Chap 12 review

1. A body oscillates with simple harmonic motion along the x axis. Its displacement varies with time according to the equation $x = 5.0 \cos(\pi t)$. The magnitude of the acceleration (in m/s^2) of the body at $t = 1.0$ s is approximately

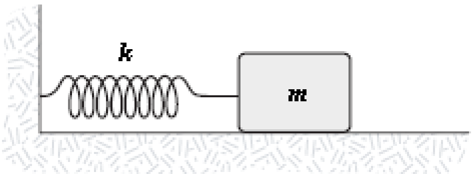
a. 3.5 b. 49 c.14 d. 43 e.4.3 ANS: B

1.B. SHM $x = 5.0 \cos \pi t \Rightarrow v = dx/dt = -5.0\pi \sin \pi t$

$\Rightarrow a = dv/dt = -5.0\pi^2 \cos \pi t$

$\therefore a(t=1) = +5\pi^2 \simeq 49$

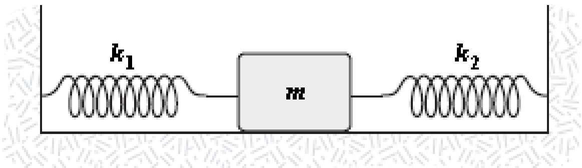
2. A mass $m = 2.0$ kg is attached to a spring having a force constant $k = 290$ N/m as in the figure. The mass is displaced from its equilibrium position and released. Its frequency of oscillation (in Hz) is approximately



a 12 b. 0.50 c.0.010 d. 1.9 e. 0.080 ANS: D

2. D. $k = 290$ N/m 2kg $\Rightarrow \omega = \sqrt{\frac{k}{m}} \Rightarrow f = \omega/2\pi = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = 1.9$ Hz

3. The mass in the figure slides on a frictionless surface. If $m = 2$ kg, $k_1 = 800$ N/m and $k_2 = 500$ N/m, the frequency of oscillation (in Hz) is approximately



a 6 b. 2 c. 4 d. 8 e. 10 ANS: C

3. C. one compress one extend. both restoring force point toward left

$k_1 = 800$ $k_2 = 500$

$\Rightarrow F = -k_1 x - k_2 x = -(k_1 + k_2)x = -Kx$

$\Rightarrow f = \frac{1}{2\pi}\sqrt{\frac{K}{m}} = \frac{1}{2\pi}\sqrt{\frac{800+500}{2}} = 4$ Hz

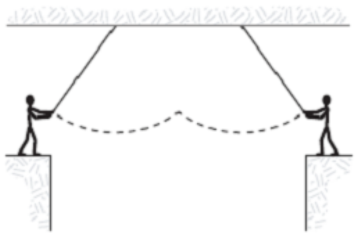
4. The oscillation of the 2.0-kg mass on a spring is described by $x = 3.0\cos(4.0t + 0.80)$ where x is in centimeters and t is in seconds. What is the force constant of the spring?

a. 4.0 N/m b. 0.80 N/m c. 16 N/m d. 32 N/m e. 2.0π N/m ANS: D

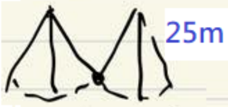
4. D. $x = 3.0\cos(4t + 0.80)$

$\omega = 4 \Rightarrow \omega^2 = 16 = \frac{k}{m} \Rightarrow k = 16m = 32.$

5. Two circus clowns (each having a mass of 50 kg) swing on two flying trapezes (negligible mass, length 25 m) shown in the figure. At the peak of the swing, one grabs the other, and the two swing back to one platform. The time for the forward and return motion is



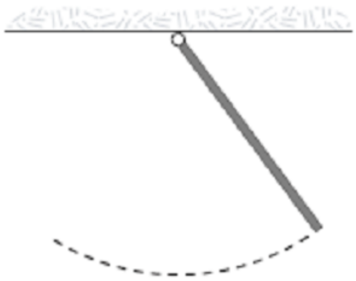
a. 10 s ANS: A
b. 50 s
c. 15 s
d. 20 s
e. 25 s



simple pendulum $T = 2\pi\sqrt{L/g}$ $\boxed{\text{indep of mass}}$

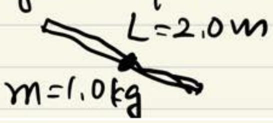
$\therefore T = 2\pi\sqrt{25/9.8} = 10s.$

6. A uniform rod (mass m = 1.0 kg and length L = 2.0 m) pivoted at one end oscillates in a vertical plane as shown below. The period of oscillation (in s) is approximately



a. 4.0 b. 1.6 c. 3.2 d. 2.3 e. 2.0 ANS: D

6. D. physical pendulum:



$$T = 2\pi\sqrt{\frac{I}{mgd}} = 2\pi\sqrt{\frac{\frac{1}{3}\cancel{m}L^2}{\cancel{m}g\,L/2}} = 2\pi\sqrt{\frac{2L}{3g}} = 2.3.$$

