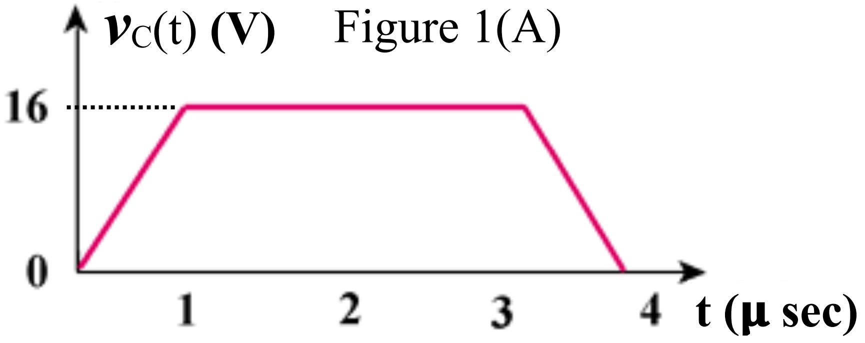
112-1
Electrical Engineering Fundamentals I

Quiz 5

Keys

1. 15% (A) The voltage $v_C(t)$ across a 10-mF capacitor is shown in Fig. 1(A), determine the current $i_C(t)$ through the capacitor.

15% (B) If the voltage $v_L(t)$ waveform in Fig. 1(B) is applied to a 10-mH inductor, find the inductor current $i_L(t)$ for $0 < t < 2$ (s). Assume $i_L(0)=0$.



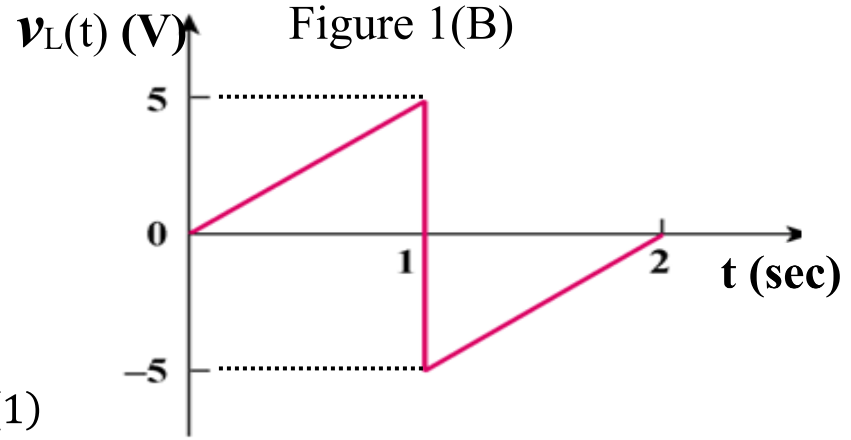
(A) C = 10 (mF)

$$v(t) = \begin{cases} 16\times10^6 \cdot t & 0 < t < 1\ \mu s \\ 16 & 1 < t < 3\ \mu s \\ 64 - 16\times10^6 \cdot t & 3 < t < 4\ \mu s \end{cases}$$

$$\frac{dv}{dt} = \begin{cases} 16x10^6, & 0 < t < 1\mu s \\ 0, & 1 < t < 3\ \mu s \\ -16x10^6, & 3 < t < 4\mu s \end{cases}$$

$$i_C(t) = C \cdot \frac{dv_C(t)}{dt} = 10\times10^{-3}\times\frac{dv_C(t)}{dt} = \begin{cases} 160(kA) & 0 < t < 1(\mu s) \\ 0 & 1 < t < 3(\mu s) \\ -160(kA) & 3 < t < 4(\mu s) \end{cases}$$

