112-1
Electrical Engineering Fundamentals I

Quiz 3

Keys

1. Apply mesh analysis to the circuit in Fig. 1 and obtain I_o.

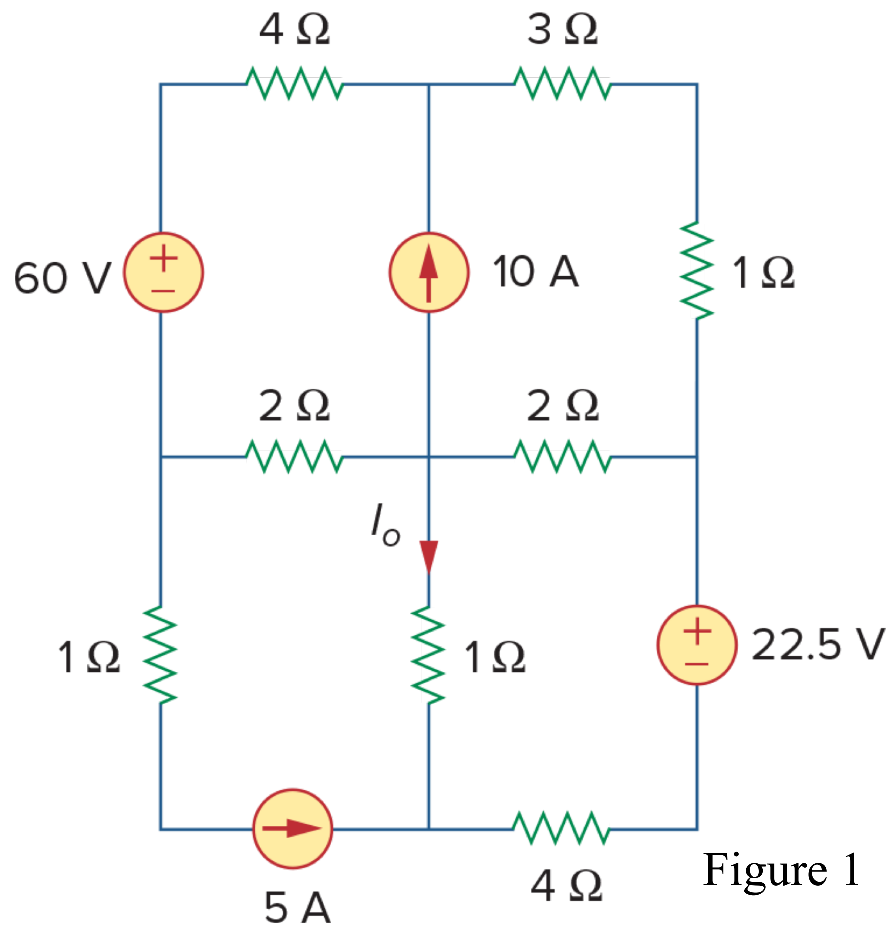
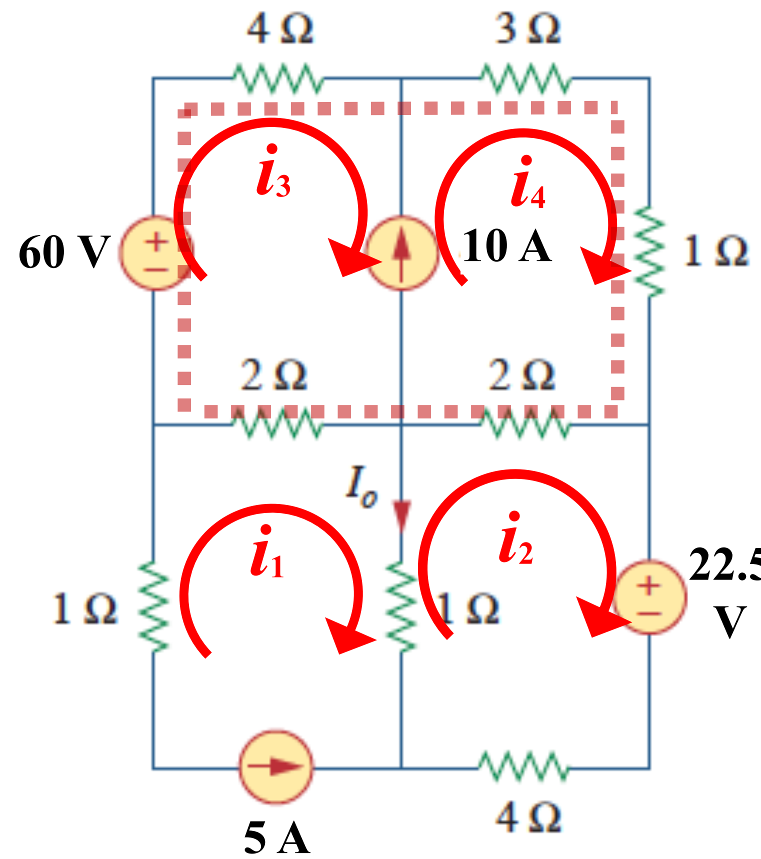


