6. Q|C Figure P21.6 represents a section of a conductor of nonuniform diameter carrying a current of $I = 5.00$ A. The radius of cross-section A_1 is $r_1 = 0.400$ cm. (a) What is the magnitude of the current density across A_1? The radius r_2 at A_2 is larger than the radius r_1 at A_1. (b) Is the current at A_2 larger, smaller, or the same? (c) Is the current density at A_2 larger, smaller, or the same? Assume $A_2 = 4A_1$. Specify the (d) radius, (e) current, and (f) current density at A_2.

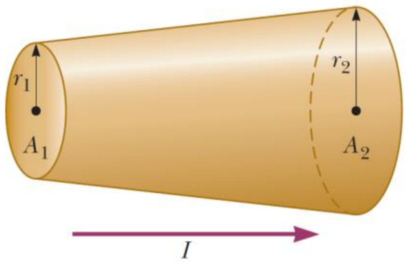


Figure P21.6

P21.6 (a) $J = \dfrac{I}{A} = \dfrac{5.00 \text{ A}}{\pi\left(4.00\times10^{-3}\text{ m}\right)^2} = \boxed{99.5 \text{ kA/m}^2}$

(b) $\boxed{\text{Current is the same.}}$

(c) The cross-sectional area is greater; therefore $\boxed{\text{the current density is smaller.}}$

(d) $A_2 = 4A_1$ or $\pi r_2^2 = 4\pi r_1^2$ so $r_2 = 2r_1 = \boxed{0.800 \text{ cm}}$.

(e) $\boxed{I = 5.00 \text{ A}}$

(f) $J_2 = \dfrac{1}{4}J_1 = \dfrac{1}{4}\left(9.95\times10^4 \text{ A/m}^2\right) = \boxed{2.49\times10^4 \text{ A/m}^2}$

9. An aluminum wire with a diameter of 0.100 mm has a uniform electric field of 0.200 V/m imposed along its entire length. The temperature of the wire is 50.0°C. Assume one free electron per atom. (a) Use the information in Table 21.1 to determine the resistivity of aluminum at this temperature. (b) What is the current density in the wire? (c) What is the total current in the wire? (d) What is the drift speed of the conduction electrons? (e) What potential difference must exist between the ends of a 2.110-m length of the wire to produce the stated electric field?

P21.9 (a) $\rho = \rho_0\left[1+\alpha\left(T-T_0\right)\right]$

$$\rho = \left(2.82\times10^{-8}\ \Omega\cdot\text{m}\right)\left[1+\left(3.9\times10^{-3}\ {}^\circ\text{C}^{-1}\right)\left(30.0^\circ\text{C}\right)\right]$$

$$= 3.15\times10^{-8}\ \Omega\cdot\text{m} = \boxed{31.5\,\text{n}\Omega\cdot\text{m}}$$

(b) From Equation 21.17, $I = \frac{E}{\rho}A$. Then,

$$J = \frac{I}{A} = \frac{EA/\rho}{A} = \frac{E}{\rho} = \frac{0.200\ \text{V/m}}{3.15\times10^{-8}\ \Omega\cdot\text{m}}$$
$$= 6.35\times10^{6}\ \text{A/m}^2 = \boxed{6.35\ \text{MA/m}^2}$$

(c)
$$I = JA = J\left(\frac{\pi d^2}{4}\right) = \left(6.35\times10^{6}\ \text{A/m}^2\right)\left[\frac{\pi\left(1.00\times10^{-4}\ \text{m}\right)^2}{4}\right]$$
$$= \boxed{49.9\ \text{mA}}$$

(d) $$n = \frac{6.02\times10^{23}\ \text{electrons}}{\left[26.98\ \text{g}/\left(2.70\times10^{6}\ \text{g/m}^3\right)\right]} = 6.02\times10^{28}\ \text{electrons/m}^3$$

$$v_d = \frac{J}{ne} = \frac{6.35\times10^{6}\ \text{A/m}^2}{\left(6.02\times10^{28}\ \text{electrons/m}^3\right)\left(1.60\times10^{-19}\ \text{C}\right)} = \boxed{659\ \mu\text{m/s}}$$

(e) $\Delta V = E\ell = (0.200\ \text{V/m})(2.00\ \text{m}) = \boxed{0.400\ \text{V}}$

14. A straight, cylindrical wire lying along the x axis has a length of 0.500 m and a diameter of 0.200 mm. It is made of a material described by Ohm's law with a resistivity of $\rho = 4.00\times10^{-8}\ \Omega\cdot\text{m}$. Assume a potential of 4.00 V is maintained at the left end of the wire at $x = 0$. Also assume $V = 0$ at $x = 0.500$ m. Find (a) the magnitude and direction of the electric field in the wire, (b) the resistance of the wire, (c) the magnitude and direction of the electric current in the wire, and (d) the current density in the wire. (e) Show that $E = \rho J$.

