7. **Q|C** Two small beads having positive charges $q_1 = 3q$ and $q_2 = q$ are fixed at the opposite ends of a horizontal insulating rod of length $d = 1.50$ m. The bead with charge q_1 is at the origin. As shown in Figure P19.7, a third small, charged bead is free to slide on the rod. (a) At what position x is the third bead in equilibrium? (b) Can the equilibrium be stable?

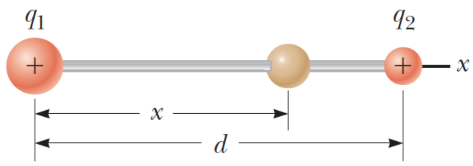


Figure P19.7 Problems 7 and 8.

P19.7 (a) Let the third bead have charge Q and be located distance x from the left end of the rod. This bead will experience a net force given by

$$\vec{\mathbf{F}} = \frac{k_e(3q)Q}{x^2}\hat{\mathbf{i}} + \frac{k_e(q)Q}{(d-x)^2}(-\hat{\mathbf{i}}), \text{ where } d = 1.50 \text{ m}$$

The net force will be zero if $\dfrac{3}{x^2} = \dfrac{1}{(d-x)^2}$, or $d - x = \dfrac{x}{\sqrt{3}}$.

This gives an equilibrium position of the third bead of

$$x = 0.634d = 0.634(1.50 \text{ m}) = \boxed{0.951 \text{ m}}$$

(b) $\boxed{\text{Yes, if the third bead has positive charge.}}$ The equilibrium would be stable because if charge Q were displaced either to the left or right on the rod, the new net force would be opposite to the direction Q has been displaced, causing it to be pushed back to its equilibrium position.

9. Three charged particles are located at the corners of an equilateral triangle as shown in Figure P19.9. Calculate the total electric force on the 7.00-μC charge.

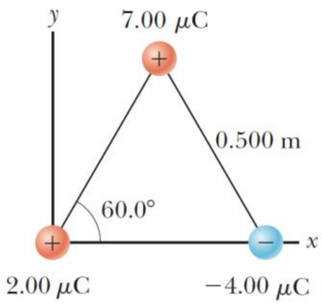


Figure P19.9

P19.9 The force exerted on the 7.00-μC charge by the 2.00-μC charge is

$$\vec{\mathbf{F}}_1 = k_e \frac{q_1 q_2}{r^2}\hat{\mathbf{r}}$$
$$= \frac{(8.99\times10^9\ \mathrm{N\cdot m^2/C^2})(7.00\times10^{-6}\ \mathrm{C})(2.00\times10^{-6}\ \mathrm{C})}{(0.500\ \mathrm{m})^2} \times(\cos 60°\hat{\mathbf{i}} + \sin 60°\hat{\mathbf{j}})$$
$$\vec{\mathbf{F}}_1 = (0.252\ \hat{\mathbf{i}} + 0.436\ \hat{\mathbf{j}})\ \mathrm{N}$$

Similarly, the force on the 7.00-μC charge by the –4.00-μC charge is

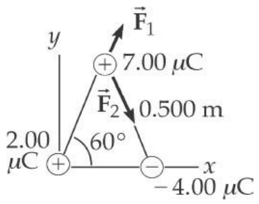
$$\mathbf{F}_2 = k_e \frac{q_1 q_3}{r^2}\hat{\mathbf{r}}$$
$$= -\frac{(8.99\times10^9\ \mathrm{N\cdot m^2/C^2})(7.00\times10^{-6}\ \mathrm{C})(-4.00\times10^{-6}\ \mathrm{C})}{(0.500\ \mathrm{m})^2} \times(\cos 60°\hat{\mathbf{i}} - \sin 60°\hat{\mathbf{j}})$$
$$\vec{\mathbf{F}}_2 = (0.503\hat{\mathbf{i}} - 0.872\ \hat{\mathbf{j}})\ \mathrm{N}$$

