112-2
Electrical Engineering Fundamentals II

Test 2

Keys

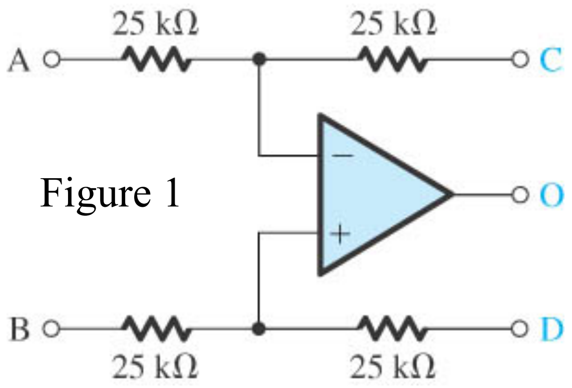
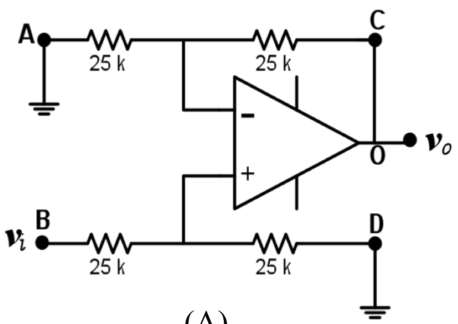


Figure 1

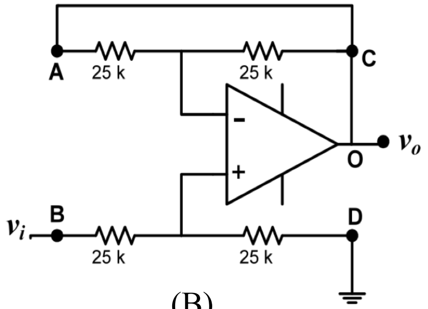
1. 10% The circuit shown in Fig. 1 is a representation of a versatile, commercially available IC, the INA 105, manufactured by Burr-Brown and known as a differential amplifier module. It consists of an OP Amp and precision, laser-trimmed, metal-film resistors. The circuit can be configured for a variety of applications by the appropriate connection of terminals, A, B, C, D, and O.

Derive the implemented gain $\frac{v_O}{v_i}$ for (A)~(D).



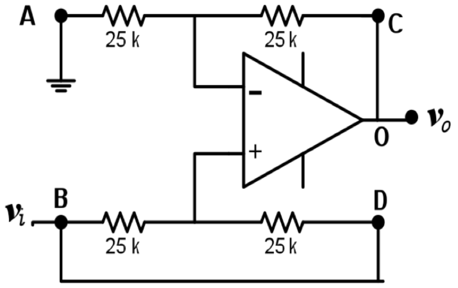
(A)

1



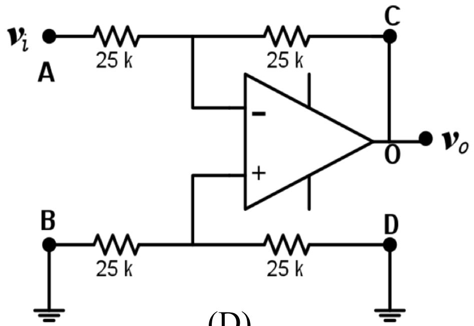
(B)

1/2



(C)

2



(D)

-1

2. (10%) Derive V_O in terms of $V_1 \sim V_4$ for the mathematical operation in the circuit of Figure 2.

$$v_+ = V_1 \times \frac{10 \parallel 20}{20 + 10 \parallel 20} + V_2 \times \frac{20 \parallel 20}{10 + 20 \parallel 20}$$
$$= \frac{1}{4} \cdot V_1 + \frac{1}{2} \cdot V_2$$

$$V_O = v_+ \times \left(1 + \frac{40}{10 \parallel 10}\right) - V_3 \times \frac{40}{10} - V_4 \times \frac{40}{10}$$
$$= \left(\frac{1}{4} \cdot V_1 + \frac{1}{2} \cdot V_2\right) \times 9 - 4 \cdot V_3 - 4 \cdot V_4$$
$$= \frac{9}{4} \cdot V_1 + \frac{9}{2} \cdot V_2 - 4 \cdot V_3 - 4 \cdot V_4$$