P21.14 (a) Assuming the change in V is uniform:

$$E_x = -\frac{dV(x)}{dx} \to E_x = -\frac{\Delta V}{\Delta x} = -\frac{(0-4.00\ \text{V})}{(0.500\ \text{m}-0)} = +8.00\ \text{V/m}$$

or $\boxed{8.00\ \text{V/m in the positive } x \text{ direction.}}$

(b) $$R = \frac{\rho\ell}{A} = \frac{\left(4.00\times10^{-8}\ \Omega\cdot\text{m}\right)(0.500\ \text{m})}{\pi\left(1.00\times10^{-4}\ \text{m}\right)^2} = \boxed{0.637\ \Omega}$$

(c) $I = \dfrac{\Delta V}{R} = \dfrac{4.00\text{ V}}{0.637\ \Omega} = \boxed{6.28\text{ A}}$

(d) $J = \dfrac{I}{A} = \dfrac{6.28\text{ A}}{\pi\left(1.00\times 10^{-4}\text{ m}\right)^2} = 2.00\times 10^{8}\text{ A/m}^2 = \boxed{200\text{ MA/m}^2}$

The field and the current are both in the x direction.

(e) $J = \dfrac{I}{A} = \dfrac{EA/\rho}{A} = \dfrac{E}{\rho}$, so

$$E = \rho J = \left(4.00\times 10^{-8}\ \Omega\cdot\text{m}\right)\left(2.00\times 10^{8}\text{ A/m}^2\right) = 8.00\text{ V/m}$$

27. Assuming the cost of energy from the electric company is \$0.110/kWh, compute the cost per day of operating a lamp that draws a current of 1.70 A from a 110-V line.

P21.27 $P = I(\Delta V) = (1.70\text{ A})(110\text{ V}) = 187\text{ W}$

Energy used in a 24-hour day = (0.187 kW)(24.0 h) = 4.49 kWh.

Therefore daily $\text{cost} = \dfrac{4.49\text{ kWh}}{\text{day}}\left(\dfrac{\$0.110\,0}{\text{kWh}}\right) = \boxed{\$0.494/\text{day}}$.

31. An all-electric car (not a hybrid) is designed to run from a bank of 12.0-V batteries with total energy storage of 2.00×10^{7} J. If the electric motor draws 8.00 kW as the car moves at a steady speed of 20.0 m/s, (a) what is the current delivered to the motor? (b) How far can the car travel before it is "out of juice"?

P21.31 (a) $P = I\Delta V$

$$I = \frac{P}{\Delta V} = \frac{8.00\times 10^{3}\text{ W}}{12.0\text{ V}} = \boxed{667\text{ A}}$$

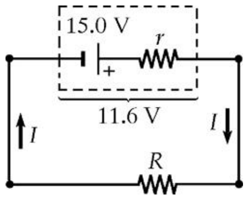
(b) $\Delta t = \dfrac{\Delta U}{P} = \dfrac{2.00\times 10^{7}\text{ J}}{8.00\times 10^{3}\text{ W}} = 2.50\times 10^{3}\text{ s}$

$$\Delta x = v\Delta t = (20.0\text{ m/s})\left(2.50\times 10^{3}\text{ s}\right) = \boxed{50.0\text{ km}}$$

33. A battery has an emf of 15.0 V. The terminal voltage of the battery is 11.6 V when it is delivering 20.0 W of power to an external load resistor R. (a) What is the value of R? (b) What is the internal resistance of the battery?

P21.33 (a) Combining Joule's law, $P = I\Delta V$, and the definition of resistance, $\Delta V = IR$, gives

$$R = \frac{(\Delta V)^2}{P} = \frac{(11.6\text{ V})^2}{20.0\text{ W}} = \boxed{6.73\ \Omega}$$



ANS. FIG. P21.33

(b) The electromotive force of the battery must equal the voltage drops across the resistances: $\mathcal{E} = IR + Ir$, where $I = \Delta V/R$.

$$r = \frac{(\mathcal{E} - IR)}{I} = \frac{(\mathcal{E} - \Delta V)R}{\Delta V}$$

$$= \frac{(15.0\text{ V} - 11.6\text{ V})(6.73\ \Omega)}{11.6\text{ V}} = \boxed{1.97\ \Omega}$$

