Content of chapter 5

1. RC network

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CHAPTER FIVE

TRANSIENT ANALYSIS

5.1 RC NETWORK

Considering the RC Network shown in Figure 5.1, we can use KCL to write Equation (5.1).

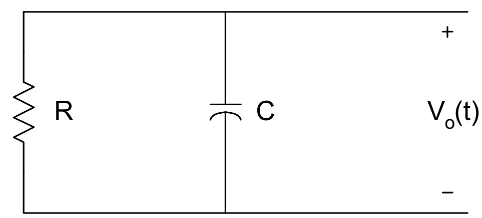


Figure 5.1 Source-free RC Network

$$C\frac{dv_o(t)}{dt} + \frac{v_o(t)}{R} = 0 \tag{5.1}$$

i.e.,

$$\frac{dv_o(t)}{dt} + \frac{v_o(t)}{CR} = 0$$

If V_m is the initial voltage across the capacitor, then the solution to Equation (5.1) is

$$v_0(t) = V_m e^{-\left(\frac{t}{CR}\right)} \tag{5.2}$$

where

CR is the time constant

Equation (5.2) represents the voltage across a discharging capacitor. To obtain the voltage across a charging capacitor, let us consider Figure 5.2.

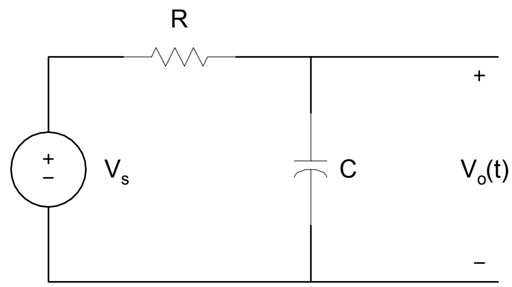


Figure 5.2 Charging of a Capacitor

Using KCL, we get

$$C\frac{dv_o(t)}{dt} + \frac{v_o(t) - V_s}{R} = 0 \tag{5.3}$$

If the capacitor is initially uncharged, that is $v_0(t) = 0$ at $t = 0$, the solution to Equation (5.3) is given as

$$v_0(t) = V_S\left(1 - e^{-\left(\frac{t}{CR}\right)}\right) \tag{5.4}$$

Examples 5.1 and 5.2 illustrate the use of MATLAB for solving problems related to RC Network.

Example 5.1

Assume that for Figure 5.2 C = 10 μF, use MATLAB to plot the voltage across the capacitor if R is equal to (a) 1.0 kΩ, (b) 10 kΩ and (c) 0.1 kΩ.

Solution

MATLAB Script

```
% Charging of an RC circuit
%
c = 10e-6;
r1 = 1e3;
```

```
tau1 = c*r1;
t = 0:0.002:0.05;
v1 = 10*(1-exp(-t/tau1));
r2 = 10e3;
tau2 = c*r2;
v2 = 10*(1-exp(-t/tau2));
r3 = .1e3;
tau3 = c*r3;
v3 = 10*(1-exp(-t/tau3));
plot(t,v1,'+',t,v2,'o', t,v3,'*')
axis([0 0.06 0 12])
title('Charging of a capacitor with three time constants')
xlabel('Time, s')
ylabel('Voltage across capacitor')
text(0.03, 5.0, '+ for R = 1 Kilohms')
text(0.03, 6.0, 'o for R = 10 Kilohms')
text(0.03, 7.0, '* for R = 0.1 Kilohms')
```

Figure 5.3 shows the charging curves.

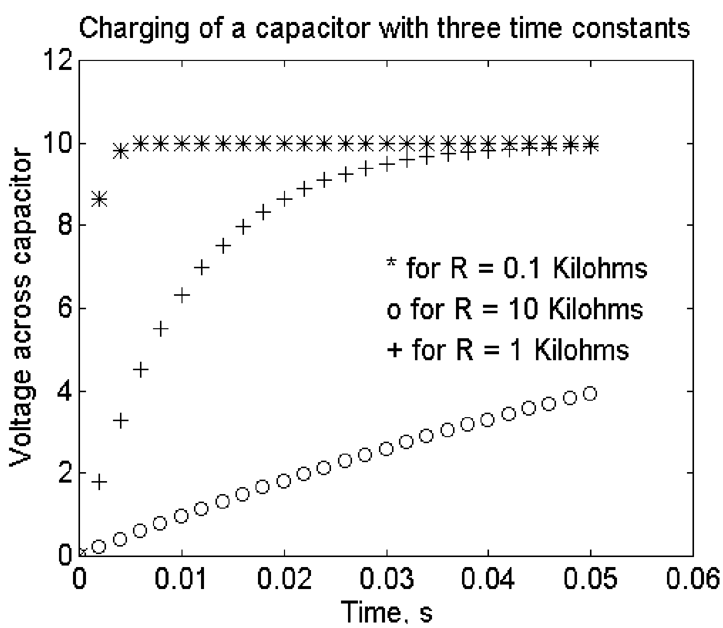


Figure 5.3 Charging of Capacitor

5.2 RL NETWORK

Consider the RL circuit shown in Figure 5.5.

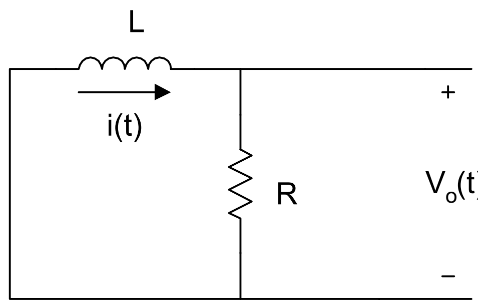


Figure 5.5 Source-free RL Circuit

Using the KVL, we get

$$L\frac{di(t)}{dt} + Ri(t) = 0 \tag{5.5}$$

If the initial current flowing through the inductor is I_m, then the solution to Equation (5.5) is

$$i(t) = I_m e^{-\left(\frac{t}{\tau}\right)} \tag{5.6}$$

where

$$\tau = L/R \tag{5.7}$$

Equation (5.6) represents the current response of a source-free RL circuit with initial current I_m, and it represents the natural response of an RL circuit.

Figure 5.6 is an RL circuit with source voltage $v(t) = V_S$.

