4. S A particle of mass m moves with momentum of magnitude p. (a) Show that the kinetic energy of the particle is $K = p^2/2m$. (b) Express the magnitude of the particle's momentum in terms of its kinetic energy and mass.

P8.4 (a) The momentum is $p = mv$, so $v = p/m$ and the kinetic energy is

$$K = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{p}{m}\right)^2 = \boxed{\frac{p^2}{2m}}$$

(b) $K = \frac{1}{2}mv^2$ implies $v = \sqrt{\frac{2K}{m}}$ so

$$p = mv = m\sqrt{\frac{2K}{m}} = \boxed{\sqrt{2mK}}.$$

7. Q|C Two blocks of masses m and $3m$ are placed on a frictionless, horizontal surface. A light spring is attached to the more massive block, and the blocks are pushed together with the spring between them (Fig. P8.7). A cord initially holding the blocks together is burned; after that happens, the block of mass $3m$ moves to the right with a speed of 2.00 m/s. (a) What is the velocity of the block of mass m? (b) Find the system's original elastic potential energy, taking $m = 0.350$ kg. (c) Is the original energy in the spring or in the cord? (d) Explain your answer to part (c). (e) Is the momentum of the system conserved in the bursting-apart process? Explain how that is possible considering (f) there are large forces acting and (g) there is no motion beforehand and plenty of motion afterward?

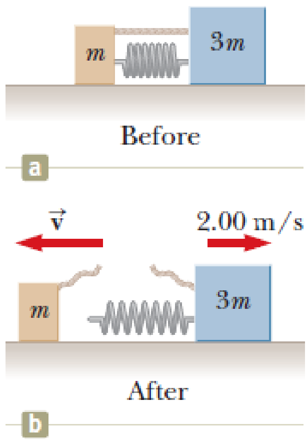


Figure P8.7

P8.7 (a) For the system of two blocks $\Delta p = 0$, or $p_i = p_f$. Therefore,

$$0 = mv_m + (3m)(2.00\text{ m/s})$$

Solving gives $v_m = \boxed{-6.00\text{ m/s}}$ (motion toward the left).

(b) $\frac{1}{2}kx^2 = \frac{1}{2}mv_M^2 + \frac{1}{2}(3m)v_{3M}^2$

$$= \frac{1}{2}(0.350\text{ kg})(-6.00\text{ m/s})^2 + \frac{3}{2}(0.350\text{ kg})(2.00\text{ m/s})^2$$

$$= \boxed{8.40\text{ J}}$$

(c) $\boxed{\text{The original energy is in the spring.}}$

(d) A force had to be exerted over a displacement to compress the spring, transferring energy into it by work.

$\boxed{\text{The cord exerts force, but over no displacement.}}$

(e) $\boxed{\text{System momentum is conserved with the value zero.}}$

(f) The forces on the two blocks are internal forces, which cannot change the momentum of the system—

$\boxed{\text{the system is isolated.}}$

(g)

Even though there is motion afterward, the final momenta are of equal magnitude in opposite directions so the final momentum of the system is still zero.

11. An estimated force-time curve for a baseball struck by a bat is shown in Figure P8.11. From this curve, determine (a) the magnitude of the impulse delivered to the ball and (b) the average force exerted on the ball.

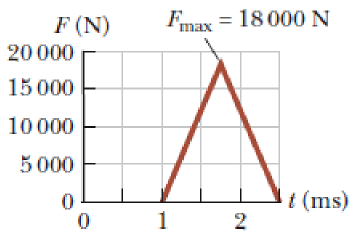
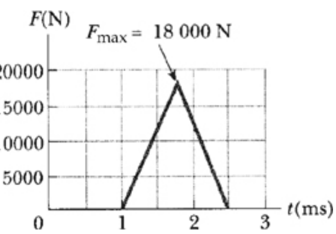


Figure P8.11

P8.11 (a) The impulse delivered to the ball is equal to the area under the F-t graph. We have a triangle and so to get its area we multiply half its height times its width:

$$I = \int F\,dt = \text{area under curve}$$



ANS. FIG. P8.11

$$I = \frac{1}{2}\left(1.50 \times 10^{-3}\ \text{s}\right)\left(18\ 000\ \text{N}\right) = \boxed{13.5\ \text{N} \cdot \text{s}}$$