36. **BIO** For the purpose of measuring the electric resistance of shoes through the body of the wearer standing on a metal ground plate, the American National Standards Institute (ANSI) specifies the circuit shown in Figure P21.36. The potential difference ΔV across the 1.00-MΩ resistor is measured with an ideal voltmeter. (a) Show that the resistance of the footwear is

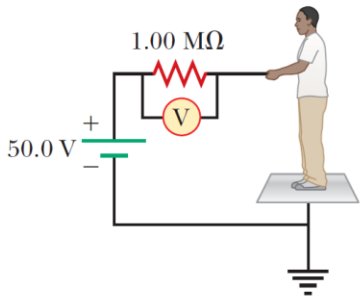


Figure P21.36

$$R_{\text{shoes}} = \frac{50.0V - \Delta V}{\Delta V}$$

(b) In a medical test, a current through the human body should not exceed 150 μA. Can the current delivered by the ANSI-specified circuit exceed 150 μA? To decide, consider a person standing barefoot on the ground plate.

P21.36 We assume that the metal wand makes low-resistance contact with the person's hand and that the resistance through the person's body is negligible compared to the resistance R_{shoes} of the shoe soles. The equivalent resistance seen by the power supply is 1.00 MΩ + R_{shoes}.

The current through both resistors is $\dfrac{50.0\text{ V}}{1.00\text{ M}\Omega + R_{\text{shoes}}}$. The voltmeter displays

$$\Delta V = I(1.00\text{ M}\Omega)$$

$$\Delta V = \frac{50.0\text{ V}}{1.00\text{ M}\Omega + R_{\text{shoes}}} = 1.00\text{ M}\Omega$$

(a) We solve to obtain

$$50.0\text{ V}(1.00\text{ M}\Omega) = \Delta V(1.00\text{ M}\Omega) + \Delta V(R_{\text{shoes}})$$

$$R_{\text{shoes}} = \frac{(1.00\text{ M}\Omega)(50.0 - \Delta V)}{\Delta V}$$

or

$$R_{\text{shoes}} = \frac{50.0 - \Delta V}{\Delta V}$$

where resistance is measured in MΩ.

(b) With $R_{\text{shoes}} \to 0$, the current through the person's body is

$$\frac{50.0\text{ V}}{1.00\text{ M}\Omega} = 50.0\ \mu\text{A}$$

$\boxed{\text{The current will never exceed } 50\ \mu\text{A.}}$

39. Consider the circuit shown in Figure P21.39. Find (a) the current in the 20.0-Ω resistor and (b) the potential difference between points *a* and *b*.

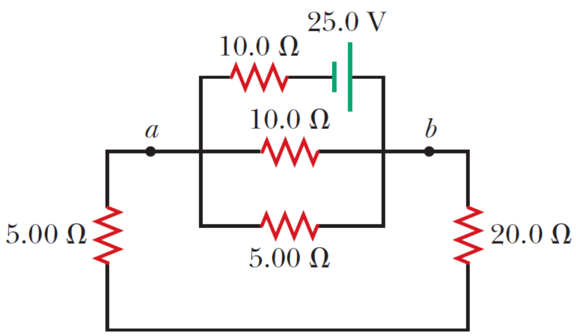


Figure P21.39

P21.39 If we turn the given diagram on its side and change the lengths of the wires, we find that it is the same as ANS. FIG. P21.39(a). The 20.0-Ω and 5.00-Ω resistors are in series, so the first reduction is shown in ANS. FIG. P21.39(b). In addition, since the 10.0-Ω, 5.00-Ω, and 25.0-Ω resistors are then in parallel, we can solve for their equivalent resistance as:

$$\frac{1}{R_{\text{eq}}} = \frac{1}{10.0\ \Omega} + \frac{1}{5.00\ \Omega} + \frac{1}{25.0\ \Omega} \rightarrow R_{\text{eq}} = 2.94\ \Omega$$

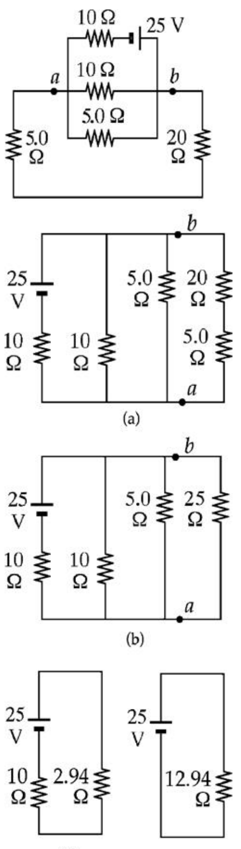
This is shown in ANS. FIG. P21.39(c), which in turn reduces to the circuit shown in ANS. FIG. P21.39(d), from which we see that the total resistance of the circuit is 12.94 Ω.