(B) L = 10 (mH)



$$i_L(t) = \frac{1}{L} \cdot \int_0^t v_L(t)dt + i_L(0)$$

For $0 < t < 1$, $v_L(t) = 5t$

$$i_L(t) = \frac{1}{10\times10^{-3}} \cdot \int_0^t 5t dt + 0$$

$$= \frac{1}{10\times10^{-3}} \cdot \times \frac{5t^2}{2}$$

$$= 250t^2\ (A)$$

For $1 < t < 2$, $v_L(t) = -10 + 5t$

$i_L(1) = 250$ (A) $= 0.25$ (kA)

$$i_L(t) = \frac{1}{10\times10^{-3}} \cdot \int_1^t (-10 + 5t)dt + i_L(1)$$

$$= \int_1^t (t/2 - 1)dt + i_L(1)\ \text{(kA)} = \left(\frac{t^2}{4} - t\right)\Big|_1^t + 0.25\ \text{(kA)} = \frac{t^2}{2} - 2t + 1\ \ (kA)$$

2. 10% (A) Express v(t) in Fig. 2(A) in terms of step functions.

10% (B) Sketch the waveform represented by $\boldsymbol{i}(t) = \boldsymbol{r}(t) - \boldsymbol{r}(t-1) - \boldsymbol{u}(t-2) - \boldsymbol{r}(t-2) + \boldsymbol{r}(t-3) + \boldsymbol{u}(t-4)$

(A)

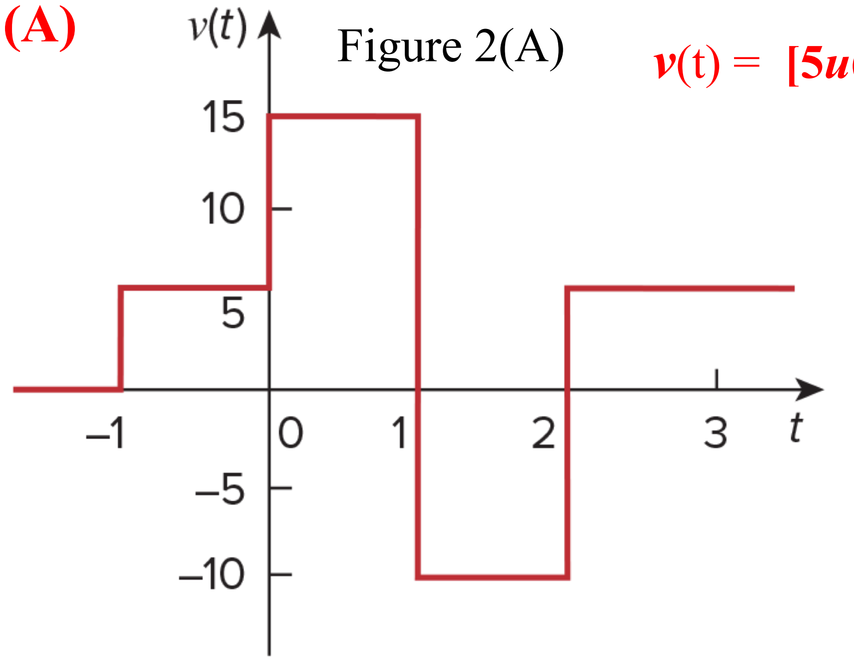
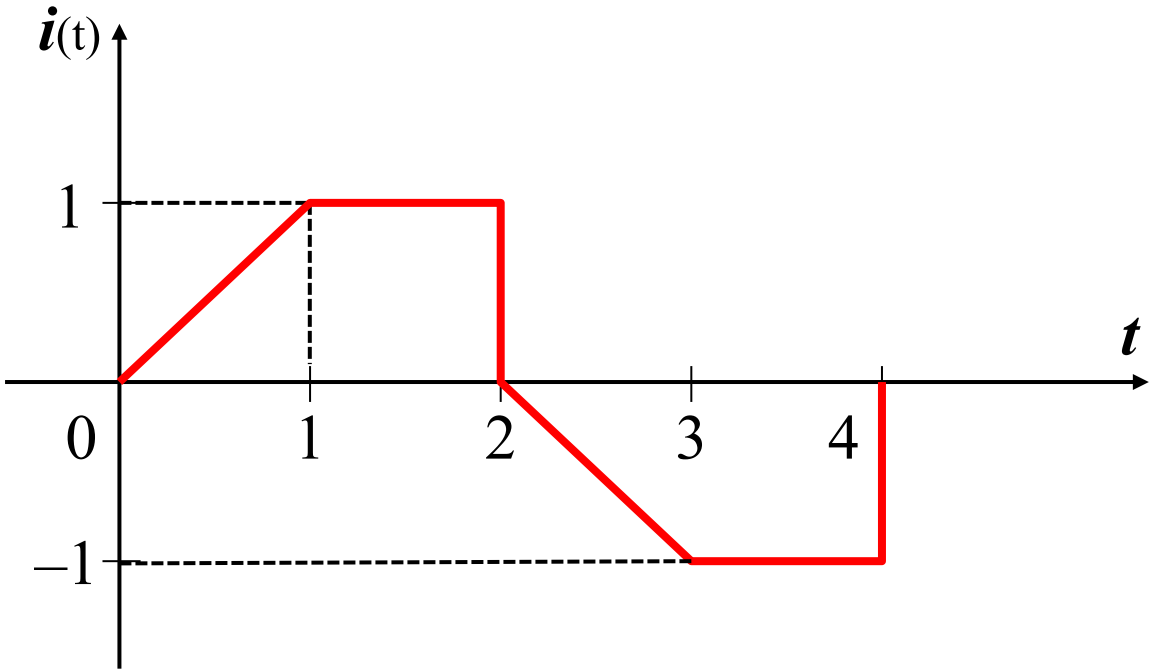