(b) $$F = \frac{13.5\ \text{N} \cdot \text{s}}{1.50 \times 10^{-3}\ \text{s}} = \boxed{9.00\ \text{kN}}$$

14. A tennis player receives a shot with the ball (0.060 0 kg) traveling horizontally at 50.0 m/s and returns the shot with the ball traveling horizontally at 40.0 m/s in the opposite direction. (a) What is the impulse delivered to the ball by the tennis racquet? (b) What work does the racquet do on the hall?

P8.14 Assume the initial direction of the ball in the –x direction.

(a) The impulse delivered to the ball is given by

$$\begin{aligned}\vec{\mathbf{I}} &= \Delta\vec{\mathbf{p}} = \vec{\mathbf{p}}_f - \vec{\mathbf{p}}_i \\ &= (0.060\ 0\ \text{kg})(40.0\ \text{m/s})\hat{\mathbf{i}} - (0.060\ 0\ \text{m/s})(50.0\ \text{m/s})(-\hat{\mathbf{i}}) \\ &= \boxed{5.40\hat{\mathbf{i}}\ \text{N} \cdot \text{s}}\end{aligned}$$

(b) The racquet does

$$\begin{aligned}\text{Work} = K_f - K_i &= \frac{1}{2}(0.060\ 0\ \text{kg})\left[(40.0\ \text{m/s})^2 - (50.0\ \text{m/s})^2\right] \\ &= \boxed{-27.0\ \text{J}}\end{aligned}$$

20. **S** As shown in Figure P8.20, a bullet of mass m and speed v passes completely through a pendulum bob of mass M. The bullet emerges with a speed of $v/2$. The pendulum bob is suspended by a stiff rod (*not* a string) of length ℓ and negligible mass. What is the minimum value of v such that the pendulum bob will barely swing through a complete vertical circle?

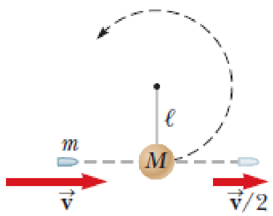
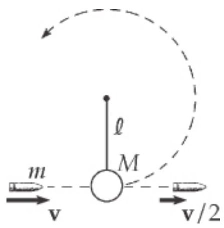


Figure P8.20

P8.20 Energy is conserved for the bob-Earth system between bottom and top of the swing. At the top the stiff rod is in compression and the bob nearly at rest.

$$K_i + U_i = K_f + U_f: \qquad \frac{1}{2}Mv_b^2 + 0 = 0 + Mg2\ell$$

$$v_b^2 = 4g\ell \qquad \text{so} \qquad v_b = 2\sqrt{g\ell}$$



ANS. FIG. P8.20

Momentum of the bob-bullet system is conserved in the collision:

$$mv = m\frac{v}{2} + M\left(2\sqrt{g\ell}\right) \quad \to \quad \boxed{v = \frac{4M}{m}\sqrt{g\ell}}$$

23. **S** Two gliders are set in motion on a horizontal air track. A spring of force constant k is attached to the back end of the second glider. As shown in Figure P8.23, the first glider, of mass m_1, moves to the right with speed v_1, and the second glider, of mass m_2, moves more slowly to the right with speed v_2. When m_1 collides with the spring attached to m_2, the spring compresses by a distance x_{max}, and the gliders then move apart again. In terms of v_1, v_2, m_1, m_2, and k, find (a) the speed v at maximum compression, (b) the maximum compression x_{max}, and (c) the velocity of each glider after m_1 has lost contact with the spring.

