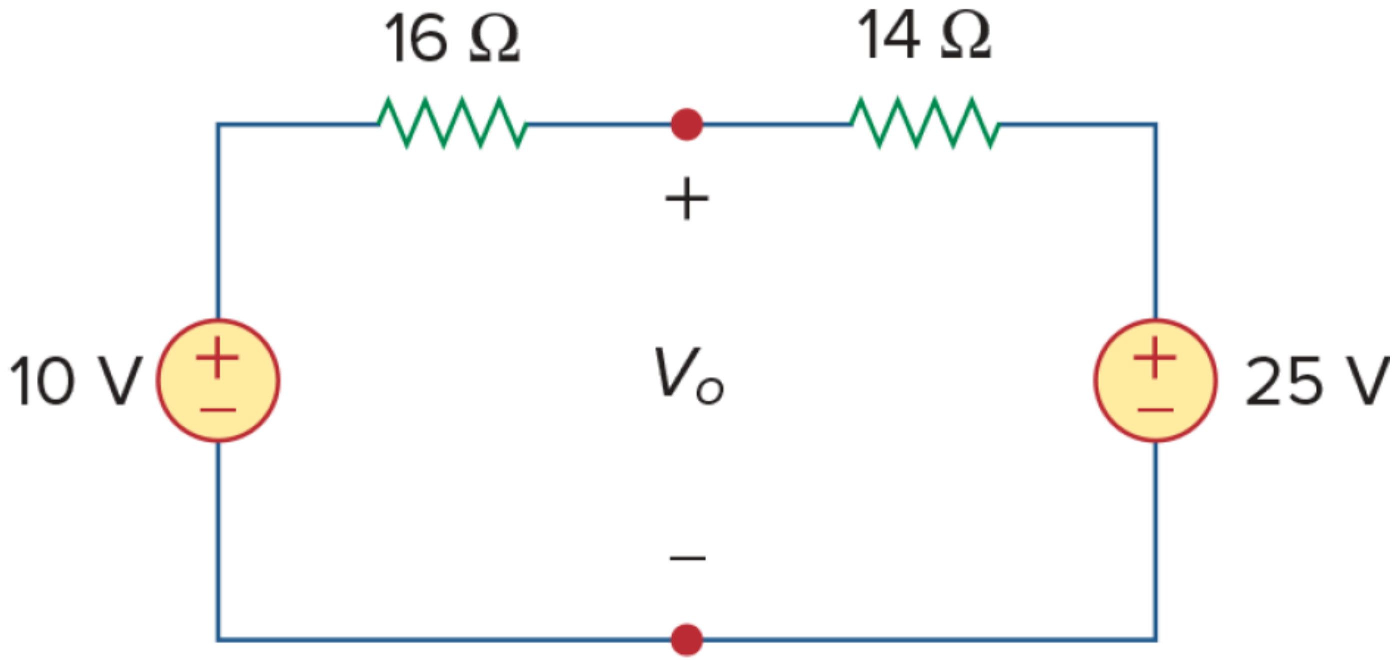
2.12 In the circuit in Fig. 2.76, obtain v_1, v_2, and v_3.



