HW Exercise

$x=3\cos\theta$ $y=4\sin\theta$ $0\le\theta\le\frac{\pi}{2}$ about y-axis. area of surface

$$S=\int_0^{\frac{\pi}{2}} 2\pi x\sqrt{\left(\frac{dx}{d\theta}\right)^2+\left(\frac{dy}{d\theta}\right)^2}\,d\theta=\int_0^{\frac{\pi}{2}} 2\pi\cdot 3\cos\theta\sqrt{(-3\sin\theta)^2+(4\cos\theta)^2}\,d\theta$$

$$=6\pi\int_0^{\frac{\pi}{2}}\cos\theta\sqrt{9\sin^2\theta+16\cos^2\theta}\,d\theta=6\pi\int_0^{\frac{\pi}{2}}\cos\theta\sqrt{9\sin^2\theta+16(1-\sin^2\theta)}\,d$$

$$=6\pi\int_0^{\frac{\pi}{2}}\cos\theta\sqrt{16-7\sin^2\theta}\,d\theta$$

$w=\sin\theta$ $\quad\theta: 0\to\frac{\pi}{2}$

$dw=\cos\theta\,d\theta$ $\quad w=0\to 1$

$$=6\pi\int_0^1\sqrt{16-7w^2}\,dw$$

$$=6\pi\cdot 4\int_0^1\sqrt{1-\left(\frac{\sqrt{7}w}{4}\right)^2}\,dw$$

$\sin t=\frac{\sqrt{7}}{4}w$

$\cos t\,dt=\frac{\sqrt{7}}{4}dw$

$\frac{4}{\sqrt{7}}\cos t\,dt=dw$

$$=24\pi\int_0^{\sin^{-1}\frac{\sqrt{7}}{4}}\sqrt{1-\sin^2 t}\,\frac{4}{\sqrt{7}}\cos t\,dt$$

$w: 0\to 1$

$t=0\to\sin^{-1}\frac{\sqrt{7}}{4}$

$$=\frac{24\pi\cdot 4}{\sqrt{7}}\int_0^{\sin^{-1}\frac{\sqrt{7}}{4}}\cos^2 t\,dt$$

$$=\frac{96\pi}{\sqrt{7}}\int_0^{\sin^{-1}\frac{\sqrt{7}}{4}}\frac{\cos 2t+1}{2}\,dt$$

$$=\frac{48\pi}{\sqrt{7}}\left[\frac{1}{2}\sin 2t+t\right]_0^{\sin^{-1}\frac{\sqrt{7}}{4}}$$

$\sin 2\left(\sin^{-1}\frac{\sqrt{7}}{4}\right)=\sin 2s$

$=2\sin s\cos s$

$s=\sin^{-1}\frac{\sqrt{7}}{4}$

$\sin s=\frac{\sqrt{7}}{4}$

$$=\frac{24\pi}{\sqrt{7}}\sin\left(2\sin^{-1}\frac{\sqrt{7}}{4}\right)+\frac{48\pi}{\sqrt{7}}\sin^{-1}\frac{\sqrt{7}}{4}$$

$$\left(=9\pi+\frac{48\pi}{\sqrt{7}}\sin^{-1}\frac{\sqrt{7}}{4}\right)$$

$$=\frac{6}{\sqrt{7}}\cdot 2\cdot\frac{\sqrt{7}}{4}\sqrt{1-\frac{7}{16}}+\frac{48\pi}{\sqrt{7}}\sin^{-1}\frac{\sqrt{7}}{4}=\frac{3}{1}\frac{24\pi}{4}\frac{3}{4}+\frac{48\pi}{\sqrt{7}}\sin^{-1}\frac{\sqrt{7}}{4}$$