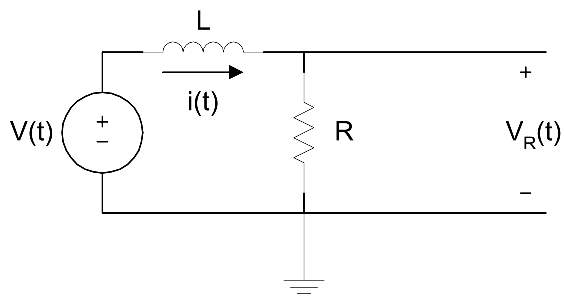


Figure 5.6 RL Circuit with a Voltage Source

Using KVL, we get

$$L\frac{di(t)}{dt} + Ri(t) = V_S \tag{5.8}$$

If the initial current flowing through the series circuit is zero, the solution of Equation (5.8) is

$$i(t) = \frac{V_S}{R}\left(1 - e^{-\left(\frac{Rt}{L}\right)}\right) \tag{5.9}$$

The voltage across the resistor is

$$\begin{aligned} v_R(t) &= Ri(t) \\ &= V_S\left(1 - e^{-\left(\frac{Rt}{L}\right)}\right) \end{aligned} \tag{5.10}$$

The voltage across the inductor is

$$\begin{aligned} v_L(t) &= V_S - v_R(t) \\ &= V_S e^{-\left(\frac{Rt}{L}\right)} \end{aligned} \tag{5.11}$$

The following example illustrates the use of MATLAB for solving RL circuit problems.

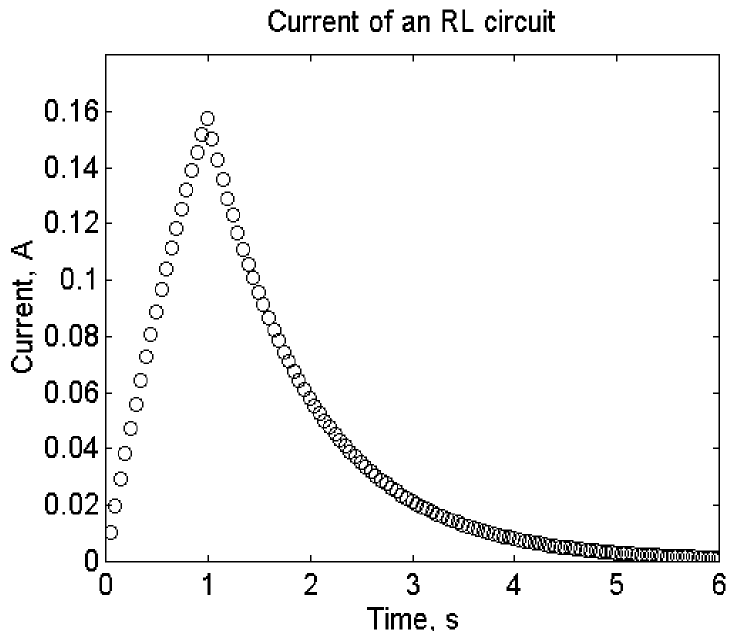


Figure 5.8 Current Flowing through Inductor

5.3 RLC CIRCUIT

For the series RLC circuit shown in Figure 5.9, we can use KVL to obtain the Equation (5.15).

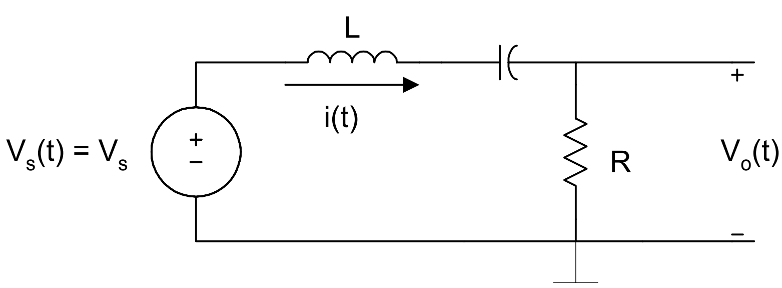


Figure 5.9 Series RLC Circuit

$$v_S(t) = L\frac{di(t)}{dt} + \frac{1}{C}\int_{-\infty}^{t} i(\tau)d\tau + Ri(t) \tag{5.15}$$

Differentiating the above expression, we get

$$\frac{dv_S(t)}{dt} = L\frac{d^2 i(t)}{dt^2} + R\frac{di(t)}{dt} + \frac{i(t)}{C}$$

i.e.,

$$\frac{1}{L}\frac{dv_S(t)}{dt} = \frac{d^2 i(t)}{dt^2} + \frac{R}{L}\frac{di(t)}{dt} + \frac{i(t)}{LC} \tag{5.16}$$

The homogeneous solution can be found by making $v_S(t)$ = constant, thus

$$0 = \frac{d^2 i(t)}{dt^2} + \frac{R}{L}\frac{di(t)}{dt} + \frac{i(t)}{LC} \tag{5.17}$$

The characteristic equation is

$$0 = \lambda^2 + a\lambda + b \tag{5.18}$$

where

$a = R/L$ and

$b = 1/LC$

The roots of the characteristic equation can be determined. If we assume that the roots are

$$\lambda = \alpha, \beta$$

then, the solution to the homogeneous solution is

$$i_h(t) = A_1 e^{\alpha_1 t} + A_2 e^{\alpha_2 t} \tag{5.19}$$

where

Example 5.5

The switch in Figure 5.10 has been opened for a long time. If the switch opens at t = 0, find the voltage $v(t)$. Assume that $R = 10\ \Omega$, $L = 1/32$ H, $C = 50\mu$F and $I_S = 2\ A$.

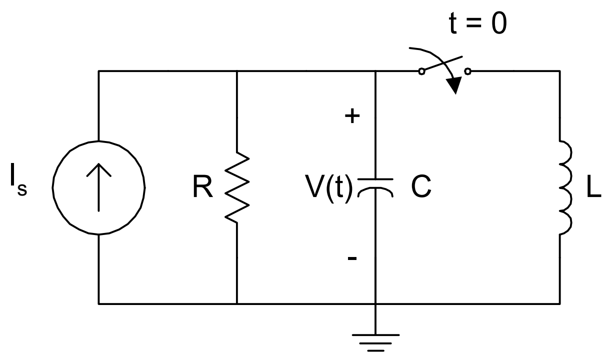


Figure 5.10 Circuit for Example 5.5

At t < 0, the voltage across the capacitor is

$$v_C(0) = (2)(10) = 20 \quad \text{V}$$

In addition, the current flowing through the inductor

$$i_L(0) = 0$$

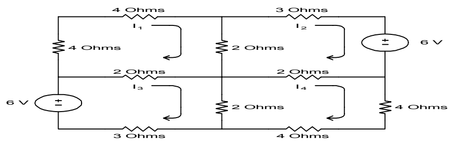
At t > 0, the switch closes and all the four elements of Figure 5.10 remain in parallel. Using KCL, we get

$$I_S = \frac{v(t)}{R} + C\frac{dv(t)}{dt} + \frac{1}{L}\int_0^t v(\tau)d\tau + i_L(0)$$