Figure 1

1. 25% Apply mesh analysis to the circuit in Fig. 1 and obtain all mesh currents and I_o.



From Mesh 1:

$$i_1 = -5 \text{ A}$$

From Mesh 2:

$$1\cdot(i_2 - i_1) + 2\cdot(i_2 - i_4) + 22.5 + 4\cdot i_2 = 0$$

$$\Rightarrow 7\cdot i_2 - 2\cdot i_4 = -27.5$$

From Supermesh:

$$-60 + 4\cdot i_3 + 3\cdot i_4 + 1\cdot i_4 + 2\cdot(i_4 - i_2) + 2\cdot(i_3 - i_1) = 0$$

$$\Rightarrow -2\cdot i_2 + 6\cdot i_3 + 6\cdot i_4 = 50$$

And: $-i_3 + i_4 = 10$

$$\begin{bmatrix} 7 & 0 & -2 \\ -2 & 6 & 6 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} -27.5 \\ 50 \\ 10 \end{bmatrix}$$

$\Delta = 80$

$\Delta_2 = -110$

$\Delta_3 = -85$

$\Delta_4 = -715$

$i_1 = -5$ (A)

$i_2 = (-110)/80 = -1.375$ (A)

$i_3 = (-85)/80 = -1.0625$ (A)

$i_4 = 715/80 = 8.9375$ (A)

$I_o = i_1 - i_2 = -5 + 1.375 = \mathbf{-3.625}$ **(A)**

2. 25% Use mesh analysis to i_o and v_o in the circuit of Figure 2.

