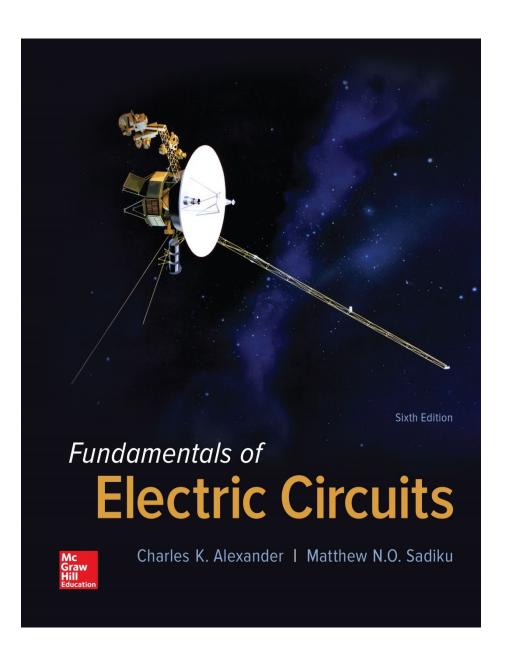
Fundamentals of Electric Circuits

Chapter 6

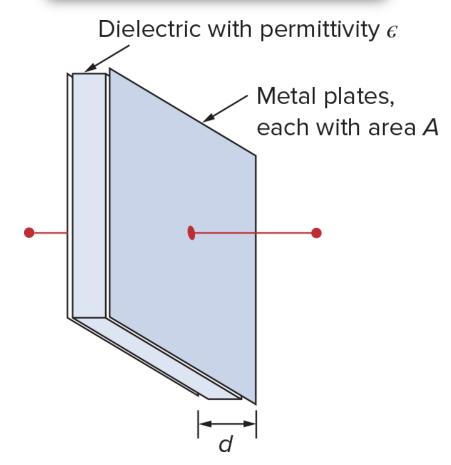
Capacitors and Inductors



6.1 Introduction

- This chapter will introduce two new linear circuit elements:
 - √The capacitor
 - √ The inductor
- Unlike resistors, these elements do not dissipate energy
- They instead store energy
- We will also look at how to analyze them in a circuit

6.2 Capacitors



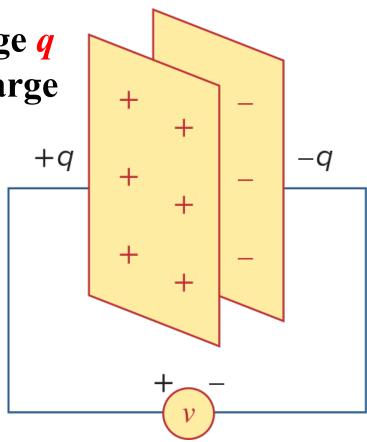
- A capacitor is a passive element that stores energy in its electric field
- It consists of two conducting plates separated by an insulator (or dielectric)
- The plates are typically aluminum foil
- The dielectric is often air, ceramic, paper, plastic, or mica

Capacitors

- When a voltage source V is connected to the capacitor, the source deposits a positive charge q on one plate and a negative charge -q on the other.
- The charges will be equal in magnitude
- The amount of charge is proportional to the voltage:

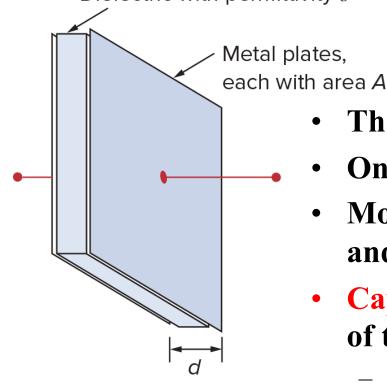
$$q = Cv$$

where C is the capacitance



Dielectric with permittivity ϵ

Capacitance



$$q = Cv$$

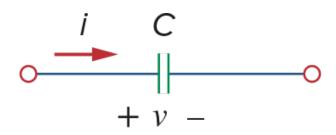
$$C = \frac{q}{v}$$

- The unit of capacitance is the *Farad* (F)
- One Farad is 1 Coulomb/Volt
- Most capacitors are rated in picofarad (pF) and microfarad (µF)
- Capacitance is determined by the geometry of the capacitor:
 - Proportional to the area of the plates (A)
 - Inversely proportional to the space between them (d)

 $C = \frac{\varepsilon A}{d}$

 ϵ is the permittivity of the dielectric

Applications for Capacitors



Fixed capacitors: (a) polyester capacitor, (b) ceramic capacitor, (c) electrolytic capacitor.