Thus, the total force on the 7.00-μC charge is

$$\vec{\mathbf{F}} = \vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2 = (0.755\ \hat{\mathbf{i}} - 0.436\ \hat{\mathbf{j}})\ \mathrm{N}$$

We can also write the total force as:

$$\vec{\mathbf{F}} = (0.755\ \mathrm{N})\hat{\mathbf{i}} - (0.436\ \mathrm{N})\hat{\mathbf{j}} = \boxed{0.872\ \mathrm{N\ at\ an\ angle\ of\ 330°}}$$



ANS. FIG. P19.9

17. In Figure P19.17, determine the point (other than infinity) at which the electric field is zero.

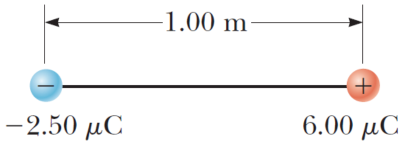
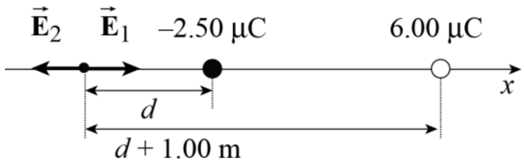


Figure P19.17

P19.17 The point is designated in ANS. FIG P19.17. The magnitudes of the electric fields, E_1 (due to the –2.50-μC charge) and E_2 (due to the 6.00-μC charge), are



ANS. FIG. P19.17

$$E_1 = \frac{k_e q}{r^2} = \frac{(8.99\times 10^9\ \mathrm{N\cdot m^2/C^2})(2.50\times 10^{-6}\ \mathrm{C})}{d^2} \qquad \textbf{[1]}$$

$$E_2 = \frac{k_e q}{r^2} = \frac{(8.99\times 10^9\ \mathrm{N\cdot m^2/C^2})(6.00\times 10^{-6}\ \mathrm{C})}{(d+1.00)^2} \qquad \textbf{[2]}$$

where d is in meters. Equate the right sides of [1] and [2] to get

$$(d+1.00)^2 = 2.40d^2$$

or $\quad d + 1.00 = \pm 1.55d$

which yields $\quad d = 1.82$ m or $d = -0.392$ m.

The negative value for d is unsatisfactory because that locates a point between the charges where both fields are in the same direction.

Thus, $d = \boxed{1.82\text{ m to the left of the } -2.50\text{-}\mu\text{C charge}}$.

18. **S** A thin rod of length ℓ and uniform charge per unit length λ lies along the x axis as shown in Figure P19.18. (a) Show that the electric field at P, a distance y from the rod along its perpendicular bisector, has no x component and is given by $E = 2k_e\lambda \sin\theta_0 / y$. (b) Using your result to part (a), show that the field of a rod of infinite length is $E = 2k_e\lambda / y$. (*Suggestion:* First, calculate the field at P due to an element of length dx, which has a charge $\lambda\, dx$. Then, change variables from x to θ, using the relationships $x = y\tan\theta$ and $dx = y\sec^2\theta\, d\theta$, and integrate over θ.)

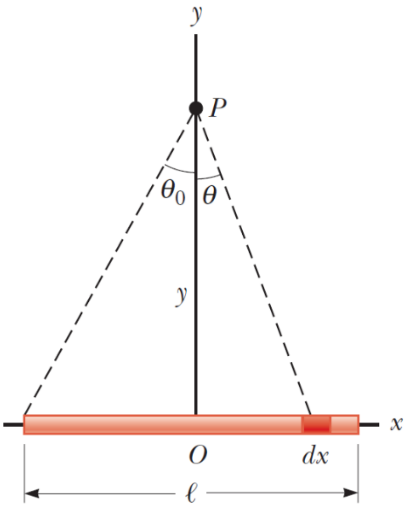
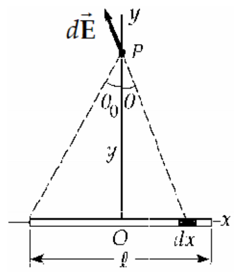


Figure P19.18

P19.18 (a) The electric field at point P, due to each element of length dx, is $dE = \dfrac{k_e dq}{x^2 + y^2}$ and is directed along the line joining the element to point P. By symmetry,

$$E_x = \int dE_x = 0$$



ANS. FIG. P19.18

and since $dq = \lambda dx$,

$$E = E_y = \int dE_y = \int dE\cos\theta$$

where $\cos\theta = \dfrac{y}{\sqrt{x^2 + y^2}}$.

