# polyval

Polynomial evaluation

## Syntax

```
y = polyval(p,x)
[y,delta] = polyval(p,x,S)
y = polyval(p,x,[],mu)
[y,delta] = polyval(p,x,S,mu)
```

## Description

`y = polyval(p,x)` evaluates the polynomial `p` at each point in `x`. The argument `p` is a vector of length `n+1` whose elements are the coefficients (in descending powers) of an `n`th-degree polynomial: example

$$p(x) = p_1 x^n + p_2 x^{n-1} + \ldots + p_n x + p_{n+1}.$$

The polynomial coefficients in `p` can be calculated for different purposes by functions like `polyint`, `polyder`, and `polyfit`, but you can specify any vector for the coefficients.

To evaluate a polynomial in a matrix sense, use `polyvalm` instead.

`[y,delta] = polyval(p,x,S)` uses the optional output structure `S` produced by `polyfit` to generate error estimates. `delta` is an estimate of the standard error in predicting a future observation at `x` by `p(x)`. example

`y = polyval(p,x,[],mu)` or `[y,delta] = polyval(p,x,S,mu)` use the optional output `mu` produced by `polyfit` to center and scale the data. `mu(1)` is `mean(x)`, and `mu(2)` is `std(x)`. Using these values, `polyval` centers `x` at zero and scales it to have unit standard deviation, example

$$\hat{x} = \frac{x - \bar{x}}{\sigma_x}.$$

This centering and scaling transformation improves the numerical properties of the polynomial.

## Examples

collapse all

### Evaluate Polynomial at Several Points

Evaluate the polynomial $p(x) = 3x^2 + 2x + 1$ at the points $x = 5, 7, 9$. The polynomial coefficients can be represented by the vector `[3 2 1]`.

Open Live Script

```
p = [3 2 1];
x = [5 7 9];
y = polyval(p,x)
```

```
y = 1×3

    86   162   262
```

### Integrate Quartic Polynomial

Evaluate the definite integral

Open Live Script

$$I = \int_{-1}^{3} (3x^4 - 4x^2 + 10x - 25)dx.$$

Create a vector to represent the polynomial integrand $3x^4 - 4x^2 + 10x - 25$. The $x^3$ term is absent and thus has a coefficient of 0.

```
p = [3 0 -4 10 -25];
```

Use `polyint` to integrate the polynomial using a constant of integration equal to `0`.

```
q = polyint(p)
```

```
q = 1×6

    0.6000         0   -1.3333    5.0000  -25.0000         0
```

Find the value of the integral by evaluating `q` at the limits of integration.

```
a = -1;
b = 3;
I = diff(polyval(q,[a b]))
```

```
I = 49.0667
```

## Linear Regression With Error Estimate

Fit a linear model to a set of data points and plot the results, including an estimate of a 95% prediction interval.

Open Live Script

Create a few vectors of sample data points *(x,y)*. Use `polyfit` to fit a first degree polynomial to the data. Specify two outputs to return the coefficients for the linear fit as well as the error estimation structure.

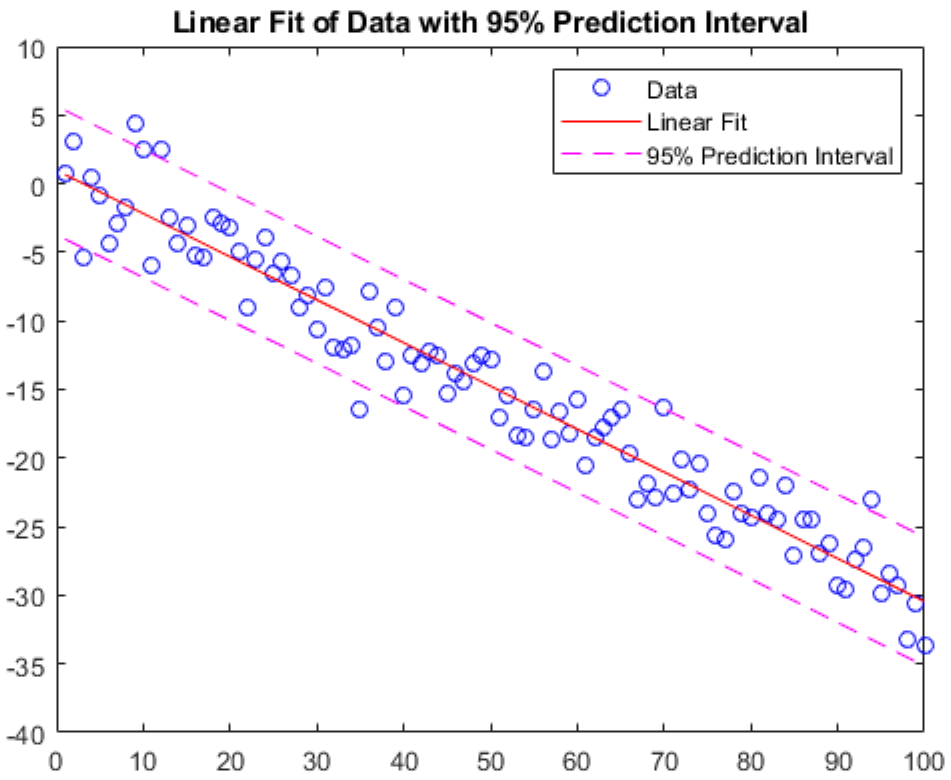
```
x = 1:100;
y = -0.3*x + 2*randn(1,100);
[p,S] = polyfit(x,y,1);
```

