11/16 Evaluate

$\iint_D xy \, dA$ where

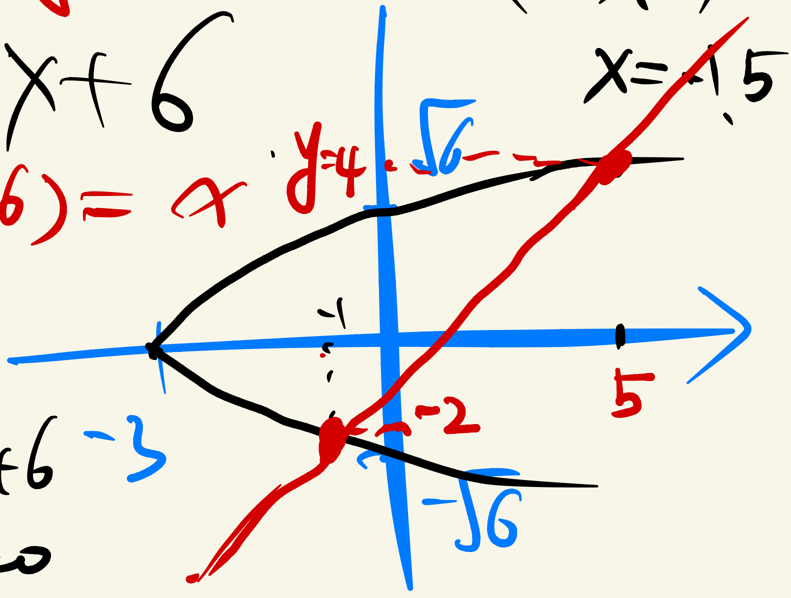
D: region bounded by $y = x - 1$ and $y^2 = 2x + 6$

$y + 1 = x$

$(x-5)(x+1) = 0$

$x = -1, 5$

$\frac{1}{2}(y^2 - 6) = x$



$(x-1)^2 = 2x + 6$

$x^2 - 2x + 1 = 2x + 6$

$x^2 - 4x - 5 = 0$

$$D = \left\{ (x, y) \,\middle|\, \begin{matrix} \frac{1}{2}y^2 - 3 \le x \le y+1 \\ -2 \le y \le 4 \end{matrix} \right\}$$

$$\iint_D xy \, dA$$

$$= \int_{-2}^{4} \int_{\frac{1}{2}y^2-3}^{y+1} xy \, dx \, dy$$

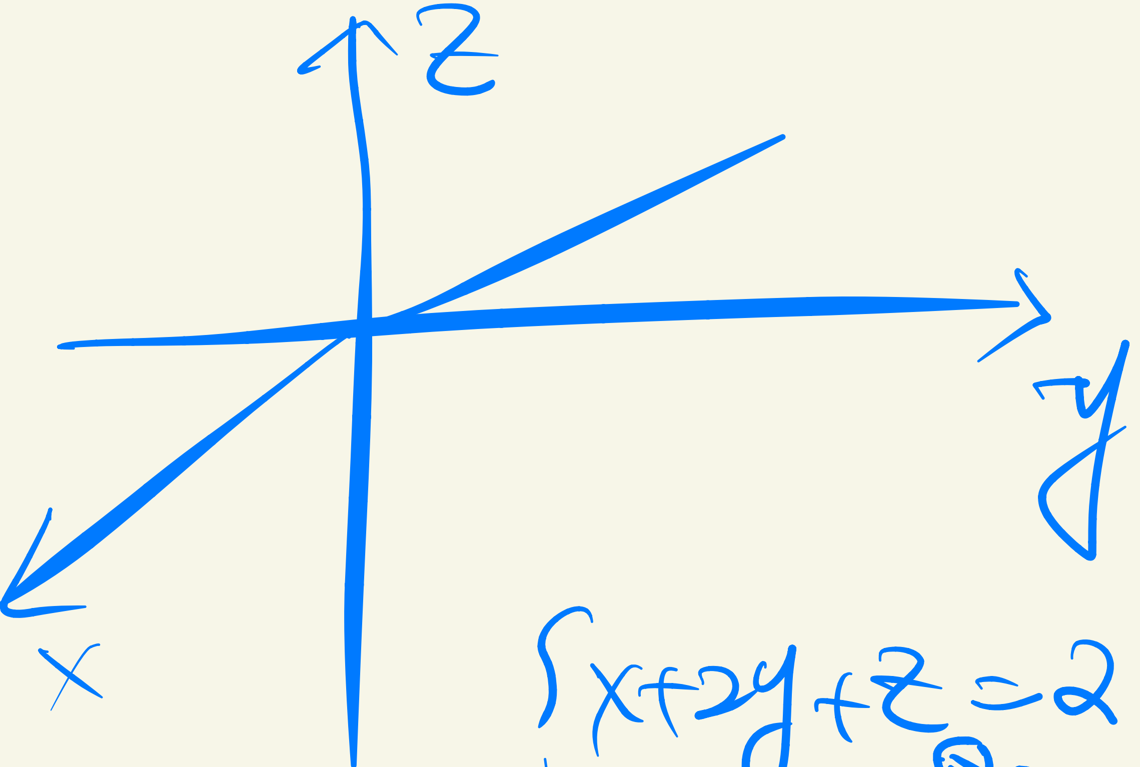
$$= \int_{-2}^{4} \left(\frac{1}{2}x^2 \Big|_{(\frac{1}{2}y^2-3)}^{y+1} \right) y \, dy$$

