EXP.5 Mass Point Balance and Rigid Body Balance

1. Section Purpose

Draw vector graphs and use vector analysis to divide the group into concurrent forces and non-concurrent forces so as to probe into what are the requirements to be met in order to be balanced when several concurrent forces and non-concurrent forces on a surface on a shared surface are in play.

2. Theory

(1) Vector

Some physical quantities can be clearly specified by quantities, such as length, duration, temperature, called pure quantity or scalar, while others need to be specified matters other than just quantities such as electric fields in which directions play a major role, and we call these invisible forces of directions "vector". Normally, vectors will be illustrated as Illustration 1 below, and the length of the line represents its intensity and the arrow symbolizes the direction of it. There are 2 methods to add or subtract vectors, known as graphing method as well as analytic reasoning.

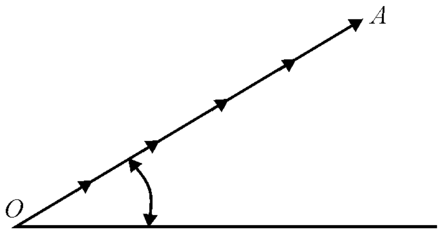


Illustration 1

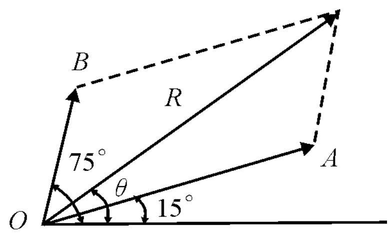


Illustration 2

1. Graphing Method

As demonstrated in Illustration 2, when two vectors, ***OA*** and ***OB*** are added together, make a line parallel to OB from point A, and make another line parallel to OA from point B, which allows to constitute a parallelogram. Its vectors along with Line R are called Parallelogram Law. When we add up together more than 2 vectors, as the same method shown above, we can make a graph as demonstrated in Illustration 3, which is also named "Polygon Method".

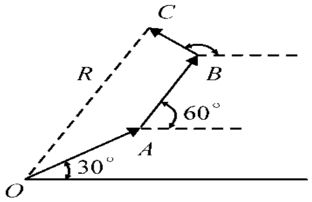


Illustration 3

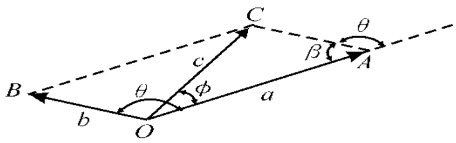


Illustration 4

2. Analytic Reasoning

(1) Approach 1

You may find it seemingly impossible to solve vector problems by using graphing method, sometimes. In such a situation, we can acquire the direction of the resultant vector by applying law of sine, and apply law of cosine to know the quantity of the resultant vector. As shown in Illustration 4, the resultant vector of vector OA and OB is OC. If the quantities of these three vectors are respectively a, b and c, the included angle is θ, and β will be the supplemental angle of θ. According to law of cosine, the resultant's quantity will be

$$c^2 = a^2 + b^2 - 2ab\cos\beta \tag{1}$$

or

$$c^2 = a^2 + b^2 + 2ab\cos\theta \tag{2}$$

its direction can be gained via law of sine

$$\frac{\sin\phi}{\sin\beta} = \frac{b}{c} \tag{3}$$

,attributing to sin β=sin θ, thereby,

$$\sin\phi = \frac{b}{c}\sin\theta \tag{4}$$

and thus c and ϕ are acquired.

