```
% 6.1.1 A concrete example
% a supply & Demand example in Table of p. 128

c = [3 12 10; 17 18 35; 7 10 24];
x = [4 0 0; 6 6 0; 0 3 5];
total = c .* x    % pointwise operation

sum(total)
sum(sum( total ))

%============================================================

% 6.1.2 Creating matrices
% 1-D subscript is column rank first
a = [1 2; 3 4];
x = [5 6];
a = [a; x]

a(3,2)
a(5)

a(3,3)

a(3,3) = 7

%============================================================

% 6.1.4 Transpose

a = [1:3; 4:6]
b = a'


%============================================================

% 6.1.5 The colon operator

a = [1:3; 4:6; 7:9]
```

```
a(2:3,1:2)

aa= floor( (rand(6,6).*10))+1
bb=aa(1:2:5,1:3) % colon generate a subscript vector

a(3,:) % ':' means all elements

a(1:2,2:3) = ones(2)    % a(1:2,2:3) is a 2*2 matrix


% To construct a table

format long
x = [0:30:180]'; % row vector transpose to a column vector
trig(:,1) = x;
trig(:,2) = sin(pi/180*x);
trig(:,3) = cos(pi/180*x);

trig
  % replaces the first and third columns of a by the fourth and second columns
% of b (a and b must have the same number of rows).

a = [1:3; 4:6; 7:9]
b = [11:14; 15:18; -4:-1]
a(:,[1 3]) = b(:,[4 2])

% A famous opertion in Linear algebra is the Gauss reduction
a=[1 -1 2; 2 1 -1; 3 0 1]
a(2,:) = a(2,:) - a(2,1)*a(1,:)

% The keyword 'end' refers to the last row or column of an array

r = ones(1,8)
sum(r(3:end)) % 'end' means till the last one.
```

```
% a(:) is different for the right or left,
% on the right: straight out to a single column vector.
a=[1 2; 3 4]
b=a(:)

% on the left, a(:) reassign a matric which is already exist, # of element
% must be the same.
a=zeros(3,2)
a(:)=[ 1:6]

  b = [1:3; 4:6]
a=zeros(3,2)
a(:) = b

  % 1-D sequence ranking of the matrix b is 1 4 2 5 3 6
  % reshape it to an 3*2 matrix note that column first
c = reshape(b,3,2)

a(:) = -1


%=============================================================
% 6.1.6 Duplicating rows and columns: tiling by 'repmat'

a = [1 2 3]
a=[ 1 2 ; 3 4];
b = repmat(a, [2 3]) % repeat matric a twice in the row & three times in the column

% alternative syntax
c = repmat(a, 2, 3)

d = repmat(a, [4 2])
e = repmat(a, 4, 2)
```

```
%=============================================================
% 6.1.7 Deleting rows and columns

a = [1:3; 4:6; 7:9]
a(:,2) = [ ] % notes that it is different from a(:,2) = 0

% You cannot delete a single element from a matrix
a(1,2) = [ ]

% You can delete a sequence of elements from a matrix
% and reshape the remaining elements into a row vector

a = [1:3; 4:6; 7:9]
a(2:2:6) = [ ]

% You can use logical vectors to extract a selection of rows or columns from a matrix,

a = [1:3; 4:6; 7:9]
cc=logical([1 0 1])
b = a(:, logical([1 0 1]))

c = a(:, [1 3]) % [1 3] is the subsript vector of the matrix

%=============================================================
% 6.1.8 Elementary matrices

a = zeros(3,4)
a = ones(3,4)
a = rand(3,4)
a = eye(3)

% As an example, eye may be used to construct a tridiagonal matrix as follows.

a = 2 * eye(5);
a(1:4, 2:5) = a(1:4, 2:5) - eye(4);
a(2:5, 1:4) = a(2:5, 1:4) - eye(4)
```

```
%============================================================
% 6.1.9 Specialized matrices

% pascal(n) generates a Pascal matrix of order n.

a = pascal(4)

b = magic(4) % equal sum along any rows or columns


%============================================================
% 6.1.10 Using MATLAB functions with matrices

% For each column of a where all the elements are true (non-zero) all returns '1', otherwise
it returns 0.
a = [1 0 1; 1 1 1; 0 0 1]
b = all(a)

% To test if all the elements of a are true, use all twice.
c = all(all(a))

d = any(a)
e = any(any(a))


%============================================================
% 6.1.11 Manipulating matrices

% diag extracts or creates a diagonal.
a = pascal(4)
b = diag(a)

% fliplr flips from left to right.
a = pascal(4)
b = fliplr(a)

% flipud flips from top to bottom.
a = pascal(4)
```

```
b = flipud(a)

% rot90 rotates.
a = pascal(4)
b = rot90(a)

% tril extracts the lower triangular part,
a = pascal(4)
b = tril(a)

% triu extracts the upper triangular part.
a = pascal(4)
b = triu(a)

%=============================================================
% 6.1.12 Pointwise (Array, element-by-element) operations on matrices

a = [1:3; 4:6]
b = a .^ 2
c = sin(a)


%=============================================================
% 6.1.13 Matrices and for

% the index v takes on the value of each column of the matrix expression a in turn.

