E&M review problems

1. A current-carrying 2.0-cm long segment of wire is inside a long solenoid (radius = 4.0 cm, n = 800 turns/m, current = 50 mA). The wire segment is oriented perpendicularly to the axis of the solenoid. If the current in the wire segment is 0.05 A, what is the magnitude of the magnetic force on this segment? in units of μN.

1. Solenoid $I_s = 50mA$, current in wire segment $= I$

$$B_s = \mu_0 n I_s = 4\pi\times10^{-7}\times800\times50\times10^{-3} = 502\times10^{-7}$$

$$F = I\ell B_s = I\times0.02\times502\times10^{-7}$$

2. A conducting rod with a square cross section (3.0 cm × 3.0 cm) carries a current of 1.2 A that is uniformly distributed across the cross section. What is the magnitude of the (line) integral $\oint \mathbf{B}\cdot d\mathbf{s}$ around a square path (1.5 cm × 1.5 cm) if the path is centered on the center of the rod and lies in a plane perpendicular to the axis of the rod? In units of μT · m.

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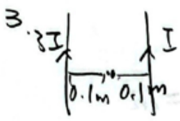
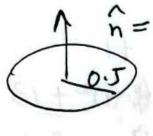
3. Two long parallel wires carry unequal currents in the same direction. The ratio of the currents is 3 to 1. The magnitude of the magnetic field at a point in the plane of the wires and 10 cm from each wire is 100 μT. What is the larger of the two currents? In units of A.

3. 

$$B = \frac{\mu_0}{2\pi}\frac{(3I-I)}{r=0.1m} = \frac{\mu_0 I}{\pi r} \to 0.1m$$

$$\Rightarrow I = B\frac{\pi r}{\mu_0} \Rightarrow \text{large} = 3I = \frac{3B\pi r}{\mu_0}$$

4. A circular loop (radius = 0.50 m) carries a current of 40 A and has unit normal vector of $(2\hat{\mathbf{i}} - \hat{\mathbf{j}} + 2\hat{\mathbf{k}})/3$. What is the x component of the torque on this loop when it is placed in a uniform magnetic field of $(2\hat{\mathbf{i}} - 6\hat{\mathbf{j}})$T? In units of N · m.

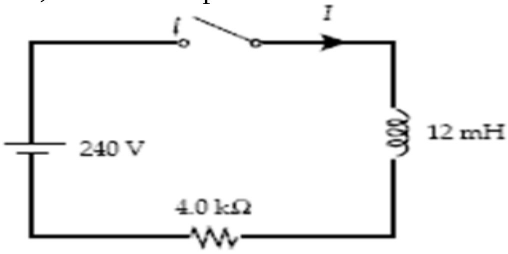
4. 

$$\hat{n} = (2\hat{i} - \hat{j} + 2\hat{k})/3, \quad \vec{A} = \pi\, 0.5^2\, \hat{n}$$

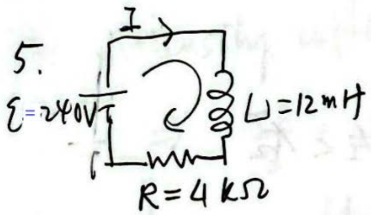
$$\vec{B} = 2\hat{i} - 6\hat{j} \Rightarrow \vec{\tau} = I\vec{A}\times\vec{B} = I\cdot\pi\times\frac{0.25}{3}\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & -1 & 2\\ 2 & -6 & 0\end{vmatrix}$$

$$\Rightarrow \tau_x = I\,\frac{\pi\times0.25}{3}\times12$$

5. The switch in the figure is closed at $t = 0$ when the current I is zero. When $I = 0.1$ mA, what is the potential difference across the inductor? In units of V.



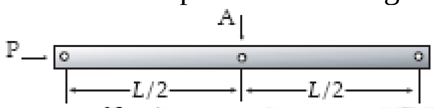
5.



$$\varepsilon - \Delta V_L - IR = 0$$

$$\Delta V_L = 240 - IR = 240 - I \times 4 \times 10^{3}$$

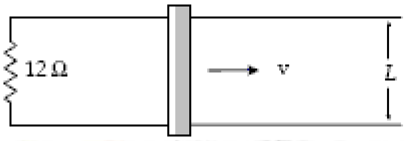
6. A conducting bar of length L rotates with a constant angular speed of +2.0 rad/s about a pivot P at one end, as shown. A uniform magnetic field (magnitude = 0.20 T) is directed into the paper. If $L = 2.0$ m, what is the potential difference, $V_A - V_P$? Be careful of the plus or minus sign. In units of mV.



