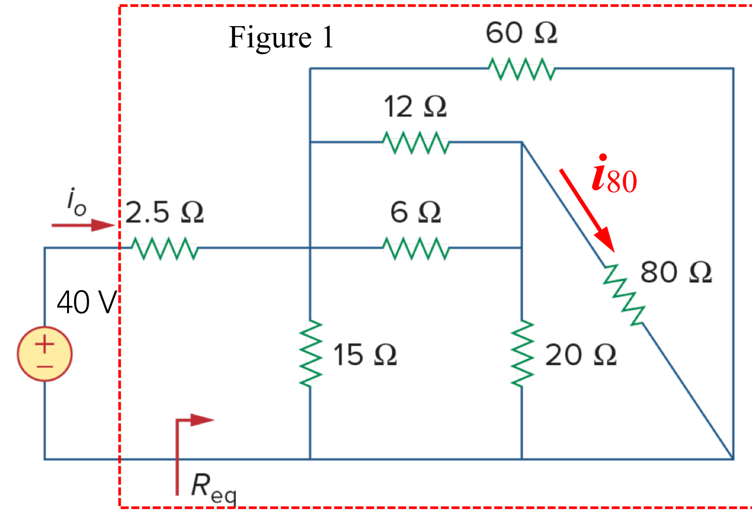
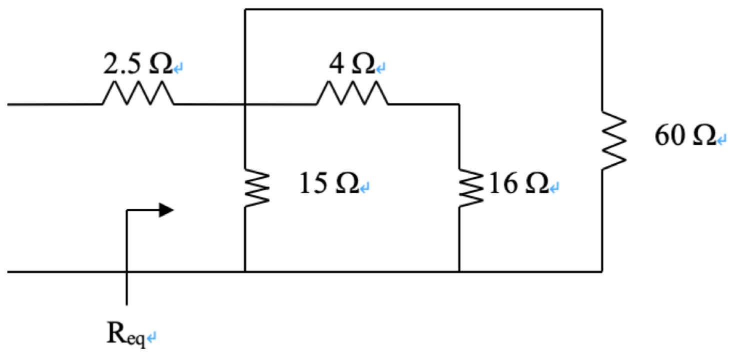
1. (15%) Find R_{eq} , i_O, and i_{80} in the circuit of Fig. 1.

Figure 1



$20//80 = 16$

$6//12 = 4$



$(4+16)//60 = 20//60 = 15$

$15//15 = 7.5$

$R_{eq} = 2.5+7.5 = 10\ (\Omega)$

$$i_O = \frac{40}{R_{eq}} = \frac{40}{10} = 4(A)$$

$$V_{15} = V_{4+16} = V_{60} = 40\times\frac{7.5}{2.5+7.5} = 30(V)$$

$$V_{80} = V_{20} = V_{4+16}\times\frac{16}{4+16} = 30\times\frac{4}{5} = 24(V)$$

$$i_{80} = \frac{24}{80} = 0.3(A)$$

2. 30% Given the circuit in Figure 2,
(A) (15%) use the **Thevenin** theorem to obtain the **Thevenin** equivalent V_{Th} and R_{Th}, and find the maximal power that can be transferred to the load as viewed from terminal **a-b**;
(B) (15%) use the **Norton** theorem to obtain the **Norton** equivalent I_N and R_N, and find the maximal power that can be transferred to the load as viewed from terminal **c-d**.

