Lab. 15, Young's modulus of metals

已註解 [t1]: Reference: http://uep.phy.ncu.edu.tw/content/general-physics/translation/first-semester/the--measurement-of-youngs-modulus-for-metal-wires/pdf1.pdf

1. Objective:

To determine the Young's modulus of metal beams by bending them.

已註解 [t2]: OPTION: The objective of this experiment is to determine...

2. Theory:

1. According to Hooke's law, the stress in any metal within its elastic range is proportional to the corresponding strain. There are three types of strain:
   1. longitudinal strain, which indicates change in the length of the material;
   2. shear strain, in which the object deforms along the tangential direction of its surface, enabling a cube to become a parallelogram;
   3. bulk strain, which means that the volume of the object changes but not its shape.

   The ratio of stress to strain in these three situations is called the elastic modulus or the modulus of elasticity.
2. Figure 1(a) shows a beam of width w and height h placed on two fulcrums, the distance between which is L. Suppose a load W is placed on the beam, causing it to bend as shown in Fig. 1(b) (the deformation in the figure is exaggerated; in reality the deformation is extremely small). As a result, the upper side of the beam is shortened due to compression, and the lower side of the beam is lengthened. In between, there must be a layer AB that remains the same length, which is referred to as the neutral surface. Both longitudinal strain and shear strain occur in this case. The deformation is so small that shear strain is negligible, so we only consider longitudinal strain. The elastic modulus for longitudinal strain is called Young’s modulus, defined as

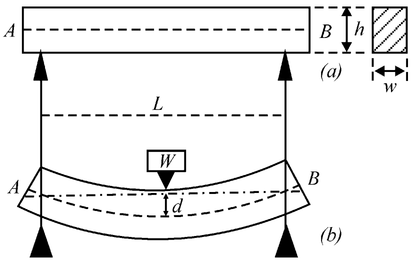


Figure 1

$$Y = \frac{(F/A)}{(\Delta L/L)} \tag{1}$$

where F is the lengthening or shortening force; A denotes the area of the cross-section; L is the original length of the metal beam, and ΔL is change in length of the beam. As shown in Fig. 1(b), the layers above and below the neutral surface either lengthen or shorten, which results in different strain in each layer. Consequently, Young's modulus cannot be directly calculated from Eq. (1). Using calculus, we can derive the formula below (based on material mechanics):

$$d = \frac{WL^3}{4wh^3Y} \tag{2}$$

where d denotes the deflection of the beam, which is the vertical displacement at the middle of the beam caused by load W. As might be expected, d is too small to be measured directly with a ruler, and for this reason, we use the optical lever method to obtain d in the experiment, the formula for which is

$$d = \frac{lD}{2S} \tag{3}$$

where D: the difference between the positions of the reflected light on the meter stick before and after the load is placed on the beam

l: the vertical distance between the hind leg of the optical lever and the line connecting the two legs of the mirror

S: the distance between the mirror of the optical lever and the screen

For an explanation of the principle behind Eq. (3), please refer to the optical lever experiment.

3. If the beam is a cylinder of radius r, then

$$d = \frac{WL^3}{12\pi r^4Y} \tag{4}$$

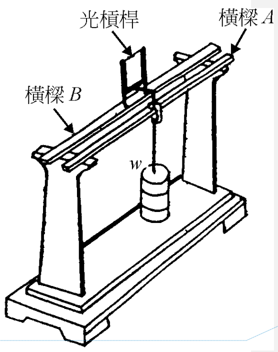
Thus, Eqs. (2) and (4) can be expressed as

$$Y = \frac{KW}{d} \tag{5}$$

where $K = L^3/4wh^3$ for a rectangular beam

$K = L^3/12\pi r^4$ for a cylindrical beam

Young's modulus Y can thus be derived.



4. Table 1 presents reference values for Young's modulus (unit: dynes/cı

Fig. 2

已註解 [s3]: I'm not sure what this should be.

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Table 1 Young's modulus

Metal	Young's modulus (dynes/cm^2)
Lead	1.6×10^{11}
Aluminum	7.0×10^{11}
Brass	9.1×10^{11}
Copper	11×10^{11}
Iron	19×10^{11}
Steel	20×10^{11}

3. Apparatus:

three-in-one stand, meter stick, hook, scale pan, weights, optical lever, vernier scale, three beams of different materials, laser source, screen

已註解 [t5]: Shown in Fig. 2, but I cannot find the term for it.

4. Procedure:

1. As shown in Fig. 2, place two beams horizontally on the two cutting edges of the three-in-one stand, where A as the beam being tested. Hang the hook from the middle of Beam A, which will be used to attach the scale pan and weights.
2. Place the front and hind legs of the optical lever on Beams *A* and *B*, respectively. Set up the laser source at an appropriate distance in front of the optical lever, and record *S*.
3. Add 200 g of weight, and record *D*. Add another 200 g, and record, and so on up to 1000 g.
4. Remove 200 g at a time down to zero weight, while recording *D*
5. Measure width *w* and height *h* of Beam *A*, the distance between the two cutting edges *L*, and the distance between the front and hind legs of the optical lever *l*.
6. Record all of the measurements in the table, and calculate *Y* and the error (percentage error, or use Excel to calculate the standard deviation).
7. Draw a graph on the graph paper with *d* as the vertical axis and *W* as the horizontal axis. Use different marks for different materials, and calculate *Y*.

已註解 [t6]: notches?

已註解 [s7]: Is this correct?

5. Questions

1. Does *D* change as the load increases or decreases? What causes this error?
2. Which property of the metal does Young's modulus represent?
3. Suppose there is a beam with *w*=1.5 cm and *h*=2.5 cm. With the same weights added, if the beam is rotated 90° about the L, what is the ratio of the deflection before rotation to the deflection after rotation?

4. What kind of geometrical relationship does the *d*-*W* graph show? Does the graph prove Hooke's law?