a = [1:3; 4:6; 7:9]

for v = a(:,1:2)
disp(v')
end

% the index v takes on the value of each row of the matrix expression a in turn.
for v = a'
disp(v')
end
```

```
%============================================================
% 6.1.15 Vectorizing nested fors: loan repayment tables
%%    forming the table of repayments for a ;oan of $1000 over 15 20 and 25 yrs
%    rate%      15 yrs       20 yrs       25 yrs
%      10        10.75         9.65         9.09
%      11        11.37        10.32         9.80
%      12        12.00        11.01        10.53
%      13        12.65        11.72        11.28
%% Exercise to write the expression in p.140 by matlab code

% Method 1:
A = 1000; % amount borrowed
n = 12; % number of payments per year
disp ([' rate%      15 yrs       20 yrs       25 yrs']);
for r = 0.1 : 0.01 : 0.2
      fprintf( '%4.0f%', 100 * r );
      for k = 15 : 5 : 25
            temp = (1 + r/n) ^ (n*k);
            P = r * A * temp / (n * (temp - 1));
            fprintf( '%10.2f', P );
      end;
      fprintf( '\n' ); % new line
end;

% The inner loop can easily be vectorized; the following code uses only one for:

% Method 2
format short
A = 1000; % amount borrowed
n = 12; % number of payments per year
disp ['    rate%           15 yrs       20 yrs       25 yrs']);
for r = 0.1 : 0.01 : 0.2
      k = 15 : 5 : 25;          % Now k is a vector 1*3
      temp = (1 + r/n) .^ (n*k); % temp is a vector with the same dim.
      P = r * A * temp / n ./ (temp - 1);
      disp( [100 * r P] ); % [100 * r P]    is a 1*4 vector
end;
```

```
  % The really tough challenge, however, is to vectorize the outer loop as well.

% Method 3
A = 1000; % amount borrowed
n = 12; % number of payments per year
r = [0.1:0.01:0.2]'    % r is a 11*1 matrix
% Now change this into a table with 3 columns each equal to r:
r = repmat(r, [1 3]) % r is 11*3 matrix

k = 15:5:25 % k is 3*1 matrix
k = repmat(k, [11 1])    % k is a 11*3 matrix
% r (11*3) matrix =
%
%         0.1000       0.1000       0.1000
%         0.1100       0.1100       0.1100
%         0.1200       0.1200       0.1200
% k (11*3) matrix =
%
%         15       20       25
%         15       20       25
%         15       20       25

% show the value of r & k

temp = (1 + r/n) .^ (n * k);
P = r * A .* temp / n ./ (temp - 1)

%% Exercise 1: Ex 6.1 in p.161
% Exercise 2: Do these methods for the Ex. 2-28 in p. 81 for L=10000:10000:50000; and
r=0.1:0.02:0.2, P=1000



%=============================================================
% 6.1.16 Multi-dimensional arrays

a = [1:2; 3:4]

% You can add a third dimension to a with
```

```
% then a is a 3-dim matrix
% with a(:,:,1)=[1:2; 3:4]
a(:,:,2) = [5:6; 7:8]

%=============================================================
% 6.2 MATRIX OPERATIONS : check Table 6.2 for the matrix operation
% 6.2.1 Matrix multiplication

a = [1 2; 3 4]
b = [5 6; 0 -1]

% Note the important difference between the pointwise operation a .* b
% and the matrix operation a * b

c = a*b
d = a.*b

e = [2 3]'
f = a * e
%=============================================================
% 6.2.2 Matrix exponentiation : check the table 6.2

a = [1 2; 3 4]
b = a^2    % it means a*a & it is different from the pointwise operation a.^2
c = a*a

% Again, note the difference between the pointwise operation a .^ 2 and the matrix
operation a ^ 2.
d = a.*2
e = a.*a

%=============================================================
% 6.3 OTHER MATRIX FUNCTIONS

a = [5 11; 11 25]

b = det(a) % determinate of a
c = eig(a)    % eigenvalues of a
```

```
d = inv(a)      % inverse of a
% Singular value decomposition.of a; if a is a square matrix, then it is
% just a eigenvalue decomposition
b=repmat(a,2,1)
[U,S,V] = svd(b)

%============================================================
% 6.4 POPULATION GROWTH: LESLIE MATRICES:    Textbook p.147

% Leslie matrix population model
n = 3;
L = zeros(n); % all elements set to zero
L(1,2) = 9;
L(1,3) = 12;
L(2,1) = 1/3;
L(3,2) = 0.5;
x = [0 0 1]'; % remember x must be a column vector!
for t = 1:24
        x = L * x;
        p(t) = sum(x);
        disp( [t x' sum(x)] ) % xi' is a row
end

figure, plot(1:15, p(1:15)), xlabel('months'), ylabel('rabbits')
hold, plot(1:15, p(1:15),'o')

[a b] = eig(L)
```

```
%================================================================
% 6.5 MARKOV PROCESSES : A random walk process
% check the textbook p. 150
% After few steps the random walk ended at either home or café
```

