

1.

8.2 There are many formulae for computing π (the ratio of a circle's circumference to its diameter). The simplest is

$$\frac{\pi}{4} = 1 - 1/3 + 1/5 - 1/7 + 1/9 - \dots \quad (8.4)$$

which comes from putting $x = 1$ in the series

$$\arctan x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \frac{x^9}{9} - \dots \quad (8.5)$$

(a) Write a program to compute π using Equation (8.4). Use as many terms in the series as your computer will reasonably allow (start modestly, with 100 terms, say, and re-run your program with more and more each time). You should find that the series converges very slowly, i.e. it takes a lot of terms to get fairly close to π .

(b) Rearranging the series speeds up the convergence:

$$\frac{\pi}{8} = \frac{1}{1 \times 3} + \frac{1}{5 \times 7} + \frac{1}{9 \times 11} \dots$$

Write a program to compute π using this series instead. You should find that you need fewer terms to reach the same level of accuracy that you got in (a).

(c) One of the fastest series for π is

$$\frac{\pi}{4} = 6 \arctan \frac{1}{8} + 2 \arctan \frac{1}{57} + \arctan \frac{1}{239}.$$

Use this formula to compute π . Don't use the MATLAB function `atan` to compute the arctangents, since that would be cheating. Rather use Equation (8.5).

2.

- 8.7 If an amount of money A is invested for k years at a nominal annual interest rate r (expressed as a decimal fraction), the value V of the investment after k years is given by

$$V = A(1 + r/n)^{nk}$$

where n is the number of compounding periods per year. Write a program to compute V as n gets larger and larger, i.e. as the compounding periods become more and more frequent, like monthly, daily, hourly, etc. Take $A = 1000$, $r = 4\%$ and $k = 10$ years. You should observe that your output gradually approaches a limit. **Hint:** use a for loop which doubles n each time, starting with $n = 1$.

Also compute the value of the formula Ae^{rk} for the same values of A , r and k (use the MATLAB function `exp`), and compare this value with the values of V computed above. What do you conclude?

Hint: As n approaches infinity, Ae^{rk} is the limit of V .

Using **`abs(Aerk - V(n))`** with **`epsilon 1e-4`** to determine the value of n .

3.

- 8.8 Write a program to compute the sum of the series $1^2 + 2^2 + 3^2 \dots$ such that the sum is as large as possible without exceeding 1000. The program should display how many terms are used in the sum.

4.

- 8.11 Use the Taylor series

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots$$

to write a program to compute $\cos x$ correct to four decimal places (x is in radians). See how many terms are needed to get 4-figure agreement with the MATLAB function `cos`. Don't make x too large; that could cause rounding error.

Note: choose **`x=3`**

5.

- 8.13 A projectile, the equations of motion of which are given in Chapter 3, is launched from the point O with an initial velocity of 60 m/s at an angle of 50° to the horizontal. Write a program which computes and displays the time in the air, and horizontal and vertical displacement from the point O every 0.5 s, as long as the projectile remains above a horizontal plane through O.