Next, we work backwards through the diagrams applying $I = \frac{\Delta V}{R}$ and $\Delta V = IR$ alternately to every resistor, real and equivalent. The total 12.94-Ω resistor is connected across 25.0 V, so the current through the battery in every diagram is

$$I = \frac{\Delta V}{R} = \frac{25.0\ \text{V}}{12.94\ \Omega} = 1.93\ \text{A}$$

In ANS. FIG. P21.39(c), this 1.93 A goes through the 2.94-Ω equivalent resistor to give a potential difference of:

$$\Delta V = IR = (1.93\ \text{A})(2.94\ \Omega) = 5.68\ \text{V}$$



ANS. FIG. P21.39

From ANS. FIG. P21.39(b), we see that this potential difference is the same as the potential difference ΔV_{ab} across the 10-Ω resistor and the 5.00-Ω resistor.

Thus we have first found the answer to part (b), which is

$$\Delta V_{ab} = \boxed{5.68\ \text{V}}$$

Since the current through the 20.0-Ω resistor is also the current through the 25.0-Ω line *ab*,

$$I = \frac{\Delta V_{ab}}{R_{ab}} = \frac{5.68\ \text{V}}{25.0\ \Omega} = 0.227\ \text{A} = \boxed{227\ \text{mA}}$$

43. Calculate the power delivered to each resistor in the circuit shown in Figure P21.43.

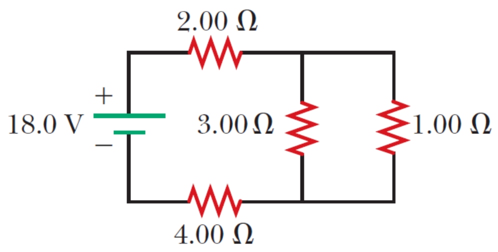


Figure P21.43

P21.43 To find the current in each resistor, we find the resistance seen by the battery. The given circuit reduces as shown in ANS. FIG. P21.43(a), since

$$\frac{1}{(1/1.00\ \Omega) + (1/3.00\ \Omega\ )} = 0.750\ \Omega$$

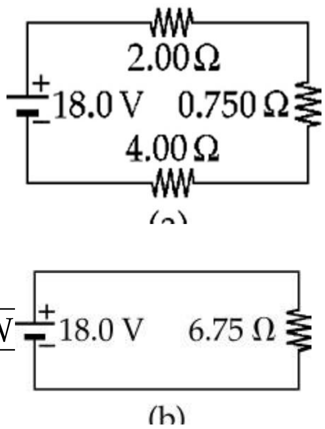
In ANS. FIG. P21.43(b),

$$I = 18.0\ \text{V}/6.75\ \Omega = 2.67\ \text{A}$$

This is also the current in ANS. FIG. P21.43(a), so the 2.00-Ω and 4.00-Ω resistors convert powers

$$P_2 = I\Delta V = I^2R = (2.67\ \text{A})^2(2.00\ \ \text{W}\ ) = \boxed{14.2\ \text{W}}$$

and $$P_4 = I^2R = (2.67\ \text{A})^2(4.00\ \ \text{W}\ \ ) = \boxed{28.4\ \text{W}}.$$



ANS. FIG. P21.43

The voltage across the 0.750-Ω resistor in ANS. FIG. P21.43(a), and across both the 3.00-Ω and the 1.00-Ω resistors in Figure P21.43, is

$$\Delta V = IR = (2.67\ \text{A})(0.750\ \ \text{W}\ ) = 2.00\ \text{V}$$

Then for the 3.00-Ω resistor,

$$I = \frac{\Delta V}{R} = \frac{2.00\ \text{V}}{3.00\ \Omega}$$

and the power is

$$P_3 = I\Delta V = \left(\frac{2.00\ \text{V}}{3.00\ \Omega}\right)(2.00\ \text{V}) = \boxed{1.33\ \text{W}}$$

For the 1.00-Ω resistor,

$$I = \frac{2.00\ \text{V}}{1.00\ \Omega} \quad \text{and} \quad P_1 = \left(\frac{2.00\ \text{V}}{1.00\ \Omega}\right)(2.00\ \text{V}) = \boxed{4.00\ \text{W}}$$

49. Taking $R = 1.00\ \text{k}\Omega$ and $\boldsymbol{\varepsilon} = 250$ V in Figure P21.49, determine the direction and magnitude of the current in the horizontal wire between *a* and *e*.

