112-2
Electrical Engineering Fundamentals II

Test 1

Keys

1. (20%) Converting the following pairs of sinusoid signals v and i into ***phasors***.

(A) $v(t) = 10\cos(4t - 60^\circ)$ $\Rightarrow$ $V = \underline{10} \measuredangle \underline{-60} = \underline{5 - j8.66}$

$i(t) = 4\sin(4t + 50^\circ)$ $\Rightarrow$ $I = \underline{4} \measuredangle \underline{-40} = \underline{3.064 - j2.571}$

$\Rightarrow$ ***i*(t) leads *v*(t) by** $\underline{20^\circ}$

(B) $v(t) = -13\cos(2t) + 5\sin(2t)$ $\Rightarrow V = \underline{13.928} \measuredangle \underline{201.038^\circ} = \underline{-13 - j5}$

$i(t) = 15\cos(2t - 40^\circ)$ $\Rightarrow I = \underline{15} \measuredangle \underline{-40^\circ} = \underline{11.491 - j9.642}$

$\Rightarrow$ ***v*(t) leads *i*(t) by** $\underline{241.038^\circ}$

(A) $v(t) = 10\cos(4t - 60^\circ)$

$V = 10\measuredangle -60^\circ = 5 - j8.66$

$i(t) = 4\sin(4t + 50^\circ) = 4\cos(4t + 50^\circ - 90^\circ) = 4\cos(4t - 40^\circ)$

$I = 4\measuredangle -40^\circ = 3.064 - j2.571$

phase difference $= -40^\circ - (-60^\circ) = 20^\circ$

Thus, ***i*(t) leads *v*(t) by 20°**.

(B) $v(t) = -13\cos(2t) + 5\sin(2t) = 13\cos(2t + 180^\circ) + 5\cos(2t - 90^\circ)$

$V = 13\measuredangle 180^\circ + 5\measuredangle -90^\circ = -13 - j5 = 13.928\measuredangle -158.962^\circ = 13.928\measuredangle 201.038^\circ$

$v(t) = 13.928\cos(2t + 201.038^\circ)$

$i(t) = 15\cos(2t - 40^\circ)$

$I = 15\measuredangle -40^\circ = 11.491 - j9.642$

phase difference $= 201.038^\circ - (-40^\circ) = 241.038^\circ$

Thus, ***v*(t) leads *i*(t) by** 241.038°.

2. (20%) Evaluate the following complex numbers and express your results in both polar and rectangular form:

(A) $\dfrac{60\measuredangle 45^\circ}{7.5 - j10} + j2 = \dfrac{60\measuredangle 45^\circ}{12.5\measuredangle - 53.13^\circ} + j2$

$= 4.8\measuredangle(98.13^\circ) + j2$

$= -0.6788 + j4.752 + j2$

$= -0.6788 + j6.752$

$= 6.786\measuredangle 95.741^\circ$

(B) $\dfrac{(10\measuredangle 30^\circ)(35\measuredangle(-50^\circ))}{(2 + j6)^* - (5 + j)} = \dfrac{350\measuredangle(-20^\circ)}{2 - j6 - 5 - j}$

$= \dfrac{350\measuredangle(-20^\circ)}{-3 - j7}$

$= \dfrac{350\measuredangle(-20^\circ)}{7.616\measuredangle(-113.199^\circ)}$

$= 45.955\measuredangle 93.199^\circ$

$= -2.564 + j45.883$

3. (15%) Solve $\frac{dv(t)}{dt} + 5v(t) + 4\int v(t)dt = 20\sin(4t + 10°)$ to get v(t) by using the phasor approach.

$$\frac{dv(t)}{dt} + 5v(t) + 4\int v(t)dt = 20\sin(4t + 10°)$$

$$j\omega\mathbf{V} + 5\mathbf{V} + \frac{4\mathbf{V}}{j\omega} = 20\angle(10° - 90°), \quad \omega = 4$$

$$\mathbf{V}\left(j4 + 5 + \frac{4}{j4}\right) = 20\angle{-80°}$$

$$\boxed{\mathbf{V} = \frac{20\angle{-80°}}{5 + j3} = 3.43\angle{-110.96°}}$$

$v(t) = \mathbf{3.43}\ \boldsymbol{cos}\mathbf{(4t - 110.96°)\ V} = \mathbf{3.43}\ \boldsymbol{sin}\mathbf{(4t - 20.96°)\ V}$