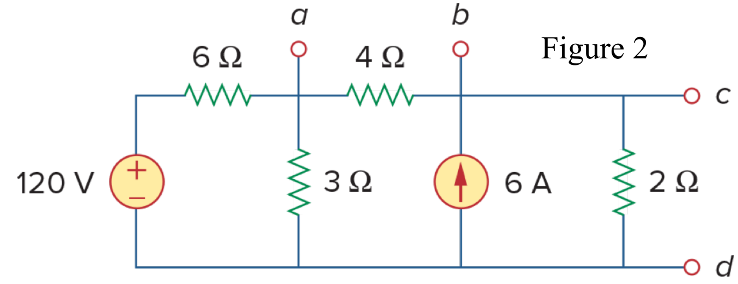
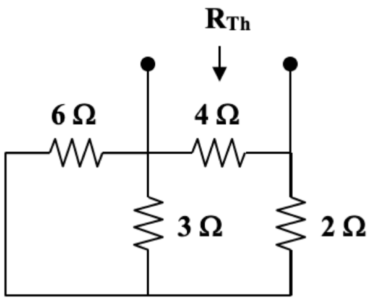
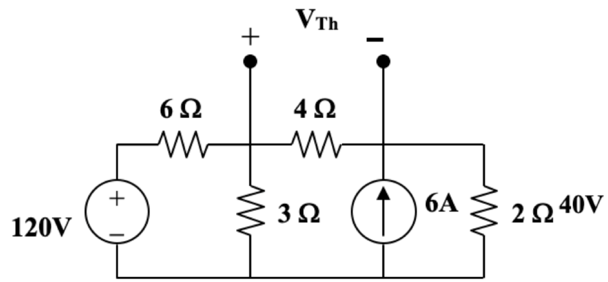
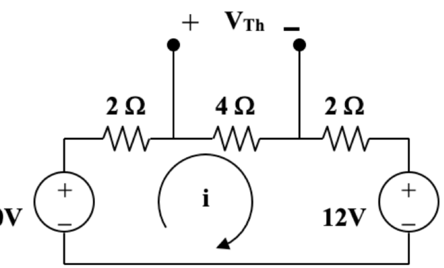


Figure 2

(A)







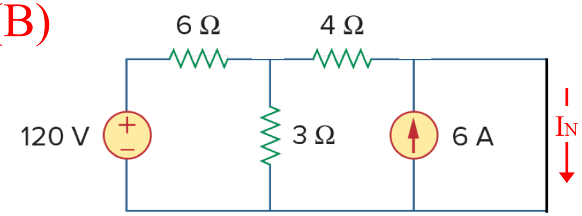
$$i = \frac{40-12}{2+4+2} = \frac{28}{8} = 3.5(A)$$

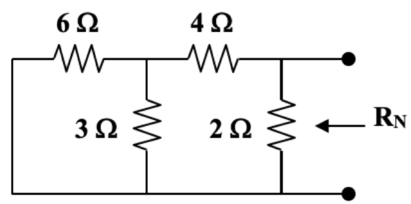
$$V_{Th} = i \cdot 4 = 14(V)$$

$$P_{max} = \frac{V_{Th}^2}{4 \cdot R_{Th}} = \frac{14^2}{4 \cdot 2} = 24.5\ (w)$$

$$R_{Th} = 4 \parallel \big((6 \parallel 3) + 2\big) = 2(\Omega)$$

(B)





$$R_N = 2 \parallel \big((6 \parallel 3) + 4\big) = 1.5(\Omega)$$

$$I_N = I_{N1} + I_{N2} = 12.6667(A)$$

By Superposition:

Due to 120V

$$R_{eq} = 6 + 4 \parallel 3 = {}^{54}/_{7}\ (\Omega)$$

$$i_{total} = \frac{120}{54/7} = 15.5556(A)$$

$$I_{N1} = i_{total} \times \frac{3}{3+4} = 6.6667(A)$$

Due to 6A

$$I_{N2} = 6(A)$$

Converted into Thevenin:

$$R_{Th} = R_N = 1.5(\Omega)$$

$$V_{Th} = I_N \times R_N = 12.6667 \times 1.5 = 19(\text{V})$$

$$P_{max} = \frac{V_{Th}^2}{4 \cdot R_{Th}} = \frac{19^2}{4 \cdot 1.5} = 60.1667\ (w)$$