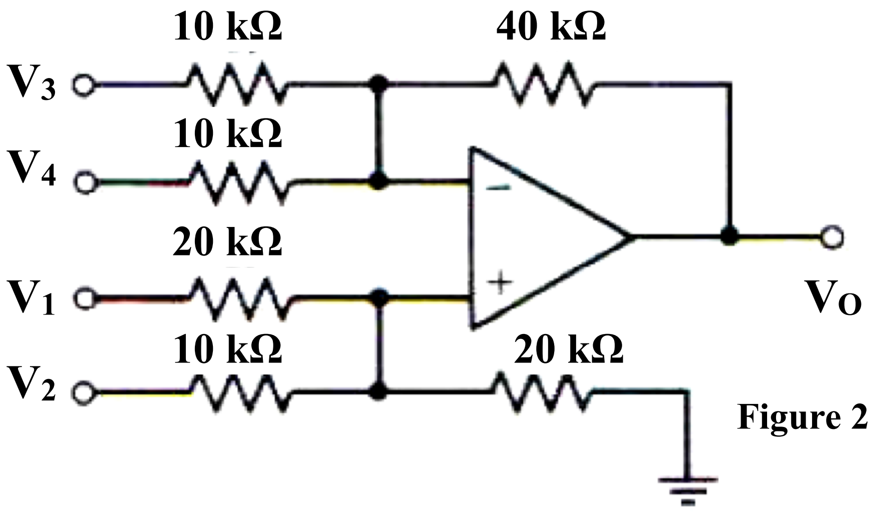


Figure 2

3. (20%) Use only one op amp to implement the function $V_O = \frac{1}{2} \cdot V_1 - \frac{1}{4} \cdot V_2 + \frac{3}{4} \cdot V_3$, with all R's of larger than 10 kΩ but as small as possible. Draw the circuit diagram with your design.

$$V_O = \tfrac{1}{2}V_1 - \tfrac{1}{4}V_2 + \tfrac{3}{4}V_3$$

$$-\frac{R_F}{R_2} = -\frac{1}{4} \Rightarrow R_2 = 4 \cdot R_F \Rightarrow \begin{cases} R_F = 10\ k\Omega \\ R_2 = 40\ k\Omega \end{cases}$$

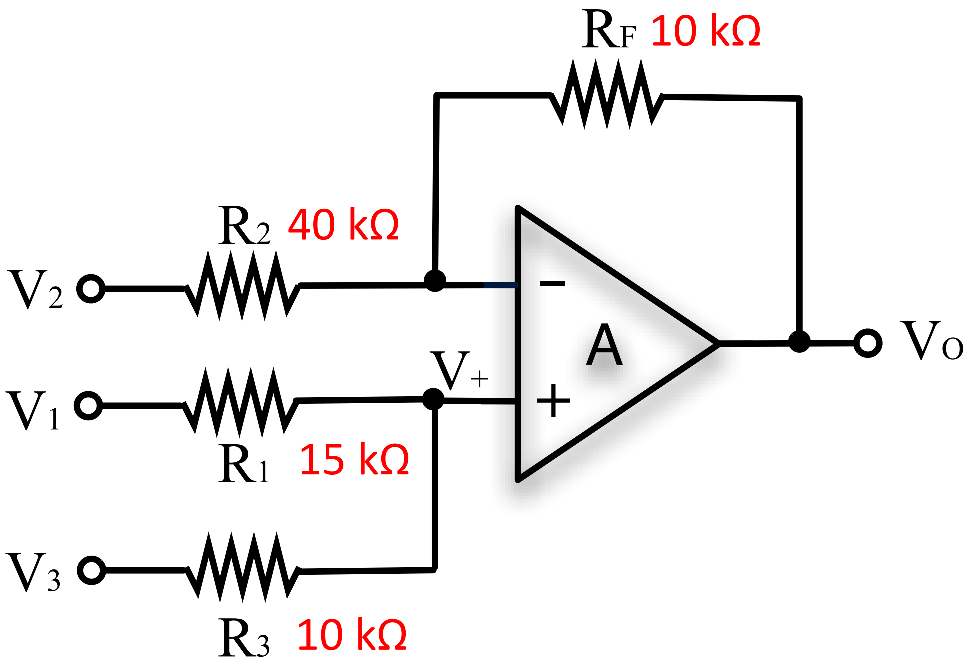
$$V^+ \cdot \left(1 + \frac{R_F}{R_2}\right) = \frac{5}{4}V^+ = \tfrac{1}{2}V_1 + \tfrac{3}{4}V_3$$