Taking the Laplace transform of the above expression, we get

$$\frac{I_S}{s} = \frac{V(s)}{R} + C[sV(s) - V_C(0)] + \frac{V(s)}{sL} + \frac{i_L(0)}{s}$$

Simplifying the above expression, we get



For I_S = 2A, C = 50μ F, R = 10Ω, L = 1/32 H, $V(s)$ becomes

$$V(s) = \frac{40000 + 20s}{s^2 + 2000s + 64 * 10^4}$$

$$V(s) = \frac{40000 + 20s}{(s+1600)(s+400)} = \frac{A}{(s+1600)} + \frac{B}{(s+400)}$$

$$A = \lim_{s \to -1600} V(s)(s+1600) = -6.67$$

$$B = \lim_{s \to -400} V(s)(s+400) = 26.67$$

$$v(t) = -6.67e^{-1600t} + 26.67e^{-400t}$$

The plot of $v(t)$ is shown in Figure 5.13.

5.4 STATE VARIABLE APPROACH

Another method of finding the transient response of an RLC circuit is the state variable technique. The later method (i) can be used to analyze and synthesize control systems, (ii) can be applied to time-varying and nonlinear systems, (iii) is suitable for digital and computer solution and (iv) can be used to develop the general system characteristics.

A state of a system is a minimal set of variables chosen such that if their values are known at the time t, and all inputs are known for times greater than t_1, one can calculate the output of the system for times greater than t_1. In general, if we designate x as the state variable, u as the input, and y as the output of a system, we can express the input u and output y as

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{5.22}$$

$$y(t) \quad = Cx(t) + Du(t) \tag{5.23}$$

where

$$u(t) = \begin{bmatrix} u_1(t) \\ u_2(t) \\ . \\ . \\ u_n(t) \end{bmatrix} \qquad x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ . \\ . \\ x_n(t) \end{bmatrix} \qquad y(t) = \begin{bmatrix} y_1(t) \\ y_2(t) \\ . \\ . \\ y_n(t) \end{bmatrix}$$

and *A, B, C*, and *D* are matrices determined by constants of a system.

For example, consider a single-input and a single-output system described by the differential equation

$$\frac{d^4 y(t)}{dt^4} + 3\frac{d^3 y(t)}{dt^3} + 4\frac{d^2 y(t)}{dt^2} + 8\frac{dy(t)}{dt} + 2y(t) \quad = \quad 6u(t) \tag{5.24}$$

We define the components of the state vector as

$$x_1(t) = y(t)$$

$$x_2(t) = \frac{dy(t)}{dt} = \frac{dx_1(t)}{dt} = \dot{x}_1(t)$$

$$x_3(t) = \frac{d^2 y(t)}{dt^2} = \frac{dx_2(t)}{dt} = \dot{x}_2(t)$$

$$x_4(t) = \frac{d^3 y(t)}{dt^3} = \frac{dx_3(t)}{dt} = \dot{x}_3(t)$$

$$x_5(t) = \frac{d^4 y(t)}{dt^4} = \frac{dx_4(t)}{dt} = \dot{x}_4(t) \tag{5.25}$$

Using Equations (5.24) and (5.25), we get

$$\dot{x}_4(t) = 6u(t) - 3x_4(t) - 4x_3(t) - 8x_2(t) - 2x_1(t) \tag{5.26}$$

From the Equations (5.25) and (5.26), we get

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \\ \dot{x}_4(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -2 & -8 & -4 & -3 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ x_4(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 6 \end{bmatrix} u(t) \tag{5.27}$$

or $$\dot{x}(t) = Ax(t) + Bu(t) \tag{5.28}$$

where

$$\dot{x} = \begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \\ \dot{x}_4(t) \end{bmatrix}; \quad A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -2 & -8 & -4 & -3 \end{bmatrix}; B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 6 \end{bmatrix} \tag{5.29}$$

Since

$$y(t) = x_1(t)$$

we can express the output $y(t)$ in terms of the state $x(t)$ and input $u(t)$ as

$$y(t) = Cx(t) + Du(t) \tag{5.30}$$

where

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \quad \text{and} \quad D = [0] \qquad (5.31)$$

In RLC circuits, if the voltage across a capacitor and the current flowing in an inductor are known at some initial time t, then the capacitor voltage and inductor current will allow the description of system behavior for all subsequent times. This suggests the following guidelines for the selection of acceptable state variables for RLC circuits:

1. Currents associated with inductors are state variables.

2. Voltages associated with capacitors are state variables.

3. Currents or voltages associated with resistors do not specify independent state variables.

4. When closed loops of capacitors or junctions of inductors exist in a circuit, the state variables chosen according to rules 1 and 2 are not independent.

Consider the circuit shown in Figure 5.11.

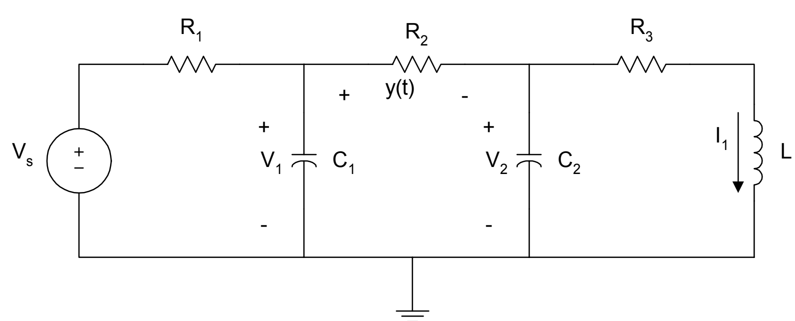


Figure 5.11 Circuit for State Analysis

Using the above guidelines, we select the state variables to be V_1, V_2, and i_1.