3. 30% For the circuit in Fig. 3, at terminals a-b,

(A) (15%) use the **Thevenin** theorem to obtain the **Thevenin** equivalent

(B) (15%) use the **Norton** theorem to obtain the **Norton** equivalent

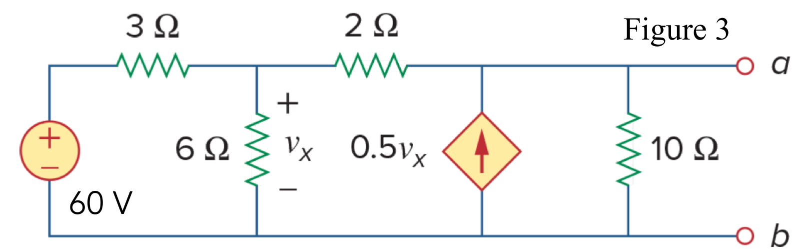
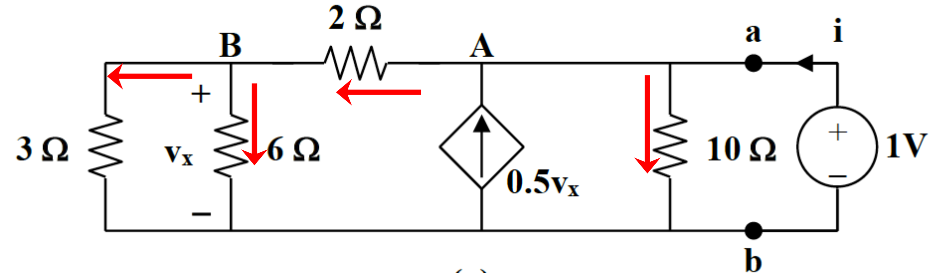


Figure 3

(A) Thevenin

R_{Th} Nodal analysis:



node A:

$$\frac{V_A - V_B}{2} + \frac{1}{10} = i + 0.5v_x$$

$$\Rightarrow i + 0.5v_x = \frac{1-v_x}{2} + \frac{1}{10}$$

$$\Rightarrow i + v_x = 0.6$$

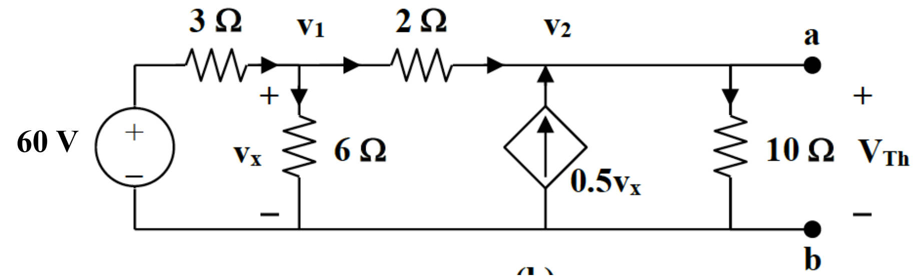
node B:

$$\frac{v_x}{3} + \frac{v_x}{6} = \frac{V_A - V_B}{2} = \frac{1 - v_x}{2}$$

$$\Rightarrow v_x = 0.5$$

$$\Rightarrow i = 0.6 - v_x = 0.1\ (A)$$

$$\Rightarrow R_{Th} = \frac{1}{i} = 10\ (\Omega)$$



V_{Th} Nodal analysis:

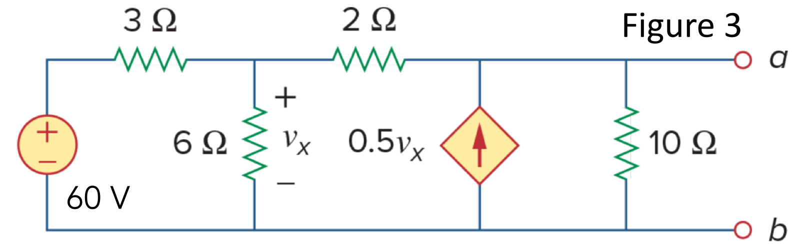
node 1: $\frac{60-V_1}{3} = \frac{V_1}{6} + \frac{V_1-V_2}{2} \Rightarrow 6 \cdot V_1 - 3 \cdot V_2 = 120$