$$\frac{5}{4}\left(\frac{R_3}{R_1 + R_3} \cdot V_1 + \frac{R_1}{R_1 + R_3} \cdot V_3\right) = \tfrac{1}{2}V_1 + \tfrac{3}{4}V_3$$

$$\Rightarrow \begin{cases} \dfrac{R_3}{R_1 + R_3} = \dfrac{4}{10} \\[2ex] \dfrac{R_1}{R_1 + R_3} = \dfrac{3}{5} = \dfrac{6}{10} \end{cases}$$

$$\Rightarrow \frac{R_1}{R_3} = \frac{6}{4} \Rightarrow 4R_1 = 6R_3 \Rightarrow R_1 = 1.5 \cdot R_3$$

$$\Rightarrow \begin{cases} R_3 = 10\ k\Omega \\ R_1 = 15\ k\Omega \end{cases}$$



4. (15%) The inverting circuit with the T network in the feedback is shown in Fig. 4. Derive the the voltage gain $\frac{v_O}{v_I}$ (10%) and the current gain $\frac{i_O}{i_1}$ (5%).

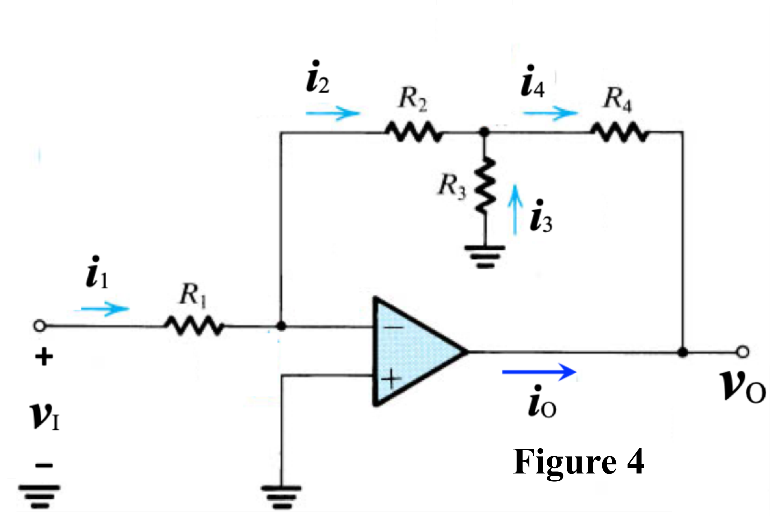


Figure 4

5. (20%) Design the circuit (R_1, R_2, R_3 = ?) to have an input resistance of 100 kΩ and a voltage gain that can be varied from –1 ~ – 100 using the 100-kΩ potentiometer R_4. What voltage gain results when the potentiometer is set exactly at its middle value?

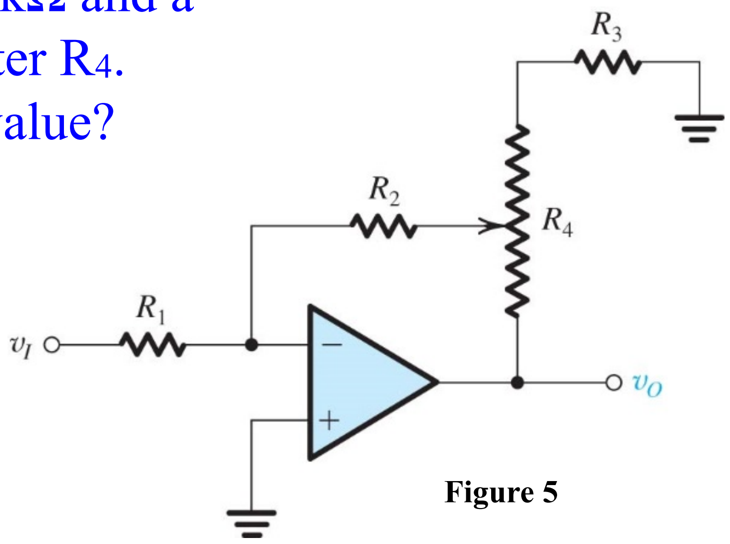


Figure 5

$$\frac{v_O}{v_I} = -\frac{R_2}{R_1}\left(1+\frac{R_4}{R_2}+\frac{R_4}{R_3}\right)$$

To obtain an input resistance of 100 kΩ, we select $R_1 = 100$ kΩ. From Example 2.2 we have

$$\frac{v_O}{v_I} = -\frac{R_2}{R_1}\left[1+\frac{(1-x)R_4}{R_2}+\frac{(1-x)R_4}{R_3+xR_4}\right]$$

The minimum gain magnitude is obtained when $x = 1$,

$$\frac{v_O}{v_I} = -\frac{R_2}{R_1} = -1$$

Thus, $R_2 = 100$ kΩ.

