

Programming Assignment 6: Linear Equation System Solver

This assignment is to design and implement a C++ project of matrices and vectors with class inheritance. The project has a basis **class Matrix** and two derived sub-classes **class Vector** and **class SMatrix**, representing vectors and square matrices as given in **matrix.h**, **smatrix.h**, and **vector.h**, respectively. Assume indices of rows and columns starting from 0. Create a project directory **Assignment6_DXXXXXXX**. Also, create a C++ project named **assignment6_DXXXXXXX.dev** to include header files **matrix.h**, **smatrix.h**, and **vector.h** and their implementation **matrix.cpp**, **smatrix.cpp**, and **vector.cpp**. Write an application program **linear_equation_system_solver.cpp** to implement a linear equation system solver. Place the project file, the header files, and the source code files in project directory **Assignment6_DXXXXXXX**.

A system of linear equations with n variables X_0, X_1, \dots, X_{n-1} is a set of n linear equations:

$$\begin{aligned} a_{0,0} X_0 + a_{0,1} X_1 + \dots + a_{0,n-2} X_{n-2} + a_{0,n-1} X_{n-1} &= c_0 \\ a_{1,0} X_0 + a_{1,1} X_1 + \dots + a_{1,n-2} X_{n-2} + a_{1,n-1} X_{n-1} &= c_1 \\ &\dots \\ a_{n-1,0} X_0 + a_{n-1,1} X_1 + \dots + a_{n-1,n-2} X_{n-2} + a_{n-1,n-1} X_{n-1} &= c_{n-1} \end{aligned}$$

These n linear equations can be written as a matrix-vector equation $AX=C$, where A is the $n \times n$ coefficient matrix, X is the solution column vector of length n , and C is the constant column vector of length n , that

$$A = \begin{bmatrix} a_{0,0} & a_{0,1} & \dots & a_{0,n-2} & a_{0,n-1} \\ a_{1,0} & a_{1,1} & \dots & a_{1,n-2} & a_{1,n-1} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ a_{n-2,0} & a_{n-2,1} & \dots & a_{n-2,n-2} & a_{n-2,n-1} \\ a_{n-1,0} & a_{n-1,1} & \dots & a_{n-1,n-2} & a_{n-1,n-1} \end{bmatrix}, X = \begin{bmatrix} X_0 \\ X_1 \\ \vdots \\ X_{n-2} \\ X_{n-1} \end{bmatrix}, \text{ and } C = \begin{bmatrix} c_0 \\ c_1 \\ \vdots \\ c_{n-2} \\ c_{n-1} \end{bmatrix}.$$

The solution of the linear equation system is $X_i = \frac{|A^i|}{|A|}$, where A^i is matrix A with replacement of the i -th column by vector C and $0 \leq i < n$. The application program should perform the following steps:

- Input n as the rank of the linear equation system, $0 < n \leq 10$,
- Randomly generate elements of coefficient matrix A and constant vector C . The values of these elements are between 0 and 1.0.
- Print the coefficient matrix A , constant vector C , and the system of linear equations.
- Solve the linear equation system to obtain the result of solution vector X .
- Print the solution of the linear equation system, i.e., the result of solution vector X .
- Verify solution of the linear equation system, i.e., compute $AX-C$ and check whether it is the zero vector, error tolerance to six digits after the decimal point.

Compress directory **Assignment6_DXXXXXXX** to obtain the compressed file **Assignment6_DXXXXXXX.YYY**, where **YYY** is **zip**, **rar**, or **7z**. Write a report **Report6_DXXXXXXX.pdf** to explain the experience of programming and development of this assignment. Submit **Assignment6_DXXXXXXX.YYY** and **Report6_DXXXXXXX.pdf** to iLearn. The assignment is due by **23:59 pm, Wednesday, May 29, 2024**.

Program execution example:

Coefficient matrix of the system of linear equations:

0.2052	0.5322	0.0611	0.2251	0.6637	0.2631
0.6520	0.2025	0.2794	0.0158	0.6411	0.8617
0.7406	0.5201	0.8131	0.4365	0.8467	0.0960
0.9960	0.5331	0.6089	0.6022	0.0760	0.1395
0.1343	0.1529	0.6589	0.5382	0.9898	0.7317
0.5145	0.0216	0.0679	0.3906	0.1191	0.1930

Constant vector of the system of linear equations:

0.7274	0.7938	0.0103	0.1642	0.3587	0.7857
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System of linear equations:

0.2052X ₀	+0.5322X ₁	+0.0611X ₂	+0.2251X ₃	+0.6637X ₄	+0.2631X ₅	=0.7274
0.6520X ₀	+0.2025X ₁	+0.2794X ₂	+0.0158X ₃	+0.6411X ₄	+0.8617X ₅	=0.7938
0.7406X ₀	+0.5201X ₁	+0.8131X ₂	+0.4365X ₃	+0.8467X ₄	+0.0960X ₅	=0.0103
0.9960X ₀	+0.5331X ₁	+0.6089X ₂	+0.6022X ₃	+0.0760X ₄	+0.1395X ₅	=0.1642
0.1343X ₀	+0.1529X ₁	+0.6589X ₂	+0.5382X ₃	+0.9898X ₄	+0.7317X ₅	=0.3587
0.5145X ₀	+0.0216X ₁	+0.0679X ₂	+0.3906X ₃	+0.1191X ₄	+0.1930X ₅	=0.7857

Solution of the linear equation system is:

0.7932	-0.2445	-1.8653	0.8745	0.7638	0.3990
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Verify solution of the linear equation system:

Equation 0	passes.
Equation 1	passes.
Equation 2	passes.
Equation 3	passes.
Equation 4	passes.
Equation 5	passes.