4. (20%) For the circuit in Fig. 4, calculate Z_{eq} and V_{ab}.

$$\mathbf{Z}_T = (20 - j5) \parallel (40 + j10) = \frac{(20 - j5)(40 + j10)}{60 + j5} = \frac{170}{145}(12 - j)$$

$$\boxed{\mathbf{Z_{eq} = 14.069 - j1.172 = 14.118\measuredangle(-4.76^\circ)\ (\Omega)}}$$

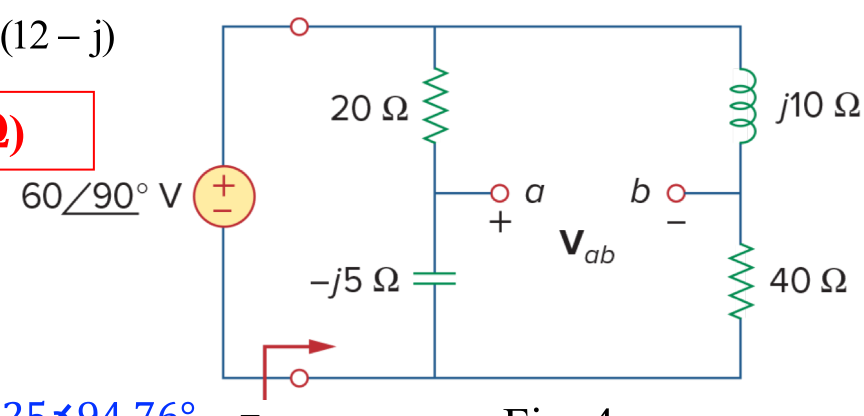


Fig. 4

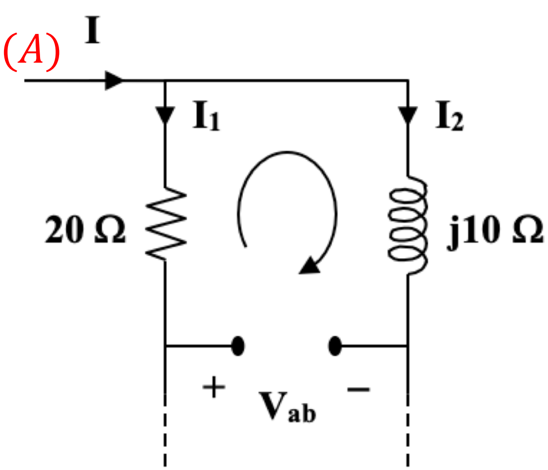
$$\mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}_T} = \frac{60\angle 90^\circ}{14.118\angle -4.76^\circ} = 4.25\angle 94.76^\circ$$

$$\mathbf{I}_1 = \frac{40 + j10}{60 + j5}\mathbf{I} = \frac{8 + j2}{12 + j}\mathbf{I}$$

$$I_1 = \frac{8.25\measuredangle 14.04^\circ}{12.04\measuredangle 4.76^\circ} \times 4.25\measuredangle 94.76^\circ$$
$$= 2.91\measuredangle 104.04^\circ = -0.707 + j2.823\ (A)$$

$$\mathbf{I}_2 = \frac{20 - j5}{60 + j5}\mathbf{I} = \frac{4 - j}{12 + j}\mathbf{I}$$

$$I_2 = \frac{4.12\measuredangle -14.04^\circ}{12.04\measuredangle 4.76^\circ} \times 4.25\measuredangle 94.76^\circ$$
$$= 1.45\measuredangle 75.96^\circ = 0.352 + j1.407\ (A)$$



$$\mathbf{V}_{ab} = -20\mathbf{I}_1 + j10\mathbf{I}_2$$

$$\mathbf{V}_{ab} = \frac{-(160 + j40)}{12 + j}\mathbf{I} + \frac{10 + j40}{12 + j}\mathbf{I}$$