6. ΔV $L/2$ $\because$ charge carrier is e^-

$\therefore \Delta V = -\int_0^{L/2} B r \omega \, dr = -B\omega \frac{L^2}{8}$, w/ $B = 0.2\,T$, $\omega = 2$ rad/s

7. A rod (length = 10 cm) moves on two horizontal frictionless conducting rails, as shown. The magnetic field in the region is directed perpendicularly to the plane of the rails and is uniform and constant. If a constant force of 60 N moves the bar at a constant velocity of 2.0 m/s, what is the current through the 12-Ω load resistor? In units of A.

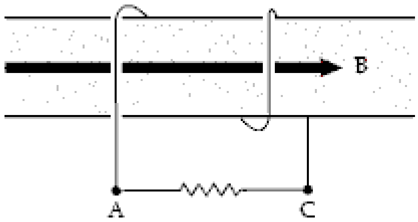


7. $F = I\ell B \Rightarrow I = F/\ell B$

$\varepsilon = d\Phi_B/dt = B\ell v = IR \Rightarrow B = IR/\ell v$

$\Rightarrow I = F/\ell \cdot \frac{IR}{\ell v} = \frac{Fv}{IR} \Rightarrow I = \sqrt{\frac{Fv}{R}} = \sqrt{\frac{F \times 2}{R}}$

8. The coil shown in the figure has 2 turns, a cross-sectional area of 0.20 m^2, and a field (parallel to the axis of the coil) with a magnitude given by $B = (4.0 + 0.1t^2)$ T, where t is in s. What is the potential difference, $V_A - V_C$, at $t = 3.0$ s? In units of V.



8. $B = (4+\beta t^2)$, $2\frac{d\Phi_B}{dt} = -2A\frac{d}{dt}(4+\beta t^2) = -4\beta A = -4\beta \times 0.2$

9. An FM radio station broadcasts at 20 MHz. What is the wavelength of the radiowaves? In units of m.

9. $\lambda = c/f = 3\times10^8/f$

10. What is the maximum radiation pressure exerted by sunlight in space ($S = 400$ W/m^2) on a highly polished silver surface? In units of Pa.

10. highly polished silver surface ⇒ total reflection

⇒ Pressure $= 2 \times S/c = 2S/3\times10^8$

11.. A solar cell has a light-gathering area of 10 cm^2 and produces 0.2 A at 0.2 V (DC) when illuminated with $S = 1\ 000$ W/m^2 sunlight. How many percent of the photonic energy has turned into electrical energy?

11. power received $P = SA = 1000\,\frac{W}{m^2} \times 10\times10^{-4}\,m^2 = 1$ W

power produced $P_{cell} = 2A \times V$

⇒ percentage $= 2V/1$

12. The magnetic field of a plane-polarized electromagnetic wave moving in the z-direction is given by $B = 120\times10^{-6}\sin\left[2\pi\left(\frac{z}{240} - \frac{t\times10^7}{8}\right)\right]$ in SI units. Find the average power per square meter carried by the EM wave. In units of W.

12. ave power $S_{avg} = \frac{c B_{max}^2}{2\mu_0} = \frac{3\times10^8\ B_{max}^2}{2\times 4\pi\times10^{-7}}$

13. If the average radiant energy from the sun comes in as a plane EM wave of intensity 100 W/m^2, calculate the peak values of E. in units of V/m.

13. $S_{avg} = \frac{E_{max}^2}{2c\mu_0}$ ⇒ $E_{max} = \sqrt{2c\mu_0 S_{avg}}$

14. A plane parallel plate capacitor has plates of 10 cm^2 area that are 1.0 mm apart. At an instant when charge is being accumulated on the plates at a rate of 150 nC/s, the displacement current between the plates is ______ A.