Therefore, $E = 2k_e\lambda y \displaystyle\int_0^{\ell/2} \frac{dx}{(x^2 + y^2)^{3/2}} = \boxed{\dfrac{2k_e\lambda \sin\theta_0}{y}}$.

(b) For a bar of infinite length, $\theta_0 = 90°$ and $E_y = \boxed{\dfrac{2k_e\lambda}{y}}$.

21. A uniformly charged insulating rod of length 14.0 cm is bent into the shape of a semicircle as shown in Figure P19.21. The rod has a total charge of $-7.50\ \mu$C. Find (a) the magnitude and (b) the direction of the electric field at O, the center of the semicircle.

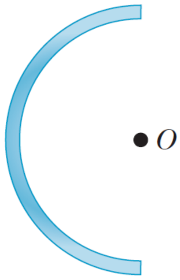
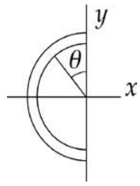


Figure P19.21

P19.21 Due to symmetry, $E_y = \int dE_y = 0$, and

$E_x = -\int dE \sin\theta = -k_e \int \frac{dq \sin\theta}{r^2}$ where $dq = \lambda ds = \lambda r d\theta$; the component E_x is negative because charge $q = -7.50\ \mu$C, causing the net electric field to be directed to the left.



ANS. FIG. P19.21

$$E_x = -\frac{k_e \lambda}{r}\int_0^\pi \sin\theta d\theta = -\frac{k_e \lambda}{r}(-\cos\theta)\Big|_0^\pi = -\frac{2k_e \lambda}{r}$$

where $\lambda = \frac{|q|}{L}$ and $r = \frac{L}{\pi}$. Thus,

$$E_x = -\frac{2k_e |q| \pi}{L^2} = -\frac{2(8.99\times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(7.50\times 10^{-6}\ \text{C})\pi}{(0.140\ \text{m})^2}$$

$$E_x = -2.16\times 10^7\ \text{N/C}$$

(a) magnitude $E = \boxed{2.16\times 10^7\ \text{N/C}}$

(b) $\boxed{\text{to the left}}$

22. **Q|C** **S** Two charged particles are located on the x axis. The first is a charge $+Q$ at $x = -a$. The second is an unknown charge located at $x = +3a$. The net electric field these charges produce at the origin has a magnitude of $2k_eQ/a^2$. Explain how many values are possible for the unknown charge and find the possible values.

P19.22 The first charge creates at the origin a field $\frac{k_eQ}{a^2}$ to the right. Both charges are on the x axis, so the total field cannot have a vertical

ANS. FIG. P19.22

component, but it can be either to the right or to the left. If the total field at the origin is to the right, then q must be negative:

$$\frac{k_e Q}{a^2}\hat{\mathbf{i}} + \frac{k_e q}{(3a)^2}\left(-\hat{\mathbf{i}}\right) = \frac{2k_e Q}{a^2}\hat{\mathbf{i}} \quad \rightarrow \quad q = -9Q$$

In the alternative, if the total field at the origin is to the left,

$$\frac{k_e Q}{a^2}\hat{\mathbf{i}} + \frac{k_e q}{9a^2}\left(-\hat{\mathbf{i}}\right) = \frac{2k_e Q}{a^2}\left(-\hat{\mathbf{i}}\right) \quad \rightarrow \quad q = +27Q$$

The field at the origin can be to the right, if the unknown charge is $-9Q$, or the field can be to the left, if and only if the unknown charge is $+27Q$.