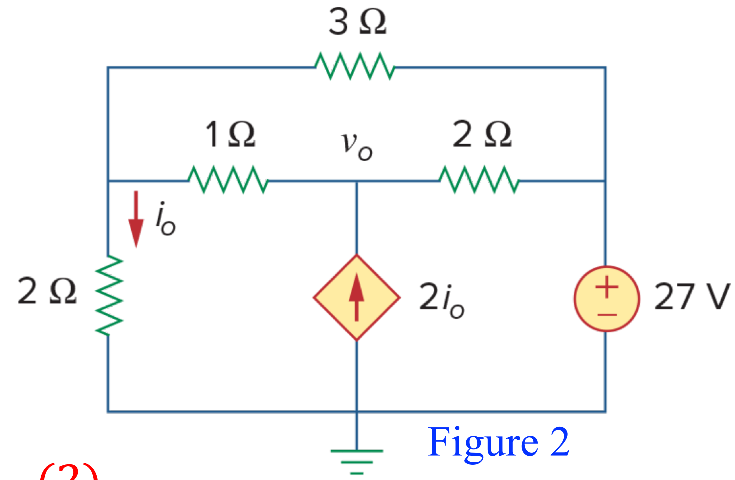
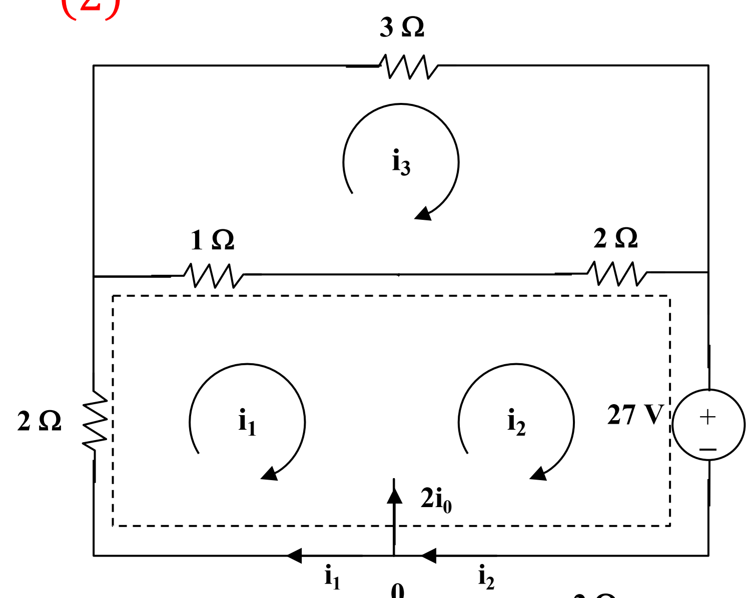


Figure 2

For Super mesh 1 & 2,

$$3 \cdot i_1 + 2 \cdot i_2 - 3 \cdot i_3 + 27 = 0$$
$$\Longrightarrow 3 \cdot i_1 + 2 \cdot i_2 - 3 \cdot i_3 = -27 \cdots\cdots (1)$$

$$i_2 - i_1 = 2i_O \qquad \Longrightarrow i_2 - i_1 = -2i_1 \Longrightarrow i_2 = -i_1 \cdots\cdots (2)$$
$$i_O = -i_1$$



For mesh 3

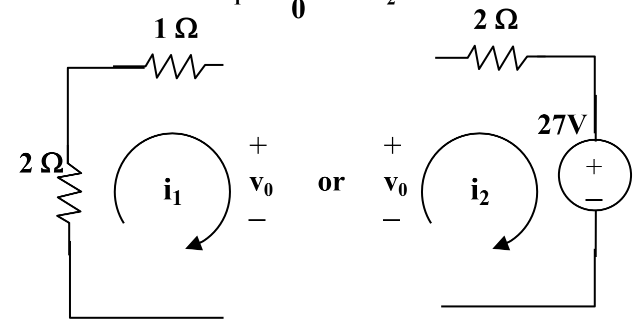
$$-i_1 - 2 \cdot i_2 + 6 \cdot i_3 = 0 \cdots\cdots (3)$$

Solving (1) to (3)