	Home	2	3	4	5	Café
Home	1.0000	0.3333	0	0	0	0
2	0	0	0.3333	0	0	0
3	0	0.6667	0	0.3333	0	0
4	0	0	0.6667	0	0.3333	0
5	0	0	0	0.6667	0	0
Café	0	0	0	0	0.6667	1.0000

```
n = 6;
P = zeros(n); % all elements set to zero
for i = 3:6
    P(i,i-1) = 2/3;
    P(i-2,i-1) = 1/3;
end
P(1,1) = 1;
P(6,6) = 1;
x = [0 1 0 0 0 0]'; % remember x must be a column vector!
for t = 1:50
    x = P * x;
    disp( [t x'] )
end
```

```
%============================================================
% 6.6 LINEAR EQUATIONS


A = [3 2 -1; -1 3 2; 1 -1 -1]
b = [10 5 -1]'
x = A \ b    % solve a linear equation Ax=b

% The same solution can be obtained by the following equation
z = inv(A) * b
% A \ b is actually more accurate and efficient

% 6.6.2 The residual
r = A*x - b

% 6.6.3 Over-determined systems
% more equations then unknowns (No solution case in L.A,)
% one can find the minimum meansquare solution (MMSE)
% which lead the rtotal = sqrt(r'*r), r = A*x - b minimum
A = [1 -1; 0 1; 1 0]
b = [0 2 1]'
x = A \ b

r = A*x - b
rtotal = sqrt(r'*r)
```