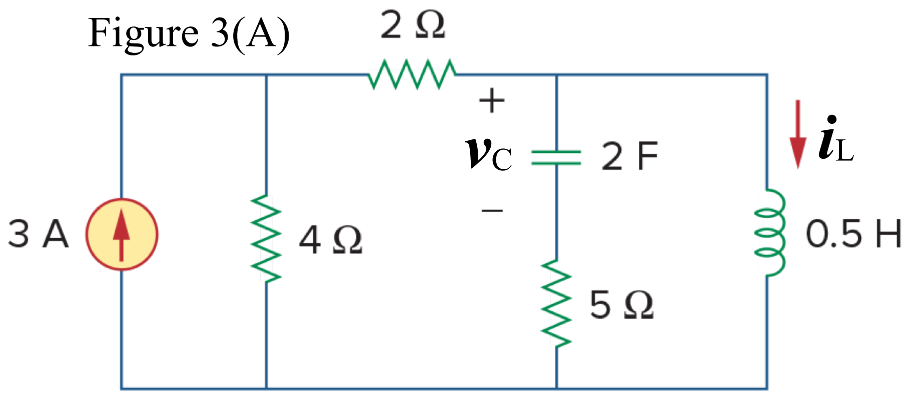
Figure 2(A)

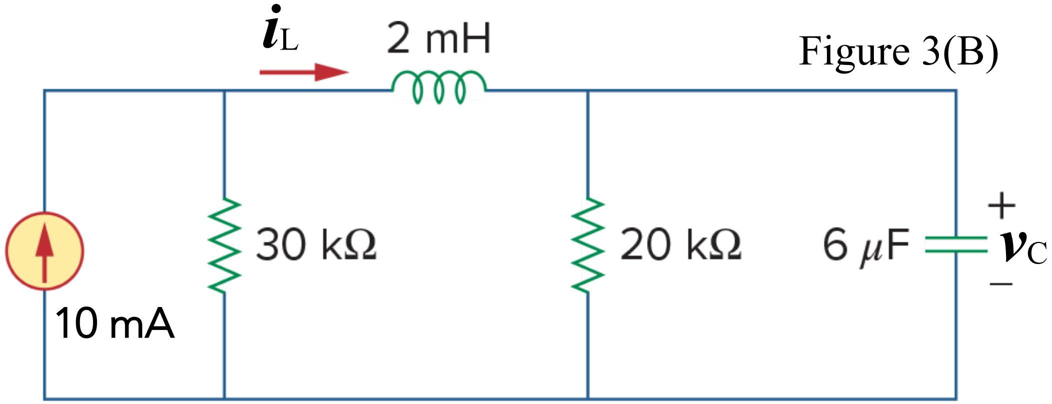
$\boldsymbol{v}(\mathrm{t}) = \mathbf{[5}\boldsymbol{u}\mathbf{(t+1) + 10}\boldsymbol{u}\mathbf{(t) - 25}\boldsymbol{u}\mathbf{(t-1) + 15}\boldsymbol{u}\mathbf{(t-2)]\ V}$

