APP_Simple Harmonic Motion

1. Objective:

The objective of this experiment is to study the simple harmonic motion of objects in a spring system and measure the force constants and periods.

2. Theory:

Suppose there is an object with mass m hanging from a spring. When the object is moved a distance x from its equilibrium position, the restoring force of the spring F is proportional to x and acts opposite to the direction of the displacement of the object. This is known as Hooke's law.

$$F = -kx \tag{1}$$

In the formula above, the constant of proportionality k is called the spring constant. Below we have Newton's second law:

$$F = ma = m\frac{d^2x}{dt^2} \tag{2}$$

From Eqs. (1) and (2), we can derive

$$m\frac{d^2x}{dt^2} = -kx \tag{3}$$

The solution to this quadratic differential equation is

$$x = A\sin(\omega t + \varphi) \tag{4}$$

$$\omega = \sqrt{\frac{k}{m}} \tag{5}$$

where A is the amplitude (maximum displacement), and ω is the angular velocity. The period of motion is

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}} \tag{6}$$

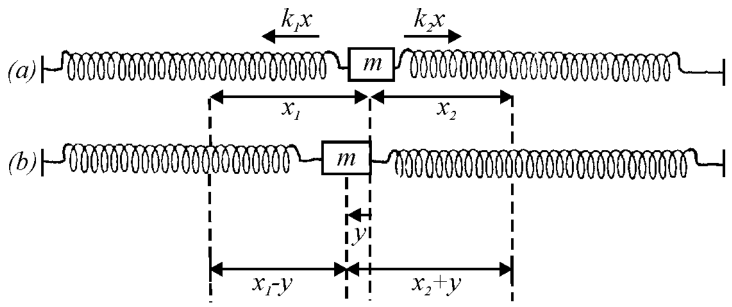

Fig. 1

Figure 1 presents a double spring system; k_1 and k_2 denote the force constants of the left and right springs, respectively, and x_1 and x_2 represent the amounts by which the springs have stretched. Figure 1(a) shows the state of equilibrium, where

$$k_1 x_1 = k_2 x_2 \tag{7}$$

If m is moved horizontally a small distance and then released, it will oscillate back and forth in simple harmonic motion. Suppose at a certain moment, the displacement of m from its equilibrium position is y. Then, as shown in Fig. 1(b), the horizontal net force F of m is

$$F = k_1(x_1 - y) - k_2(x_2 + y) \tag{8}$$

Substituting Eq. (7) into Eq. (8) gives

$$F = -(k_1 + k_2)y \tag{9}$$

Eq. (9) can be understood physically in the following way. In Fig. 1(a), the object is in a force-free (the net force equals 0) state of static equilibrium, whereas in Fig. 1(b), the object has moved to the left a distance y from its equilibrium position. The spring on the left will therefore push the object right with a force of $k_1 y$, and the spring on the right will pull the object right with a force of $k_2 y$. As a result, the object is subject to a net force of ($k_1 y + k_2 y$) towards the right.

In Eq. (3), we can define the force constant $\boldsymbol{K}$ of the spring system as

$$\boldsymbol{K} = k_1 + k_2 \tag{10}$$

Since the spring system follows Hooke's law,

$$F = -\boldsymbol{K}y \tag{11}$$

Similarly, we can derive the period T of the system as

$$T = 2\pi\sqrt{\frac{m}{\boldsymbol{K}}} = 2\pi\sqrt{\frac{m}{k_1 + k_2}} \tag{12}$$

After measuring the restoring force F in the two springs at different displacements x, we can use Eq. (1) to obtain the force constant k. The theoretical period of the double spring system can be derived from Eq. (12) and will be used to verify the periods measured in the experiment.

3. Apparatus:

air track, air pump, sliding car, springs, scale, smartphone, magnet, stand, weights

4. Procedure

4.1 Find the force constant k of each spring

(1) Setup a stand, a smartphone and a spring with a weight and a magnet attached to it as shown in Fig. 2.

Fig. 2

(2) Open the APP and use the magnet to move over the smartphone to find a point where the magnetic field reading was the highest. This position is where the magnetic sensor is located.

(3) Stretch a spring with 2 cm amplitude and use a smartphone to detect the magnetic flux density. We can get the chart of magnetic flux density versus time from APP and use it to calculate the period T. After measuring the weight of the whole spring set M, we can get the force constant K.

(4) Repeat above steps with another spring to get the force constant K of another spring.

4.2 Find the period T of the simple harmonic motion in the double spring system.

(1) Setup a track, a smartphone and a cart with a magnet attached to it as shown in Fig. 3

(2) Measure and record the weight of the entire sliding cart (including the magnet).

Fig. 3

(3) Turn on the air pump, and adjust the air track so that it is horizontally level. (One approach is to place the cart in the middle of the track and see if it slides towards either end.)

(4) Connect one end of the two springs to the sliding cart and the other to the hooks at either end of the track. The point at which the cart is not moving on the track is the equilibrium point.

(5) Let the cart move back and forth with 3 cm amplitude and use a smartphone to detect the magnetic flux density. We can get the chart of magnetic flux density versus time from APP and use it to calculate the period T_{ex}. We can get the theoretical period T_{th} from equation (12). Compare this experimental period T_{ex} to the theoretical period T_{th} and calculate the percent error

(6) Place different weights on the sliding cart, and repeat Steps (5) to derive the experimental data and statistics for three more cart masses.

5. Questions

1. What are the errors between the measured T values and the theoretical T values in the experiments for the sliding cart and for the sliding cart with weights? What causes these errors?
2. How does the period change as the mass of the weights increases or decreases? Please explain.
3. How does the period change when the amplitude changes?