Evaluate the first-degree polynomial fit in `p` at the points in `x`. Specify the error estimation structure as the third input so that `polyval` calculates an estimate of the standard error. The standard error estimate is returned in `delta`.

```
[y_fit,delta] = polyval(p,x,S);
```

Plot the original data, linear fit, and 95% prediction interval $y \pm 2\Delta$.

```
plot(x,y,'bo')
hold on
plot(x,y_fit,'r-')
plot(x,y_fit+2*delta,'m--',x,y_fit-2*delta,'m--')
title('Linear Fit of Data with 95% Prediction Interval')
legend('Data','Linear Fit','95% Prediction Interval')
```




## Use Centering and Scaling to Improve Numerical Properties

Create a table of population data for the years 1750 - 2000 and plot the data points.

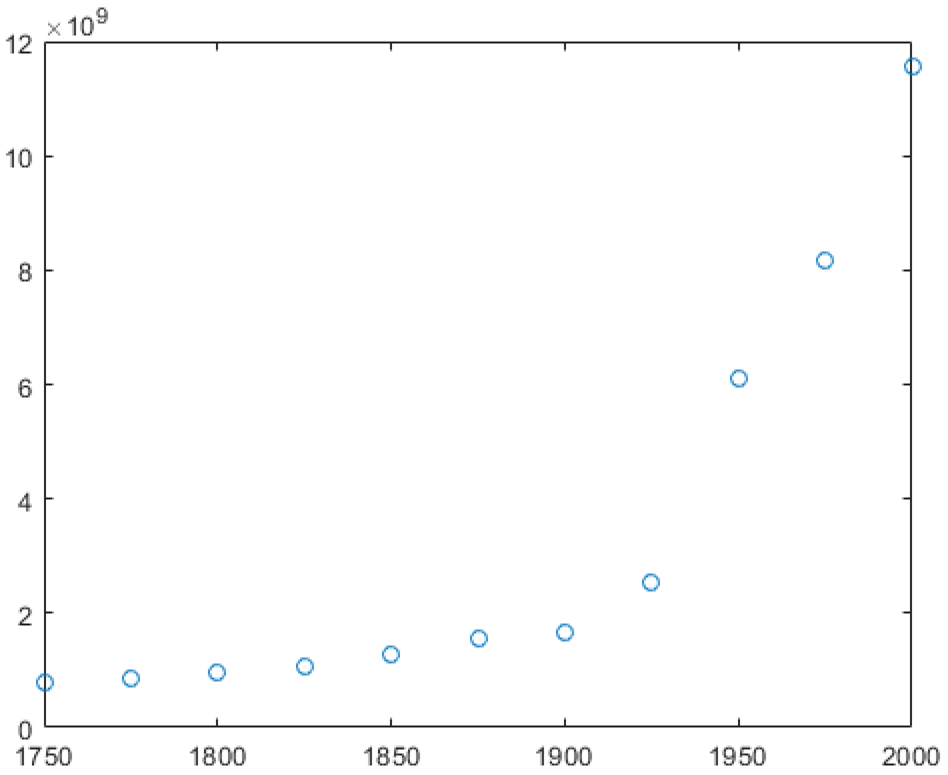
Open Live Script

```
year = (1750:25:2000)';
pop = 1e6*[791 856 978 1050 1262 1544 1650 2532 6122 8170 11560]';
T = table(year, pop)
```

T=*11×2 table*

| year | pop |
|---|---|
| 1750 | 7.91e+08 |
| 1775 | 8.56e+08 |
| 1800 | 9.78e+08 |
| 1825 | 1.05e+09 |
| 1850 | 1.262e+09 |
| 1875 | 1.544e+09 |
| 1900 | 1.65e+09 |
| 1925 | 2.532e+09 |
| 1950 | 6.122e+09 |
| 1975 | 8.17e+09 |
| 2000 | 1.156e+10 |

```
plot(year,pop,'o')
```