30. Figure P19.30 shows the electric field lines for two charged particles separated by a small distance. (a) Determine the ratio q_1/q_2. (b) What are the signs of q_1 and q_2?

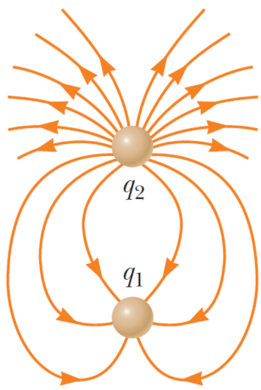


Figure P19.30

P19.30 Field lines emerge from positive charge and enter negative charge.

(a) The number of field lines emerging from positive q_2 and entering negative charge q_1 is proportional to their charges:

$$\frac{q_1}{q_2} = \frac{-6}{18} = \boxed{-\frac{1}{3}}$$

(b) From above, $\boxed{q_1 \text{ is negative, } q_2 \text{ is positive}}$.

38. **S** An infinitely long line charge having a uniform charge per unit length λ lies a distance d from point O as shown in Figure P19.38. Determine the total electric flux through the surface of a sphere of radius R centered at O resulting from this line charge. Consider both cases, where (a) $R < d$ and (b) $R > d$.

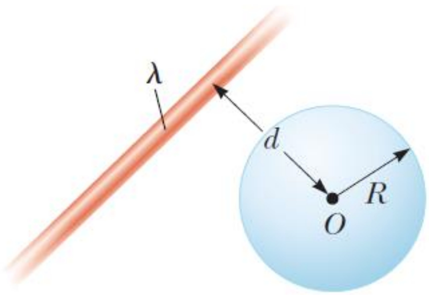


Figure P19.38

P19.38 (a) If $R \le d$, the sphere encloses no charge and $\Phi_E = \frac{q_{\text{in}}}{\epsilon_0} = \boxed{0}$.

(b) If $R > d$, the length of line falling within the sphere is

$2\sqrt{R^2 - d^2}$,

$$\text{so} \quad \Phi_E = \boxed{\frac{2\lambda\sqrt{R^2 - d^2}}{\epsilon_0}}.$$

42. **S** A particle with charge Q is located at the center of a cube of edge L. In addition, six other identicala charged particles q are positioned symmetrically around Q as shown in Figure P19.41. For each of these particles, q is a negative number. Determine the electric flux through one face of the cube.

P19.42 The total charge is $Q - 6|q|$. The total outward flux from the cube is

$\frac{Q - 6|q|}{\epsilon_0}$, of which one-sixth goes through each face:

$$(\Phi_E)_{\text{one face}} = \boxed{\frac{Q - 6|q|}{6\,\epsilon_0}}$$

52. **GP** **S** A solid, insulating sphere of radius a has a uniform charge density throughout its volume and a total charge Q. Concentric with this sphere is an uncharged, conducting, hollow sphere whose inner and outer radii are b and c as shown in Figure P19.52. We wish to understand completely the charges and electric fields at all locations. (a) Find the charge contained within a sphere of radius $r < a$. (b) From this value, find the magnitude of the electric field for $r < a$.

(c) What charge is contained within a sphere of radius r when $a < r < b$? (d) From this value, find the magnitude of the electric field for r when $a < r < b$. (e) Now consider r when $b < r < c$. What is the magnitude of the electric field for this range of values of r ? (f) From this value, what must be the charge on the inner surface of the hollow sphere? (g) From part (f), what must be the charge on the outer surface of the hollow sphere? (h) Consider the three spherical surfaces of radii a, b, and c. Which of these surfaces has the largest magnitude of surface charge density?