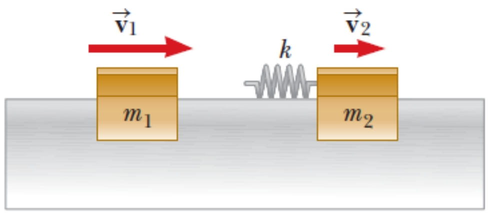


Figure P8.23

P8.23 (a) When the spring is fully compressed, each cart moves with same velocity v. Apply conservation of momentum for the system of two gliders

$$p_i = p_f: \qquad m_1v_1 + m_2v_2 = (m_1 + m_2)v \quad \to \quad \boxed{v = \frac{m_1v_1 + m_2v_2}{m_1 + m_2}}$$

(b) Only conservative forces act; therefore, $\Delta E = 0$.

$$\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_1v_2^2 = \frac{1}{2}(m_1 + m_2)v^2 + \frac{1}{2}kx_m^2$$

Substitute for v from (a) and solve for x_m.

$$x_m^2 = \left(\frac{1}{k(m_1+m_2)}\right)[(m_1+m_2)m_1v_1^2 + (m_1+m_2)m_2v_2^2$$
$$-(m_1v_1)^2 - (m_2v_2)^2 - 2m_1m_2v_1v_2]$$
$$x_m = \sqrt{\frac{m_1m_2\left(v_1^2+v_2^2-2v_1v_2\right)}{k(m_1+m_2)}} = \boxed{(v_1-v_2)\sqrt{\frac{m_1m_2}{k(m_1+m_2)}}}$$

(c) $m_1v_1 + m_2v_2 = m_1v_{1f} + m_2v_{2f}$

Conservation of momentum: $m_1\left(v_1 - v_{1f}\right) = m_2\left(v_{2f} - v_2\right)$ **[1]**

Conservation of energy: $\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 = \frac{1}{2}m_1v_{1f}^2 + \frac{1}{2}m_2v_{2f}^2$

which simplifies to: $m_1\left(v_1^2 - v_{1f}^2\right) = m_2\left(v_{2f}^2 - v_2^2\right)$

Factoring gives

$$m_1\left(v_1 - v_{1f}\right)\left(v_1 + v_{1f}\right) = m_2\left(v_{2f} - v_2\right)\cdot\left(v_{2f} + v_2\right)$$

and with the use of the momentum equation (equation [1]),

this reduces to $\left(v_1 + v_{1f}\right) = \left(v_{2f} + v_2\right)$

or $v_{1f} = v_{2f} + v_2 - v_1$ **[2]**

Substituting equation [2] into equation [1] and simplifying yields:

$$\boxed{v_{2f} = \frac{2m_1v_1 + (m_2 - m_1)v_2}{m_1 + m_2}}$$

Upon substitution of this expression for into equation [2], one finds

$$\boxed{v_{1f} = \frac{(m_1 - m_2)v_1 + 2m_2v_2}{m_1 + m_2}}$$

Observe that these results are the same as two equations given in the chapter text for the situation of a perfectly elastic collision in one dimension. Whatever the details of how the spring behaves, this collision ends up being just such a perfectly elastic collision in one dimension.

29. S Two particles with masses m and $3m$ are moving toward each other along the x axis with the same initial speeds v_i. Particle m is traveling to the left, and particle $3m$ is traveling to the right. They undergo an elastic glancing collision such that particle m is moving in the negative y direction after the collision at a right angle from its initial direction. (a) Find the final speeds of the two particles in terms of v_i. (b) What is the angle θ at which the particle $3m$ is scattered?

P8.29 x component of momentum for the system of the two objects:

$$p_{1ix} + p_{2ix} = p_{1fx} + p_{2fx}$$

$$-mv_i + 3mv_i = 0 + 3mv_{2x}$$

y component of momentum of the system:

$$0 + 0 = -mv_{1y} + 3mv_{2y}$$

by conservation of energy of the system:

$$+\frac{1}{2}mv_i^2 + \frac{1}{2}3mv_i^2 = \frac{1}{2}mv_{1y}^2 + \frac{1}{2}3m\left(v_{2x}^2 + v_{2y}^2\right)$$

