Chap. 10 Review

1. At $t = 0$, a wheel rotating about a fixed axis at a constant angular acceleration of $-0.40\ \text{rad/s}^2$ has an angular velocity of 1.5 rad/s and an angular position of 2.3 rad. What is the angular position of the wheel at $t = 2.0$ s?

A. 4.9 rad B. 4.7 rad C. 4.5 rad D. 4.3 rad E. 4.1 rad

1. Const ∡ acc^n $\alpha = -0.40\ \text{rad}/s^2$

at $t=0$. $\theta_i = 2.3$ rad., $\omega_i = 1.5$ rad/s

$\Rightarrow \theta(t=2) = \theta_i + \Delta\theta = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2$

$= 2.3 + 1.5\times 2 - \frac{1}{2}\, 0.40 \times 2^2 = 4.5\ \text{rad}$, C

2. A wheel starts from rest and rotates with a constant angular acceleration about a fixed axis. It completes the first revolution 6.0 s after it started. How long after it started will the wheel complete the second revolution?

A. 9.9 s B. 7.8 s C. 8.5 s D. 9.2 s E. 6.4 s

2. the first revolution $\Rightarrow \Delta\theta_1 = 2\pi$ in 6 s, w/ $\omega_i = 0$

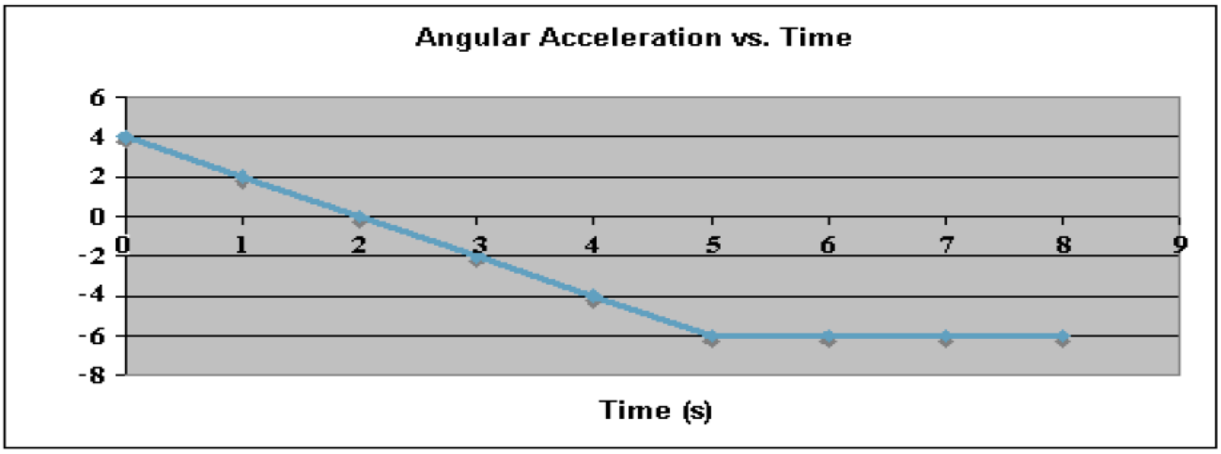
$\Rightarrow 2\pi = \frac{1}{2}\alpha \cdot t^2 = \frac{1}{2}\alpha\cdot 6^2 \Rightarrow \alpha = \frac{4\pi}{36}\ \text{rad}/s^2 = \frac{\pi}{9}\ \text{rad}/s^2$

to complete 2 revolution $\Rightarrow \Delta\theta = 2\times 2\pi = 4\pi$.

$\Rightarrow \Delta\theta = 4\pi = \frac{1}{2}\alpha t_2^2 = \frac{1}{2}\cdot\frac{\pi}{9} t_2^2$

$\Rightarrow t_2^2 = 18\times 4 \Rightarrow t_2 = \sqrt{72} = 8.5\ \text{s}$ C

3. The graph below shows a plot of angular acceleration in rad/s^2 versus time from $t = 0$ s to $t = 8$ s. The change in angular velocity, $\Delta\omega$, during this 8-second period is



A. $18\,\frac{\text{rad}}{\text{s}}$, CW (clockwise). B. $18\,\frac{\text{rad}}{\text{s}}$, CCW (counterclockwise).

C. $23\,\frac{\text{rad}}{\text{s}}$, CW. D. $23\,\frac{\text{rad}}{\text{s}}$, CCW. E. $31\,\frac{\text{rad}}{\text{s}}$, CW.