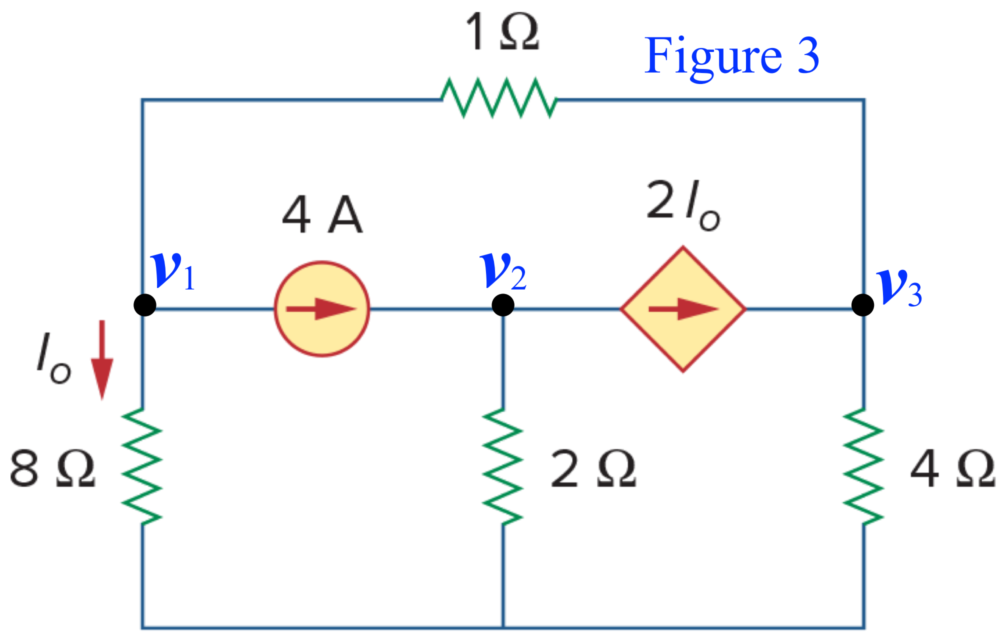
$$(1) \Longrightarrow i_1 - 3 \cdot i_3 = -27$$
$$(3) \Longrightarrow i_1 + 6 \cdot i_3 = 0$$

$$\Longrightarrow i_3 = 3\ (A)$$
$$\Longrightarrow i_1 = -18\ (A)$$
$$\Longrightarrow i_2 = 18\ (A)$$

$$\Longrightarrow i_O = -i_1 = 18\ (A)$$
$$\Longrightarrow v_O = 2 \cdot (i_2 - i_3) + 27$$
$$= 2 \times (18 - 3) + 27 (V)$$
$$= 57 (V)$$



3. (25%) Find Io and node voltage $\boldsymbol{v}_1$, $\boldsymbol{v}_2$, and $\boldsymbol{v}_3$ in the circuit of Fig. 3 using nodal analysis.



At node 1:

$$[(v_1-0)/8] + [(v_1-v_3)/1] + 4 = 0$$

At node 2:

$$[(v_2-0)/2] + 2\,i_O = 4$$

At node 3:

$$2\,i_O + [(v_1-v_3)/1] = [(v_3-0)/4]$$

And:

$$i_o = v_1/8$$

This produces,

$1.125\,v_1 - v_3 = -4$ (1)

$0.25\,v_1 + 0.5\,v_2 = 4$ (2)

$-1.25\,v_1 + 1.25\,v_3 = 0$ or $v_1 = v_3$ (3)

Substituting (3) into (1)

$(1.125-1)\,v_1 = -4$

$\boldsymbol{v}_1 = -4/0.125 = -32$ (V)

$\boldsymbol{v}_3 = v_1 = -32$ (V)

Substituting into (2)

$v_2 = [4-0.25\text{x}(-32)]/0.5 = 24$ (V)

$i_o = 32/8 =$ **–4(A)**

4. (25%) For the circuit in Fig. 4, find $\boldsymbol{i}$ and node voltage $\boldsymbol{v}_1$, $\boldsymbol{v}_2$, and $\boldsymbol{v}_3$ using nodal analysis.