we have $v_{2x} = \dfrac{2v_i}{3}$

also $v_{1y} = 3v_{2y}$

So the energy equation becomes

$$4v_i^2 = 9v_{2y}^2 + \frac{4v_i^2}{3} + 3v_{2y}^2$$

$$\frac{8v_i^2}{3} = 12v_{2y}^2$$

or $v_{2y} = \dfrac{\sqrt{2}v_i}{3}$

(a) The object of mass m has final speed

$$v_{1y} = 3v_{2y} = \boxed{\sqrt{2}v_i}$$

and the object of mass $3m$ moves at

$$\sqrt{v_{2x}^2 + v_{2y}^2} = \sqrt{\frac{4v_i^2}{9} + \frac{2v_i^2}{9}}$$

$$\sqrt{v_{2x}^2 + v_{2y}^2} = \boxed{\sqrt{\frac{2}{3}}v_i}$$

(b) $\theta = \tan^{-1}\left(\frac{v_{2y}}{v_{2x}}\right)$

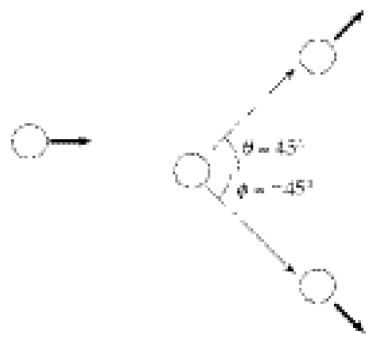
$$\theta = \tan^{-1}\left(\frac{\sqrt{2}v_i}{3}\frac{3}{2v_i}\right) = \boxed{35.3°}$$

32. **S** A proton, moving with a velocity of $v_i\hat{\mathbf{i}}$, collides elastically with another proton that is initially at rest. Assuming that the two protons have equal speeds after the collision, find (a) the speed of each proton after the collision in terms of v_i and (b) the direction of the velocity vectors after the collision.

P8.32 (a) The vector expression for conservation of momentum,

$\vec{\mathbf{p}}_i = \vec{\mathbf{p}}_f$:, gives $p_{xi} = p_{xf}$ and

$p_{yi} = p_{yf}$



$$mv_i = mv\cos\theta + mv\cos\phi \qquad \textbf{[1]}$$

$$0 = mv\sin\theta + mv\sin\phi \qquad \textbf{[2]}$$

From [2], $\sin\theta = -\sin\phi$ so $\theta = -\phi$

ANS. FIG. P8.32

Furthermore, energy conservation for the system of two protons requires

$$\frac{1}{2}mv_i^2 = \frac{1}{2}mv^2 + \frac{1}{2}mv^2$$

so

$$\boxed{v = \frac{v_i}{\sqrt{2}}}$$

(b) Hence, [1] gives

$$v_i = \frac{2v_i \cos\theta}{\sqrt{2}}$$

with $\theta = \boxed{45.0°}$ and $\phi = \boxed{-45.0°}$

36. A water molecule consists of an oxygen atom with two hydrogen atoms bound to it (Fig. P8.36). The angle between the two bonds is 106°. If the bonds are 0.100 nm long, where is the center of mass of the molecule?

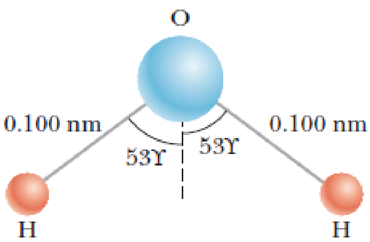
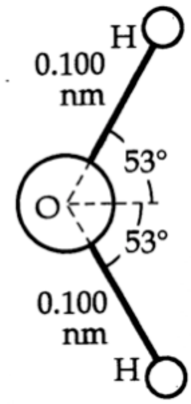


Figure P8.36

P8.36 Take the x axis starting from the oxygen nucleus and pointing toward the middle of the V.



Then

$$\boxed{y_{CM} = 0}$$

and

$$x_{CM} = \frac{\sum m_i x_i}{\sum m_i}$$

$$x_{\text{CM}} = \left(\frac{1}{15.999\text{ u} + 1.008\text{ u} + 1.008\text{ u}}\right)$$
$$[0 + 1.008\text{ u}(0.100\text{ nm})\cos 53.0°$$
$$+ 1.008\text{ u}(0.100\text{ nm})\cos 53.0°]$$

ANS. FIG. P8.36

$$\boxed{x_{\text{CM}} = 0.006\ 73\text{ nm from the oxygen nucleus}}$$

38. A rod of length 30.0 cm has linear density (mass per length) given by

$$\lambda = 50.0 + 20.0x$$

where x is the distance from one end, measured in meters, and λ is in grams/meter, (a) What is the mass of the rod? (b) How far from the $x = 0$ end is its center of mass?