7. A damped oscillator is released from rest with an initial displacement of 10.00 cm. At the end of the first complete oscillation the displacement reaches 9.05 cm. When 4 more oscillations are completed, what is the displacement reached? ANS: C

a. 7.41 cm
b. 6.71 cm
c. 6.07 cm
d. 5.49 cm
e. 5.25 cm

7. C. damped oscillation

$x(t) = Ae^{-(b/2m)t}\cos(\omega t + \varphi)$. $A = 10$

$\Rightarrow 9.5 = 10\,e^{-(b/2m)T} \Rightarrow e^{-\frac{b}{2m}T} = \frac{9.5}{10} = 0.95$

also for $t = 5T$. $Ae^{-(b/2m)5T} = 10(e^{-\frac{b}{2m}T})^5 = 10 \times 0.95^5 = 6.07$

8. A mass-spring system is oscillating with amplitude A. The kinetic energy will equal the potential energy only when the displacement is

A. 0 B. $\pm A/4$ C. $\pm A/\sqrt{2}$ D. $\pm A/2$

E. anywhere between $-A$ and $+A$ ANS: C

8. C. total energy $E = \frac{1}{2}kA^2 = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = 2\times\frac{1}{2}kx^2 = kx^2$

$\Rightarrow x^2 = A^2/2 \Rightarrow x = \pm A/\sqrt{2}$

9. A particle is in simple harmonic motion along the x axis. The amplitude of the motion is x_m. At one point in its motion its kinetic energy is $K = 5$ J and its potential energy (measured with $U = 0$ at $x = 0$) is $U = 3$ J. When it is at $x = x_m$, the kinetic and potential energies are:

A. $K = 5$ J and $U = 3$ J B. $K = 5$ J and $U = -3$ J C. $K = 8$ J and $U = 0$ J

D. $K = 0$ J and $U = 8$ J E. $K = 0$ J and $U = -8$ J ANS: D

9. D. total energy $E = K + U = 5 + 3 = 8$J

at $x = x_m$. oscillator stops $\Rightarrow K = 0$. $U = 8$J

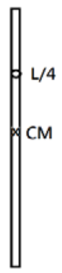
10. At sea level, at a latitude where $g = 9.80\,\frac{\text{m}}{\text{s}^2}$, a pendulum that takes 2.00 s for a complete swing back and forth has a length of 0.993 m. What is the value of g in m/s^2 at a location where the length of such a pendulum is 0.970 m?

A. 0.0983 B. 3.05 C. 9.57 D. 10.0 E. 38.3 ANS: C

10. C. $T = 2\pi\sqrt{L/g}$ ∴ $2 = 2\pi\sqrt{0.970/g} \Rightarrow \frac{1}{\pi^2} = \frac{0.970}{g}$

∴ $g = \pi^2 \times 0.970 = 9.57$

11. The rotational inertia of a uniform thin rod about its center of mass is $ML^2/12$, where M is the mass and L is the length. Such a rod is hung vertically at $L/4$ from one end and set into small amplitude oscillation. Find the angular frequency.



A. $\sqrt{\frac{g}{L}}$ B. $\sqrt{\frac{3g}{2L}}$ C. $\sqrt{\frac{12g}{7L}}$ D. $\sqrt{\frac{2g}{3L}}$ E. $\sqrt{\frac{7g}{12L}}$

ANS: E

11. E. according to $\parallel$-axis Thm $I_{L/4} = M(L/4)^2 + \frac{1}{12}ML^2 = \frac{7}{48}ML^2$

∡ freq $\omega = \sqrt{Mgd/I} = \sqrt{\frac{MgL/4}{7/48\,ML^2}} = \sqrt{\frac{12g}{7L}}$

12. The position of an harmonic oscillator is given as $x(t) = x_m \cos(\omega t)$. If starts at $t = 0$, how long it take the oscillator to get to the position $x = \frac{x_m}{2}$ for the first time?

Denoting T as the period of the oscillation.

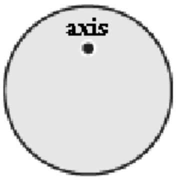
A. $T/6$ B. $T/4$ C. $T/3$ D. $T/2$ E. $2T/3$ ANS: A

12. A. $x(t) = x_m \cos\omega t \Rightarrow x(0) = x_m$

$\Rightarrow x(t) = \frac{\cancel{x_m}}{2} = \cancel{x_m} \cos(\frac{2\pi}{T} t) \Rightarrow \cos\frac{2\pi}{T}t = \frac{1}{2} \Rightarrow \frac{2\cancel{\pi}t}{T} = \frac{\cancel{\pi}}{3}$

$\Rightarrow t = T/6$

13. In the figure below, a disk (radius $R = 1.0$ m, mass = 2.0 kg) is suspended from a pivot a distance $d = 0.25$ m above its center of mass. For a circular disk, $I_{cm} = \frac{1}{2}mR^2$. The angular frequency (in rad/s) for small oscillations is approximately



a. 4.2
b. 2.1
c. 1.5
d. 1.0
e. 3.8

ANS: B

13. B. from // -axis Thm.

$I = M(\frac{1}{4})^2 + I_{cM} = 2\times(\frac{1}{4})^2 + \frac{1}{2}\times 2\times 1^2 = 9/8$

$\Rightarrow \omega = \sqrt{mgd/I} = \sqrt{\frac{2\times 9.8\times 1/4}{9/8}} = 2.1$