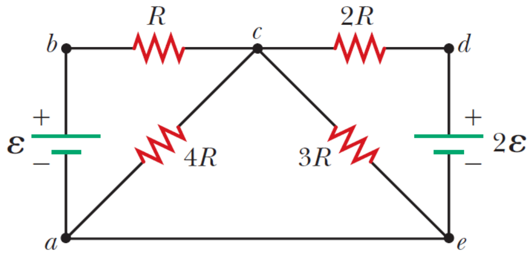
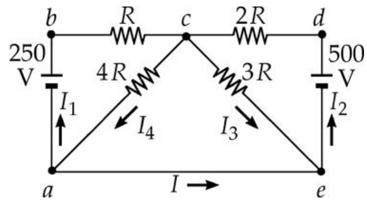
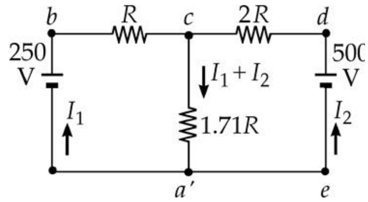


Figure P21.49

P21.49 Label the currents in the branches as shown in ANS. FIG. P21.49(a). Reduce the circuit by combining the two parallel resistors as shown in ANS. FIG. P21.49(b).



ANS. FIG. P21.49(a)



ANS. FIG. P21.49(b)

Apply Kirchhoff's loop rule to both loops in ANS. FIG. P21.49(b) to obtain:

$$(2.71R)I_1 + (1.71R)I_2 = 250\ \text{V}$$

$$(1.71R)I_1 + (3.71R)I_2 = 500\ \text{V}$$

With $R = 1\ 000\ \Omega$, simultaneous solution of these equations yields:

$$I_1 = 10.0\ \text{mA}$$

$$I_2 = 130.0\ \text{mA}$$

From ANS. FIG. P21.49(b), $V_c - V_a = (I_1 + I_2)(1.71R) = 240$ V.

Thus, from ANS. FIG. P21.49(a), $I_4 = \dfrac{V_c - V_a}{4R} = \dfrac{240\ \text{V}}{4\ 000\ \Omega} = 60.0\ \text{mA}.$

Finally, applying Kirchhoff's point rule at point *a* in ANS. FIG. P21.49(a) gives:

$$I = I_4 - I_1 = 60.0\ \text{mA} - 10.0\ \text{mA} = +50.0\ \text{mA}$$

or $\quad I = \boxed{50.0\ \text{mA from point } a \text{ to point } e}$.

50. GP Q|C For the circuit shown in Figure P21.50, we wish to find the currents I_1, I_2, and I_3. Use Kirchhoff's rules to obtain equations for (a) the upper loop, (b) the lower loop, and (c) the junction on the left side. In each case, suppress units for clarity and simplify, combining the terms. (d) Solve the junction equation for I_3. (e) Using the equation found in part (d), eliminate I_3 from the equation found in part (b). (f) Solve the equation found in parts (a) and (e) simultaneously for the two unknowns I_1 and I_2. (g) Substitute the answers found in part (f) into the junction equation found in part (d), solving for I_3. (h) What is the significance of the negative answer for I_2?

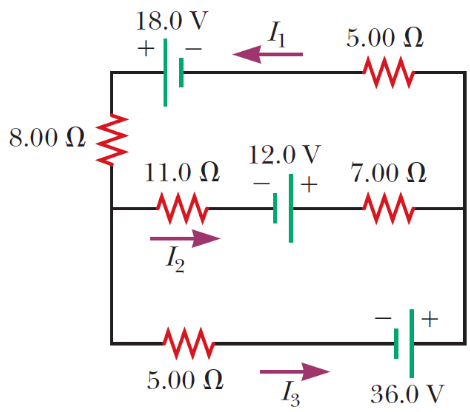
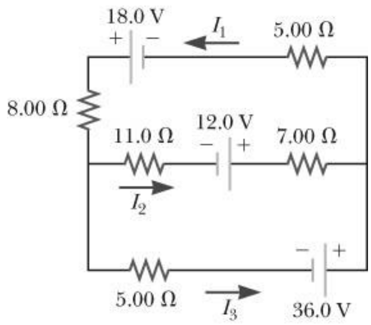