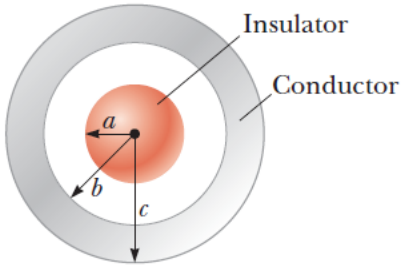
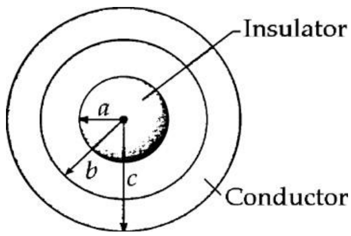


Figure P19.52

P19.52 Choose as each gaussian surface a concentric sphere of radius r. The electric field will be perpendicular to its surface, and will be uniform in strength over its surface. The density of charge in the insulating sphere is

$$\rho = Q/\left(\tfrac{4}{3}\pi a^3\right)$$



ANS. FIG. P19.52

(a) The sphere of radius $r < a$ encloses charge

$$q_{\text{in}} = \rho\left(\frac{4}{3}\pi r^3\right) = \left(\frac{Q}{\frac{4}{3}\pi R^3}\right)\left(\frac{4}{3}\pi r^3\right) = \boxed{Q\left(\frac{r}{R}\right)^3}$$

(b) Applying Gauss's law to this sphere reveals the magnitude of the field at its surface.

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{q_{\text{in}}}{\epsilon_0}$$

$$E\left(4\pi r^2\right) = \frac{Q}{\epsilon_0}\left(\frac{r}{a}\right)^3 \to E = \frac{1}{4\pi\epsilon_0}\frac{Qr}{a^3} = \boxed{k_e\frac{Qr}{a^3}}$$

(c) For a sphere of radius r with $a < r < b$, the whole insulating sphere is enclosed, so the charge within is Q: $q_{in} = \boxed{Q}$.

(d) Gauss's law for this sphere becomes:

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{q_{in}}{\epsilon_0}$$

$$E\left(4\pi r^2\right) = \frac{Q}{\epsilon_0} \rightarrow E = \frac{1}{4\pi \epsilon_0} \frac{Q}{r^2} = \boxed{k_e \frac{Q}{r^2}}$$

(e) For $b \le r \le c$, $\boxed{E = 0}$ because there is no electric field inside a conductor.

(f) For $b \le r \le c$, we know $E = 0$. Assume the inner surface of the hollow sphere holds charge Q_{inner}. By Gauss's law,

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{q_{in}}{\epsilon_0}$$

$$0 = \frac{Q + Q_{inner}}{\epsilon_0} \rightarrow Q_{inner} = \boxed{-Q}$$

(g) The total charge on the hollow sphere is zero; therefore, charge on the outer surface is opposite to that on the inner surface:

$Q_{outer} = -Q_{inner} = \boxed{+Q}$.

(h) A surface of area A holding charge Q has surface charge $\sigma = q/A$. The solid, insulating sphere has small surface charge because its total charge Q is uniformly distributed throughout its volume. The $\boxed{\text{inner surface of radius } b}$ has smaller surface area, and therefore larger surface charge, than the outer surface of radius c.

55. A solid conducting sphere of radius 2.00 cm has a charge 8.00 μC. A conducting spherical shell of inner radius 4.00 cm and outer radius 5.00 cm is concentric with the solid sphere and has a total charge −4.00 μC. Find the electric field at (a) r = 1.00 cm, (b) r = 3.00 cm, (c) r = 4.50 cm, and (d) r = 7.00 cm from the center of this charge configuration.

P19.55 (a) $\vec{\mathbf{E}} = \boxed{0}$

(b)

$$E = \frac{k_e Q}{r^2} = \frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(8.00\times10^{-6}\ \text{C}\right)}{\left(0.030\,0\ \text{m}\right)^2} = 7.99\times10^7\ \text{N/C}$$

$$\vec{\mathbf{E}} = \boxed{79.9\ \text{MN/C radially outward}}$$

(c) $\vec{\mathbf{E}} = \boxed{0}$

(d)

$$E = \frac{k_e Q}{r^2} = \frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(4.00\times10^{-6}\ \text{C}\right)}{\left(0.070\,0\ \text{m}\right)^2} = 7.34\times10^6\ \text{N/C}$$

$$\vec{\mathbf{E}} = \boxed{7.34\ \text{MN/C radially outward}}$$