3. $\Delta\omega =$ area under α-t curve

$$= \underbrace{\frac{4\times 2}{2}}_{(0-2s)} - \underbrace{(6+3)\times\frac{6}{2}}_{(2-8s)} = 4 - 27 = \underset{\text{CW}}{-}23 \text{ rad/s} \quad \text{C}$$

4. A particle located at the position vector $\hat{\mathbf{r}} = \left(\hat{\mathbf{i}} + \hat{\mathbf{j}}\right)$ m has a force $\vec{\mathbf{F}} = \left(2\hat{\mathbf{i}} + 3\hat{\mathbf{j}}\right)$ N acting on it. The torque about the origin is

A. $(1\hat{\mathbf{k}})$N·m B. $(5\hat{\mathbf{k}})$N·m C. $(-1\hat{\mathbf{k}})$N·m D. $(-5\hat{\mathbf{k}})$N·m E. $(2\hat{\mathbf{i}} + 3\hat{\mathbf{j}})$N·m

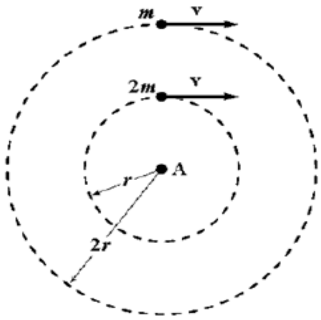
$\because \hat i \times \hat i = \hat j \times \hat j = 0$

4. $\vec r = \hat i + \hat j$, $\vec F = 2\hat i + 3\hat j$

$$\vec\tau = \vec r \times \vec F = (\hat i + \hat j)\times(2\hat i + 3\hat j) = \hat i \times 3\hat j + \hat j \times 2\hat i$$

$$= 3\hat k - 2\hat k = \hat k \quad \text{A}$$

5. Two objects of mass $m_1 = 2m$ and $m_2 = m$ move around a rotation axis A in parallel circles of radii $r_1 = r$ and $r_2 = 2r$ with equal tangential speeds. As they rotate, forces of equal magnitude are applied opposite to their velocities to stop them. Which statement is correct?



A. m_2 will stop first because it has the larger initial angular velocity.
B. m_1 will stop first because it has the smaller radius.
C m_2 will stop first because the torque on it is greater.
D. m_1 will stop first because it has the smaller moment of inertia.
E. Both objects will stop at the same time because the angular accelerations are equal.

5. $\because$ const forces are always along tangent direction

$\Rightarrow$ torque $= \tau =$ const

for $m_1 = 2m$, $\tau_1 = rF$ (out of paper)

$m_2 = m$ $\tau_2 = 2rF$ (out of paper)

initial angular momenta $\begin{cases} L_1 = 2mv\cdot r \\ L_2 = mv\cdot 2r \end{cases} \Rightarrow L_1 = L_2$

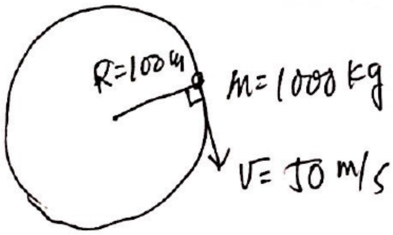
but $\because \tau_2 > \tau_1$ 而 $\frac{\Delta L}{\Delta t} = \tau \Rightarrow \Delta t = \frac{\Delta L}{\tau} \Rightarrow \Delta t_2 < \Delta t_1$

$\Rightarrow m_2$ stops first

6. A car of mass 1 000 kg moves with a speed of 50 m/s on a circular track of radius 100 m. What is the magnitude of its angular momentum (in kg·m²/s) relative to the center of the race track?

A. 5.0×10^2 B. 5.0×10^6 C. 2.5×10^4 D. 2.5×10^6 E. 5.0×10^3

6.