Figure P21.50

P21.50 (a) Going counterclockwise around the upper loop and suppressing units, Kirchhoff's loop rule gives

$$-11.0I_2 + 12.0 - 7.00I_2 - 5.00I_1 + 18.0 - 8.00I_1 = 0$$

or $\boxed{13.0I_1 + 18.0I_2 = 30.0}$. **[1]**



ANS. FIG. P21.50

(b) Going counterclockwise around the lower loop:

$$-5.00I_3 + 36.0 + 7.00I_2 - 12.0 + 11.0I_2 = 0$$

or $\boxed{18.0I_2 - 5.00I_3 = -24.0}$. **[2]**

(c) Applying the junction rule at the node in the left end of the circuit gives

$\boxed{I_1 - I_2 - I_3 = 0}$ **[3]**

(d) Solving equation [3] for I_3 yields $\boxed{I_3 = I_1 - I_2}$ **[4]**

(e) Substituting equation [4] into [2] gives

$$5.00(I_1 - I_2) - 18.0I_2 = 24.0$$

or $\boxed{5.00I_1 - 23.0I_2 = 24.0}$. **[5]**

(f) Solving equation [5] for I_1 yields $I_1 = (24.0 + 23.0I_2)/5$.

Substituting this into equation [1] gives

$$13.0I_1 + 18.0I_2 = 30.0$$
$$13.0\frac{(24.0 + 23.0I_2)}{5.00} + 18.0I_2 = 30.0$$
$$13.0(24.0 + 23.0I_2) + 5.00(18.0I_2) = 5.00(30.0)$$
$$389I_2 = -162 \quad \to \quad I_2 = -162/389 \quad \to \quad \boxed{I_2 = -0.416 \text{ A}}$$

Then, from equation [2], $I_1 = (30 - 18I_2)/13$ which yields

$$\boxed{I_1 = 2.88 \text{ A}}$$

(g) Equation [4] gives

$$I_3 = I_1 - I_2 = 2.88 \text{ A} - (-0.416 \text{ A}) \quad \to \quad \boxed{I_3 = 3.30 \text{ A}}$$

(h) The negative sign in the answer for I_2 means that this current flows in the opposite direction to that shown in the circuit diagram and assumed during the solution. That is, the actual current in the middle branch of the circuit flows from right to left and has a magnitude of 0.416 A.

52. **QC** Jumper cables are connected from a fresh battery in one car to charge a dead battery in another car. Figure P21.52 shows the circuit diagram for this situation. While the cables are connected, the ignition switch of the car with the dead battery is closed and the starter is activated to start the engine. Determine the current in (a) the starter and (b) the dead battery. (c) Is the dead battery being charged while the starter is operating?

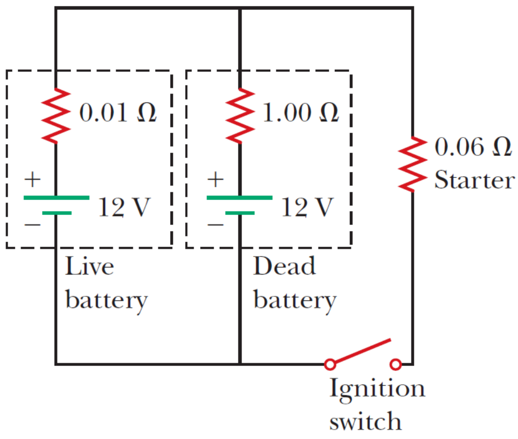


Figure P21.52

P21.52 Using Kirchhoff's rules and suppressing units,

$$12.0-(0.01)I_1-(0.06)I_3=0 \quad \textbf{[1]}$$

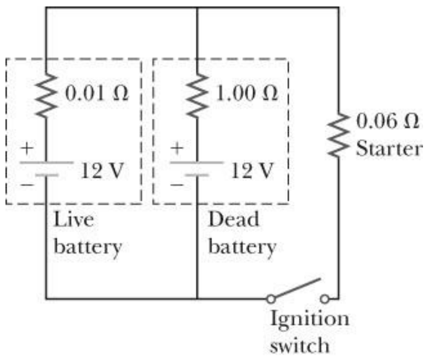
$$12.0+(1.00)I_2-(0.06)I_3=0 \quad \textbf{[2]}$$

and $I_1 = I_2 + I_3$. **[3]**

Substitute [3] into [1]:

$$12.0-(0.01)(I_2+I_3)-(0.06)I_3=0$$

$$12.0-(0.01)I_2-(0.07)I_3=0 \quad \textbf{[4]}$$



ANS. FIG. P21.52

Solving [4] and [2] simultaneously gives

(a) $I_3 = 172\text{ A} = \boxed{172\text{ A downward}}$ in the starter.