The maximum gain magnitude is obtained when $x = 0$,

$$\frac{v_O}{v_I} = -\frac{R_2}{R_1}\left[1+\frac{R_4}{R_2}+\frac{R_4}{R_3}\right] = -100$$

$$\Rightarrow 1+\frac{100}{100}+\frac{100}{R_3} = 100$$

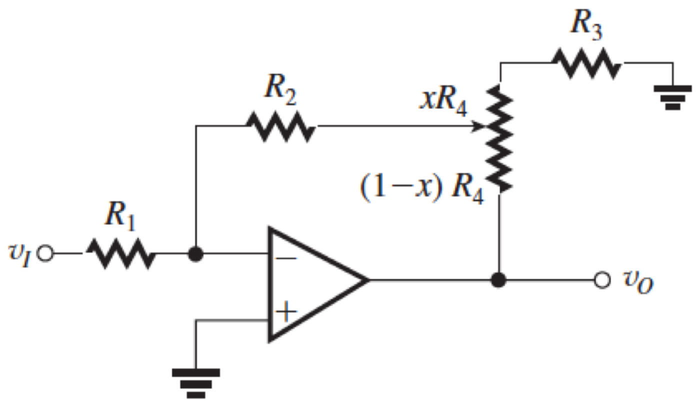
$$\Rightarrow R_3 = \frac{100}{98} = 1.02 \text{ k}\Omega$$

When the potentiometer is set exactly in the middle, $x = 0.5$ and

$$\frac{v_O}{v_I} = -\frac{R_2}{R_1}\left[1+\frac{0.5R_4}{R_2}+\frac{0.5R_4}{R_3+0.5R_4}\right]$$

$$= -\frac{100}{100}\left[1+\frac{0.5\times 100}{100}+\frac{0.5\times 100}{1.02+0.5\times 100}\right]$$

$$= -2.48 \text{ V/V}$$



6. (20%) Derive v_O and i_O in the circuit of Figure 6.A and 6.B.