$$= 36.$$

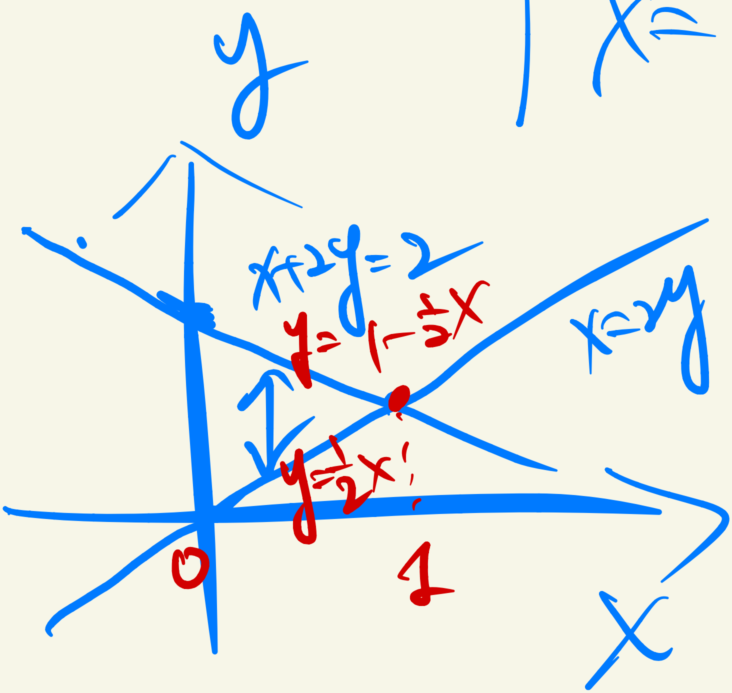
Ex4 Find the volume of tetrahedron bounded by the planes.

$x+2y+z=2$, $x=2y$

$x=0$ (yz plane) $z=0$ (xy plane)



$$\begin{cases} x+2y+\underset{\geq 0}{z}=2 \\ x=2y \end{cases}$$



$\boxed{x+2y=2}$

$x=2y$

$4y=2$

$x=1$ $\quad y=\frac{1}{2}$

$$z = 2 - x - 2y$$

$\underline{\qquad\qquad\qquad}$ function as height

$$V = \iint_D 2 - x - 2y \, dA$$

$$= \int_0^1 \int_{y=\frac{1}{2}x}^{y=1-\frac{1}{2}x} 2 - x - 2y \, dy \, dx$$

$$= \int_0^1 \left((2-x)y - y^2 \Big|_{\frac{x}{2}}^{1-\frac{x}{2}} \right) dx$$

$$= \int_0^1 (2-x)(1-x) - \left[\left(1-\frac{x}{2}\right)^2 - \left(\frac{x}{2}\right)^2 \right] dx$$

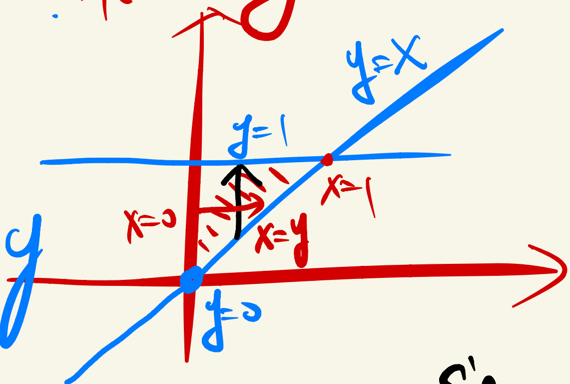
$$= \int_0^1 2 - 3x + x^2 - (1-x) \, dx$$