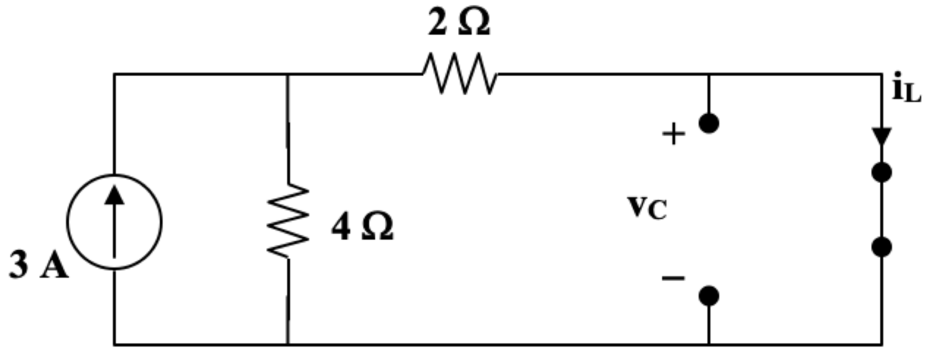
(B)

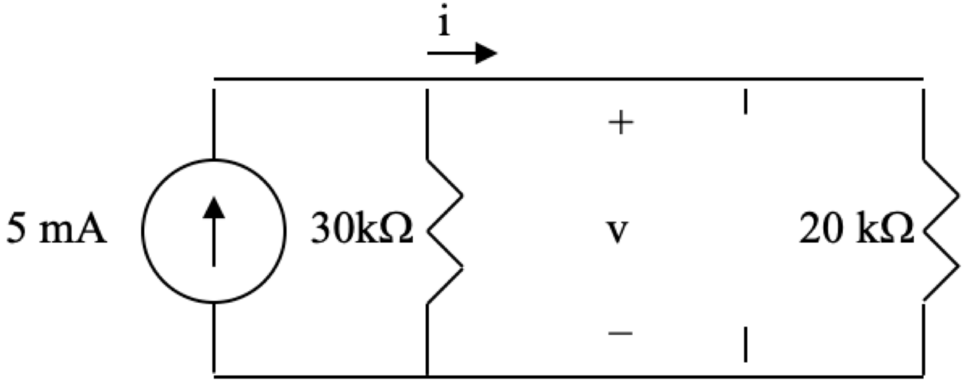


3. 30% Under steady-state dc conditions, find i_L , v_C and the energy stored in the capacitor and inductor in the circuit in Fig. 3(A) and Fig.3(B).









$$i_L = 3 \cdot \frac{4}{4+2} = 2\ (A)$$

$$v_C = 0(V)$$

$$w_L = \tfrac{1}{2} \cdot L \cdot i_L^2 = \tfrac{1}{2} \times 0.5 \cdot 2^2 = 1(J)$$

$$w_C = \tfrac{1}{2} \cdot C \cdot v_C^2 = \tfrac{1}{2} \times 2 \cdot 0^2 = 0(J)$$

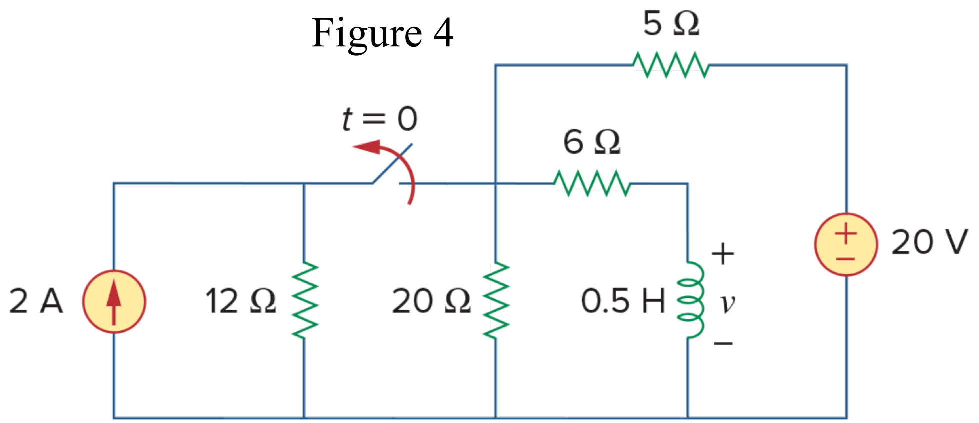
$$i_L = 10 \cdot \frac{30}{30+20} = 6\ (mA)$$