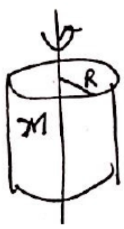
$L = |\vec{r} \times m\vec{v}| \underset{\Updownarrow}{=} mvr = 1000 \times 50 \times 100$

$\boxed{\because \vec{v} \perp \vec{r}} = 5\times 10^6$ (B)

7. A solid cylinder of radius R = 1.0 m and mass 10 kg rotates about its axis. When its angular velocity is 10 rad/s, its angular momentum (in kg·m²/s) is

A. 50. B. 20 C. 40 D. 25 E. 70

7.



cylinder about its axis $I = \frac{1}{2} mR^2 = \frac{1}{2} \times 10 \times 1^2 = 5$

$L = I\omega = 5 \times 10 = 50$ (A)

8. A merry-go-round of radius R = 2.0 m has a moment of inertia I = 250 kg·m², and is rotating at 10 rpm. A child whose mass is 25 kg jumps onto the edge of the merry-go-round, heading directly toward the center at 6.0 m/s. The new angular speed (in rpm) of the merry-go-round is approximately

A. 10 B. 9.2 C. 8.5 D. 7.1 E. 6.4

8.

merry-go-around of $I = 250\ kg\,m^2$, $\omega_i = 10$ rpm

round per minute.

kid of m = 25 kg jump onto the wheel along radial direction

Note that $\vec{\tau}_{ext} = \frac{d\vec{L}}{dt}$, for the kid-wheel system No external torque.

$\Rightarrow \vec{L}$ conserved.

$\Rightarrow \vec{L}_i = I\omega = 250 \times 10 = 2500 \ \text{kg}\,\text{m}^2\,\text{rpm}$

$\because \vec{L}_{kid} = \vec{r} \times \vec{p} = 0 \ (\because \vec{r} \parallel \vec{p})$

$\Rightarrow$ total 角 momentum $\vec{L}_f = \vec{L}_i + \vec{L}_{kid} = \vec{L}_i = I_f \omega_f$

$\Rightarrow 2500 = (250 + mR^2)\omega_f = (250 + 25 \times 2^2)\omega_f$

$\Rightarrow \omega_f = \dfrac{2500}{350} = 7.143$ rpm (D)

本題和完全非彈性碰撞 (perfectly inelastic collision)

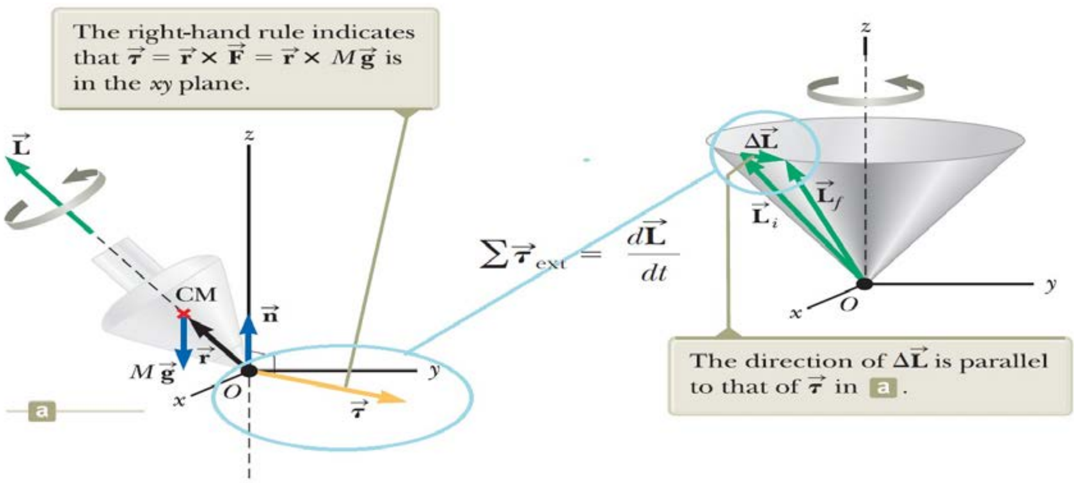
$m_1 \xrightarrow{v_1} \quad m_2 \xrightarrow{v_2} \quad \Rightarrow \quad m_1 m_2 \longrightarrow v$

for $m_1 + m_2$ system 在碰撞前後 $F_{ext} = 0 \Rightarrow$ momentum conserva

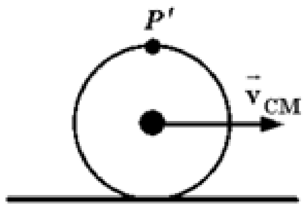
$\Rightarrow \vec{P}_1 + \vec{P}_2 = \vec{P} \Rightarrow m_1\vec{v}_1 + m_2\vec{v}_2 = (m_1 + m_2)\vec{v}$