Example of overdetermined system. For the least square fitting of a straight line.

When $Ax = b$ has no solution, multiply by A^T and solve $A^TA\hat{x} = A^Tb$.

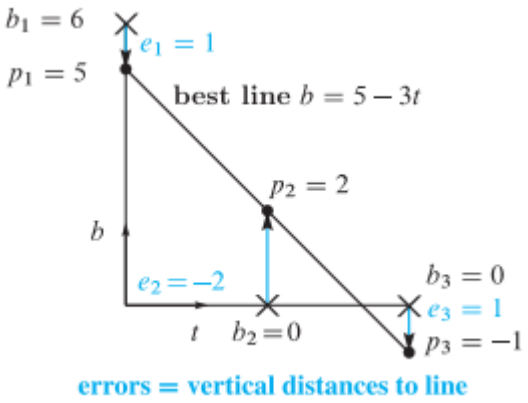
Example 1 A crucial application of least squares is fitting a straight line to m points. Start with three points: *Find the closest line to the points* $(0, 6)$, $(1, 0)$, *and* $(2, 0)$.

No straight line $b = C + Dt$ goes through those three points. We are asking for two numbers C and D that satisfy three equations. Here are the equations at $t = 0, 1, 2$ to match the given values $b = 6, 0, 0$:

$t = 0$	The first point is on the line $b = C + Dt$ if	$C + D \cdot 0 = 6$
$t = 1$	The second point is on the line $b = C + Dt$ if	$C + D \cdot 1 = 0$
$t = 2$	The third point is on the line $b = C + Dt$ if	$C + D \cdot 2 = 0$.

This 3 by 2 system has *no solution*: $\boldsymbol{b} = (6, 0, 0)$ is not a combination of the columns $(1, 1, 1)$ and $(0, 1, 2)$. Read off A, $\boldsymbol{x}$, and $\boldsymbol{b}$ from those equations:

$$A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 2 \end{bmatrix} \quad \boldsymbol{x} = \begin{bmatrix} C \\ D \end{bmatrix} \quad \boldsymbol{b} = \begin{bmatrix} 6 \\ 0 \\ 0 \end{bmatrix} \quad A\boldsymbol{x} = \boldsymbol{b} \text{ is } \textit{not} \text{ solvable.}$$



% 6.6.4 Under-determined systems (Infinity many solutions)

```
A = [1 -1 5; 3 1 2] % A 2*3 matrix; two equations, 3 unknown;
b = [2 1]'
x = A \ b
```

% 6.6.5 Ill conditioning

```
A = [10 7 8 7; 7 5 6 5; 8 6 10 9; 7 5 9 10]
b = [32 23 33 31]'
x = A \ b

% perturbed b vector
b = [32.1 22.9 32.9 31.1]'
x = A \ b

% rcond ; is an estimate for the reciprocal of the
%         condition of X in the 1-norm obtained by the LAPACK
%         condition estimator. If X is well conditioned, rcond(X)
%         is near 1.0. If X is badly conditioned, rcond(X) is
%         near EPS.
rcond(A)
```

you should always compute the residual when solving a system of equations.

- If you work with large matrices with relatively few non-zero entries you should consider using MATLAB's sparse matrix facilities.

EXERCISES

6.1 Set up any 3×3 matrix `a`. Write some command-line statements to perform the following operations on `a`:
(a) interchange columns 2 and 3;
(b) add a fourth column (of 0s);
(c) insert a row of 1s as the new second row of `a` (i.e. move the current second and third rows down);
(d) remove the second column.

6.2 Compute the limiting probabilities for the student in Section 6.5 when he starts at each of the remaining intersections in turn, and confirm that the closer he starts to the cafe, the more likely he is to end up there.
Compute `P^50` directly. Can you see the limiting probabilities in the first row?

6.3 Solve the equations

$$2x - y + z = 4$$
$$x + y + z = 3$$
$$3x - y - z = 1$$

using the left division operator. Check your solution by computing the residual. Also compute the determinant (`det`) and the condition estimator (`rcond`). What do you conclude?

6.4 This problem, suggested by R.V. Andree, demonstrates ill conditioning (where small changes in the coefficients cause large changes in the solution). Use the left division operator to show that the solution of the system

$$x + 5.000y = 17.0$$
$$1.5x + 7.501y = 25.503$$

is $x = 2$, $y = 3$. Compute the residual.
Now change the term on the right-hand side of the second equation to 25.501, a change of about one part in 12 000, and find the new solution and the residual. The solution is completely different. Also try changing this term to 25.502, 25.504, etc. If the coefficients are subject to experimental errors, the solution is clearly meaningless. Use `rcond` to find the condition estimator and `det` to compute the determinant. Do these values confirm ill conditioning?
Another way to anticipate ill conditioning is to perform a *sensitivity analysis* on the coefficients: change them all in turn by the same small percentage, and observe what effect this has on the solution.

6.5 Use `sparse` to represent the Leslie matrix in Section 6.4. Test your representation by projecting the rabbit population over 24 months.