Capacitors have a wide range of applications,

some of which are:

- Blocking DC
- Passing AC
- Shift phase
- Store energy
- Suppress noise
- Start motors



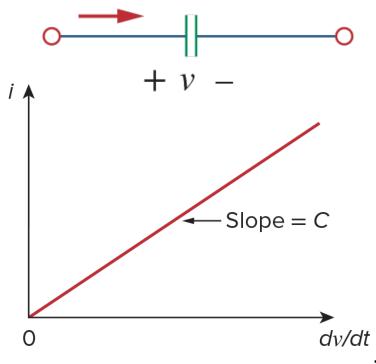
Current Voltage Relationship

 Using the formula for the charge stored in a capacitor, we can find the current voltage relationship

Take the first derivative with respect to time gives:

$$i = \frac{dq}{dt}$$
 $i = C\frac{dv}{dt}$

 This assumes the passive sign convention



Stored Charge



• Similarly, the voltage-current relationship is: $^+$

$$i = C \frac{dv}{dt} \qquad v(t) = \frac{1}{C} \int_{t_0}^{t} i(\tau) d\tau + v(t_0)$$

- This shows the capacitor has a memory, which is often exploited in circuits
- The instantaneous power delivered to the capacitor is

$$p = vi = Cv \frac{dv}{dt}$$

The energy stored in a capacitor is:

$$w = \int_{-\infty}^{t} p(\tau) d\tau = C \int_{-\infty}^{t} v \frac{dv}{d\tau} d\tau = C \int_{v(-\infty)}^{v(t)} v dv = \frac{1}{2} C v^2 \Big|_{v(-\infty)}^{v(t)}$$
 $w = \frac{1}{2} C v^2$
 $w = \frac{q^2}{2C}$

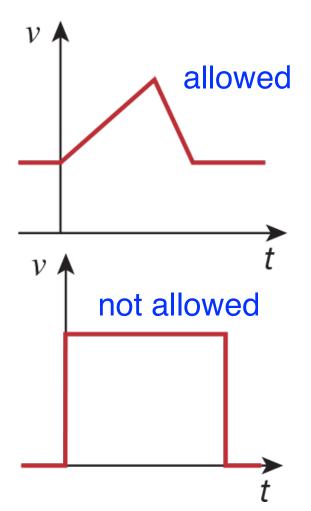
Properties of Capacitors

Ideal capacitors all have these characteristics:

$$i = C \frac{dv}{dt}$$

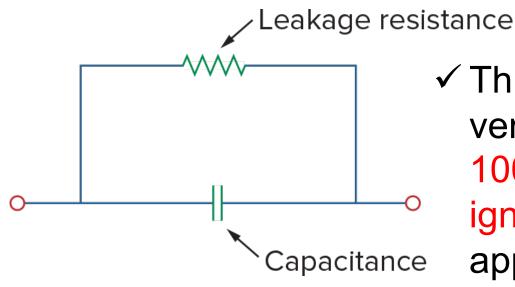
Voltage across a capacitor:

- ✓ When the voltage is not changing, the current through the cap is zero.
- ✓ This means that with DC applied to the terminals no current will flow.
- ✓ Except, the voltage on the capacitor's plates can't change instantaneously.
- ✓ An abrupt change in voltage would require an infinite current!
- ✓ This means if the voltage on the cap
 does not equal the applied voltage,
 charge will flow and the voltage will
 finally reach the applied voltage.



Properties of capacitors

- ✓ An ideal capacitor does not dissipate energy, meaning stored energy may be retrieved later
- ✓ A real capacitor has a parallel-model leakage resistance, leading to a slow loss of the stored energy internally



This resistance is typically very high, on the order of 100 MΩ and thus can be ignored for many circuit applications.

(a) Calculate the charge stored on a 3-pF capacitor with 20 V across it.

$$q = Cv = 3 imes 10^{-12} imes 20 = 60 \; \mathrm{pC}$$

(b) Find the energy stored in the capacitor.

Find the energy stored in the capacitor
$$w = \frac{1}{2} Cv^2 = \frac{1}{2} \times 3 \times 10^{-12} \times 400 = 600 \text{ pJ}$$

Example 6.2

The voltage across a $5-\mu F$ capacitor is $v(t) = 10\cos 6000t \,\mathrm{V}$

Calculate the current through it.