Using nodal analysis, we have

$$C_1 \frac{dv_1(t)}{dt} + \frac{V_1 - V_s}{R_1} + \frac{V_1 - V_2}{R_2} = 0 \tag{5.32}$$

$$C_2 \frac{dv_2(t)}{dt} + \frac{V_2 - V_1}{R_2} + i_1 = 0 \tag{5.33}$$

Using loop analysis

$$V_2 = i_1 R_3 + L \frac{di_1(t)}{dt} \tag{5.34}$$

The output $y(t)$ is given as

$$y(t) = v_1(t) - v_2(t) \tag{5.35}$$

Simplifying Equations (5.32) to (5.34), we get

$$\frac{dv_1(t)}{dt} = -(\frac{1}{C_1 R_1} + \frac{1}{C_1 R_2})V_1 + \frac{V_2}{C_1 R_2} + \frac{V_s}{C_1 R_1} \tag{5.36}$$

$$\frac{dv_2(t)}{dt} = \frac{V_1}{C_2 R_2} - \frac{V_2}{C_2 R_2} - \frac{i_1}{C_2} \tag{5.37}$$

$$\frac{di_1(t)}{dt} = \frac{V_2}{L} - \frac{R_3}{L} i_1 \tag{5.38}$$

Expressing the equations in matrix form, we get

$$\begin{bmatrix} \dot{V}_1 \\ \dot{V}_2 \\ \dot{i}_1 \end{bmatrix} = \begin{bmatrix} -(\frac{1}{C_1 R_1} + \frac{1}{C_1 R_2}) & \frac{1}{C_1 R_2} & 0 \\ \frac{1}{C_2 R_2} & -\frac{1}{C_2 R_2} & -\frac{1}{C_2} \\ 0 & \frac{1}{L} & -\frac{R_3}{L} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ i_1 \end{bmatrix} + \begin{bmatrix} \frac{1}{C_1 R_1} \\ 0 \\ 0 \end{bmatrix} V_s \tag{5.39}$$

and the output is

$$y = \begin{bmatrix} 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ i_1 \end{bmatrix} \tag{5.40}$$

MATLAB functions for solving ordinary differential equations are ODE functions. These are described in the following section.

5.4.1 MATLAB Ode Functions

MATLAB has two functions, **ode23 and ode45**, for computing numerical solutions to ordinary differential equations. The **ode23** function integrates a system of ordinary differential equations using second- and third-order Runge-Kutta formulas; the **ode45** function uses fourth- and fifth-order Runge-Kutta integration equations.

The general forms of the ode functions are

Check ode23 in matlab help for the new version

[t,x] = ode23 (xprime, tstart, tfinal, xo, tol,trace)

or

[t,x] = ode45 (xprime, tstart, tfinal, xo, tol, trace)

where

xprime is the name (in quotation marks) of the MATLAB function or m-file that contains the differential equations to be integrated. The function will compute the state derivative vector $\dot{x}(t)$ given the current time *t*, and state vector $x(t)$. The function must have 2 input arguments, scalar t (time) and column vector x (state) and the function returns the output argument $xdot$, $(\dot{x})$, a column vector of state derivatives

$$\dot{x}(t_1) = \frac{dx(t_1)}{dt}$$

tstart	is the starting time for the integration
tfinal	is the final time for the integration
xo	is a column vector of initial conditions
tol	is optional. It specifies the desired accuracy of the solution.

Let us illustrate the use of MATLAB ode functions with the following two examples.

Example 5.6

For Figure 5.2, V_S = 10V, R = 10,000 Ω, C = 10μF. Find the output voltage $v_0(t)$, between the interval 0 to 20 ms, assuming $v_0(0) = 0$ and by (a) using a numerical solution to the differential equation; and (b) analytical solution.

Solution

From Equation (5.3), we have

$$C\frac{dv_o(t)}{dt} + \frac{v_o(t) - V_s}{R} = 0$$

thus

$$\frac{dv_o(t)}{dt} = \frac{V_s}{CR} - \frac{v_o(t)}{CR} = 100 - 10v_0(t)$$

From Equation(5.4), the analytical solution is

$$v_0(t) = 10\left(1 - e^{-\left(\frac{t}{CR}\right)}\right)$$

MATLAB Script

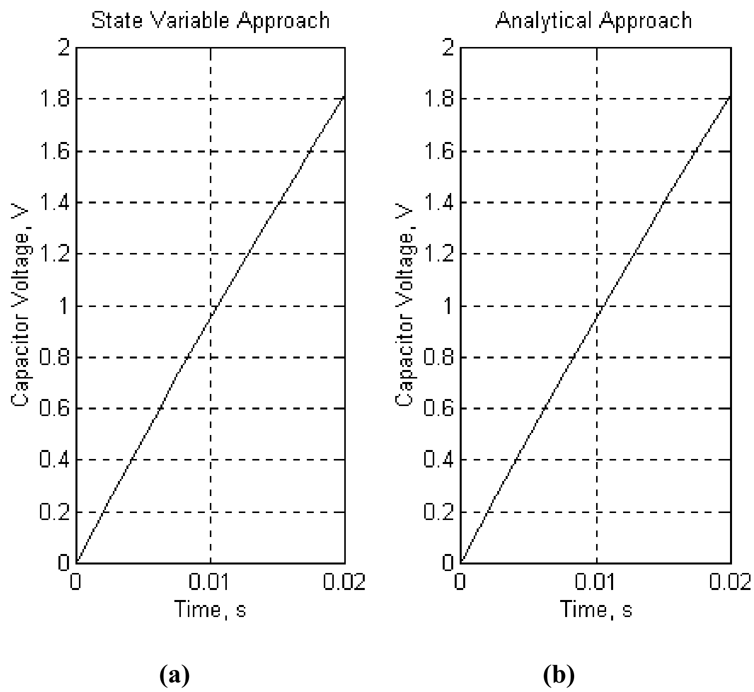


Figure 5.12 Output Voltage $v_0(t)$ Obtained from (a) State Variable Approach and (b) Analytical Method

Example 5.7

For Figure 5.10, if R = 10Ω, L = 1/32 H, C = 50μF, use a numerical solution of the differential equation to solve $v(t)$. Compare the numerical solution to the analytical solution obtained from Example 5.5.

Solution

From Example 5.5, $v_C(0)$ = 20V, $i_L(0) = 0$, and

$$L\frac{di_L(t)}{dt} = v_C(t)$$

$$C\frac{dv_C(t)}{dt} + i_L + \frac{v_C(t)}{R} - I_S = 0$$

Simplifying, we get

$$\frac{di_L(t)}{dt} = \frac{v_C(t)}{L}$$
$$\frac{dv_C(t)}{dt} = \frac{I_S}{C} - \frac{i_L(t)}{C} - \frac{v_C(t)}{RC}$$

Assuming that

$$x_1(t) = i_L(t)$$
$$x_2(t) = v_C(t)$$

We get

$$\dot{x}_1(t) = \frac{1}{L}x_2(t)$$

$$\dot{x}_2(t) = \frac{I_S}{C} - \frac{1}{C}x_1(t) - \frac{1}{RC}x_2(t)$$

We create function m-file containing the above differential equations.