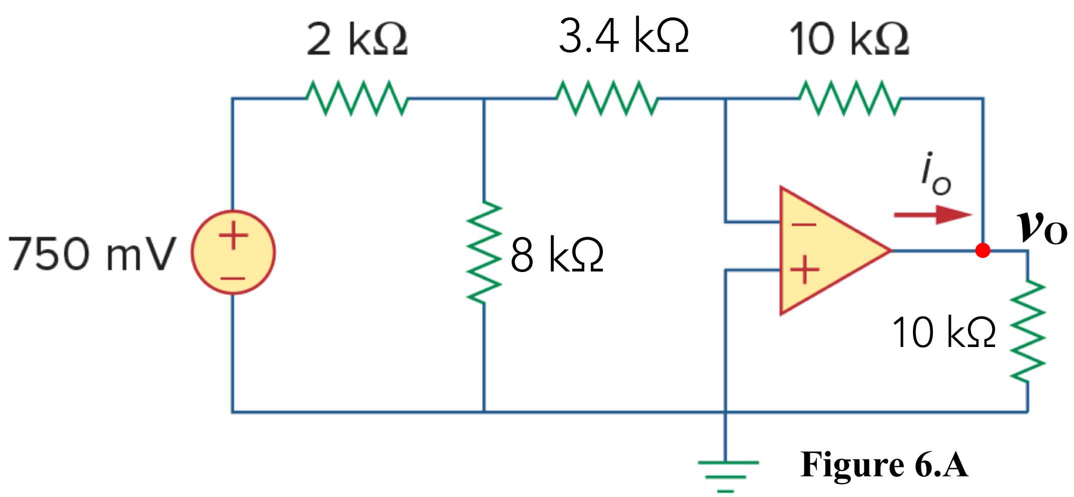


Figure 6.A

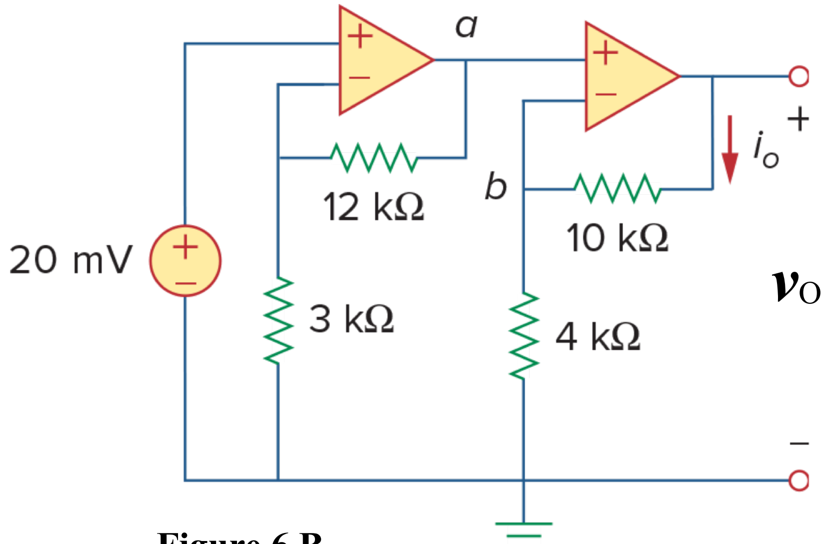


Figure 6.B

$$V_{Th} = 750\times\frac{8}{2+8} = 600(mV)$$

$$R_{Th} = 2k \parallel 8k = 1.6(k\Omega)$$

$$V_O = -\frac{10}{R_{Th}+3.4}\times V_{Th} = -\frac{10}{1.6+3.4}\times 600 = -1200(mV) = -1.2(V)$$

$$i_O = \frac{V_O}{10k}+\frac{V_O}{10k} = -0.24(mA) = -240(\mu A)$$

$$v_a = \left(1+\frac{12}{3}\right)(20) = 100\text{ mV}$$

$$\boxed{v_o = \left(1+\frac{10}{4}\right)v_a = (1+2.5)100 = 350\text{ mV}}$$

$$i_o = \frac{v_o - v_b}{10}\text{ mA}$$

$$v_b = v_a = 100\text{ mV}$$

$$\boxed{i_o = \frac{(350-100)\times 10^{-3}}{10\times 10^{3}} = 25\ \mu\text{A}}$$

7. (20%) Derive v_O in Fig. 7.A and voltage gain $\frac{v_O}{v_I}$ in Fig 7.B for the OP Amp circuits

$$V_2 = 15.5 \times \frac{60 \parallel (20+80)}{40 + 60 \parallel (20+80)}$$
$$= 15.5 \times \frac{37.5}{40+37.5}$$
$$= 7.5\ (mV)$$

