13/6 §5-1 Area between curves

$y = f(x)$

$f(x) \geq g(x)$

≥ 0

$y = g(x)$

a b

$y = f(x)$

$0 \geq f(x) \geq g(x)$

$y = g(x)$

$f(x) - g(x) \geq 0$

If $f(x) \geq 0$ on $[a, b]$
then $\int_a^b f(x)\,dx$

$= $ area under the
graph of $y = f(x)$
over $[a, b]$

$f(x) \geq g(x)$ on $[a, b]$

$$\int_a^b f(x) - g(x) \, dx$$

$=$ area between

$y = f(x)$ and $y = g(x)$

over $[a, b]$

$$\lim_{n \to \infty} \sum_{i=1}^{n} \left[f(x_i^*) - g(x_i^*) \right] \Delta x$$

exists

$$= \int_a^b \boxed{f(x) - g(x)} \, dx \quad \Longrightarrow$$

c , b

area bounded
by $y = f(x)$, $y = g(x)$
and $x = a$, $x = b$

$y = f(x)$

$y = g(x)$

x

$f(x) = g(x)$
to solve
for x

a
bounded
by $y = f(x)$, $y = g(x)$
and $x = a$

a   b

area enclosed
by $y = f(x)$
and $y = g(x)$

let $f(x) = g(x)$
to solve for a and b.

Ex1 p365

area of region bounded
by $y = x^2 + 1$ and $y = x$
and bounded by $x = 0$
and $x = 1$.

$$A = \int_0^1 (x^2 + 1) - x \, dx$$

$$= \frac{1}{3}x^3 + x - \frac{x^2}{2} \Big|_0^1$$

$$= \frac{1}{3} + 1 - \frac{1}{2}$$

$$= \frac{1}{3} + \frac{1}{2} = \frac{5}{6} \; \#$$

Ex.2 Area enclosed by $y = x^2$ and $y = 2x - x^2$

Sol:
$$x^2 = 2x - x^2$$
$$2x^2 - 2x \implies$$
$$2x(x-1) = 0$$
$$x = 0 \ . \ x = 1$$

$$A = \int_0^1 (2x - x^2) - x^2 \, dx$$

$$= \int_0^1 2x - 2x^2 \, dx$$

$$= x^2 - \frac{2}{3}x^3 \Big|_0^1$$

$$= 1 - \frac{2}{3} = \frac{1}{3}$$

$y = 2x - x^2$

$y = x^2$

$$y = 2x - x^2 \qquad \text{X-intercepts}$$
$$x = 0, 2$$

$$= -\left(\underline{x^2 - 2x + 1} - 1 \right)$$

$$= \ominus (x-1)^2 + 1$$

downward

$y = f(x)$

$y = g(x)$

$= x_0$ x_1 x_2

$f(x) = g(x)$

to find intercepts .

$x_0 , x_1 , x_2 . —$

$$\int_{x_0}^{x_2} |f(x) - g(x)| \, dx$$

$$= \int_{x_0}^{x_1} |f(x) - g(x)| \, dx + \int_{x_1}^{x_2} |f(x) - g(x)| \, dx$$

$$= \int_{x_0}^{x_1} f(x) - g(x) \, dx + \int_{x_1}^{x_2} g(x) - f(x) \, dx$$

area between curves

$$\int_L^R (Top - bottom) \, dx$$

x function x function

$$= \int_L^R |f_T - g_B| \, dx$$

Wed

$X = h(y)$

$X = k(y)$

$X = h(y)$

$X = k(y)$

$k(y) \geq h(y)$