9. A top is set spinning so that the rotation is counterclockwise around its axis when viewed from above. When the top is placed on a level surface it happens that its axis of rotation is not quite vertical. Viewed from above, which way does the rotational axis of the top precess? Hint: $\vec{\tau} = \dfrac{d\vec{L}}{dt}$

A. clockwise
B. counterclockwise
C. It's random, if it starts clockwise it will continue clockwise, and vice versa, i.e., a 50% chance either way.
D. The direction depends on the little shove given to the axis when the top is placed on the surface.
E. In the northern hemisphere it will be clockwise, in the southern hemisphere it will be counterclockwise.



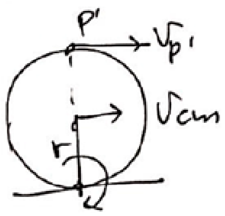
10. When the center of a bicycle wheel has linear velocity $\vec{\mathbf{v}}_{CM}$ relative to the ground, the velocity relative to the ground of point P' at the top of the wheel is



A. 0.
B. $\vec{\mathbf{v}}_{CM}$.
C. $2\vec{\mathbf{v}}_{CM}$.
D. $-\vec{\mathbf{v}}_{CM}$.
E. $-2\vec{\mathbf{v}}_{CM}$.

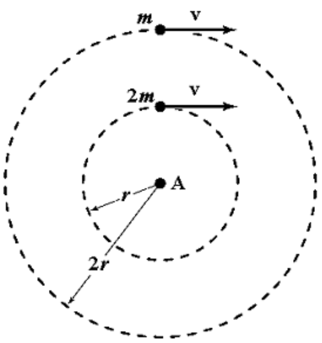
10. 

rolling: the contact point w/ the ground = momentarily stationary
P'



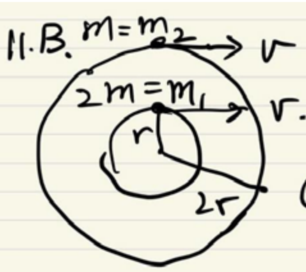
$v_{cm} = r\omega$
$v_{p'} = 2r\omega = 2v_{cm}$ (C)

11. Two objects of mass $m_1 = 2m$ and $m_2 = m$ move around a rotation axis A in parallel circles of radii $r_1 = r$ and $r_2 = 2r$ with equal tangential speeds. As they rotate, forces of equal magnitude are applied opposite to their velocities to stop them. Find the ratio α_1/α_2, where α_1 and α_2 are angular accelerations of m_1 and m_2, respectively?



A. 1/2
B. 1
C. 3/2
D. 2
E. 5/2 ANS: B

11. B.

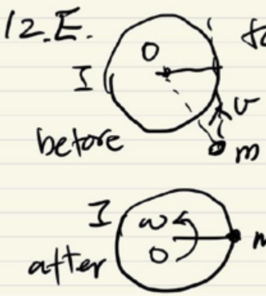


$\tau_1 = rF = I_1\alpha_1 = 2mr^2\alpha_1$ —①
$\tau_2 = 2rF = I_2\alpha_2 = m(2r)^2\alpha_2$ —②
$\frac{①}{②} = \frac{\cancel{r}F}{2\cancel{r}F} = \frac{2\cancel{m}\cancel{r^2}\alpha_1}{\cancel{m}(2\cancel{r})^2\alpha_2} \Rightarrow \frac{\alpha_1}{\alpha_2} = 1$

12. A playground merry-go-round has a radius R and a rotational inertia I. When the merry-go-round is at rest, a child with mass m runs with speed v along a line tangent to the rim and jumps on. The angular velocity of the merry-go-round is then:

A) mv/I B) v/R C) mRv/I D) $2mRv/I$ E) $mRv/(mR^2 + I)$ Ans: E

12.E.