P8.38 This object can be made by wrapping tape around a light, stiff, uniform rod.

(a) $$M = \int_0^{0.300\text{ m}} \lambda\, dx = \int_0^{0.300\text{ m}} [50.0 + 20.0x]dx$$

$$M = \left[50.0x + 10.0x^2\right]_0^{0.300\text{ m}} = \boxed{15.9\text{ g}}$$

(b) $$x_{\text{CM}} = \frac{\int_{\text{all mass}} x\, dm}{M} = \frac{1}{M}\int_0^{0.300\text{ m}} \lambda\, x\, dx = \frac{1}{M}\int_0^{0.300\text{ m}} \left[50.0x + 20.0x^2\right]dx$$

$$x_{\text{CM}} = \frac{1}{15.9\text{ g}}\left[25.0x^2 + \frac{20x^3}{3}\right]_0^{0.300\text{ m}} = \boxed{0.153\text{ m}}$$

41. Romeo (77.0 kg) entertains Juliet (55.0 kg) by playing his guitar from the rear of their boat at rest in still water, 2.70 m away from Juliet, who is in the front of the boat. After the serenade, Juliet carefully moves to the rear of the boat (away from shore) to plant a kiss on Romeo's cheek. How far does the 80.0-kg boat move toward the shore it is facing?

P8.41 No outside forces act on the boat-plus-lovers system, so its momentum is conserved at zero and the center of mass of the boat-passengers system stays fixed:

$$x_{\text{CM},i} = x_{\text{CM},f}$$

ANS. FIG. P8.41

Define K to be the point where they kiss, and Δx_J and Δx_b as shown in the figure. Since Romeo moves with the boat (and thus $\Delta x_{\text{Romeo}} = \Delta x_b$), let m_b be the combined mass of Romeo and the boat. The front of the boat and the shore are to the right in this picture, and we take the positive x direction to the right. Then,

$$m_J \Delta x_J + m_b \Delta x_b = 0$$

Choosing the x axis to point toward the shore,

$$(55.0\text{ kg})\Delta x_J + (77.0\text{ kg} + 80.0\text{ kg})\,\Delta x_b = 0$$

and $\Delta x_J = -2.85\Delta x_b$

As Juliet moves away from shore, the boat and Romeo glide toward the shore until the original 2.70-m gap between them is closed. We describe the relative motion with the equation

$$|\Delta x_J| + \Delta x_b = 2.70\text{ m}$$

Here the first term needs absolute value signs because Juliet's change in position is toward the left. An equivalent equation is then

$$-\Delta x_J + \Delta x_b = 2.70\text{ m}$$

Substituting, we find $+2.85\,\Delta x_b + \Delta x_b = 2.70$ m

so

$$\Delta x_b = 2.70\text{ m}/3.85 = \boxed{0.700\text{ m}} \text{ towards the shore.}$$

46. A jet aircraft is traveling at 223 m/s in horizontal flight. The engine takes in air at a rate of 80.0 kg/s and burns fuel at a rate of 3.00 kg/s. The exhaust gases are ejected at 600 m/s relative to the aircraft. Find the thrust of the jet engine and the delivered power.

P8.46 The aircraft's thrust is given by

$$T = \frac{dm}{dt}(\Delta v_{\text{fuel}}) + \frac{dm}{dt}(\Delta v_{\text{air}})$$

or

$$T = (3.00 \text{ kg/s})(600 \text{ m/s}) + (80.0 \text{ kg/s})(600 \text{ m/s} - 223 \text{ m/s})$$

$$T = 1\,800 \text{ N} + 3.02 \times 10^4 \text{ N} = \boxed{3.20 \times 10^4 \text{ N}}$$

The delivered power is the force (or thrust) multiplied by the velocity,

$$P = Tv = \left(3.20 \times 10^4 \text{ N}\right)(223 \text{ m/s}) = \boxed{7.13 \times 10^6 \text{ W}}$$