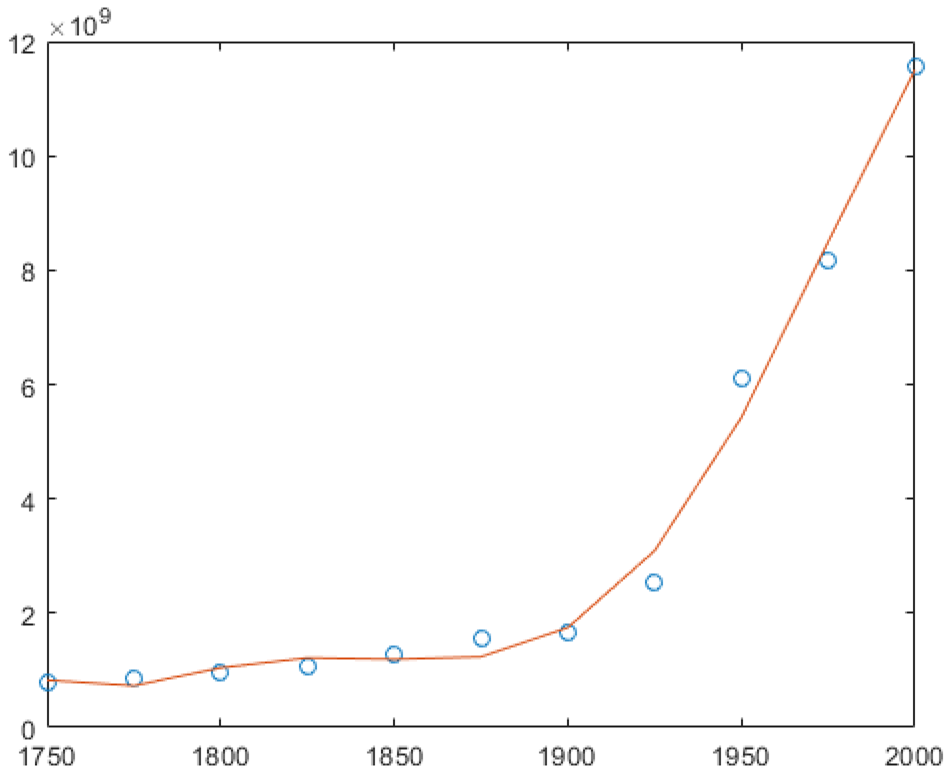



Use polyfit with three outputs to fit a 5th-degree polynomial using centering and scaling, which improves the numerical properties of the problem. polyfit centers the data in year at 0 and scales it to have a standard deviation of 1, which avoids an ill-conditioned Vandermonde matrix in the fit calculation.

```
[p,~,mu] = polyfit(T.year, T.pop, 5);
```

Use polyval with four inputs to evaluate p with the scaled years, (year-mu(1))/mu(2). Plot the results against the original years.

```
f = polyval(p,year,[],mu);
hold on
plot(year,f)
hold off
```




## Input Arguments

collapse all

**p — Polynomial coefficients**
vector

Polynomial coefficients, specified as a vector. For example, the vector `[1 0 1]` represents the polynomial $x^2 + 1$, and the vector `[3.13 -2.21 5.99]` represents the polynomial $3.13x^2 - 2.21x + 5.99$.

For more information, see Create and Evaluate Polynomials.

**Data Types:** `single` | `double`
**Complex Number Support:** Yes

**x — Query points**
vector

Query points, specified as a vector. `polyval` evaluates the polynomial p at the points in `x` and returns the corresponding function values in y.

**Data Types:** `single` | `double`
**Complex Number Support:** Yes

**S — Error estimation structure**
structure

Error estimation structure. This structure is an optional output from `[p,S] = polyfit(x,y,n)` that can be used to obtain error estimates. `S` contains the following fields:

| Field | Description |
|---|---|
| R | Triangular factor from a QR decomposition of the Vandermonde matrix of x |
| df | Degrees of freedom |
| normr | Norm of the residuals |

If the data in y is random, then an estimate of the covariance matrix of p is (Rinv*Rinv')*normr^2/df, where Rinv is the inverse of R.

**mu — Centering and scaling values**
two-element vector

Centering and scaling values, specified as a two-element vector. This vector is an optional output from [p,S,mu] = polyfit(x,y,n) that is used to improve the numerical properties of fitting and evaluating the polynomial p. The value mu(1) is mean(x), and mu(2) is std(x). These values are used to center the query points in x at zero with unit standard deviation.

Specify mu to evaluate p at the scaled points, (x - mu(1))/mu(2).

## Output Arguments

collapse all

**y — Function values**
vector

Function values, returned as a vector of the same size as the query points x. The vector contains the result of evaluating the polynomial p at each point in x.

**delta — Standard error for prediction**
vector

Standard error for prediction, returned as a vector of the same size as the query points x. Generally, an interval of $y \pm \Delta$ corresponds to a roughly 68% prediction interval for future observations of large samples, and $y \pm 2\Delta$ a roughly 95% prediction interval.

If the coefficients in p are least-squares estimates computed by polyfit, and the errors in the data input to polyfit are independent, normal, and have constant variance, then $y \pm \Delta$ is at least a 50% prediction interval.

## Extended Capabilities

### Tall Arrays
Calculate with arrays that have more rows than fit in memory.

### C/C++ Code Generation
Generate C and C++ code using MATLAB® Coder™.

### GPU Arrays
Accelerate code by running on a graphics processing unit (GPU) using Parallel Computing Toolbox™.

### Distributed Arrays

Partition large arrays across the combined memory of your cluster using Parallel Computing Toolbox™.

## See Also

`polyder` | `polyfit` | `polyint` | `polyvalm`

### Topics

Create and Evaluate Polynomials

Programmatic Fitting

**Introduced before R2006a**