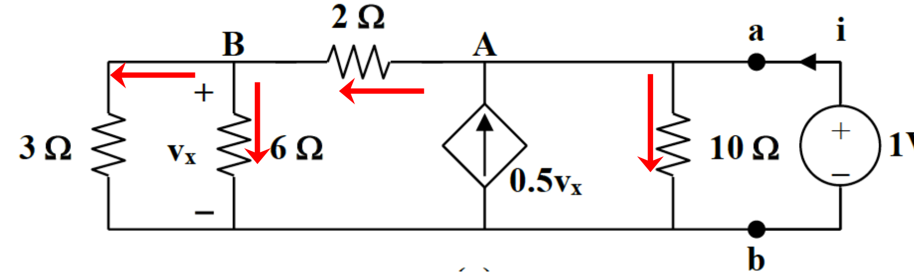
node 2: $\frac{V_1-V_2}{2} + 0.5 \cdot v_x = \frac{V_2}{10} \Rightarrow V_1 - \frac{6}{10}V_2 = 0 \Rightarrow V_1 = 0.6 \cdot V_2$

$$\Rightarrow V_2 = \frac{120}{0.6} = 200\ (V)$$

$$\Rightarrow V_1 = 0.6 \cdot V_2 = 120\ (V)$$

$$V_{Th} \Rightarrow V_2 = 200(V)$$





(B) Norton theorem

R_N Nodal analysis:

node A:

$$\frac{V_A - V_B}{2} + \frac{1}{10} = i + 0.5v_x$$

$$\Rightarrow i + 0.5v_x = \frac{1-v_x}{2} + \frac{1}{10}$$

$$\Rightarrow i + v_x = 0.6$$

node B:

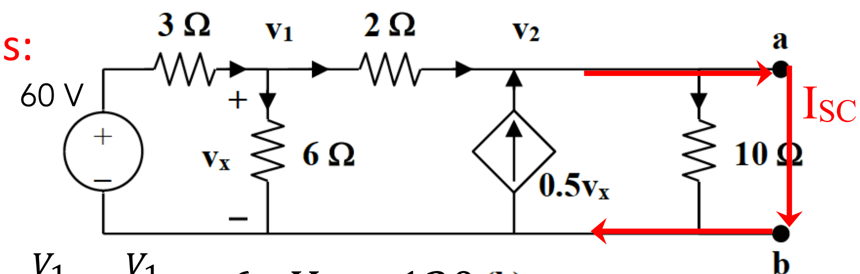
$$\frac{v_x}{3} + \frac{v_x}{6} = \frac{V_A - V_B}{2} = \frac{1 - v_x}{2}$$

$$\Rightarrow v_x = 0.5$$

$$\Rightarrow i = 0.6 - v_x = 0.1\ (A)$$

$$\Rightarrow R_N = \frac{1}{i} = 10\ (\Omega)$$

I_N Nodal analysis:



node 2: $V_2 = 0$

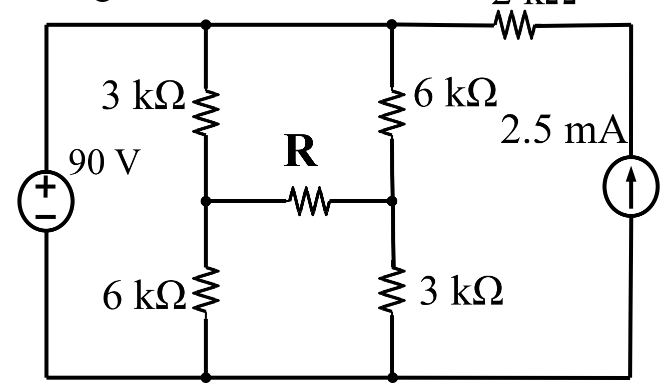
node 1: $\frac{60-V_1}{3} = \frac{V_1}{6} + \frac{V_1}{2} \Rightarrow 6 \cdot V_1 = 120$