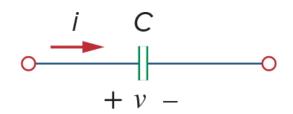
$$egin{aligned} i(t) &= C rac{dv}{dt} = 5 imes 10^{-6} rac{d}{dt} \left(10 \cos 6000t
ight) \ &= -5 imes 10^{-6} imes 6000 imes 10 \sin 6000t \ &= -0.3 \sin 6000t ext{ A} \end{aligned}$$

$$v(t) = \frac{1}{C} \int_{t_0}^{t} i(\tau) d\tau + v(t_0)$$

Determine the voltage across a $2-\mu F$ capacitor

if the current through it is

$$i(t)=6e^{-3000t} \mathrm{\ mA}$$



Assume that the initial capacitor voltage is zero. $i = C \frac{dv}{dt}$

$$v(t) = \frac{1}{C} \int_0^t i \, d\tau + v(0)$$

$$= \frac{1}{2 \times 10^{-6}} \int_0^t 6e^{-3000t} \, d\tau$$

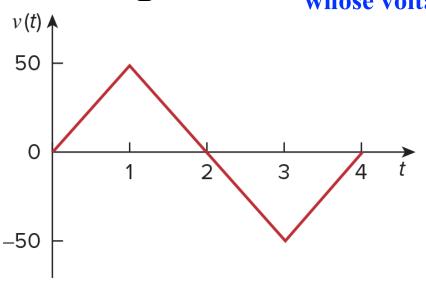
$$= \frac{6 \times 10^6}{2 \times (-3000)} e^{-3000\tau} \Big|_0^t$$

$$= -10^3 (e^{-3000t} - 1)$$

$$= (1 - e^{-3000t}) \times 10^3 (mV)$$

$$= (1 - e^{-3000t}) (V)$$

Determine the current through a 200- μF capacitor whose voltage is shown in the figure.



$$i = C \frac{dv}{dt}$$

$$v(t) = \begin{cases} 50t \text{ (V)} & 0 < t < 1\\ 100 - 50t \text{ (V)} & 1 < t < 3\\ -200 + 50t \text{ (V)} & 3 < t < 4\\ 0 & \text{otherwise} \end{cases}$$

$$i(t) = C \frac{dv(t)}{dt}$$

$$= 200 \times 10^{-6} \begin{cases} 50 \text{ (A)} & 0 < t < 1 \\ -50 \text{ (A)} & 1 < t < 3 \\ 50 \text{ (A)} & 3 < t < 4 \\ 0 & \text{otherwise} \end{cases}$$

$$= \begin{cases} 10 \text{ (mA)} & 0 < t < 1 \\ -10 \text{ (mA)} & 1 < t < 3 \\ 10 \text{ (mA)} & 3 < t < 4 \\ 0 & \text{otherwise} \end{cases}$$

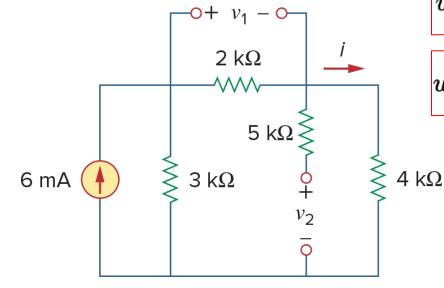
Obtain the energy stored in each

capacitor under dc conditions

$$i = rac{3}{3+2+4} \, (6 ext{ mA}) = 2 ext{ mA}$$

$$v_1 = 2000i = 4 \text{ V}$$

$$v_2=4000i=8~\mathrm{V}$$



$$w_1 = rac{1}{2} \, C_1 v_1^2 = rac{1}{2} \, igl(2 imes 10^{-3} igr) igl(4 igr)^2 = 16 \ \mathrm{mJ}$$

2 mF

 $2 \text{ k}\Omega$

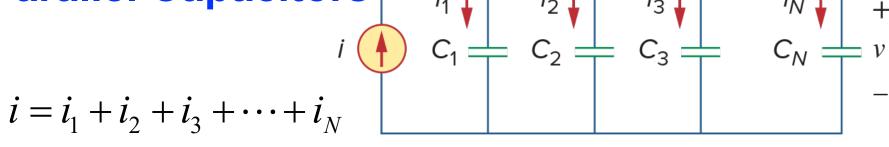
 $i_C = C \frac{dv_C}{dt}$

$$w_2 = rac{1}{2} \, C_2 v_2^2 = rac{1}{2} \, ig(4 imes 10^{-3} ig) ig(8 ig)^2 = 128 \ \mathrm{mJ}$$



