

[SLM03] - Slit diffraction and interference

1. Objectives :

Understanding wave property of light by diffraction pattern and interference pattern of light.

2. Introduction :

(1) Double-slit interference

Young's double-slit interference experiment was the earliest strong evidence of wave phenomena of light. From then on, the wave phenomena of light theory was superior than the particle phenomena of light theory gradually. Due to the lacking knowledge of coherent source in the early stage, scientists put a single slit between light source and double-slit in order to ensure they got spatial coherent length during experiment. The disadvantage of this way was weakening the light intensity of source, and hard to observe clear interference pattern. Nowadays, scientists use coherent and very bright laser light as light source in experiment instead of single-slit, so the contrast between light and dark interference pattern on screen becomes much larger, and make the experiment much easier to achieve.

In double-slit interference experiment, we use slits with very small width and we could treat each slit as one point light source (The diffraction effect of slit itself can be ignored.). Then we superposition two electric fields of light waves on screen from both point light source.

Figure 1 shows parallel light incidents double-slit, with spacing of two slits is d and distance between slits and screen is L . Two slits are treated as two point light sources S_1 and S_2 . Spherical waves are emitted from S_1 and S_2 , and produce interference pattern on arbitrary observable point P on screen.

Because L is much larger than d , two lights which are from S_1 and S_2 reach point P could be seen as two parallel lights. So the light path difference is

$$r_2 - r_1 \cong d \sin \theta \quad (1)$$

$$\text{If } d \sin \theta_n = n\lambda \quad (2)$$

$n = 0, 1, 2, 3, \dots$, and constructive

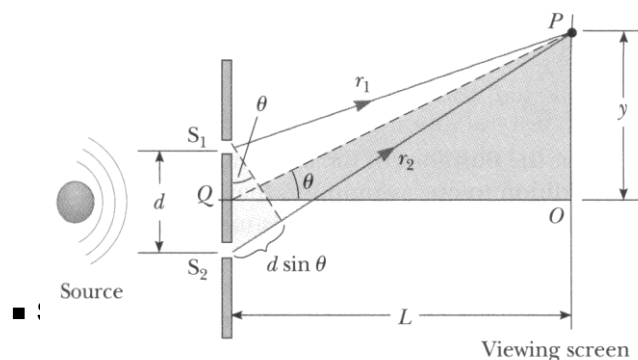


Figure 1

interference occurs on point P , which is the location of light fringe.

$$\text{Similarity} \quad d \sin \theta_n = \left(n + \frac{1}{2}\right) \lambda \quad (3)$$

$n = 0, 1, 2, 3, \dots$, and destructive interference occurs on point P , which is the location of dark fringe.

In experimental setup, we usually have $L \gg y_n$, with describes the distance between n th light fringe and central light fringe. So we have very small angle θ_n and $\tan \theta_n \cong \sin \theta_n$.

$$\text{According to Figure 1, we have} \quad \tan \theta_n = \frac{y_n}{L} \cong \sin \theta_n \quad (4)$$

$$\text{and} \quad d \sin \theta = d \frac{y_n}{L} = n \lambda \quad n = 0, 1, 2, \dots \quad (5)$$

$$\text{or} \quad y_n = n \frac{L \lambda}{d} \quad n = 0, 1, 2, \dots, \quad (6)$$

so the spacing Δy_n between two light fringes is

$$\Delta y_n = y_{n+1} - y_n = L \lambda / d . \quad (7)$$

With L, λ, d are constants in equation (7), which are independent of n , we get the spacing between two arbitrary light fringes is the same,

$$\text{ie.} \quad \Delta y = \frac{L \lambda}{d}$$

In this experiment, we placed a lens in front of the CCD. Assuming the distance between $+1$ and -1 dark fringes Δy_d , double slit spacing d , lens focal length f , then we can use Fourier transform to get the wavelength equal to

$$\lambda = \frac{d \Delta y_d}{f} \quad (8)$$

(2) Single-slit diffraction

Each point on slit can be treated as one point light source which emits spherical wave when light passes through slit. Different spherical waves produce interference when they arrive screen due to light path difference. Light from all point sources reach screen cause interference, called diffraction. So we call diffraction as interference of many light paths. When width of slit is much larger than wavelength, diffraction phenomena is not obvious. However, when width of slit is compatible with or smaller than wavelength, diffraction phenomena is magnificent.