14. displacement current = actual current in the wire

$$\Rightarrow I_d = I = \frac{dq}{dt} = x\ nC/s = x\ nA = x \times 10^{-9} A.$$

15. The magnetic field of a plane-polarized electromagnetic wave moving in the z-direction is given by $B = 2.4\times10^{-6}\sin\left[2\pi\left(\frac{z}{5} - \frac{t\times10^{7}}{8}\right)\right]$ in SI units. What is the wavelength of the EM wave? In units of m.

15. $B = 2.4\times10^{-6} \sin\left[2\pi\left(\frac{z}{k} - \frac{t\times10^7}{8}\right)\right] \Rightarrow \omega = 2\pi f \Rightarrow f = \frac{10^7}{8}$.

$$\Rightarrow \lambda = c/f = 240\ m$$

16. An unpolarized beam of light has intensity I_0. It is incident on two ideal polarizing sheets. The angle between the transmission axes of these sheets is θ. Find the value of $\cos\theta$ if the emerging light has intensity $I_0/20$:

16. intensity after passing thru the first polarizer = $I_0/2$.

$$\Rightarrow \frac{I_0}{2} \times \cos^2\theta = I_0/20 \Rightarrow \cos^2\theta = \frac{2}{20} \Rightarrow \cos\theta = \sqrt{1/10}$$

17. In a stack of three polarizing sheets the transmission axes of the first and third are crossed while the middle one has its axis at 5° to the that of the first one. The fraction of the intensity of an incident unpolarized beam of light that is transmitted by the stack is: (in units of %).

17. intensity after the first polarizer - $I_0/2$

after the 2^{nd} : $\frac{I_0}{2}\cdot\cos^2\theta$,

$\Rightarrow$ after the 3^{rd} polarizer $I = \frac{I_0}{2}\cos^2\theta\cos^2(\pi/2-\theta)$

18. A sinusoidal electromagnetic wave with an electric field amplitude of 5 V/m is incident normally on a surface with an area of 1 cm^2 and is completely absorbed. The energy absorbed in 10 s is: (in units of mJ)

18. $S_{av} = \frac{E_{max}^2}{2c\mu_0} = \frac{E_{max}^2}{2\times3\times10^8\times4\pi\times10^{-7}}$

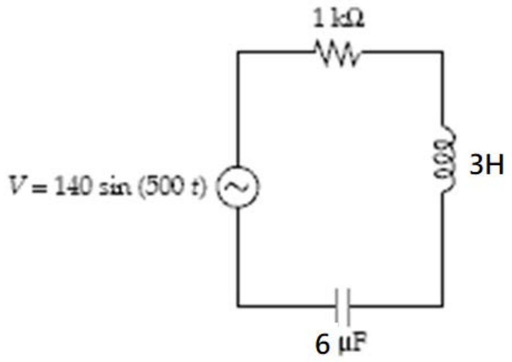
complete absorption energy = $1\times S_{av}\times area\times t$

$$= \frac{E_{max}^2}{2.4\pi}\times10^{-4}\ m^2\times10s$$

19. An *RLC* series circuit has *R* = 100 ohms, *C* = 40 μF, and *L* = 0.1 H. For what angular frequency of an ac voltage is the current flow maximum? In units of rad/s.

19. $\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{0.1 \times C}}$

20. Determine the rms current for the circuit. In units of mA.



20. $Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{1000^2 + (500L - 1/500C)^2}$

$V_{rms} = \frac{V_{max}}{\sqrt{2}} = 140/\sqrt{2}$

$I_{rms} = V_{rms}/Z$

21. A current *I* = 0.5 sin (400 *t*) amperes flows in a series *RL* circuit in which *L* = 1 mH and *R* = 100Ω. What is the average power loss? In units of W.

21. $P_{av} = I_{rms}^2 \times R = \frac{I_{max}^2}{2} \cdot 100$

22. An ideal step-down transformer has 200 primary turns and 50 secondary turns. If 440 volts (rms) is placed across the primary, what is the rms current in the secondary when the load resistance is 50.0 ohms? In units of A.

22. $\Delta V_2 \, rms = 440 \, \frac{\text{primary turns}}{\text{secondary turns}}$

$I_{rms} = \frac{\Delta V_2 \, rms}{R}$