(b) $I_2 = -1.70\text{ A} = \boxed{1.70\text{ A upward}}$ in the dead battery.

(c) $\boxed{\text{No, the current in the dead battery is upward in Figure P21.52, so it is not being charged. The dead battery is providing a small amount of power to operate the starter, so it is not really "dead."}}$

57. In the circuit of Figure P21.57, the .switch S has been open for a long time. It is then suddenly closed. Take $\varepsilon = 10.0$ V, $R_1 = 50\,\text{k}\Omega$, $R_2 = 100\,\text{k}\Omega$, and $C = 10.0\ \mu\text{F}$. Determine the time constant (a) before the switch is closed and (b) after the switch is closed. (c) Let the switch be closed at $t = 0$. Determine the current in the switch as a function of time.

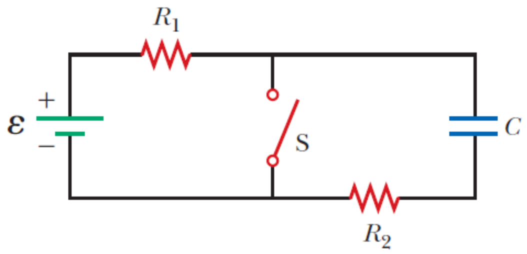


Figure P21.57

P21.57 (a) Before the switch is closed, the two resistors are in series. The time constant is

$$\tau = (R_1 + R_2)C = (1.50 \times 10^5\ \Omega)(10.0 \times 10^{-6}\ \text{F}) = \boxed{1.50\ \text{s}}$$

(b) After the switch is closed, the capacitor discharges through resistor R_2. The time constant is

$$\tau = (1.00 \times 10^5\ \Omega)(10.0 \times 10^{-6}\ \text{F}) = \boxed{1.00\ \text{s}}$$

(c) Before the switch is closed, there is no current in the circuit because the capacitor is fully charged, and the voltage across the capacitor is $\mathcal{E}$. After the switch is closed, current flows clockwise from the battery to resistor R_1 and down through the switch, and current from the capacitor flows counterclockwise and down through the switch to resistor R_2; the result is that the total current through the switch is $I_1 + I_2$.

Going clockwise around the left loop,

$$\mathcal{E} - I_1R_1 = 0 \rightarrow I_1 = \frac{\mathcal{E}}{R_1}$$

so the battery carries current $\qquad I_1 = \dfrac{10.0\ \text{V}}{50.0 \times 10^3\ \Omega} = 200\ \mu\text{A}.$

Going counterclockwise around the right loop,

$$\frac{q}{C} - I_2R_2 = 0 \rightarrow I_2 = \frac{q}{R_2C} = \frac{\mathcal{E}}{R_2}e^{-t/(R_2C)}$$

so the 100-kΩ resistor carries current of magnitude

$$I_2 = \frac{\mathcal{E}}{R_2}e^{-t/RC} = \left(\frac{10.0\ \text{V}}{100 \times 10^3\ \Omega}\right)e^{-t/1.00\ \text{s}}$$

So the switch carries downward current

$$I_1 + I_2 = \boxed{200\ \mu\text{A} + (100\ \mu\text{A})e^{-t/1.00\ \text{s}}}$$

58. The values of the components in a simple series RC circuit containing a switch (Fig. P21.53) are $C = 1.00\ \mu\text{F}$, $R = 2.00 \times 10^6\ \Omega$, and $\varepsilon = 10.0$ V. At the instant 10.0 s after the switch is closed, calculate (a) the charge on the capacitor, (b) the current in the resistor, (c) the rate at which energy is being stored in the capacitor, and (d) the rate at which energy is being delivered by the battery.

P21.58 (a) $q = C\Delta V\left(1 - e^{-t/RC}\right)$

$$q = \left(1.00 \times 10^{-6}\ \text{F}\right)(10.0\ \text{V})\left[1 - e^{-10.0/\left[\left(2.00\times10^6\ \Omega\right)\left(\ .00\times10^{-6}\ \right)\right]}\right]$$

$$= \boxed{9.93\ \mu\text{C}}$$

(b) $I = \dfrac{dq}{dt} = \left(\dfrac{\Delta V}{R}\right)e^{-t/RC}$

$$I = \left(\frac{10.0\ \text{V}}{2.00 \times 10^6\ \Omega}\right)e^{-5.00} = 3.37 \times 10^{-8}\ \text{A} = \boxed{33.7\ \text{nA}}$$