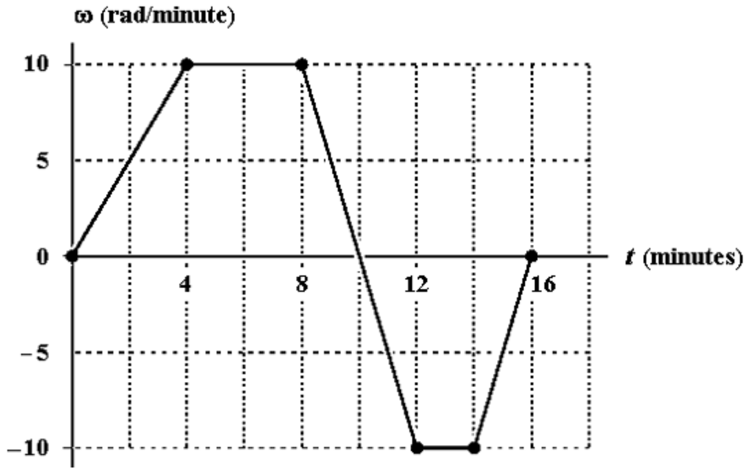
for the merry-go-around + child ⇒ angular momentum conserved

before $L_i = mvR$

after $L_f = (I + mR^2)\omega$ $\Big\}\ L_i = L_f$

$\therefore mvR = (I + mR^2)\omega \Rightarrow \omega = \dfrac{mvR}{I + mR^2}$

13. The figure below shows a graph of angular velocity as a function of time for a car driving around a circular track. Through how many radians does the car travel in the first 10 minutes?

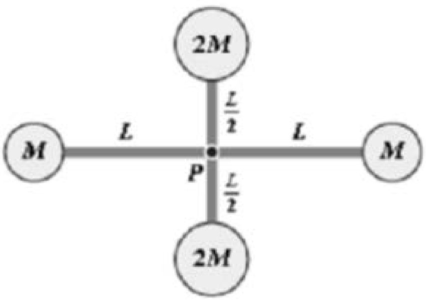


a. 30
b. 50
c. 70
d. 90
e. 100

ANS: C

13.C. $\Delta\theta = \int_0^{10} \omega\, dt = (4+10)\frac{10}{2} = 70$ rad

14. The rigid object shown is rotated about an axis perpendicular to the paper and through point P. The total kinetic energy of the object as it rotates is equal to 1.4 J. If $M = 1.3$ kg and $L = 0.50$ m, what is the angular velocity of the object? Neglect the mass of the connecting rods and treat the masses as particles.



a. 1.3 rad/s
b. 1.5 rad/s
c. 1.7 rad/s
d. 1.2 rad/s
e. 2.1 rad/s

ANS: C

14. C. $K_R = \Sigma \frac{1}{2} I_i \omega^2 = \frac{1}{2}\left[2ML^2 + 2\cdot 2M\left(\frac{L}{2}\right)^2\right]\omega^2$

$= \frac{3}{2}ML^2\omega^2 = \frac{3}{2}\times 1.3\times 0.5^2\,\omega^2 = 1.4\,J$

$\Rightarrow \omega = 1.7\ \text{rad/s}$

15. A solid sphere of mass M and radius R rolls without sliding along the floor (moment of inertia about the axis is $\frac{2}{5}MR^2$). The ratio of its translational kinetic energy to its rotational kinetic energy (about an axis through its center of mass) is:

A) 5/2 B) 2 C) 2/5 D) 25/4 E) 1/3 Ans: A

15. A. $K_T = \frac{1}{2}Mv_{CM}^2$, $K_R = \frac{1}{2}I\omega^2 = \frac{1}{2}I\frac{v_{CM}^2}{R^2} = \frac{1}{5}Mv_{CM}^2$

$\therefore K_T/K_R = \frac{\frac{1}{2}Mv_{CM}^2}{\frac{1}{5}Mv_{CM}^2} = 5/2$