6.3 Series and Parallel Capacitors

Parallel Capacitors



$$i = C_1 \frac{dv}{dt} + C_2 \frac{dv}{dt} + C_3 \frac{dv}{dt} + \dots + C_N \frac{dv}{dt}$$
$$= \left(\sum_{k=1}^{N} C_k\right) \frac{dv}{dt} = C_{eq} \frac{dv}{dt}$$

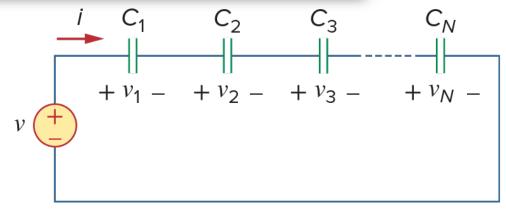
$$C_{eq} = C_1 + C_2 + C_3 + \dots + C_N$$



6.3 Series and Parallel Capacitors

Series Capacitors

$$v = v_1 + v_2 + v_3 + \dots + v_N$$



$$v = \frac{1}{C_{1}} \int_{t_{0}}^{t} i(\tau) d\tau + v_{1}(t_{0}) + \frac{1}{C_{2}} \int_{t_{0}}^{t} i(\tau) d\tau + v_{2}(t_{0}) + \frac{1}{C_{3}} \int_{t_{0}}^{t} i(\tau) d\tau + v_{3}(t_{0}) + \dots + \frac{1}{C_{N}} \int_{t_{0}}^{t} i(\tau) d\tau + v_{N}(t_{0})$$

$$= \left(\frac{1}{C_{1}} + \frac{1}{C_{2}} + \frac{1}{C_{3}} + \dots + \frac{1}{C_{N}}\right) \int_{t_{0}}^{t} i(\tau) d\tau + v_{1}(t_{0}) + v_{2}(t_{0}) + v_{3}(t_{0}) + \dots + v_{N}(t_{0})$$

$$= \frac{1}{C_{eq}} \int_{t_{0}}^{t} i(\tau) d\tau + v(t_{0})$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \dots + \frac{1}{C_N}$$

series combination of capacitors resembles the parallel combination of resistors.



Series and Parallel Caps $C = \frac{\mathcal{E}A}{d}$

$$C = \frac{\mathcal{E}A}{d}$$

- Another way to think about the combinations of capacitors is this:
 - A parallel combining is equivalent to
 - increasing the surface area of the capacitors:
 - This would lead to an increased overall capacitance $C_{eq} = C_1 + C_2 + C_3 + \dots + C_N$

- increasing the total plate separation
- This would result in a decrease in capacitance

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \dots + \frac{1}{C_N}$$

The $20-\mu F$ and $5-\mu F$ capacitors are in series:

$$rac{20 imes5}{20+5}\,=4~\mu\mathrm{F}$$

This 4- μ F capacitor is in parallel with the 6- μ F and 20- μ F capacitors:

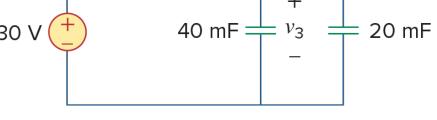
$$4+6+20=30~\mu {
m F}$$

This 30- μ F capacitor is in series with the 60- μ F capacitor

$$C_{
m eq} = \, rac{30 imes 60}{30 + 60} \, = 20 \, \mu {
m F}$$

Example 6.7 find the voltage across each

$$C_{
m eq} = rac{1}{rac{1}{60} + rac{1}{30} + rac{1}{20}}
m mF = 10
m mF$$



20 mF 30 mF

total charge $q = C_{\rm eq}v = 10 \times 10^{-3} \times 30 = 0.3 \ {\rm C}$

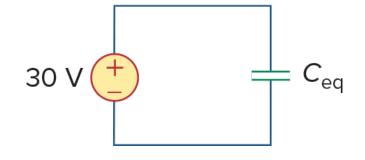
Charge acts like current, since i = dq/dt