(c) $\dfrac{dU}{dt} = \dfrac{d}{dt}\left(\dfrac{1}{2}\dfrac{q^2}{C}\right) = \left(\dfrac{q}{C}\right)\dfrac{dq}{dt} = \left(\dfrac{q}{C}\right)I$

$$\frac{dU}{dt} = \left(\frac{9.93\times10^{-6}\ \text{C}}{1.00\times10^{-6}\ \text{C/V}}\right)\left(3.37\times10^{-8}\ \text{A}\right) = 3.34\times10^{-7}\ \text{W} = \boxed{334\ \text{nW}}$$

(d)

$$P_{\text{battery}} = I\mathcal{E} = \left(3.37\times10^{-8}\ \text{A}\right)\left(10.0\ \text{V}\right) = 3.37\times10^{-7}\ \text{W} = \boxed{337\ \text{nW}}$$

The battery power could also be computed as the sum of the instantaneous powers delivered to the resistor and to the capacitor:

$$I^2R + dU/dt = (33.7\times10^{-9}\ \text{A})^2(2.00\times10^{6}\ \Omega) + 334\ \text{nW} = 337\ \text{nW}$$

59. The circuit in Figure P21.59 has been connected for a long time. (a) What is the potential difference across the capacitor? (b) If the battery is disconnected from the circuit, over what time interval does the capacitor discharge to one-tenth its initial voltage?

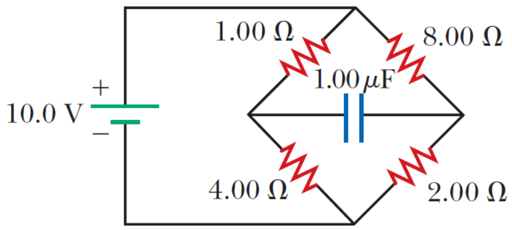
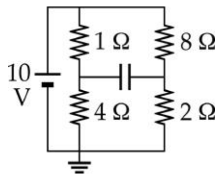


Figure P21.59

P21.59 (a) Call the potential at the left junction V_L and at the right V_R. After a "long" time, the capacitor is fully charged.



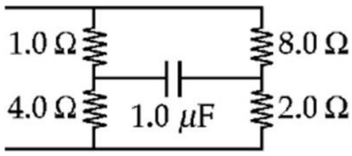
ANS. FIG. P21.59(a)

$$I_L = \frac{10.0\ \text{V}}{5.00\ \Omega} = 2.00\ \text{A}$$

$$V_L = 10.0\ \text{V} - (2.00\ \text{A})(1.00\ \Omega) = 8.00\ \text{V}$$

$$I_R = \frac{10.0\ \text{V}}{10.0\ \Omega} = 1.00\ \text{A}$$

$$V_R = (10.0\ \text{V}) - (8.00\ \Omega)(1.00\ \text{A}) = 2.00\ \text{V}$$

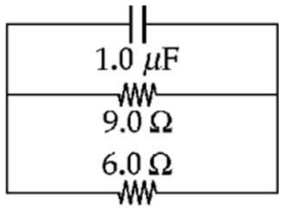


ANS. FIG. P21.59(b)

Therefore, $\Delta V = V_L - V_R = 8.00 - 2.00 = \boxed{6.00\ \text{V}}$

(b) We suppose the battery is pulled out leaving an open circuit. We are left with ANS. FIG. P21.59(b), which can be reduced to the equivalent circuits shown in ANS. FIG. P21.59(c) and ANS. FIG. P21.59(d).

From ANS. FIG. P21.59(d), we can see that the capacitor discharges through a 3.60-Ω equivalent resistance.

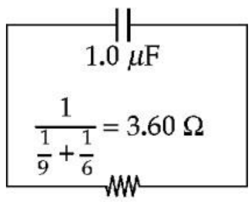


ANS. FIG. P21.59(c)

According to $q = Qe^{-t/RC}$,

we calculate that $qC = QCe^{-t/RC}$

and $\Delta V = \Delta V_i e^{-t/RC}$.



ANS. FIG. P21.59(d)

We proceed to solve for *t:*

$$V/V_i = e^{-t/RC} \quad \text{or} \quad V_i/V = e^{+t/RC}$$

Take natural logarithms of both sides:

$$\ln(V_i/V) = +t/RC$$

so

$$\begin{aligned} t &= RC\ln\left(\frac{V_i}{V}\right) \\ &= (3.60\ \Omega)\left(1.00\times 10^{-6}\ \text{F}\right)\ln\left(\frac{V_i}{0.100V_i}\right) \\ &= \left(3.60\times 10^{-6}\ \text{s}\right)\ln 10 \\ &= \boxed{8.29\ \mu\text{s}} \end{aligned}$$