MATLAB Script

```
%  Solution of second-order differential equation
% The function diff2(x,y) is created to evaluate the diff. equation
%  the name of the m-file is diff2.m
%  the function is defined as:
%
function xdot = diff2(t,x)
is = 2;
c = 50e-6;  L = 1/32;  r = 10;
k1 = 1/c ;   %  1/C
k2 =  1/L ;   %  1/L
k3 = 1/(r*c);   % 1/RC

xdot(1) = k2*x(2);
xdot(2) = k1*is - k1*x(1) - k3*x(2);
end
```


$$\dot{x}_1(t) = \frac{1}{L}x_2(t)$$

$$\dot{x}_2(t) = \frac{I_S}{C} - \frac{1}{C}x_1(t) - \frac{1}{RC}x_2(t)$$

To simulate the differential equation defined in diff2 in the interval $0 \le t \le 30$ ms, we note that

$$x_1(0) = i_L(0) = 0 \text{ V}$$

$$x_2(0) = v_C(0) = 20$$

Using the MATLAB ode23 function, we get

```
% solution of second-order differential equation
% the function diff2(x,y) is created to evaluate
% the differential equation
% the name of m-file is diff2.m
%
% Transient analysis of RLC circuit using ode function
% numerical solution

t0 = 0;
tf = 30e-3;
x0 = [0 20]; % Initial conditions
[t,x] = ode23('diff2',t0,tf,x0);

% Second column of matrix x represent capacitor voltage
subplot(211), plot(t,x(:,2))
xlabel('Time, s'), ylabel('Capacitor voltage, V')
text(0.01, 7, 'State Variable Approach')

% Transient analysis of RLC circuit from Example 5.5
t2 =0:1e-3:30e-3;
vt = -6.667*exp(-1600*t2) + 26.667*exp(-400*t2);
subplot(212), plot(t2,vt)
xlabel('Time, s'), ylabel('Capacitor voltage, V')
text(0.01, 4.5, 'Results from Example 5.5')
```

The plot is shown in Figure 5.13.

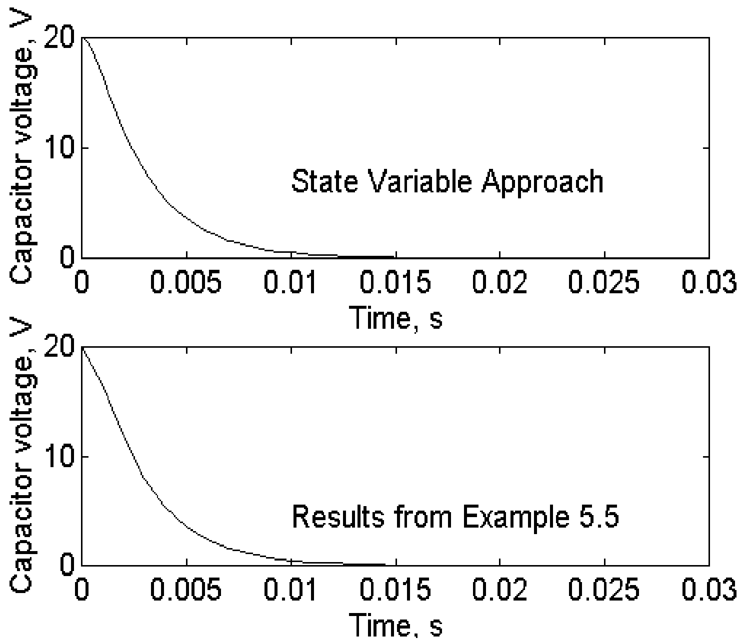


Figure 5.13 Capacitor Voltage $v_0(t)$ Obtained from Both State Variable Approach and Laplace Transform

The results from the state variable approach and those obtained from Example 5.5 are identical.

Example 5.8

For Figure 5.11, if $v_S(t) = 5u(t)$ where $u(t)$ is the unit step function and $R_1 = R_2 = R_3 = 10\ K\Omega$, $C_1 = C_2 = 5\ \mu F$, and L = 10 H, find and plot the voltage $v_0(t)$ within the intervals of 0 to 5 s.

Solution

Using the element values and Equations (5.36) to (5.38), we have

$$\frac{dv_1(t)}{dt} = -40v_1(t) + 20v_2(t) + 20V_s$$

$$\frac{dv_1(t)}{dt} = -40v_1(t) + 20v_2(t) + 20V_s$$

$$\frac{dv_2(t)}{dt} = 20v_1(t) - 20v_2(t) - i_1(t)$$

$$\frac{di_1(t)}{dt} = 0.1v_2(t) - 1000i_1(t)$$

We create an m-file containing the above differential equations.

MATLAB Script

```
%
% solution of a set of first order differential equations
% the function diff3(t,v) is created to evaluate
% the differential equation
% the name of the m-file is diff3.m
%

function vdot = diff3(t,v)

vdot(1) = -40*v(1) + 20*v(2) + 20*5;
vdot(2) = 20*v(1) - 20*v(2) - v(3);
vdot(3) = 0.1*v(2) -1000*v(3);
end
```

To obtain the output voltage in the interval of $0 \le t \le 5$ s, we note that the output voltage

$$v_0(t) = v_1(t) - v_2(t)$$

Note that at t < 0, the step signal is zero so

$$v_0(0) = v_2(0) = i_1(0) = 0$$

Using ode45 we get

```
% solution of a set of first-order differential equations
% the function diff3(t,v) is created to evaluate
% the differential equation
% the name of the m-file is diff3.m
%
% Transient analysis of RLC circuit using state
```

```
% variable approach

t0 = 0;
tf = 2;
x0 = [0 0 0]; % initial conditions

[t,x] = ode23('diff3', t0, tf, x0);

tt = length(t);

for i = 1:tt
   vo(i) = x(i,1) - x(i,2);
end

plot(t, vo)
title('Transient analysis of RLC')
xlabel('Time, s'), ylabel('Output voltage')
```

The plot of the output voltage is shown in Figure 5.14.