$V_1 = 20\ (V)$,

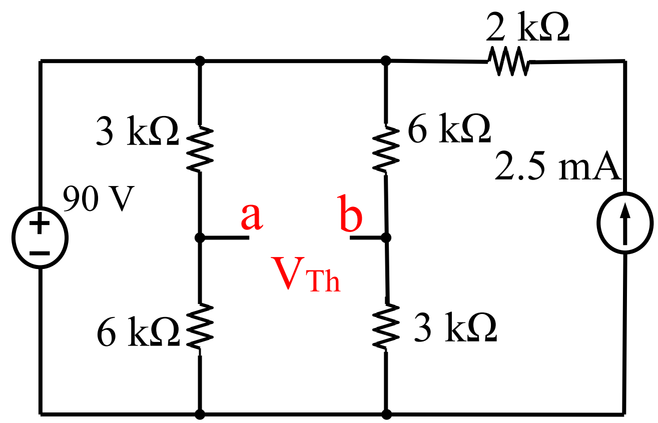
$$I_N = I_{SC} = \frac{V_1}{2} + 0.5v_x = \frac{V_1}{2} + 0.5V_1 = V_1 = 20\ (A)$$

4. (20%) For the circuit shown in Figure 4, if the current passing through the unknown resistor **R** is 0.5 mA, find the value of **R**.

Figure 4



Thevenin's

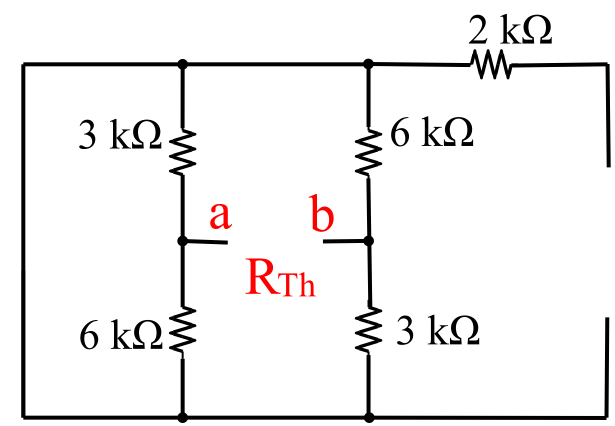


$$V_{Th} = V_{ab} = V_a - V_b$$

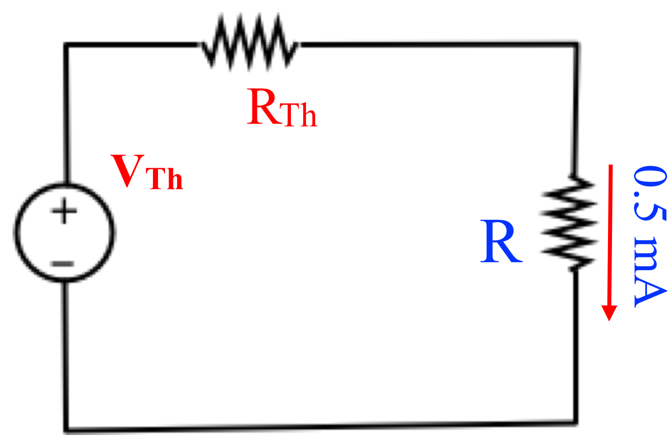
$$V_a = 90\times\frac{6}{3+6} = 60\ (\text{V})$$

$$V_b = 90\times\frac{3}{6+3} = 30\ (\text{V})$$

$$V_{Th} = V_{ab} = V_a - V_b = 60 - 30 = 30\ (\text{V})$$



$$R_{Th} = R_{ab} = 3//6 + 6//3 = 2 + 2 = 4\ (k\Omega)$$



$$V_{Th} = i\times R_{Th} + i\times R$$

$$R = \frac{V_{Th} - 0.5\times R_{Th}}{0.5} = \frac{30-0.5\times4}{0.5} = 56(k\Omega)$$

5. 20% Use superposition to solve for v_x in the circuit of Figure 5.