$$v_1 = rac{q}{C_1} = rac{0.3}{20 imes 10^{-3}} = 15 \, ext{V}$$
 30 V

$$v_2 = rac{q}{C_2} = rac{0.3}{30 imes 10^{-3}} = 10 ext{ V}$$

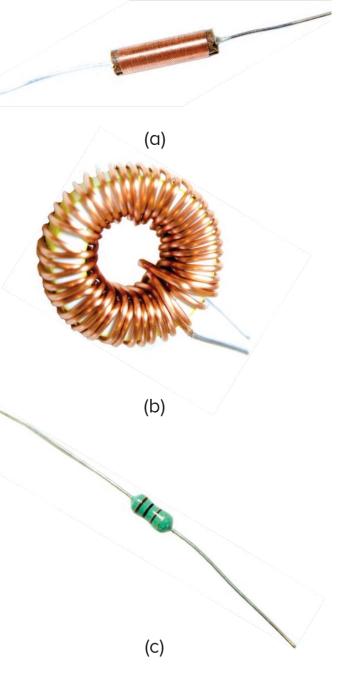
$$v_3 = 30 - v_1 - v_2 = 5 \text{ V}$$

$$v_3 = rac{q}{60 ext{ mF}} = rac{0.3}{60 imes 10^{-3}} = 5 ext{ V}$$



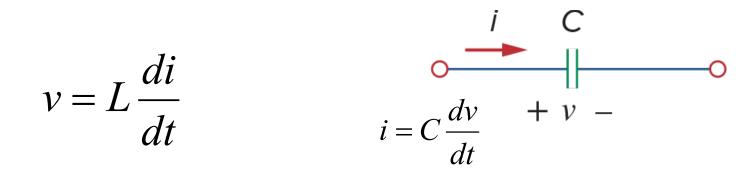
6.4 Inductors

- An inductor is a passive element that stores energy in its magnetic field
- They have applications in power supplies, transformers, radios, TVs, radars, and electric motors.
- Any conductor has inductance, but the effect is typically enhanced by coiling the wire up.



Inductors

 If a current is passed through an inductor, the voltage across it is directly proportional to the time rate of change in current



where, *L*, is the unit of inductance, measured in Henries, H.

- On Henry is 1 volt-second per ampere (V-s/A).
- The voltage developed tends to oppose a changing flow of current.

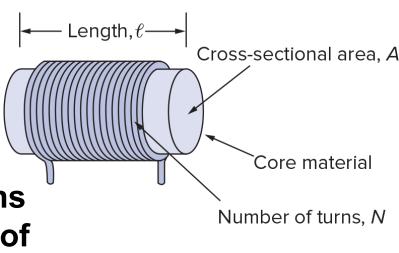
Inductors

- Calculating the inductance depends on the geometry:
- For example, for a solenoid the inductance is:

$$L = \frac{N^2 \mu A}{l}$$

here *N* is the number of turns of the wire around the core of cross sectional area *A* and length *I*.

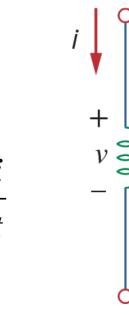
 The material used for the core has a magnetic property called the permeability, µ.



Current in an Inductor

The current voltage relationship for

an inductor is:
$$I = \frac{1}{L} \int_{t_0}^{t} v(\tau) d\tau + i(t_0) \qquad v = L \frac{di}{dt}$$



The power delivered to the inductor

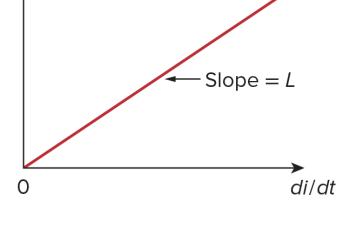
is:

$$p = vi = \left(L\frac{di}{dt}\right)i$$

The energy stored is:

$$w = \int_{-\infty}^{t} p(\tau) d\tau = L \int_{-\infty}^{t} \frac{di}{d\tau} i d\tau$$
$$= L \int_{-\infty}^{t} i di = \frac{1}{2} Li^{2}(t) - \frac{1}{2} Li^{2}(-\infty) 0$$

$$w = \frac{1}{2}Li^2$$

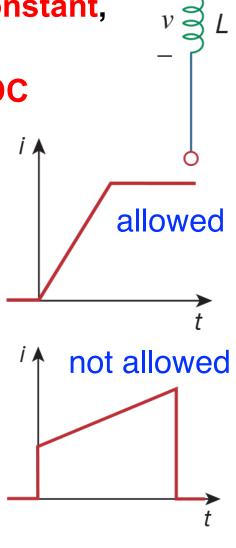


Properties of Inductors

- If the current through an inductor is constant, the voltage across it is zero
- Thus an inductor acts like a short for DC

$$v = L \frac{di}{dt} \qquad I = \frac{1}{L} \int_{t_0}^{t} v(\tau) d\tau + i(t_0)$$

- The current through an inductor cannot change instantaneously
- If this did happen, the voltage across the inductor would be infinity!
- This is an important consideration if an inductor is to be turned off abruptly; it will produce a high voltage



Properties of Inductors

- Like the ideal capacitor, the ideal inductor does not dissipate energy stored in it.
- Energy stored will be returned to the circuit later
- In reality, inductors do have internal resistance due to the wiring used to make them.
- A real inductor thus has a winding resistance in series with it. R_w

 There is also a small winding capacitance due to the closeness of the windings

Circuit model

inductor

for a practical

 These two characteristics are typically small, though at high frequencies, the capacitance may matter.