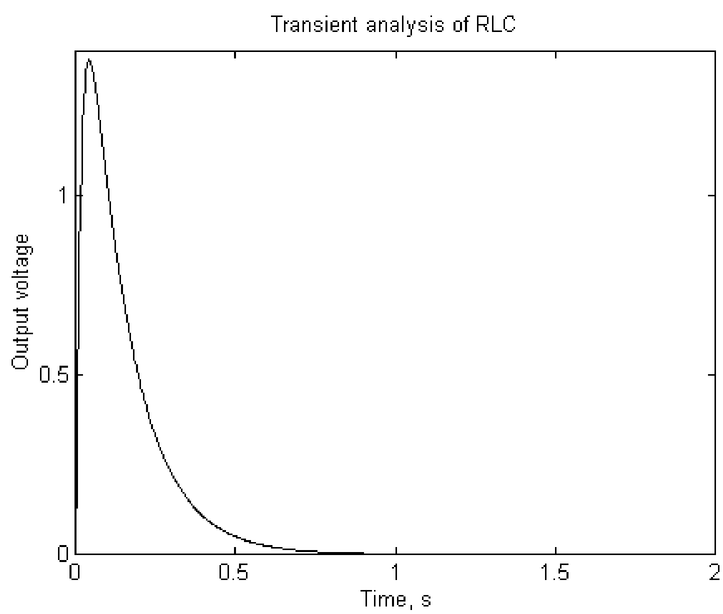
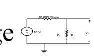


Figure 5.14 Output Voltage 

5.2 The switch is close at t = 0; find $i(t)$ between the intervals 0 to 10 ms. The resistance values are in ohms.

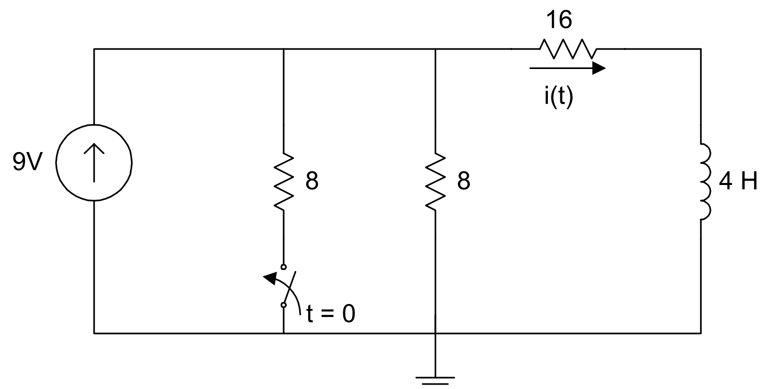


Figure P5.2 Figure for Exercise 5.2

5.3 For the series RLC circuit, the switch is closed at t = 0. The initial energy in the storage elements is zero. Use MATLAB to find $v_0(t)$.

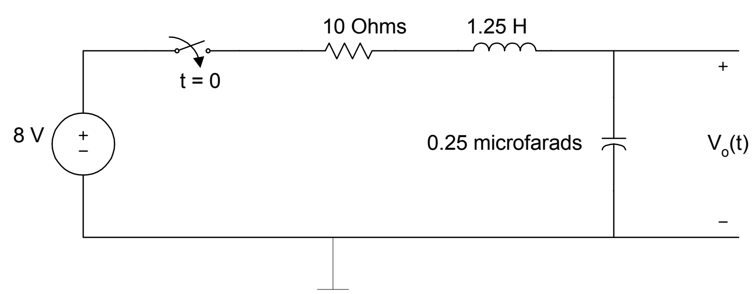


Figure P5.3 Circuit for Exercise 5.3

5.4 Use MATLAB to solve the following differential equation

$$\frac{d^3y(t)}{dt^3} + 7\frac{d^2y(t)}{dt^2} + 14\frac{dy(t)}{dt} + 12y(t) = 10$$

$x_1(t) = y(t)$

$x_2(t) = \dot{y}(t)$

$x_3(t) = \ddot{y}(t)$

$$\begin{pmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -12 & -14 & -7 \end{pmatrix} \begin{pmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{pmatrix} + \begin{pmatrix} \\ \\ -10 \end{pmatrix}$$

xdot(1) = 1 * x(2)
xdot(2) = 1 * x(3)
xdot(3) = -12 * x(1) - 14 * x(2) - 7 * x(3)
-10

with initial conditions

$$y(0) = 1, \qquad \frac{dy(0)}{dt} = 2, \qquad \frac{d^2y(0)}{dt^2} = 5$$

Plot y(t) within the intervals of 0 and 10 s.

5.5 For Figure P5.5, if $V_S = 5u(t)$, determine the voltages $V_1(t)$, $V_2(t)$, $V_3(t)$ and $V_4(t)$ between the intervals of 0 to 20 s. Assume that the initial voltage across each capacitor is zero.

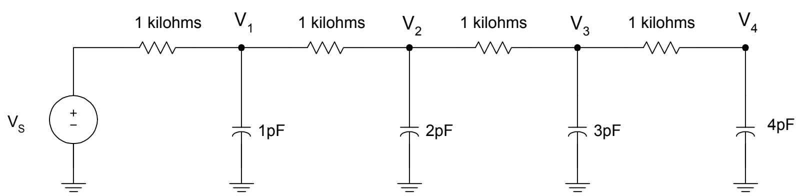


Figure P5.5 RC Network

5.6 For the differential equation

$$\frac{d^2y(t)}{dt^2} + 5\frac{dy(t)}{dt} + 6y(t) = 3\sin(t) + 7\cos(t)$$

with initial conditions $y(0) = 4$ and $\dfrac{dy(0)}{dt} = -1$

(a) Determine $y(t)$ using Laplace transforms.

(b) Use MATLAB to determine $y(t)$.

(c) Sketch $y(t)$ obtained in parts (a) and (b).

(d) Compare the results obtained in part c.

AC circuit

Voltages and currents of a network can be obtained in the time domain. This normally involves solving differential equations. By transforming the differential equations into algebraic equations using phasors or complex frequency representation, the analysis can be simplified.

Ac circuit → Differential equation → phase representation → MATLAB to solve equation → obatin the time domain response for the output

(1) Phase representation

When the voltage is purely sinusoidal, that is

$$v_2(t) = V_{m2}\cos(wt + \theta_2)$$

Is the real part of $\mathrm{Re}\left(e^{j(\omega t+\theta_2)}\right)$ then the phasor

$$V_2 = V_{m2}e^{j\theta_2} = V_{m2}\angle\theta_2 \qquad (6.19)$$

and complex frequency is purely imaginary, that is,

$$s = jw \qquad (6.20)$$

To analyze circuits with sinusoidal excitations, we convert the circuits into the s-domain with $s = jw$. Network analysis laws, theorems, and rules are

(2) Example 6.2

Step 1: Figure 6.2 RLC Circuit with Sinusoidal Excitation in Time Domain (a)

(b) Frequency Domain Equivalent

Step 2: Using nodal analysis, obtain the following equations for the circuit loops

Step 3: MATLAB solution

Step 4: the output & transfer to the time domain representation

$$\mathrm{V}_3(\mathrm{t}) = \mathrm{V}_{\mathrm{m3}}\cos(\omega t + \theta_3) = \mathrm{Re}\left(\mathrm{V}_{\mathrm{m3}}e^{j(\omega t+\theta_2)}\right)$$

```
function pt = inst_pr(t)
% inst_pr   This function is used to define
%           instantaneous power obtained by multiplying
%           sinusoidal voltage and current
it = 6*cos(120*pi*t + 30.0*pi/180);
vt = 10*cos(120*pi*t + 60*pi/180);
pt = it.*vt;
end
```

The results obtained are

```
Average power, analytical 25.980762
Average power, numerical: 25.980762
rms voltage, analytical: 7.071068
rms voltage, numerical: 7.071076
power factor, analytical: 0.866025
power factor, numerical: 0.866023
```

From the results, it can be seen that the two techniques give almost the same answers.