$$= \int_0^1 x^2 - 2x + 1 \, dx = \frac{x^3}{3} - x^2 + x \Big|_0^1$$

$$= \frac{1}{3}$$

Ex5. Evaluate the iterated integral $\displaystyle\int_{x=0}^{x=1}\int_{y=x}^{y=1} \sin(y^2)\,dy\,dx$

$$= \int_{y=0}^{y=1}\int_{x=0}^{x=y} \sin(y^2)\,dx\,dy$$



$$= \int_0^1\int_0^y \sin(y^2)\,dx\,dy$$

$$= \int_0^1 \left(x\Big|_0^y\right) \sin y^2\,dy$$

sin

$-\cos x$ cos

$-\sin x$

$$= \int_0^1 y \sin y^2\,dy = -\frac{1}{2}\cos(y^2)\Big|_0^1$$

$$= -\frac{1}{2}\cos 1 + \frac{1}{2}\cos 0 = \frac{1}{2}(1-\cos 1)$$

	#
G1	55
2	57
3	59
4	56
5	58
6	60

of the solid.

51. Under the surface $z = x^3y^4 + xy^2$ and above the region bounded by the curves $y = x^3 - x$ and $y = x^2 + x$ for $x \geqslant 0$

52. Between the paraboloids $z = 2x^2 + y^2$ and $z = 8 - x^2 - 2y^2$ and inside the cylinder $x^2 + y^2 = 1$

53. Enclosed by $z = 1 - x^2 - y^2$ and $z = 0$

54. Enclosed by $z = x^2 + y^2$ and $z = 2y$

55–60 Sketch the region of integration and change the order of integration.

55. $\int_0^1 \int_0^y f(x, y)\, dx\, dy$

56. $\int_0^2 \int_{x^2}^4 f(x, y)\, dy\, dx$

57. $\int_0^{\pi/2} \int_{\sin x}^1 f(x, y)\, dy\, dx$

58. $\int_{-2}^2 \int_0^{\sqrt{4-y^2}} f(x, y)\, dx\, dy$

59. $\int_1^2 \int_0^{\ln x} f(x, y)\, dy\, dx$

60. $\int_0^1 \int_{\arctan x}^{\pi/4} f(x, y)\, dy\, dx$

61–66 Evaluate the integral by reversing the order of integration.

61. $\int_0^1 \int_{3y}^3 e^{x^2}\, dx\, dy$

62. $\int_0^1 \int_{x^2}^1 \sqrt{y} \sin y\, dy\, dx$

63. $\int_0^1 \int_{\sqrt{x}}^1 \sqrt{y^3 + 1}\, dy\, dx$

64. $\int_0^2 \int_{y/2}^1 y \cos(x^3 - 1)\, dx\, dy$

65. $\int_0^1 \int^{\pi/2} \cos x \sqrt{1 + \cos^2 x}\, dx\, dy$

#59

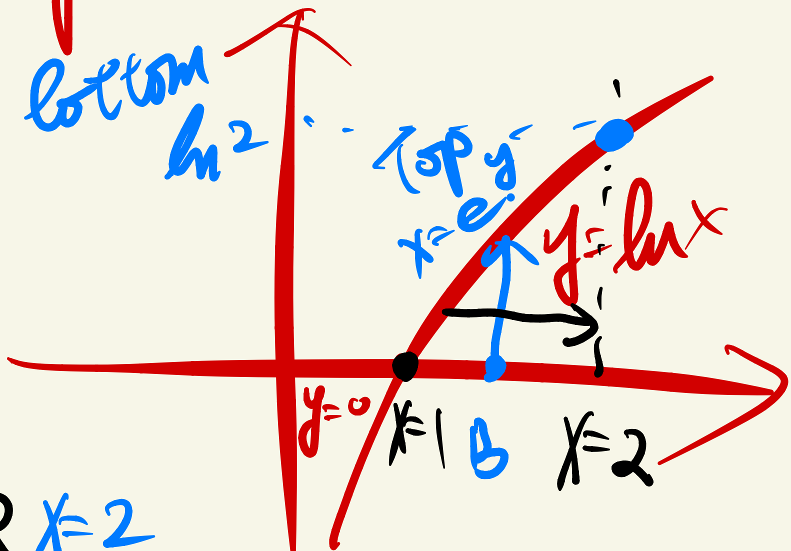
Top

$$\int_{x=1}^{x=2} \int_{y=0}^{y=\ln x} f(x,y)\, dy\, dx$$

R

L

bottom



T R

$$\int_{y=0}^{y=\ln 2} \int_{x=e^y}^{x=2} f(x,y)\, dx\, dy$$

B L

61 $\int_0^1 \int_{3y}^3 e^{x^2}\,dx\,dy$