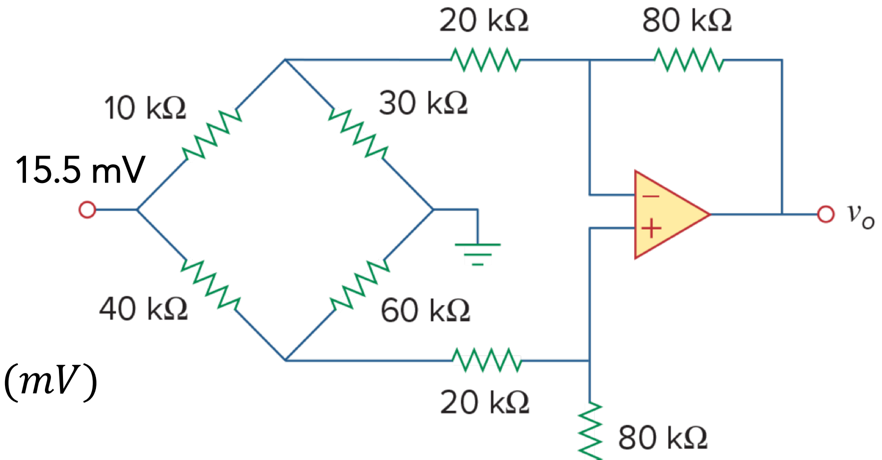
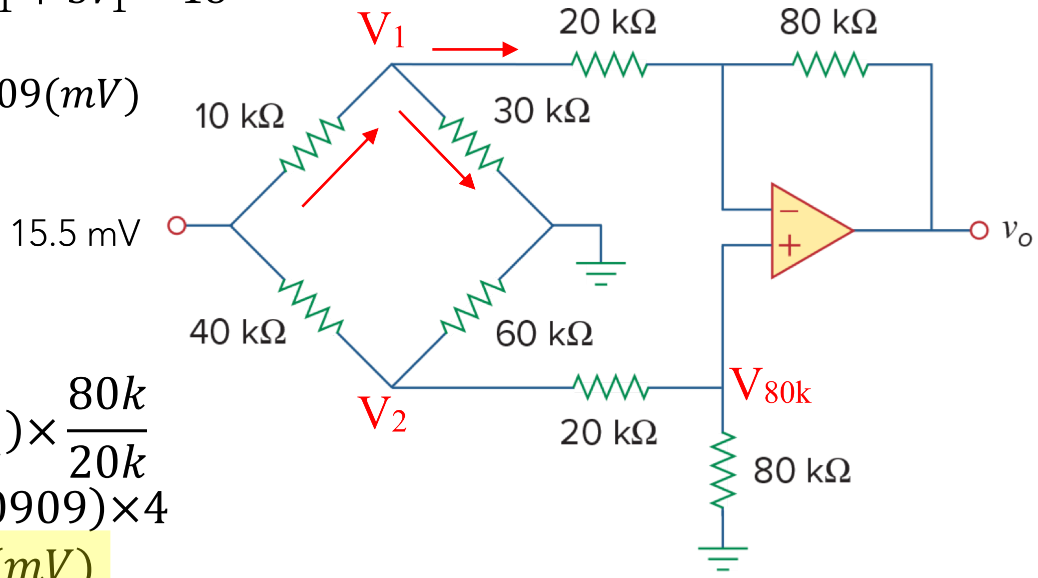


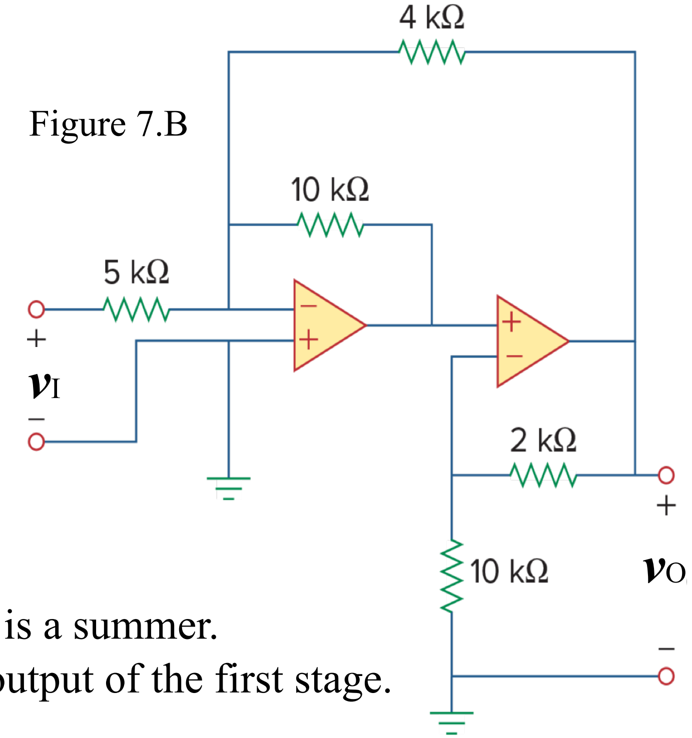
Figure 7.A

$$V_{80k} = V_2 \times \frac{80}{20+80} = 7.5 \times 0.8 = 6\ (mV)$$

$$\frac{15.5 - V_1}{10k} = \frac{V_1}{30k} + \frac{V_1 - V_{80k}}{20k}$$
$$93 - 6V_1 = 2V_1 + 3V_1 - 18$$
$$11V_1 = 111$$
$$\Rightarrow V_1 = 10.0909 (mV)$$



$$V_0 = (V_2 - V_1) \times \frac{80k}{20k}$$
$$= (7.5 - 10.0909) \times 4$$
$$= -10.3636 (mV)$$



The first stage is a summer.
Let V_1 be the output of the first stage.

$$v_1 = -\frac{10}{5}v_i - \frac{10}{4}v_o \longrightarrow v_1 = -2v_i - 2.5v_o \qquad (1)$$