$$v_C = 6(m) \times 20(k) = 120\ (V)$$

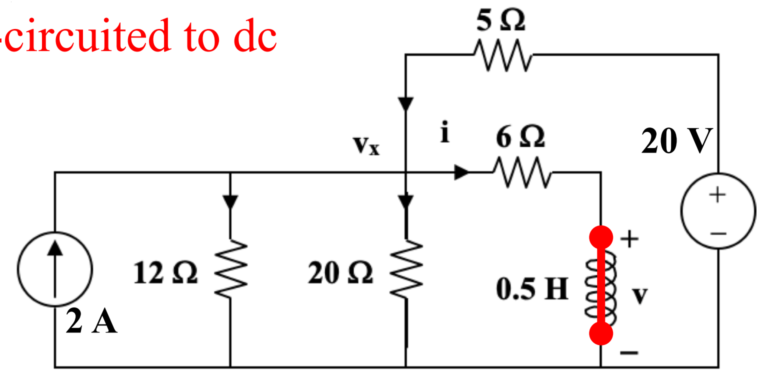
$$w_L = \tfrac{1}{2} \cdot L \cdot i_L^2 = \tfrac{1}{2} \times 2 \times 10^{-3} \times (6 \times 10^{-3})^2 = 36(nJ)$$

$$w_C = \tfrac{1}{2} \cdot C \cdot v_C^2 = \tfrac{1}{2} \times 6 \times 10^{-6} \times 120^2 = 0.0432(J)$$

4. 30% For the network shown in Fig. 4, find $\boldsymbol{v}(t)$ for $t > 0$.



$t < 0$, L short-circuited to dc



Nodal analysis:

$$2+\frac{20-v_x}{5}=\frac{v_x}{12}+\frac{v_x}{20}+\frac{v_x}{6} \longrightarrow v_x=12$$

$$i(0)=\frac{v_x}{6}=2\text{ A}$$

$t \to \infty$, L short-circuited to dc :

$$i(\infty)=\frac{20\times\frac{20\parallel 6}{5+20\parallel 6}}{6}=1.6\ (A)$$

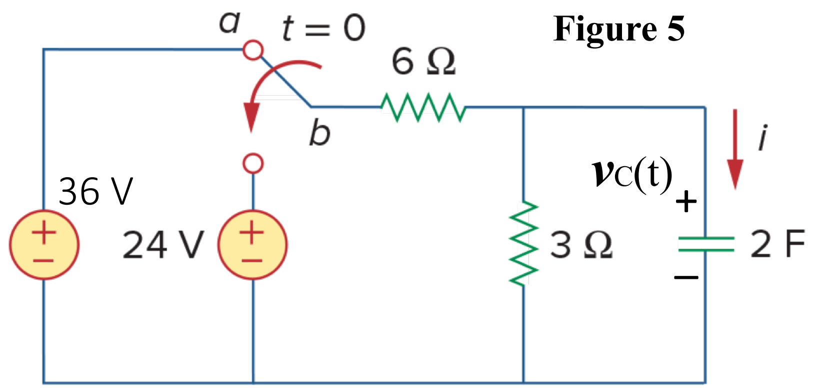
$t > 0$, switch open:

$$R_{eq}=6+20\,||\,5=10\,\Omega \qquad \tau=\frac{L}{R}=0.05$$

$$i(t)=i(\infty)+(i(0)-i(\infty))e^{-t/\tau}$$
$$=1.6+(2-1.6)e^{-\frac{t}{0.05}}$$
$$=1.6+0.4e^{-20t}$$

$$v(t)=L\cdot\frac{di(t)}{dt}$$
$$=0.5\times\frac{d(1.6+0.4e^{-20t})}{dt}$$
$$=0.5\times0.4\times(-20)\cdot e^{-20t}$$
$$=-4\cdot e^{-20t}$$