After the light is incident on the slit with width a , the diffraction pattern formed by the light on the screen S is discussed below:

- (1) For the central point P_0 on the screen, the light is concentrated at this point through the lens and each light has the same optical path, so it is extremely bright. Since the slit has a length, it actually forms a bright line in the center of the screen.
- (2) For any point on the screen, if the optical path difference bb' of the two rays emitted from the upper end and the center of the slit equals to $\lambda/2$, it is destructive interference. When any of the rays between r_1 and r_2 reaches the point P_1 , a light can be found Correspondingly below r_2 to destructively interfere with it. Therefore, the point P_1 is completely destructive interference. In

Figure 2

$$bb' = \frac{a}{2} \sin \theta = \frac{\lambda}{2}$$

$$\text{or} \quad a \sin \theta = \lambda \quad (9)$$

is the condition of dark fringe. In general, when $a \sin \theta_m = \pm m\lambda$ ($m = 1, 2, \dots$), dark fringes occur.

- (3) When

$$a \sin \theta_{m'} = \pm \left(m' + \frac{1}{2} \right) \lambda \quad (10)$$

($m' \approx 1, 2, \dots$), other bright fringes occur. The Figure 3 shows intensity versus $\sin \theta$. When θ is small,

$$\theta \cong \sin \theta \cong \tan \theta \cong \frac{y_1}{L} \quad (11)$$

$$\tan \theta_1 = \frac{y_1}{L}, \quad \theta_1 = \tan^{-1} \frac{y_1}{L} \quad (12)$$

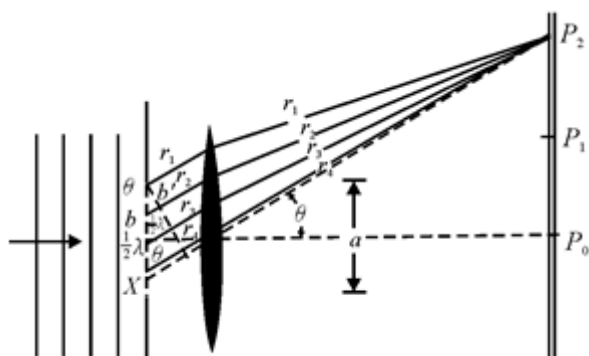


Figure2

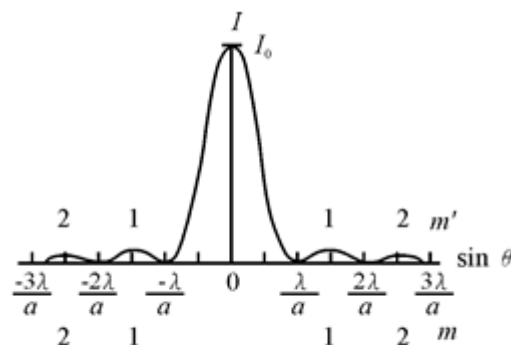


Figure3

$$\sin \theta_1 = \sin \left(\tan^{-1} \frac{y_1}{L} \right) = \frac{\lambda}{a} \quad (13)$$

$$\lambda = a \sin \left(\tan^{-1} \frac{y_1}{L} \right) \quad (14)$$

$$\text{In general,} \quad \lambda = \frac{a}{m} \sin \left(\tan^{-1} \frac{y_m}{L} \right) \quad (15)$$

In this experiment, we placed a lens in front of the CCD. Assuming the distance between +1 and -1 dark fringes Δy_s , single slit width a , lens focal length f , then we can use Fourier transform to get the wavelength equal to

$$\lambda = \frac{a \Delta y_{\text{單}}}{2f} \quad (16)$$

3. Materials :

Laser 、 Mirror 、 Spatial filter 、 Lens 、 Beam splitter 、 Spatial Light Modulator (SLM) 、 Target 、 Polarizer 、 Charge-Coupled Device (CCD) 、 Screen.

4. Procedure :

(1) Calibration

1. Setup instruments as shown in Figure 4. Make sure that the light emitted by the laser runs parallel to the surface of the table. Also make sure that the light beam passes through the center of each component and it enters each component perpendicularly.
2. Place the block between the mirror and the beam splitter.
3. Place the cross target in front of the screen.
4. Adjust 2 screws behind SLM until the dark cross is in the center of the rectangle as shown in Figure 5.