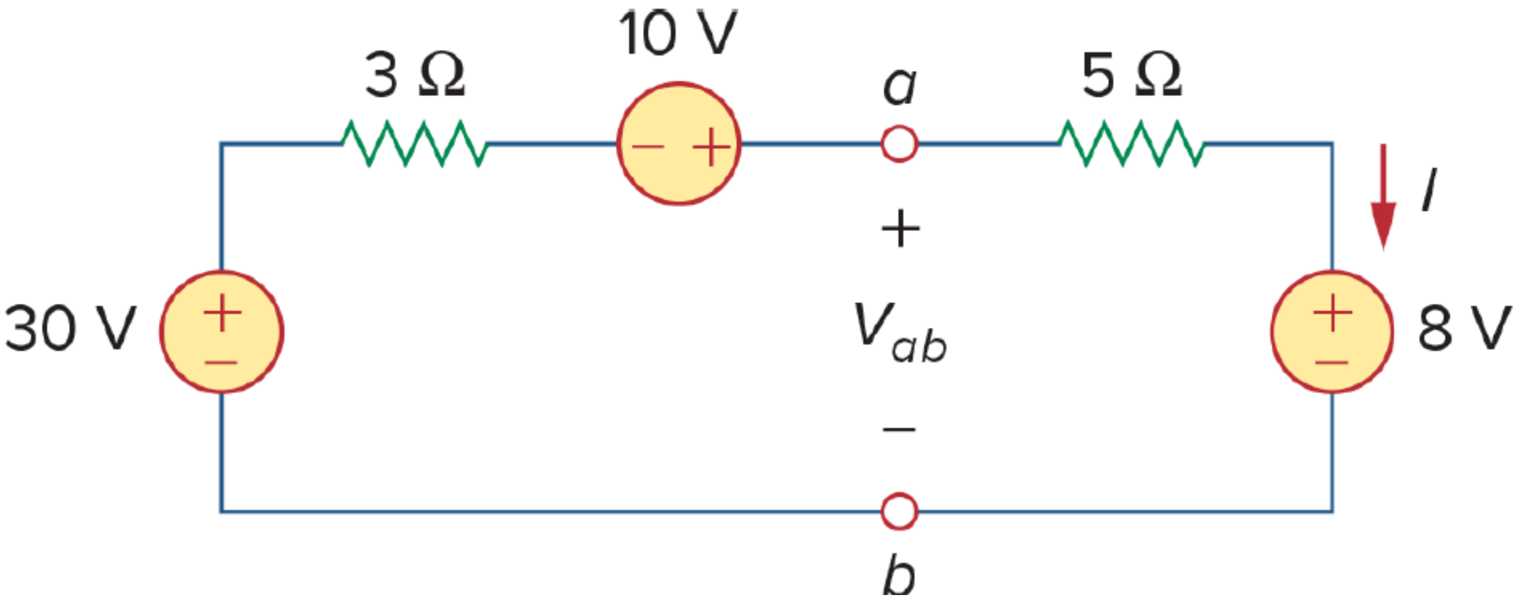
2.13 For the circuit in Fig. 2.77, use KCL to find the branch currents I_1 to I_4.



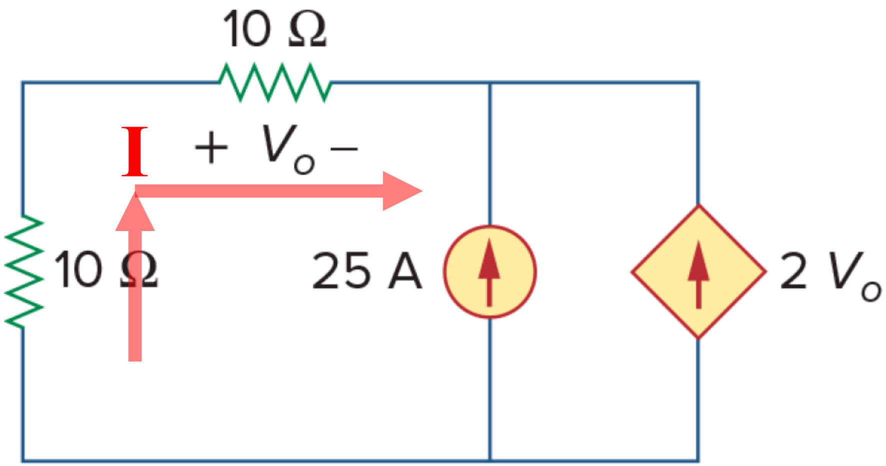
2.16 Determine V_o in the circuit



2.18 Find I and V_{ab} in the circuit



2.22 Find V_o in the circuit in Fig. 2.86 and the power absorbed by the dependent source.



KCL:

$$I + 25 + 2V_O = 0$$

$$\Rightarrow I = -25 - 2V_O$$

$$V_O = I \cdot 10 = -250 - 20V_O$$

$$\Rightarrow 21 \cdot V_O = -250$$

$$\Rightarrow V_O = \frac{-250}{21} = -11.9048(V)$$

Current of the dependent source:

$$\Rightarrow 2 \cdot V_O = -23.8096(A)$$

Voltage Across the dependent source:

$$\Rightarrow -I \cdot (10 + 10)$$

$$= -20 \cdot I$$

$$= 500 + 40V_O$$

$$= 500 - 40 \times 11.9048$$

$$= 23.808(V)$$

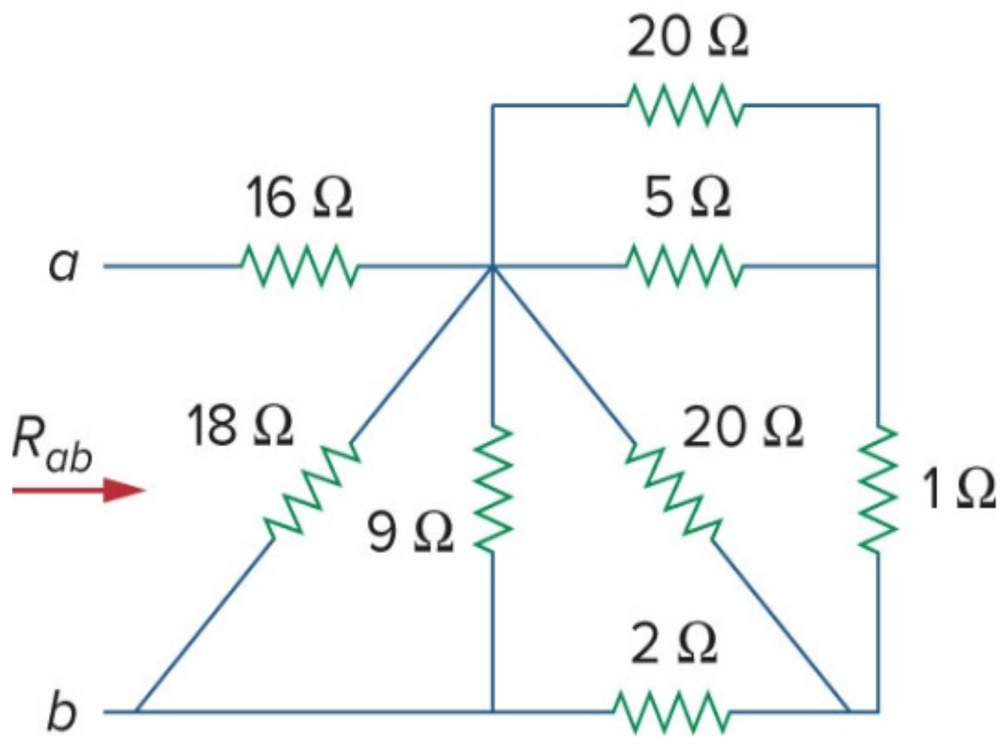
Power absorbed by the dependent source:

$$\Rightarrow P = V \cdot (-2 \cdot V_O)$$

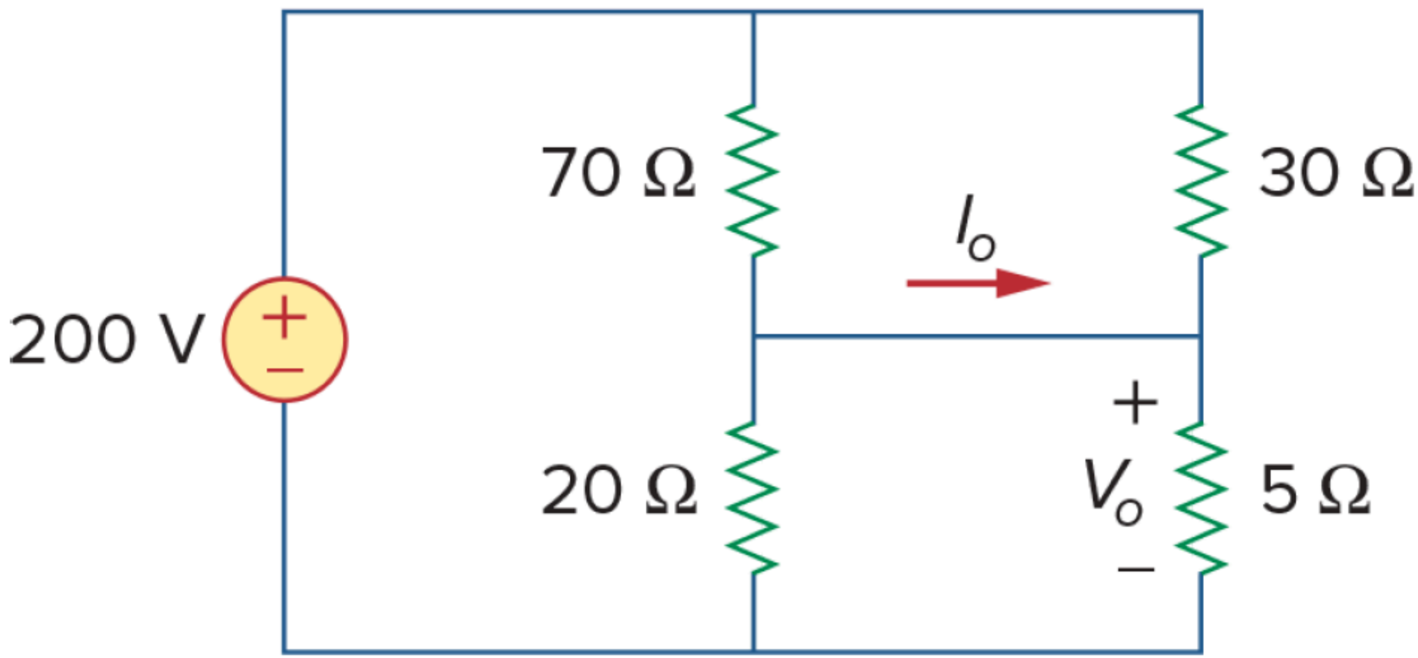
$$= 23.808 \times 23.8096$$

$$= 566.8590(W)$$

Find R_{ab} for the circuit

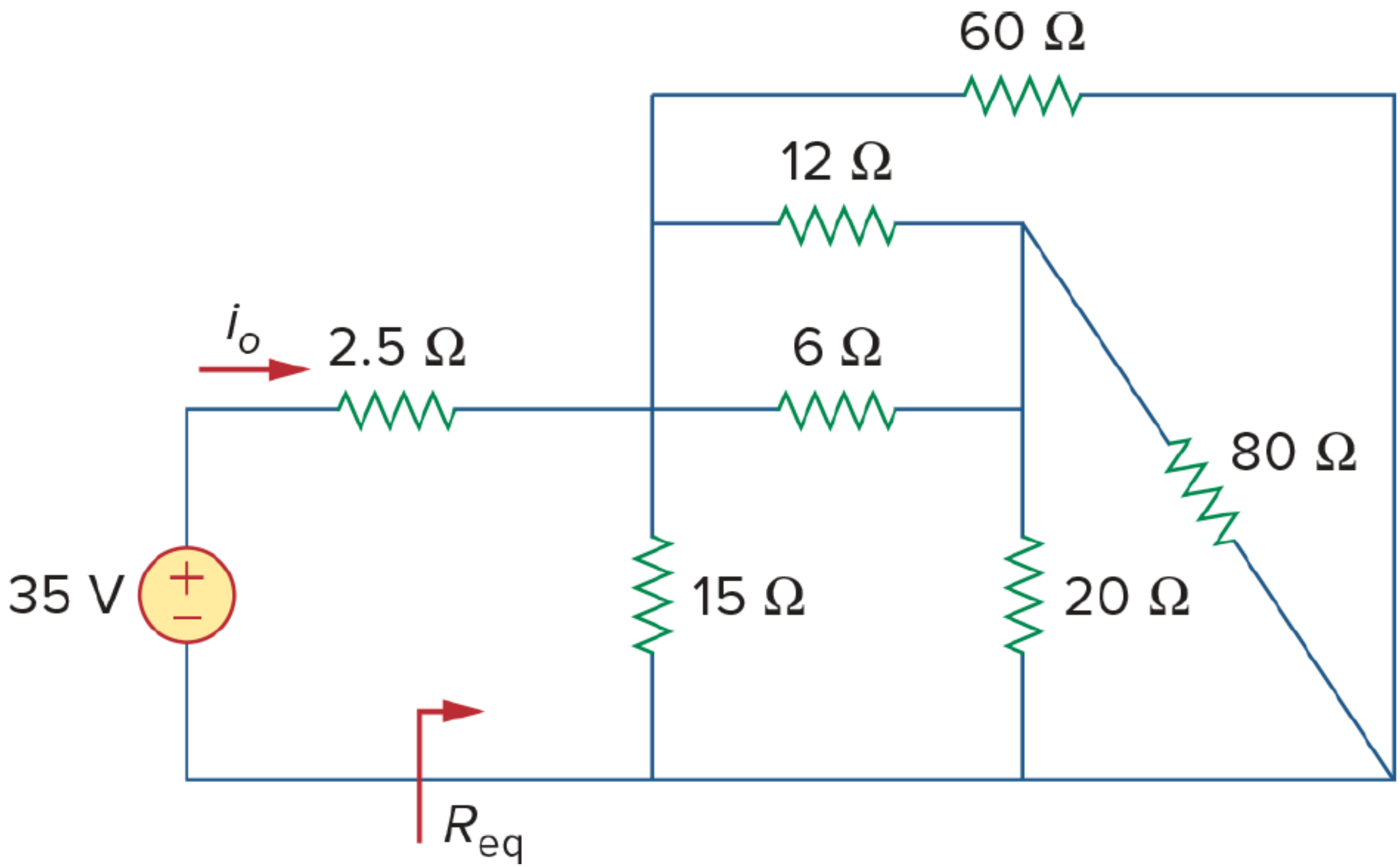


2.35 Calculate V_o and I_o in the circuit



V_O = **32 (V)** and I_O = **0.8(A)**

2.38 Find R_{eq} and i_o in the circuit



2.31 For the circuit determine i_1 to i_5.