(2) Approach 2

The other method is to divide a vector into two separated components of a Cartesian coordinate. As shown in Illustration 5, $R_x = R\cos\theta$ and $R_y = R\sin\theta$ ◦ When more than 2 vectors are put together, adding up each vector components will do. As shown in Illustration 6,

$$R_x = A_x + B_x + C_x \qquad (B_x，C_x, negatives) \qquad (5)$$

$$R_y = A_y + B_y + C_y \qquad (C_y, negative) \qquad (6)$$

Thus, in Illustration 5,

$$R = \sqrt{R_x^2 + R_y^2} \qquad (7)$$

$$\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right) \qquad (8)$$

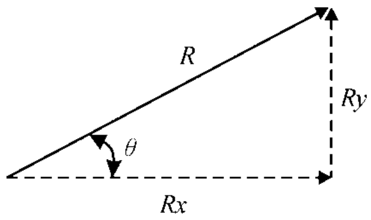


Illustration 5

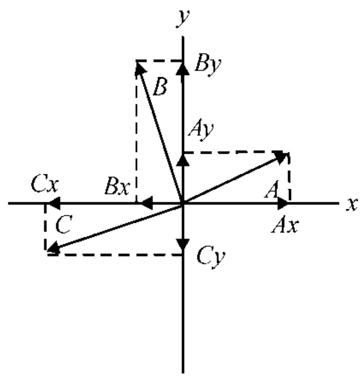


Illustration 6

(2) Requirements to be balanced

When the linear acceleration and angle acceleration are both 0, we call the object staying balanced.

1. Balance of Concurrent Forces

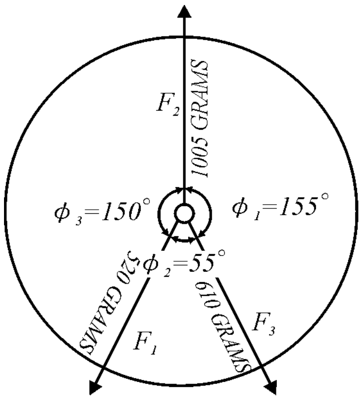
From Newton’s second law of motion,

$$\Sigma \boldsymbol{F}_{\text{ext}} = m\mathbf{a}_{\text{cm}} \qquad (9)$$

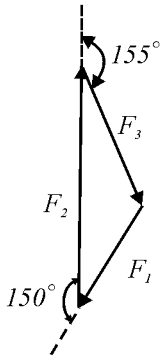
is the linear acceleration of the rigid body's barycenter, while $\Sigma \boldsymbol{F}_{\text{ext}}$ is the resultant forces of the foreign force. The only requirement for a concurrent force to make an object balanced is $\Sigma \boldsymbol{F}_{\text{ext}} = 0$. If three different forces are in play in a concurrent point and reach its balance, it will be exactly the same as Illustration 7 below. Analyze it by applying graphing method and the result is indicated in Illustration 8 below, while the analytic reasoning is presented in Illustration 9. Therefore, we set $\boldsymbol{F}_4=\boldsymbol{F}_1+\boldsymbol{F}_2$. And based on law of cosine, we can infer that

And based on law of sine, the following equation is gained

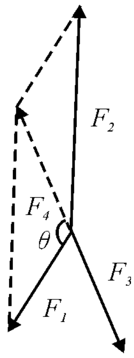
from which we know that xxx and aaa can be neutralized. Generally speaking, we regard the balance of a concurrent force as mass point balance.



圖七



圖八



圖九

$$F_4^2 = 520^2 + 1005^2 + 2\times 520\times 1005\times \cos 150^\circ \tag{10}$$

And based on law of sine, the following equation is gained

$$\sin\phi = \frac{1005}{F_4}\sin 150^\circ \tag{11}$$

, from which we know that F_4=612.6 gw and ϕ=124.9° can be neutralized by each other. Generally speaking, we regard the balance of a concurrent force as mass point balance.