5. 30% The switch in Fig. 3 has been in position ***a*** for a long time. At t = 0, it moves to position ***b***. (A) 20% Derive v_C(t) and i(t) for all t > 0. (B) 10% Sketch the waveform of v_C(t) for t > 0, and label the time constant on the waveform.



(A) $v_C(t) = v(\infty) + [v(0) - v(\infty)] \cdot e^{-\frac{t}{\tau}}$

$t < 0$, switch at ***a***; capacitor →open to 36 V dc.

$$v_C(0) = v(0^-) = 36\times\frac{3}{6+3} = 12\ (V)$$

$$v(0) = v(0^+) = v(0^-) = 12\ (V)$$ 3%

$t > 0$, switch at ***b***; → 24 V RC circuit

$$R_{eq} = 6 /\!/ 3 = 2\Omega$$

$$\tau = R_{eq} \cdot C = 2\times 2 = 4\ (s)$$ 3%

$$v(\infty) = 24\times\frac{3}{6+3} = 8\ (V)$$ 3%

$$v_C(t) = v(\infty) + [v(0) - v(\infty)] \cdot e^{-\frac{t}{\tau}}$$

$$= 8 + (12 - 8) \cdot e^{-\frac{t}{4}}$$

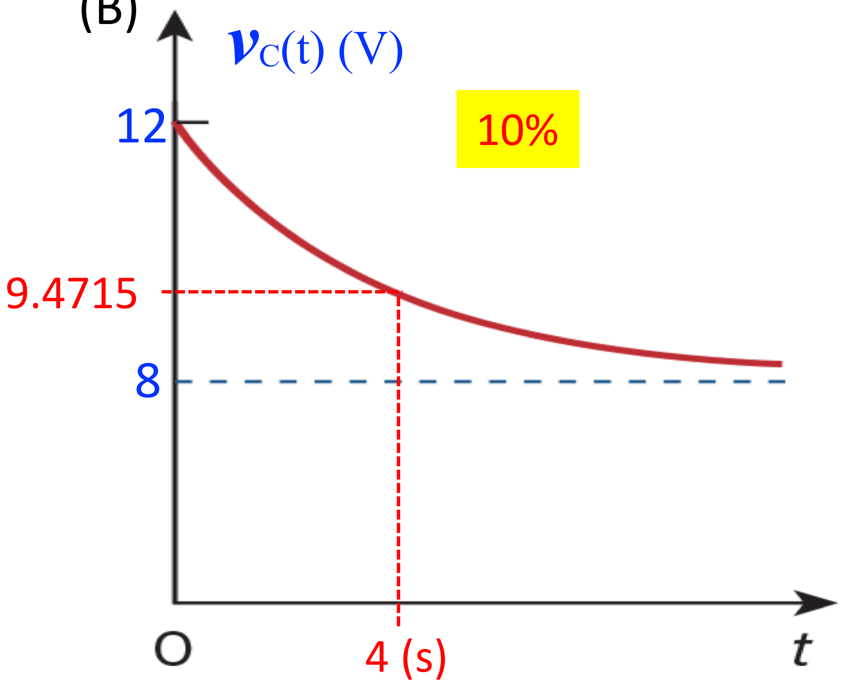
$$= 8 + 4 \cdot e^{-0.25t}\ (V)$$ 6%

$$i(t) = C\frac{dv_C(t)}{dt} = 2\times\frac{d(8 + 4 \cdot e^{-0.25t})}{dt}$$

$$= 2\times 4\times(-0.25) \cdot e^{-0.25t}$$

$$= -2 \cdot e^{-0.25t}\ (A)$$ 5%

(B)



10%