Proof of Sectors in Square

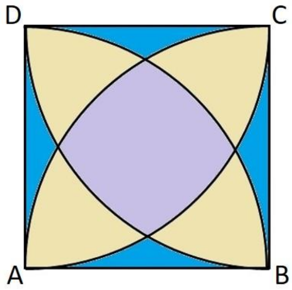

Figure 1. Three regions, R_1: purple, R_2: yellow, and R_3: blue

Let ABCD be a square of side a that four sectors are drawn by treating A, B, C, and D as centers and side a as the radius. Let the region of the three colors be R_1 for the purple color area, R_2 for the yellow color area, and R_3 for the blue color area as shown in Figure 1.

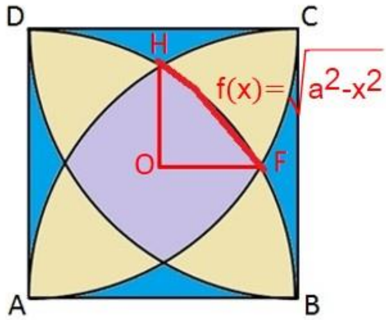

Figure 2. Area of OHF, area(R_2)/4

We first consider region R_1 as shown in Figure 2. The area covered by the red curves is a quarter of R_1. If we make point A as the origin and the curve HF is the curve $f(x) = \sqrt{a^2\text{-}x^2}$.

Consider Figure 3, line segments AG and OG have length a/2. Hence, line OF is of function g(x)=a/2. For triangle AEF, line segment AF is of length a, the radius of the sector. The length of line segment FE is the same as OG and it is a/2. Hence, the length of line segment $AE=\sqrt{a^2\text{-}(\frac{a}{2})^2}=\sqrt{3}a/2$.

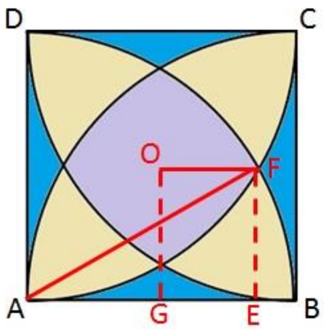

Figure 3 Length AG and length AE

As the result, the quarter area of region R_1 is the area covered by two vertical line $x=a/2$ and $x=\sqrt{3}a/2$ and the two curves f(x) and g(x). Hence the area of the read curve in Figure 2 is:

$$\text{area}(R1)/4=\int_{a/2}^{\sqrt{3}a/2}\left(\sqrt{a^2-x^2}-a/2\right)dx.$$

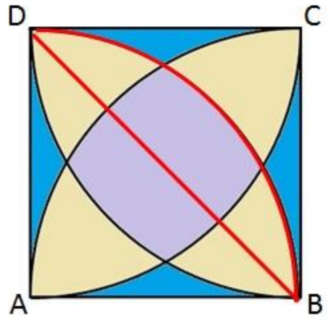

Figure 4. area(R_1)/2+area(R_2)/4

From Figure 4, we obtain the equations area(R_1)/2+area(R_2)/4=(πa^2–$2a^2$)/4. Furthermore area(R_1)+area(R_2)+area(R_3)=a^2. Hence, we can compute the areas of regions: R_1, R_2, and R_3.

Q.E.D.