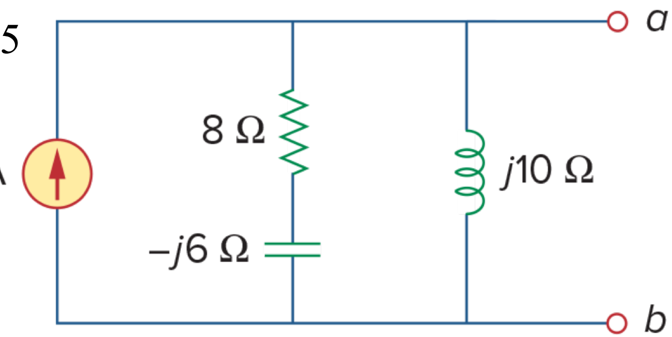
$$\mathbf{V}_{ab} = \frac{-150}{12 + j}\mathbf{I} = \frac{(-12 + j)(150)}{145}\mathbf{I}$$

$$V_{ab} = (12.457\measuredangle 175.24^\circ)(4.25\measuredangle 94.76^\circ)$$
$$= 52.94\measuredangle 270^\circ = 52.94\measuredangle(-90^\circ)$$

$$\boxed{V_{ab} = 52.94\measuredangle 270^\circ = 52.94\measuredangle(-90^\circ) = -j\,52.867\ (V)}$$

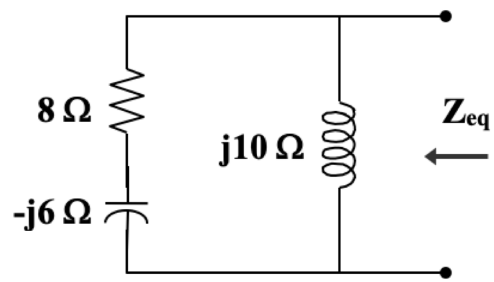
5. (15%) For the circuit depicted in Figure 5, find the Thevenin equivalent circuit at terminals a-b.

Figure 5



$$Z_{Th} = j10 \,\|\, (8 - j6) = \frac{(j10)(8 - j6)}{8 + j4} = 5(2 + j)$$

$$= \mathbf{11.18\angle 26.56^\circ\ \Omega}$$

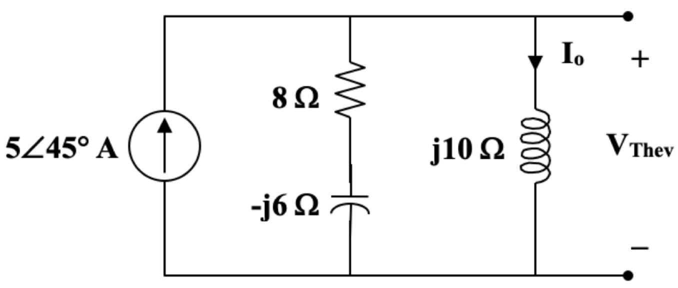


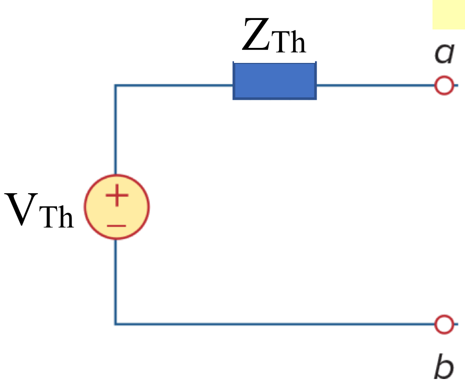
$$\mathbf{I}_o = \frac{8 - j6}{8 - j6 + j10}(5\angle 45^\circ) = \frac{4 - j3}{4 + j2}(5\angle 45^\circ)$$

$$\mathbf{V}_{Thev} = j10\,\mathbf{I}_o = \frac{(j10)(4 - j3)(5\angle 45^\circ)}{(2)(2 + j)}$$

$$= \mathbf{55.9\angle 71.56^\circ\ V}$$

$$= 17.68 + j\ 53.03\ (\mathrm{V})$$





6. (20%) In the circuit of Fig. 6, determine the mesh currents $\boldsymbol{i}_1$ and $\boldsymbol{i}_2$ with mesh analysis. Let $\boldsymbol{v}_1 = 10\cos 4t$ (V) and $\boldsymbol{v}_2 = 20\cos(4t - 60^\circ)$ (V).