6.6 If you are familiar with *Gauss reduction* it is an excellent programming exercise to code a Gauss reduction directly with operations on the rows of the augmented coefficient matrix. See if you can write a function

```
x = mygauss(a, b)
```

to solve the general system $\mathbf{Ax} = \mathbf{b}$. Skillful use of the colon operator in the row operations can reduce the code to a few lines!
Test it on $\mathbf{A}$ and $\mathbf{b}$ with random entries, and on the systems in Section 6.6 and Exercise 16.4. Check your solutions with left division.

```
%6.1
a=magic(3) %設 a 為一個 3*3 矩陣
b=a(:,2);c=a(:,3);%將 2,3 col 設為 a,b
a(:,2)=c;a(:,3)=b;%將 c 帶回  col2,b 帶回  col3
disp(a);
a(:,4)=[0 0 0]'%加入第 4column
a=[a(1,:) ; [1 1 1 1] ; a(2:end,:)]%插入第 2row
a(:,2)=[] %remove the second column

%6.2

n = 6;
P = zeros(n); % all elements set to zero
for i = 3:6
    P(i,i-1) = 2/3;
    P(i-2,i-1) = 1/3;
end
P(1,1) = 1;
P(6,6) = 1;
x0 = [1 0 0 0 0 0]'; % initial position of the student, remember x0 must be a column
vector!
% x0 = [0 1 0 0 0 0]'; % Try each x0 to see the result
% x0 = [0 0 1 0 0 0]';
% x0 = [0 0 0 1 0 0]';
% x0 = [0 0 0 0 1 0]';
% x0 = [0 0 0 0 0 1]';

x = x0;
for t = 1:50
    x = P*x;
    disp([t x'])
end



% Anoter way to compute the final x from the initial position x0
Pfinal = P^50 % Note the limiting probabilities in the first and the last rows
```

```
xfinal = Pfinal*x0 % Anoter way to compute the final x from the initial position x0

%6.3
A=[2 -1 1;1 1 1;3 -1 -1]
b=[4 3 1]'
x=A\b;
r=A*x-b%residual
det(A)
rcond(A)
%6.4
A=[1 5;1.5 7.501]
b=[17 25.503]'
x=A\b;%得 x=2 y=3
r=A*x-b%residual

det(A)
rcond(A)
b1=[17 25.501]' %改變方程式的值
x1=A\b1 %解答發生很大變化
r1=A*x1-b1 %residual

b2=[17 25.502]' %改變方程式的值
x2=A\b2 %解答發生很大變化
r1=A*x2-b2 %residual

b3=[17 25.504]'%改變方程式的值
x3=A\b3 %解答發生很大變化
r1=A*x3-b3 %residual

%6.6
a=[2 1 -1;-3 -1    2;-2 1 2];
b=[8 -11 -3];
x = mygauss(a, b) %x=[2 3 -1]'
% Function file mygauss.m
function x = mygauss(a, b)
n = length(a);
a(:,n+1) = b;
for k = 1:n
```

```
        a(k,:) = a(k,:)/a(k,k); % pivot element must be 1
        for i = 1:n
            if i ~= k
                a(i,:) = a(i,:) - a(i,k) * a(k,:);
            end
        end
end
% solution is in column n+1 of a:
x = a(:,n+1);
```

1. In MATLAB, input the following matrix，and use this matrix to answer the following questions :

$$\mathbf{A} = \begin{bmatrix} 3 & 7 & -4 & 12 \\ -5 & 9 & 10 & 2 \\ 6 & 13 & 8 & 11 \\ 15 & 5 & 4 & 1 \end{bmatrix}$$

a. Construct a 4 × 3 matrix **B**，its elements is the second column through 4-th column of A.
b. Construct a 3 × 4 matrix **C**，its elements is the second row through 4-th row of A.
c. Construct a 2 × 3 matrix **D**，its elements is the first two rows and last three columns of A.

2. In MATLAB, find the length and absolute value of the following vectors:
   a. **x** = [2, 4, 7]
   b. **y** = [2, –4, 7, -6]

3. Construct following matrix A and let B=ln(A'):

$$\mathbf{A} = \begin{bmatrix} 1 & 4 & 2 \\ 2 & 4 & 100 \\ 7 & 9 & 7 \\ 3 & \pi & 42 \end{bmatrix}$$

   Use MATLAB to find the following:
   a. Construct a matrix **C**，it is the transpose of A.
   b. Construct a matrix **D,** deleting **2-nd row of A.**
   c. Add a column with values 1 to the 2-nd column of D.
   d. Extracting 1st and 3th column of A and put it into the matrix E.
   e. Construct vector **x,** its elements is the only second row of **B**.
   f. Calculate the sum of all the elements of **x.**
   g. Pointwise multiplication of the 2-nd row of A and 3-th column of B.
   h. Pointwise multiplication of the 1st row of A and 3-th column of B.
   i. Find the maximum, minimum and summation values of the resulting vector in h.