Find the voltage across the inductor and the energy stored in it.

The current through a 0.1-H inductor is $i(t) = 10te^{-5t} A$.

$$v = L \frac{di}{dt}$$

$$v = 0.1 \frac{d}{dt} (10te^{-5t})$$

$$= e^{-5t} + t(-5)e^{-5t} = e^{-5t} (1 - 5t) V$$

$$w = \frac{1}{2} Li^{2}$$

$$= \frac{1}{2} (0.1)100t^{2}e^{-10t} = 5t^{2}e^{-10t} J$$

ed in it.

ote^{-5t} A.

$$v = L \frac{di}{dt}$$

$$I = \frac{1}{L} \int_{t_0}^{t} v(\tau) d\tau + i(t_0)$$

$$p = vi = \left(L \frac{di}{dt}\right)i$$

$$w = \frac{1}{2} Li^2$$

Find the current through a 5-H inductor

if the voltage across it is
$$v(t) = \begin{cases} 30t^2 \text{ (V)} & t > 0 \\ 0 & t < 0 \end{cases}$$

Find the energy stored at t = 5 s. Assume i(v) > 0.

$$i(t) = \frac{1}{L} \int_{t_0}^{t} v(\tau) d\tau + i(t_0)$$

$$= \frac{1}{5} \int_{0}^{t} 30\tau^2 d\tau + i(0)$$

$$= \frac{30}{5} \left(\frac{t^3}{3}\right) + 0$$

$$= 2t^3 \text{ (A)}$$

$$p = v \times i$$

$$= 30t^2 \times 2t^3$$

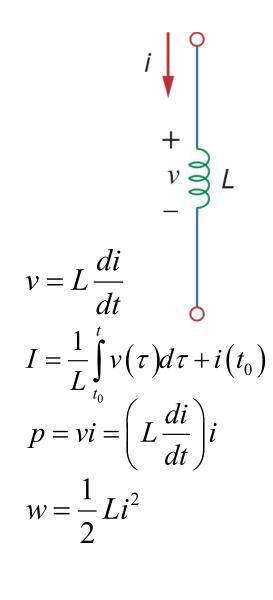
$$= 60t^5$$

$$w = \int_{0}^{t} p dt$$

$$= \int_{0}^{5} 60t^5 dt$$

$$p = v \times i$$
= 30t²×2t³
= 60t⁵

$$w = \int_{0}^{t} p \, dt$$
=
$$\int_{0}^{5} 60t^{5} dt$$
=
$$60 \left(\frac{t^{6}}{6}\right)\Big|_{0}^{t=5}$$
= 156,250 (J)
= 156.25 (kJ)



Consider the circuit under dc conditions, find:

- (a) \boldsymbol{i} , $\boldsymbol{v}_{\mathrm{C}}$, and $\boldsymbol{i}_{\mathrm{L}}$,
- (b) the energy stored in the capacitor and inductor.

$$i = C \frac{dv}{dt}$$
 DC $\Rightarrow i \rightarrow 0 \Rightarrow$ C open circuit

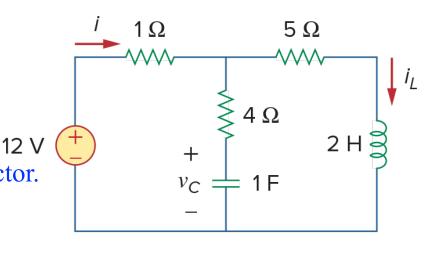
$$v = L \frac{di}{dt}$$
 DC $\Rightarrow v \rightarrow 0 \Rightarrow L$ short circuit