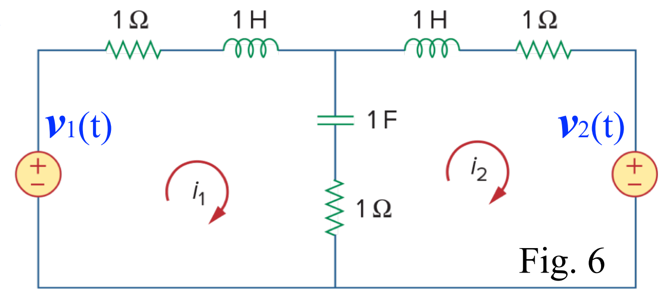
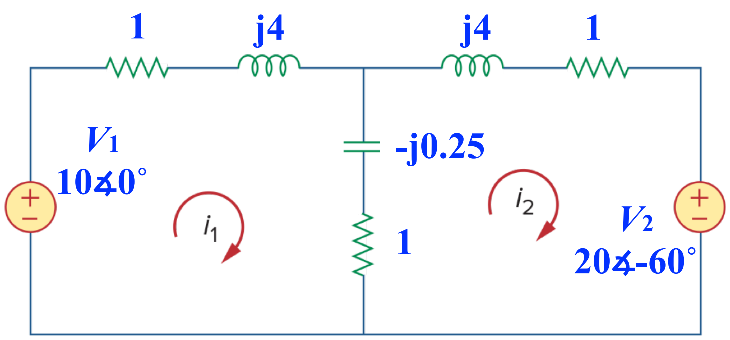


Fig. 6



$V_2 = 20\angle -60^\circ = 10 - j17.321$

mesh analysis

$$(2 + j3.75)I_1 - (1 - j0.25)I_2 = 10$$
$$-(1 - j0.25)I_1 + (2 + j3.75)I_2 = -10 + j17.32$$

$$\begin{bmatrix} 2 + j3.75 & -(1 - j0.25) \\ -(1 - j0.25) & 2 + j3.75 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 10 \\ -10 + j17.321 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 2 + j3.75 & -(1 - j0.25) \\ -(1 - j0.25) & 2 + j3.75 \end{vmatrix}$$
$$= -10.0625 + j15 - (0.9375 - j0.5)$$
$$= -11 + j15.5 = 19.01\angle 125.36^\circ$$

$$\Delta_1 = \begin{vmatrix} 10 & -(1 - j0.25) \\ -10 + j17.32 & 2 + j3.75 \end{vmatrix}$$
$$= 14.33 + j57.32 = 59.08\angle 75.96^\circ$$

$$\Delta_2 = \begin{vmatrix} 2 + j3.75 & 10 \\ -(1 - j0.25) & -10 + j17.32 \end{vmatrix}$$
$$= -74.95 - j5.36 = 75.15\angle -175.9^\circ$$

$$I_1 = \frac{\Delta_1}{\Delta} = \frac{14.33 + j57.32}{-11 + j15.5} = \frac{59.08\angle 75.96^\circ}{19.01\angle 125.36^\circ} = 3.108\angle -49.4^\circ = 2.023 - j2.36$$

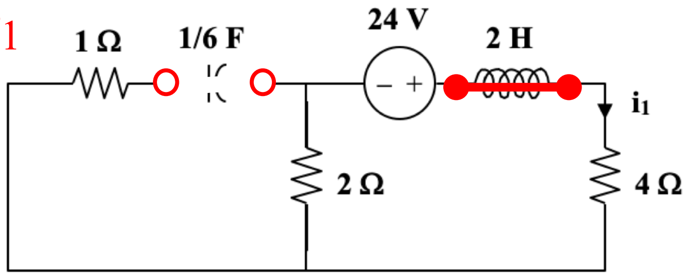
$$I_2 = \frac{\Delta_2}{\Delta} = \frac{-74.95 - j5.36}{-11 + j15.5} = \frac{75.15\angle -175.9^\circ}{19.01\angle 125.36^\circ} = 3.948\angle 58.74^\circ = 2.049 + j3.375$$

$\boldsymbol{i}_1(t) = 3.108\,\boldsymbol{cos}(4t - 49.4^\circ)$ (A)

$\boldsymbol{i}_2(t) = 3.948\,\boldsymbol{cos}(4t + 58.74^\circ)$ (A)

7. (20%) Determine i_O in the circuit of Fig. 7. Let $\boldsymbol{v}_1(t) = 24$ (V), $\boldsymbol{v}_2(t) = 10\ sin\ (t + 60^\circ)$ (V), $\boldsymbol{i}(t) = 2\ cos\ (2t)$ (A).