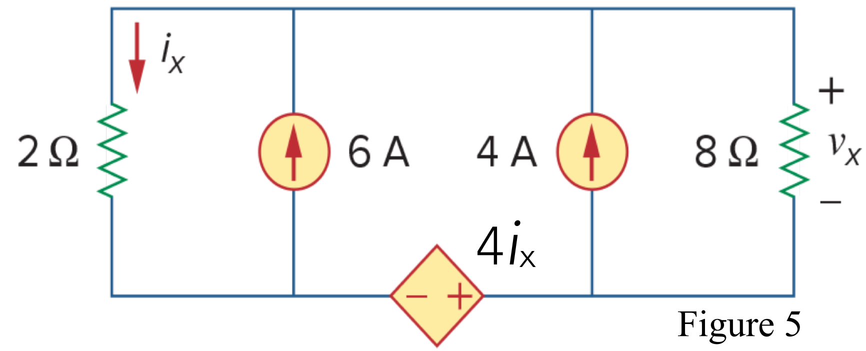
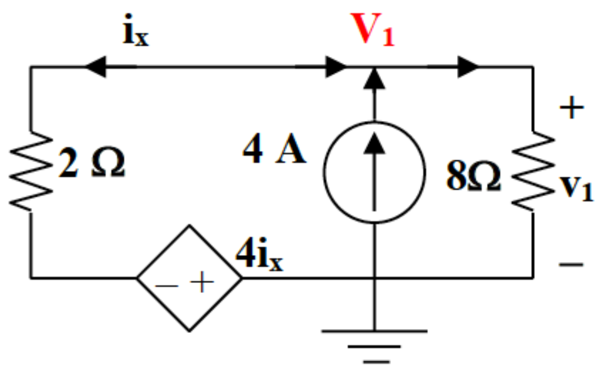


Figure 5



with 4A:

$V_1/8 - 4 + (V_1 - (-4i_x))/2 = 0$

$(0.125+0.5)V_1 = 4 - 2i_x$

$V_1 = 6.4 - 3.2i_x$

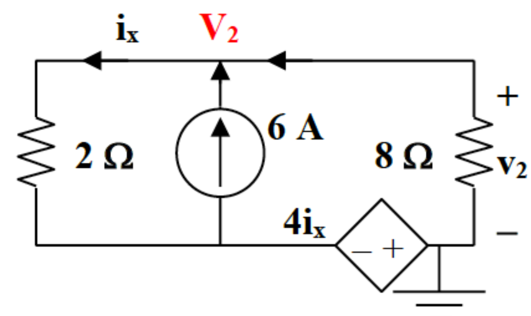
But,

$i_x = (V_1 - (-4i_x))/2$

$i_x = -0.5v_1$

$V_1 = 6.4 + 3.2(0.5v_1)$

$V_1 = -6.4/0.6 = -10.6667\ (V)$



with 6A:

$V_2/8 - 6 + (V_2 - (-4i_x))/2 = 0$

$V_2 + 3.2i_x = 9.6$

But

$i_x = -0.5V_2$

$V_2 + 3.2(-0.5V_2) = 9.6$

$V_2 = -9.6/0.6 = -16\ (V)$

$$\boxed{v_x = V_1 + V_2 = -10.6667 - 16 = -26.6667\ (V)}$$

6. 20% Use mesh analysis and apply Cramer's rule to obtain $\boldsymbol{i}_o$ in the circuit of Fig 6.

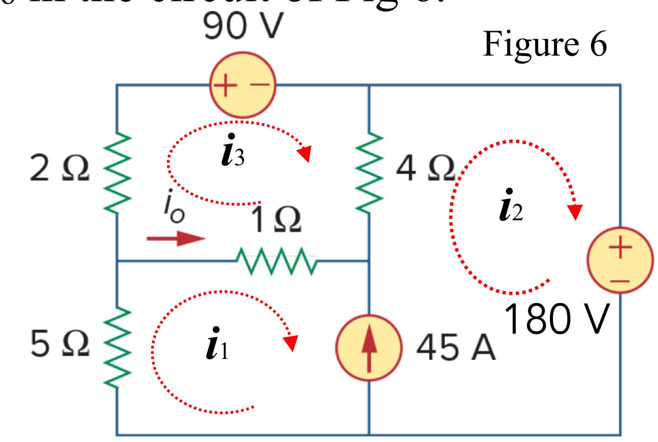


Figure 6

Loop 1 and 2 form a supermesh.