4. The following table shows the cost of a product and the output of the four seasons for each business year. Using MATLAB to find (*a*) the cost of materials, labor, and transportation for each season. (b) the total cost of materials, labor, and transportation for each year. And (*c*) total costs of each season.

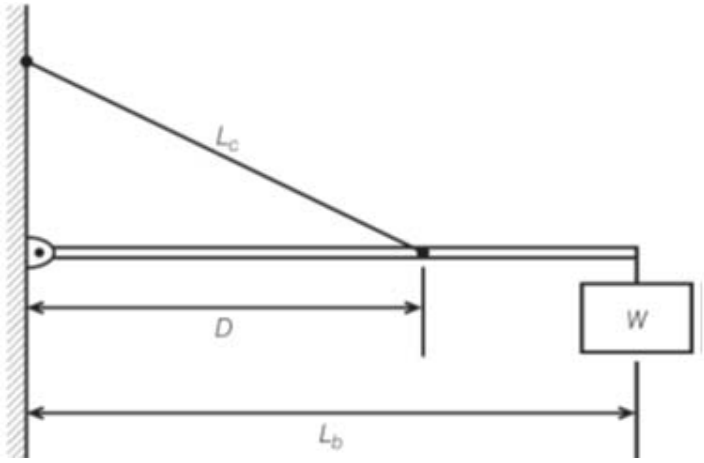
	Cost of a product ($*1000)		
product	materials	labor	transportation
1	7	3	2
2	3	1	3
3	9	4	5
4	2	5	4
5	6	2	1

	Total number of product for each season			
Product	1st season	2nd season	3th season	4th season
1	16	14	10	12
2	12	15	11	13
3	8	9	7	11
4	14	13	15	17
5	13	16	12	18

5. Cables of length L_c support a beam column of length L_b that remains level when the end of the beam column hangs an object with weight W. According to the law of statics, the tension T of this cable can be calculated

$$T = \frac{L_b L_c W}{D\sqrt{L_b^2 - D^2}}$$

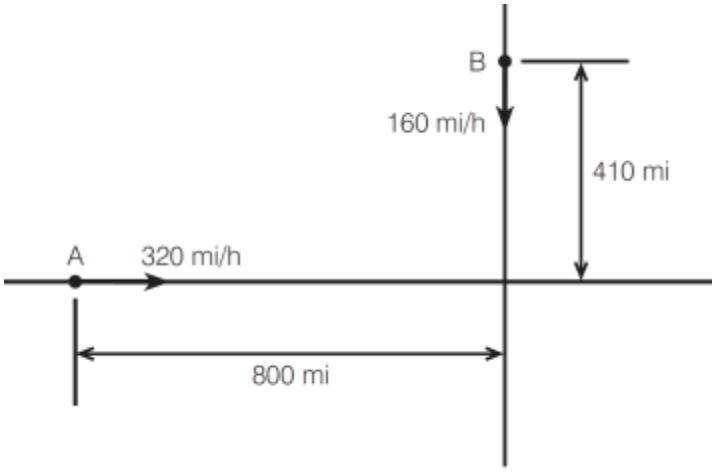
Where D is the distance from the connection cable to the beam column. Refer to the following figure



(a) Given that $W = 400$ N，$L_b = 3$ m, and $L_c = 5$ m，use pointwise operation and 'min' function to find the distance D which lead to the minimum tension T.

(b) Plot the relationship between the tension T and the distance D.

6. Aircraft **A** flew east at a rate of 320 mi/hr and aircraft **B** flew south at a rate of 160 mi/hr. The position of the aircraft at 1 pm is shown in the following Figure.



(a) Find the distance D between two airplanes as a function of time. D is plotted against time until D reaches its minimum value.

7. Given the following matrices

$$\mathbf{A} = \begin{bmatrix} 4 & -2 & 1 \\ 6 & 8 & -5 \\ 7 & 9 & 10 \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} 6 & 9 & -4 \\ 7 & 5 & 3 \\ -8 & 2 & 1 \end{bmatrix} \quad \mathbf{C} = \begin{bmatrix} -4 & -5 & 2 \\ 10 & 6 & 1 \\ 3 & -9 & 8 \end{bmatrix}$$

Use the MATLAB to verify the following properties:

(a) The association property:

$$A(B + C) = AB + AC$$

(b) The distribution property :

$$(AB)C = A(BC)$$