v_1



$$\Rightarrow \boxed{i_1 = \frac{24}{4+2} = 4\ \text{A}}$$

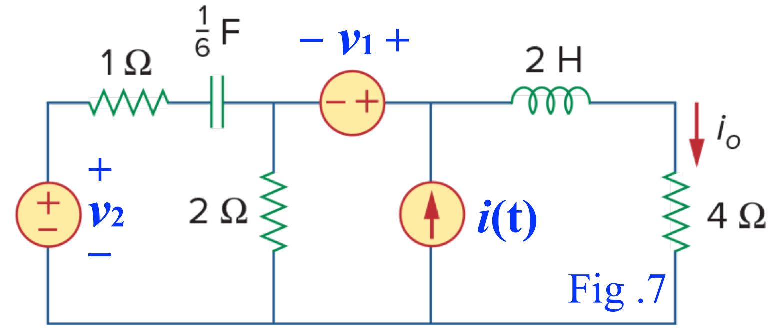
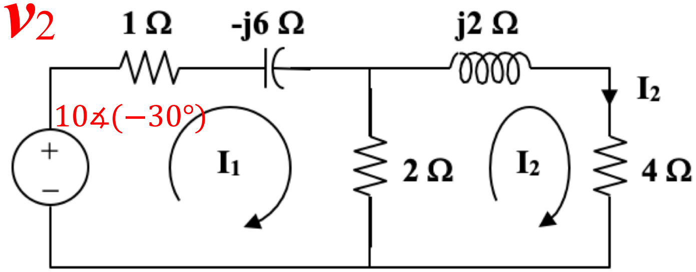
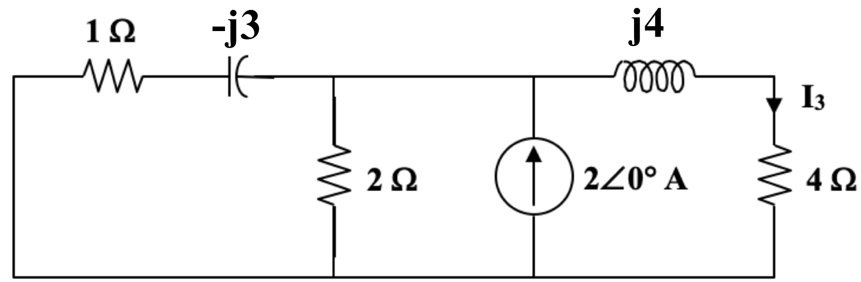


Fig .7

v_2



i



$$2 \mathbin{/\!/} (1-j3) = \frac{2(1-j3)}{3-j3} = \frac{6.325\measuredangle-71.57^\circ}{4.243\measuredangle-45^\circ} = 1.49\measuredangle-26.57^\circ = 1.333 - j0.666$$

Mesh 1,

$$-10\angle -30^\circ + (3 - j6)\mathbf{I}_1 - 2\mathbf{I}_2 = 0$$

$$10\angle -30^\circ = 3(1-2j)\mathbf{I}_1 - 2\mathbf{I}_2 \quad \cdots\cdots ①$$

Mesh 2,

$$0 = -2\mathbf{I}_1 + (6 + j2)\mathbf{I}_2$$

$$\mathbf{I}_1 = (3 + j)\mathbf{I}_2 \quad \cdots\cdots ②$$

$$① \Rightarrow 10\angle -30^\circ = 13 - j15\mathbf{I}_2$$

$$\Rightarrow I_2 = \frac{10\measuredangle-30^\circ}{13-j15} = \frac{10\measuredangle-30^\circ}{19.85\measuredangle-49.09^\circ} = 0.476 + j19.09 = 0.504\measuredangle19.09^\circ$$

$$\Rightarrow \boxed{\begin{aligned} i_2 &= 0.504\cos(t + 19.1^\circ) \\ &= 0.504\sin(t + 109.1^\circ) \end{aligned}}$$

Current division: $$I_3 = 2\measuredangle0^\circ \times \frac{1.49\measuredangle-26.57^\circ}{1.333 - j0.666 + 4 + j4}$$

$$= 2\measuredangle0^\circ \times \frac{1.49\measuredangle-26.57^\circ}{5.333 + j3.334} = 2\measuredangle0^\circ \times \frac{1.49\measuredangle-26.57^\circ}{6.29\measuredangle32.01^\circ}$$

$$= 0.4737\measuredangle-58.58^\circ = 0.247 - j0.404$$

$$\Rightarrow \boxed{\boldsymbol{i}_3 = 0.4737\cos(2t - 58.58^\circ)}$$

$$\boxed{\begin{aligned} i_O &= i_1 + i_2 + i_3 \\ &= 4 + 0.504\sin(t + 109.1^\circ) + 0.4737\cos(2t - 58.58^\circ)\ (A) \end{aligned}}$$