6.2 SINGLE- AND THREE-PHASE AC CIRCUITS

Voltages and currents of a network can be obtained in the time domain. This normally involves solving differential equations. By transforming the differential equations into algebraic equations using phasors or complex frequency representation, the analysis can be simplified. For a voltage given by

$$v(t) = V_m e^{\sigma t} \cos(wt + \theta)$$

$$v(t) = \mathrm{Re}\left[V_m e^{\sigma t} \cos(wt + \theta)\right] \tag{6.15}$$

the phasor is

$$V = V_m e^{j\theta} = V_m \angle \theta \tag{6.16}$$

and the complex frequency s is

$$s = \sigma + jw \qquad (6.17)$$

When the voltage is purely sinusoidal, that is

$$v_2(t) = V_{m2}\cos(wt + \theta_2) \qquad (6.18)$$

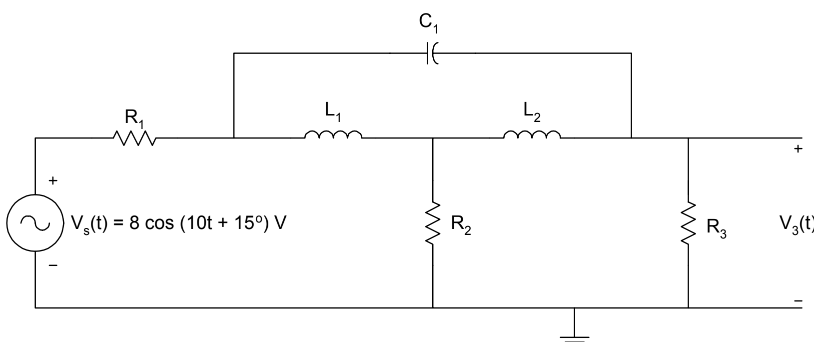
then the phasor

$$V_2 = V_{m2}e^{j\theta_2} = V_{m2}\angle\theta_2 \qquad (6.19)$$

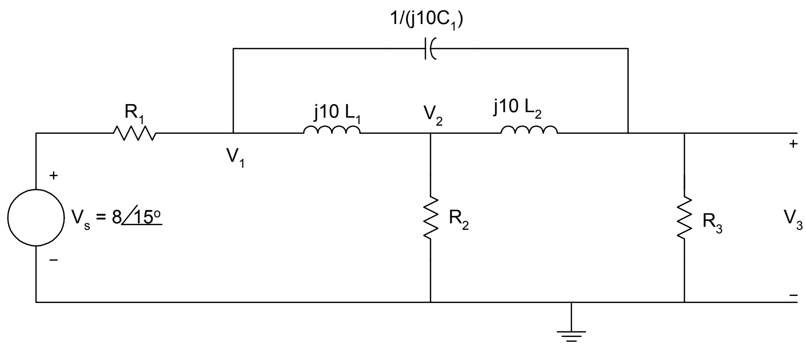
and complex frequency is purely imaginary, that is,

$$s = jw \qquad (6.20)$$

To analyze circuits with sinusoidal excitations, we convert the circuits into the s-domain with $s = jw$. Network analysis laws, theorems, and rules are used to solve for unknown currents and voltages in the frequency domain. The solution is then converted into the time domain using inverse phasor transformation. For example, Figure 6.2 shows an RLC circuit in both the time and frequency domains.



(a)



(b)

Figure 6.2 RLC Circuit with Sinusoidal Excitation (a) Time Domain (b) Frequency Domain Equivalent

If the values of R_1, R_2, R_3, L_1, L_2 and C_1 are known, the voltage V_3 can be obtained using circuit analysis tools. Suppose V_3 is

$$V_3 = V_{m3} \angle \theta_3,$$

then the time domain voltage *V_3 (t)* is

$$v_3(t) = V_{m3} \cos(wt + \theta_3)$$

The following two examples illustrate the use of MATLAB for solving one-phase circuits.

Example 6.2

In Figure 6.2, if R_1 = 20 Ω, R_2 = 100Ω, R_3 = 50 Ω, and L_1 = 4 H, L_2 = 8 H and C_1 = 250μ*F*, find $v_3(t)$ when $w = 10$ rad/s.

Solution

Using nodal analysis, we obtain the following equations.

At node 1,

$$\frac{V_1 - V_s}{R_1} + \frac{V_1 - V_2}{j10L_1} + \frac{V_1 - V_3}{1/(j10C_1)} = 0 \qquad (6.21)$$

At node 2,

$$\frac{V_2 - V_1}{j10L_1} + \frac{V_2}{R_2} + \frac{V_2 - V_3}{j10L_2} = 0 \qquad (6.22)$$

At node 3,

$$\frac{V_3}{R_3} + \frac{V_3 - V_2}{j10L_2} + \frac{V_3 - V_1}{1/(j10C_1)} = 0 \qquad (6.23)$$

Substituting the element values in the above three equations and simplifying, we get the matrix equation

$$\begin{bmatrix} 0.05 - j0.0225 & j0.025 & -j0.0025 \\ j0.025 & 0.01 - j0.0375 & j0.0125 \\ -j0.0025 & j0.0125 & 0.02 - j0.01 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 0.4\angle 15^0 \\ 0 \\ 0 \end{bmatrix}$$

The above matrix can be written as

$$[Y][V] = [I].$$

We can compute the vector [v] using the MATLAB command

$$V = inv(Y) * I$$

where

$inv(Y)$ is the inverse of the matrix $[Y]$.