2. Balance of a Non-concurrent Force

If a force on a plane of constraint is in play at different spots, then even though the combined vector of each forces is zero, with linear acceleration also being zero, angle acceleration can be still detected on the plane of constraint, meaning that a rotation will occur. 2 major factors contribute to the rotation occurring on an axis, and they are the arm of force and forces. The vertical distance from the axis and the line of force is called an arm of force or A moment (torque) arm, symbolized as r. The exterior product of an arm of force and a certain force is referred to as torque or torsional moment, which is as well considered a vector, and it's definition is as below.

$$\tau = \mathbf{r} \times \mathbf{F} \tag{12}$$

When the concurrent torque or moment is zero to any axis, that means the object is lack of angle acceleration, namely, the object possesses rotational balance. When the sum of torques are calculated, a torque making an object spin counterclockwise is called a positive torque and when it is the other way around, it is called a negative torque. The point which is tangent with an axis is usually thought of as the center of a torque. If an object shows no acceleration on an axis of a vertical plane of constraint, it won't cause any acceleration on any axis for a vertical plane of constraint. Shown in Illustration 10 are 3 forces in play at spot A, B and C. If it is analyzed with graphing method, the concurrent force of the 3 is zero as shown in Illustration 11. If we measure the vertical distance of any forces in play at any torque center with rules set with our right hand (with four fingers stretching outward and the thumb pointing upward, this position represents positive, namely, it is turned counterclockwise, and if it is done in the other way around, it is negative). Shown in Illustration 12 is an example, P, when turned counterclockwise, the $\boldsymbol{F}_1$ and $\boldsymbol{F}_2$ are its action, and the total sum of moment (torque) is

$$19.9 \times 1040 + 8.1 \times 600 = 25556\,(gw \cdot cm) \tag{13}$$

$\boldsymbol{F}_3$ is the force in action when it is turned clockwise.

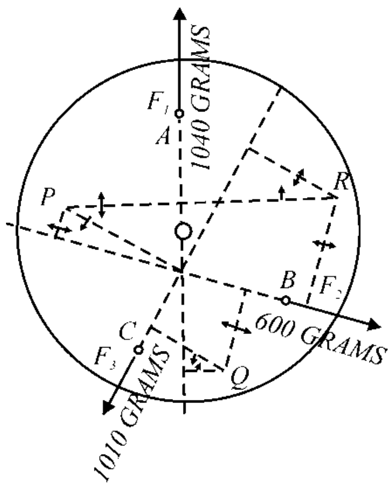
$$-25.3 \times 1010 = -25553\,(gw \cdot cm) \tag{14}$$

If we don't take into account the error in the experiment, the total torque will be zero; namely, it represents the balanced status of rotation. From the following illustrations, we can discover

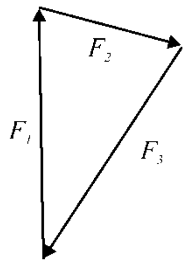
$$\sum \boldsymbol{F} = \boldsymbol{F}_1 + \boldsymbol{F}_2 + \boldsymbol{F}_3 = 0 \tag{15}$$

$$\sum \tau = \boldsymbol{r}_1 \times \boldsymbol{F}_1 + \boldsymbol{r}_2 \times \boldsymbol{F}_2 + \boldsymbol{r}_3 \times \boldsymbol{F}_3 = 0 \tag{16}$$

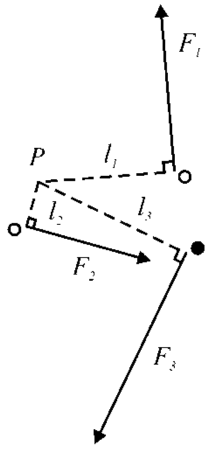
The balance of different forces on a plane of constraint is exactly the balance of a rigid body.