By voltage division,

$$v_1 = \frac{10}{10+2}v_o = \frac{5}{6}v_o \qquad (2)$$

Combining (1) and (2),

$$\frac{5}{6}v_o = -2v_i - 2.5v_o \longrightarrow \frac{10}{3}v_o = -2v_i$$

$$\frac{v_o}{v_i} = -6/10 = \underline{-0.6}$$

8. (25%) Figure 8 shows a circuit that is known as a first-order, low-pass active filter.

a. (10%) Derive the transfer function $\frac{V_o}{V_i}$ (in s and jω), and show its dc gain and 3-dB frequency ω_b.

b. (10%) Design the circuit (R_1, R_2 and C) to obtain an input resistance of 10 kΩ, a dc gain of 40 dB, and a 3-dB frequency (f_b) of 1 kHz. What is its unity frequency (f_t = ?)?

c. (5%) Construct the Bode plot (frequency response of |Vo/Vi|(dB)) with the design.

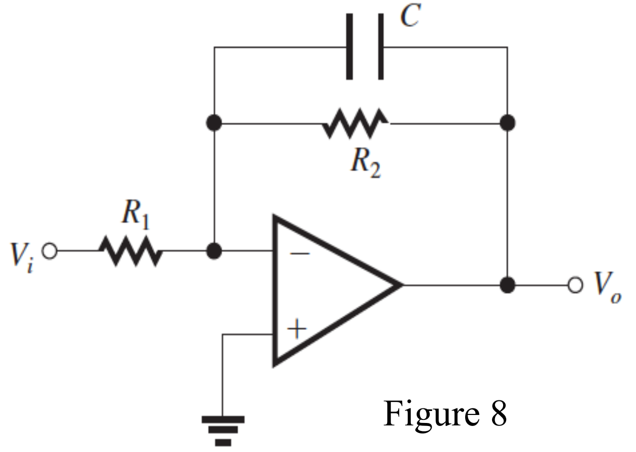


Figure 8

Let $Z_2 = R_2 \parallel \frac{1}{sC}$ and $Z_1 = R_1$

$$\frac{V_o}{V_i} = -\frac{Z_2}{Z_1} = -\frac{Y_1}{Y_2} = -\frac{1/R_1}{\frac{1}{R_2} + sC}$$

$$= -\frac{(R_2/R_1)}{1 + sCR_2}$$ 5%

This function is of the STC low-pass type, having a dc gain of $-\frac{R_2}{R_1}$ and a 3-dB frequency

5%

$$\omega_0 = \frac{1}{CR_2}$$ 5%

$$R_{in} = R_1 = 10\ \text{k}\Omega$$ 5%

dc gain = 40 dB = 100

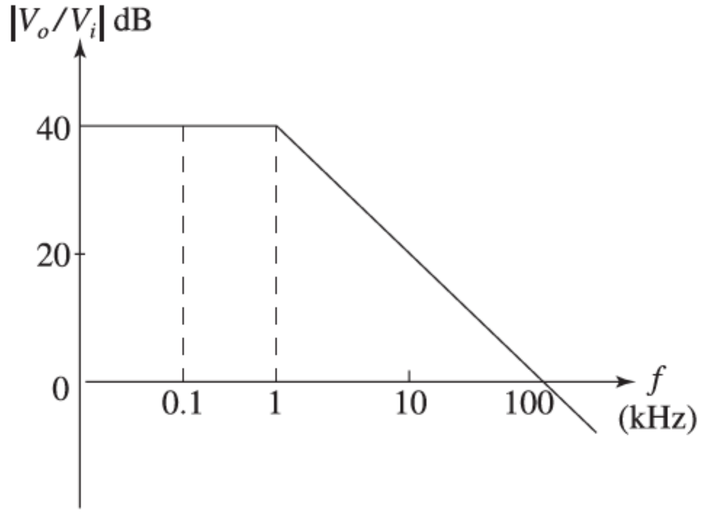
$$\therefore\ 100 = \frac{R_2}{R_1} \Rightarrow R_2 = 100R_1 = 1\ \text{M}\Omega$$

5%

3-dB frequency at 1 kHz

$$\therefore \omega_0 = 2\pi \times 1 \times 10^3 = \frac{1}{CR_2}$$

$$C = \frac{1}{2\pi \times 1 \times 10^3 \times 10^6} = 0.16\ \text{nF}$$ 5%



From the Bode plot shown in previous column, the unity-gain frequency is 100 kHz.