For the supermesh:

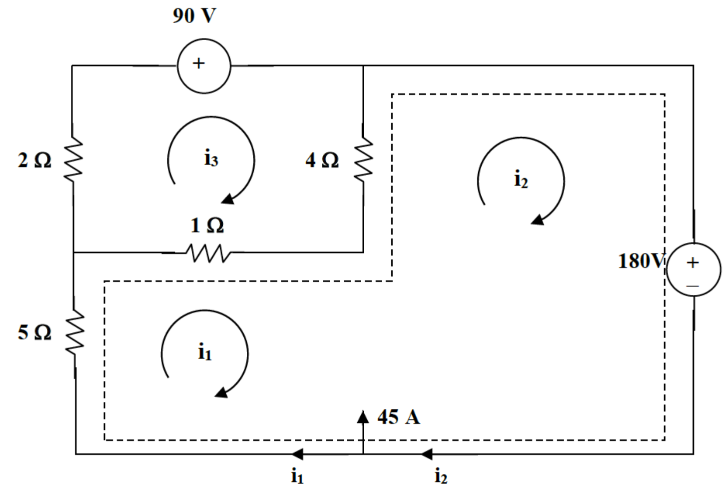
$$5 \cdot i_1 + 1 \cdot (i_1 - i_3) + 4 \cdot (i_2 - i_3) + 180 = 0$$

$$6 \cdot i_1 + 4 \cdot i_2 - 5 \cdot i_3 = -180 \cdots\cdots (1)$$

For Loop 3:

$$-i_1 - 4 \cdot i_2 + 7 \cdot i_3 = -90 \cdots\cdots (2)$$

Also: $-i_1 + i_2 = 45 \cdots\cdots (3)$



$$\begin{bmatrix} 6 & 4 & -5 \\ -1 & -4 & 7 \\ -1 & 1 & 0 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} -180 \\ -90 \\ 45 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 6 & 4 & -5 \\ -1 & -4 & 7 \\ -1 & 1 & 0 \end{vmatrix}$$

$$= 6\times(-4)\times 0 + 4\times 7\times(-1) + (-5)\times 1\times(-1) - (-1)\times(-4)\times(-5) - (-1)\times 4\times 0 - 6\times 7\times 1$$

$$= -28 + 5 + 20 - 42 = -45$$

$$\Delta_1 = \begin{vmatrix} -180 & 4 & -5 \\ -90 & -4 & 7 \\ 45 & 1 & 0 \end{vmatrix} = 0 + 1260 + 450 - (900 + 0 - 1260) = 2070$$

$$\Delta_2 = \begin{vmatrix} 6 & -180 & -5 \\ -1 & -90 & 7 \\ -1 & 45 & 0 \end{vmatrix} = 0 + 1260 + 225 - (-450 + 0 + 1890) = 45$$

$$\Delta_3 = \begin{vmatrix} 6 & 4 & -180 \\ -1 & -4 & -90 \\ -1 & 1 & 45 \end{vmatrix} = -1080 + 360 + 180 - (-720 - 180 - 540) = 900$$

$$i_1 = \frac{\Delta_1}{\Delta} = \frac{2070}{-45} = -46\ (A)$$

$$i_2 = \frac{\Delta_2}{\Delta} = \frac{45}{-45} = -1\ (A)$$

$$i_3 = \frac{\Delta_3}{\Delta} = \frac{900}{-45} = -20\ (A)$$

$$\boxed{\begin{aligned} i_O &= i_1 - i_3 \\ &= -46 + 20 = -26(A) \end{aligned}}$$