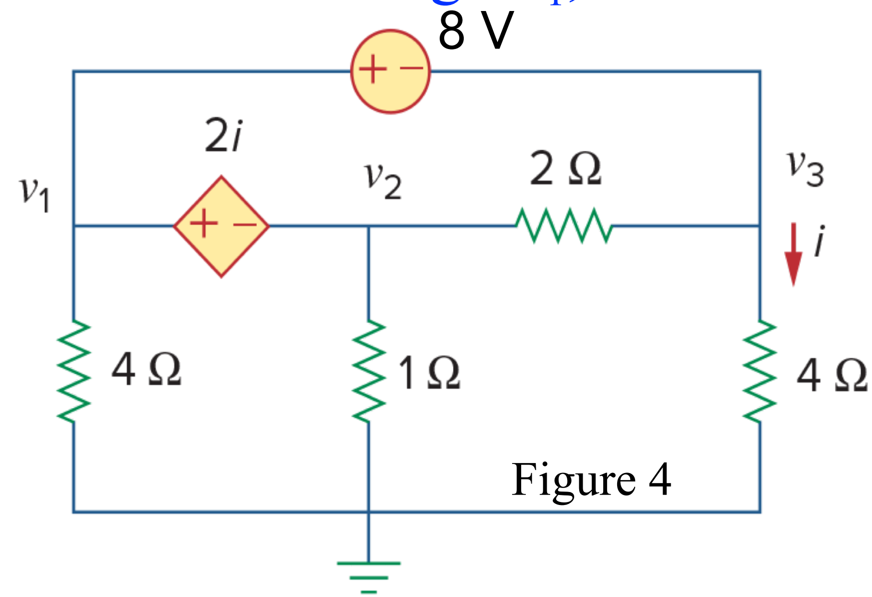


Figure 4

Nodes 1 and 2 form a supernode;
so do nodes 1 and 3

Supernode 1 & 2 & 3

$$i_1 + i_2 + i = 0$$
$$\Rightarrow \frac{v_1}{4} + \frac{v_2}{1} + \frac{v_3}{4} = 0$$
$$\Rightarrow v_1 + 4v_2 + v_3 = 0 \cdots (1)$$

Supernode 1 & 3,

$$v_1 - v_3 = 8$$
$$\Rightarrow v_3 = v_1 - 8 \cdots (2)$$

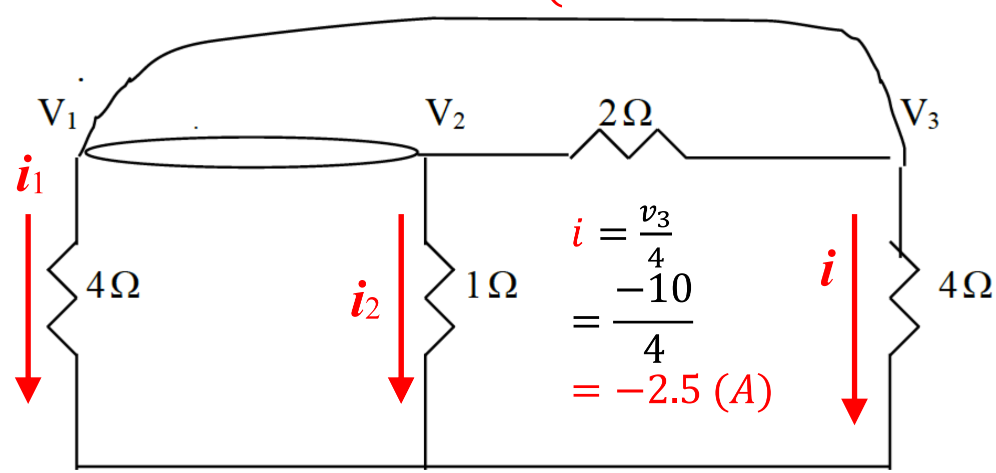
Supernode 1 & 2,

$$2i = v_1 - v_2$$
$$i = \frac{v_3}{4}$$
$$\Rightarrow \frac{v_3}{2} = v_1 - v_2$$
$$\Rightarrow \frac{v_1 - 8}{2} = v_1 - v_2$$
$$\Rightarrow v_2 = \frac{v_1}{2} + 4 \cdots (3)$$

Substitute (2) & (3) into (1)

$$\Rightarrow v_1 + 4\left(\frac{v_1}{2} + 4\right) + v_1 - 8 = 0$$
$$\Rightarrow 4\, v_1 = -8$$

$$\Rightarrow \begin{cases} v_1 = -2\ (V) \\ v_2 = 3\ (V) \\ v_3 = \ -10\ (V) \end{cases}$$



$$i = \frac{v_3}{4} = \frac{-10}{4} = -2.5\ (A)$$