A MATLAB program for solving V_3 is as follows:

MATLAB Script

```
diary ex6_2.dat
% This program computes the nodal voltage v3 of circuit Figure 6.2
```

```
% Y is the admittance matrix; % I is the current matrix
% V is the voltage vector

Y = [0.05-0.0225*j   0.025*j        -0.0025*j;
     0.025*j        0.01-0.0375*j   0.0125*j;
    -0.0025*j       0.0125*j       0.02-0.01*j];

c1 = 0.4*exp(pi*15*j/180);
I = [c1
     0
     0];  % current vector entered as column vector

V = inv(Y)*I;  % solve for nodal voltages
v3_abs = abs(V(3));
v3_ang = angle(V(3))*180/pi;

fprintf('voltage V3, magnitude: %f \n voltage V3, angle in degree: %f', v3_abs, v3_ang)
diary
```

The following results are obtained:

```
voltage V3, magnitude: 1.850409
voltage V3, angle in degree: -72.453299
```

From the MATLAB results, the time domain voltage $v_3(t)$ is

$$v_3(t) = 1.85\cos(10t - 72.45^0) \text{ V}$$

Example 6.3

For the circuit shown in Figure 6.3, find the current $i_1(t)$ and the voltage $v_C(t)$.

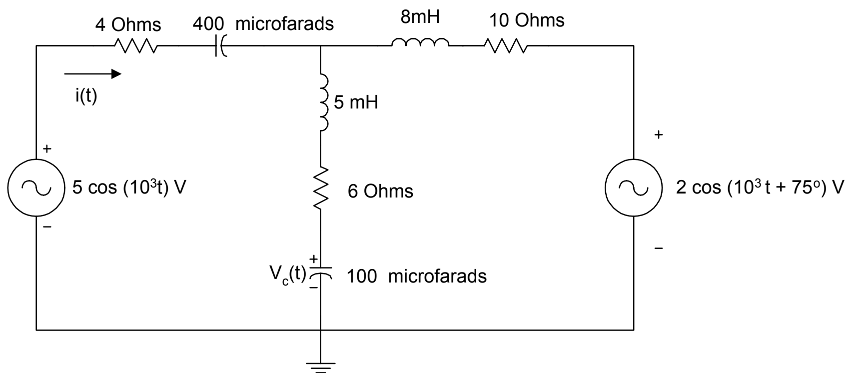


Figure 6.3 Circuit with Two Sources

Solution

Figure 6.3 is transformed into the frequency domain. The resulting circuit is shown in Figure 6.4. The impedances are in ohms.

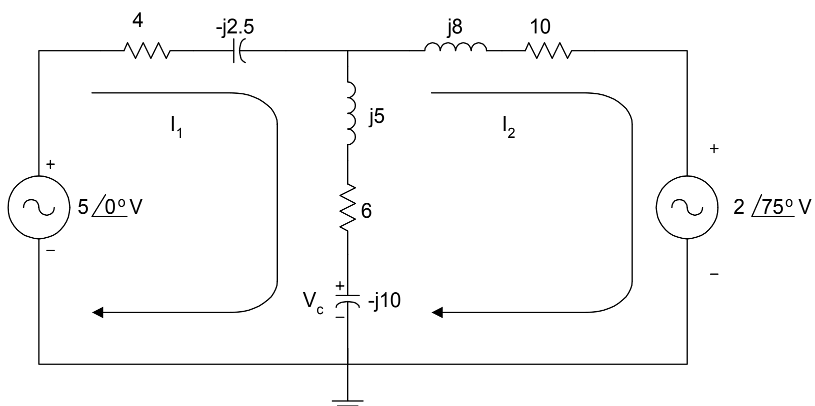


Figure 6.4 Frequency Domain Equivalent of Figure 6.3

Using loop analysis, we have

$$-5\angle 0^0 + (4 - j2.5)I_1 + (6 + j5 - j10)(I_1 - I_2) = 0 \quad (6.24)$$

$$(10 + j8)I_2 + 2\angle 75^0 + (6 + j5 - j10)(I_2 - I_1) = 0 \quad (6.25)$$

Simplifying, we have

$$(10 - j7.5)I_1 - (6 - j5)I_2 = 5\angle 0^0$$

$$-(6 - j5)I_1 + (16 + j3)I_2 = -2\angle 75^0$$

In matrix form, we obtain

$$\begin{bmatrix} 10 - j7.5 & -6 + j5 \\ -6 + j5 & 16 + j3 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 5\angle 0^0 \\ -2\angle 75^0 \end{bmatrix}$$

The above matrix equation can be rewritten as

$$[Z][I] = [V].$$

We obtain the current vector $[I]$ using the MATLAB command

$$I = inv(Z) * V$$

where $inv(Z)$ is the inverse of the matrix $[Z]$.

The voltage V_C can be obtained as

$$V_C = (-j10)(I_1 - I_2)$$

A MATLAB program for determining I_1 and V_a is as follows:

MATLAB Script

```
diary ex6_3.dat
% This programs calculates the phasor current I1 and
% phasor voltage Va.
% Z is impedance matrix
% V is voltage vector
% I is current vector

Z = [10-7.5*j   -6+5*j;
     -6+5*j   16+3*j];

b = -2*exp(j*pi*75/180);
```

```
V = [5
     b];  % voltage vector in column form

I = inv(Z)*V; % solve for loop currents
i1 = I(1);
i2 = I(2);

Vc = -10*j*(i1 - i2);
i1_abs = abs(I(1));
i1_ang = angle(I(1))*180/pi;
Vc_abs = abs(Vc);
Vc_ang = angle(Vc)*180/pi;

%results are printed
fprintf('phasor current i1, magnitude: %f \n phasor current i1, angle in
degree: %f \n', i1_abs,i1_ang)
fprintf('phasor voltage Vc, magnitude: %f \n phasor voltage Vc, angle
in degree: %f \n',Vc_abs,Vc_ang)
diary
```

The following results were obtained:

```
phasor current i1, magnitude: 0.387710
phasor current i1, angle in degree: 15.019255
phasor voltage Vc, magnitude: 4.218263
phasor voltage Vc, angle in degree: -40.861691
```

The current $i_1(t)$ is

$$i_1(t) = 0.388\cos(10^3 t + 15.02^0) \text{ A}$$

and the voltage $v_C(t)$ is

$$v_C(t) = 4.21\cos(10^3 t - 40.86^0) \text{ V}$$

Power utility companies use three-phase circuits for the generation, transmission and distribution of large blocks of electrical power. The basic structure of a three-phase system consists of a three-phase voltage source connected to a three-phase load through transformers and transmission lines. The three-phase voltage source can be wye- or delta-connected. Also the three-phase load can be delta- or wye-connected. Figure 6.5 shows a 3-phase system with wye-connected source and wye-connected load.