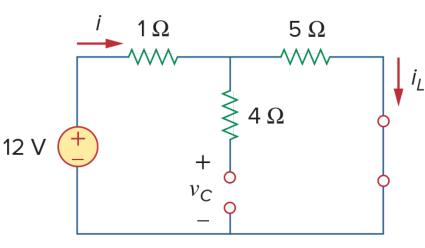
$$i = i_L = \frac{12}{1+5} = 2 A$$

$$v_C = 5i = 10 \text{ V}$$

$$w_C = \frac{1}{2}Cv_C^2 = \frac{1}{2}(1)(10^2) = 50 J$$

$$w_L = \frac{1}{2}Li_L^2 = \frac{1}{2}(2)(2^2) = 4J$$





6.5 Series and Parallel Inductors

Series Inductors

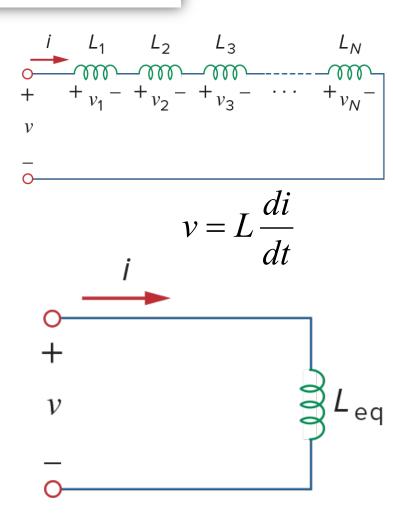
Applying KVL to the loop:

$$v = v_1 + v_2 + v_3 + \dots + v_N$$

$$v = L_1 \frac{di}{dt} + L_2 \frac{di}{dt} + L_3 \frac{di}{dt} + \dots + L_N \frac{di}{dt}$$

$$= \left(\sum_{k=1}^{N} L_k\right) \frac{di}{dt} = L_{eq} \frac{di}{dt}$$

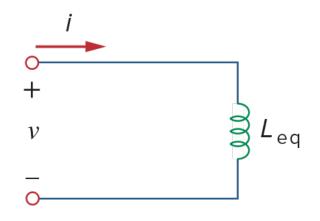
$$L_{eq} = L_1 + L_2 + L_3 + \dots + L_N$$



Here we can see that the inductors have the same behavior as resistors

Parallel Inductors

$$v = L \frac{di}{dt}$$



Applying KCL to the circuit:

$$i = i_1 + i_2 + i_3 + \dots + i_N$$

$$i = \left(\sum_{k=1}^{N} \frac{1}{L_k}\right) \int_{t_0}^{t} v dt + \sum_{k=1}^{N} i_k \left(t_0\right) = \frac{1}{L_{eq}} \int_{t_0}^{t} v dt + i\left(t_0\right)$$

$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \dots + \frac{1}{L_N}$$

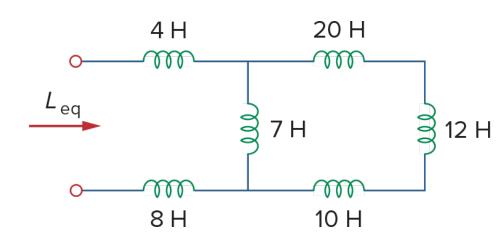
$$i(t_0) = i_1(t_0) + i_2(t_0) + \dots + i_N(t_0)$$

- Once again, the parallel combination resembles that of resistors
- On a related note, the Δ-Y transformation can also be applied to inductors and capacitors in a similar manner, as long as all elements are the same type.

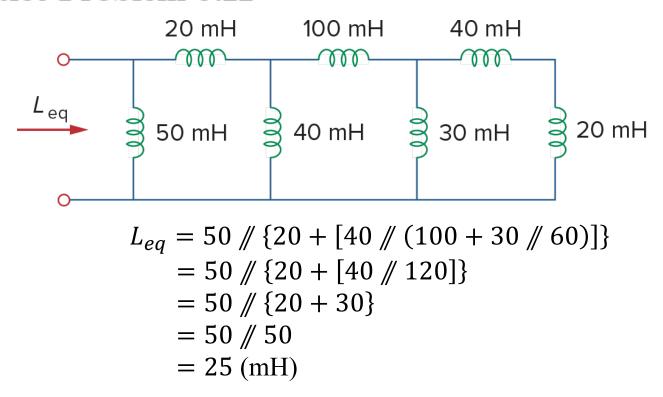
Find the equivalent inductance

$$\frac{7 \times 42}{7 + 42} = 6 \text{ H}$$

$$L_{eq} = 4 + 6 + 8 = 18 \text{ H}$$



Practice Problem 6.11

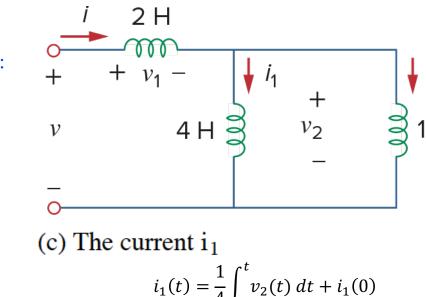


For the, $\mathbf{i}(t) = 4 \cdot (2 - \mathbf{e}^{-10t})$ mA. If $\mathbf{i}_2(0) = -1$ mA, find:

- (a) $i_1(0)$
- (b) v(t), $v_1(t)$, and $v_2(t)$;
- (c) $i_1(t)$ and $i_2(t)$.
- (a) From $i(t) = 4(2 e^{-10t}) \text{ mA},$ i(0) = 4(2 - 1) = 4 mA
 - $i = i_1 + i_2$
- \Rightarrow $i_1(0) = i(0) i_2(0) = 4 (-1) = 5 \text{ mA}$
- (b) The equivalent inductance

$$L_{eq} = 2 + 4 \parallel 12 = 2 + 3 = 5 H$$

- $v(t) = L_{eq} \frac{d i(t)}{dt}$ $= 5 \times \frac{d[4 \cdot (2 e^{-10t})]}{dt}$
 - $dt = 5 \times [4 \times 10e^{-10t}] = 200e^{-10t} \text{ (mV)}$
 - $v_1(t) = 2\frac{d i(t)}{dt} = 2 \times [4 \times 10e^{-10t}]$ = 80e^{-10t} (mV)
 - $v_2(t) = v(t) v_2(t)$ = $(200 - 80)e^{-10t}$ = $120e^{-10t}$ (mV)



$$i_{1}(t) = \frac{1}{4} \int_{0}^{t} v_{2}(t) dt + i_{1}(0)$$

$$= \frac{1}{4} \int_{0}^{t} 120e^{-10t} dt + 5$$

$$= \frac{120}{4} \left(\frac{e^{-10t}}{-10} \right) \Big|_{0}^{t} + 5$$

$$= -3(e^{-10t} - e^{0}) + 5$$

$$= -3e^{-10t} + 8 \text{ (mA)}$$

$$= \frac{1}{12} \int_0^t 120e^{-10t} dt - 1$$
$$= \frac{120}{12} \left(\frac{e^{-10t}}{-10} \right) \Big|_0^t - 1$$

$$= -(e^{-10t} - e^{0}) - 1$$
$$= -e^{-10t} \text{ (mA)}$$

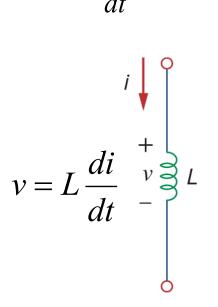
Check if
$$i(t) = 4 \cdot (2 - e^{-10t})$$
?

Summary of Capacitors & Inductors

TABLE 6.1

Important characteristics of the basic elements.[†]

The second secon			
Relation	Resistor (R	Capacitor (C)	Inductor (L)
<i>v-i</i> :	v = iR	$v = \frac{1}{C} \int_{t_0}^t i(\tau) d\tau + v(t_0)$	$v = L \frac{di}{dt}$
i-v:	i = v/R	$i = C \frac{dv}{dt}$	$i = \frac{1}{L} \int_{t_0}^t v(\tau) d\tau + i(t_0)$
<i>p</i> or <i>w</i> :	$p = i^2 R = \frac{v^2}{R}$	$w = \frac{1}{2}Cv^2$	$w = \frac{1}{2}Li^2$
Series:	$R_{\rm eq} = R_1 + R_2$	$C_{\rm eq} = \frac{C_1 C_2}{C_1 + C_2}$	$L_{\rm eq} = L_1 + L_2$
Parallel:	$R_{\rm eq} = \frac{R_1 R_2}{R_1 + R_2}$	$C_{\rm eq} = C_1 + C_2$	$L_{\rm eq} = \frac{L_1 L_2}{L_1 + L_2}$
At dc:	Same	Open circuit	Short circuit



Circuit variable

that cannot

change abruptly: Not applicable v i

Applications

Inductors

- Due to their bulky size, inductors are less frequently used as compared to capacitors, however they have some applications where they are best suited.
- They can be used to create a large amount of current or voltage for a short period of time.
- Their resistance to sudden changes in current can be used for spark suppression.
- Along with capacitors, they can be used for frequency discrimination.