圖十



圖十一



圖十二

3. Instruments and Materials Required

force table x 1/ round acrylic plate x 1/ sliding wheel x 3/ bubble level x 1/ a weight set kit (should contain 10g, 50g, 20g copper weight x 3/ 10g, 5g aluminum weights x 3/ weights-hoist x 3/ steel beads x 3/ closures x 3/ ring x 3), several cotton threads/ protractor x1/ electo-scale x1

4. Steps to the experiment

(1) 3 Forces in Action at 1 Spot

1. Put the bubble level on a force table and adjust the legs of it until the bubbles in the level are positioned in its center and then change the direction of the level again, if you find that the bubbles are not at the level's center, turn the level adjusting knob to make the bubbles get back to the center until the bubbles in the level stay at the center no matter which direction you put it. Now, the force table's level is horizontally secured.
2. Place three sliding wheels around the force table with each of them not being vertical to one another.
3. Connect the 3 weights-hoist with a cotton thread and make them go across the sliding wheels and attach them onto the ring, and make the central column go through the ring as demonstrated in Illustration 13.
4. Hang weights of different masses onto the weights-hoist and fix two of them in place and only change the wheel position above the third weights-hoist, and add or reduce the weight

hung on the third weights-hoist to make the central column positioned at the middle of the ring.

5. Measure and record the mass of the weights, hoists included and then also measure the included angles of the sliding wheels placed on the force table. Next, draw a circle as the plan surface of the force table and draw the position of each cotton threads and directions of each hoists and specify the quantities of each forces in action. If the mass of the weight is measured by an electro-scale, do remember to gently put the weights to be measured onto it. Carelessly dropping weights onto the scale will harm the accuracy of it.
6. Explain mass point balance with vector graphing method.
7. Explain mass point balance with vector analytic reasoning method.

(2) 3 Forces in Action at Different Spots

1. Leave three steel beads on the force table to roughly form a regular triangle, with the round acrylic plate’s smooth side down, and then pierce a hole in the middle to put it through the central column until it is evenly placed on the steel beads.
2. Lay a sheet of paper with smooth surface onto the round acrylic plate.
3. Insert the smooth-surfaced sheet into the 3 closures prepared with
4. Attach the closures with cotton threads and respectively connect the sliding wheels placed on the force table onto the weights-hoist. As shown in Illustration 14, the directions of each thread shall not go beyond the center of the force table.
5. Hang weights of different masses onto the weights-hoists and fix two of them in place and only change the position of the wheel above the third weights-hoist, and add or reduce the weight hung on the third weights-hoist to make the central column positioned at the middle of the hole at the center of the round acryl tic plate..
6. To reduce friction, please gently push the acrylic round plate to make it roll for a bit after adding weights and make sure that the central column is at the center of the plate. If you discover that the position has deviated, please adjust it to make it positioned at its right place.
7. Trace the position of each cotton thread on a sheet of paper and use “arrows” to specify the directions of weights’ positions.
8. Record the masses of each weights-hoists and weights and draw a circle as the force table surface, and then draw the positions of closures and cotton threads as well as the directions of weights-hoists. At last, specify the quantities of all the forces in action.
9. Pick three random spots as the centers of torques on a force table and specify their positions in step 8.

10. Apply vector analytic reasoning method to these three spots to explain the requirements to reach rigid body balance.

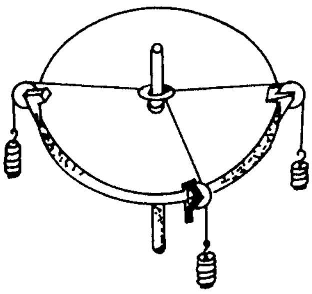


Illustration 13

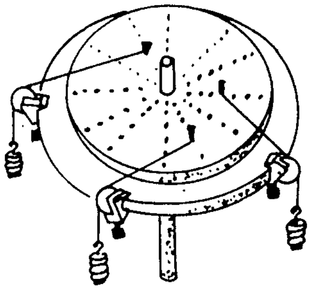


Illustration 14

5. Questions:

(1) If the force table is not horizontally secured, how will it affect the experiment?

(2) If we force two different forces into a straight line in the experiment, how will it affect the result?

(3) If the central column is not positioned at the center of the ring or the central opening on the round acrylic plate, how will it affect the experiment and what kinds of error will be brought into the experiment?