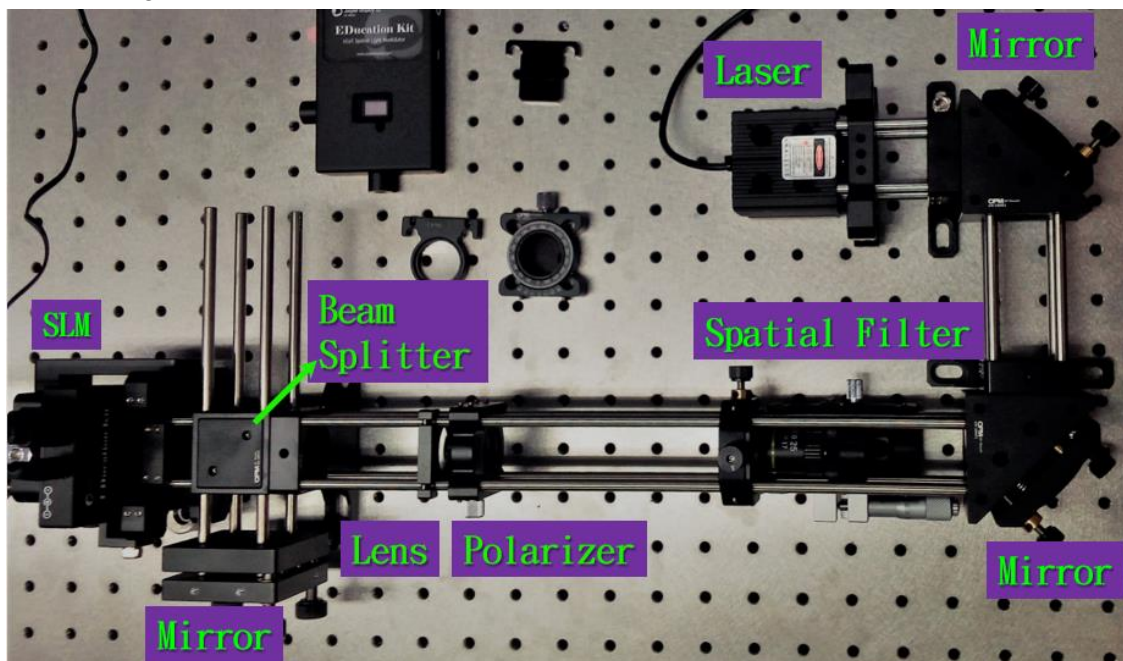


Figure 4

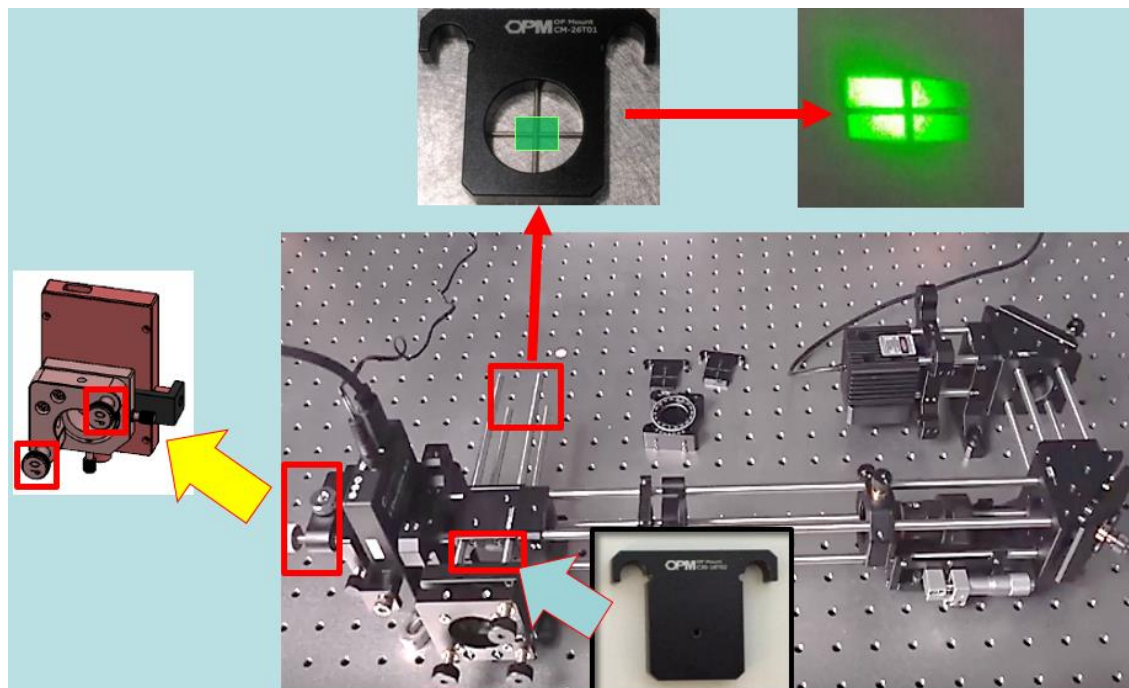


Figure 5

(2) Single-slit

1. Remove the cross target and install lens and analyzer.
2. Adjust the angles of polarizer and analyzer to amplitude modulation mode.
3. Open the EDK software. Select 1 for the Monitor Index in the upper right corner. Select experiment to 5. Diffraction and Interference and set the Slit size. The grayscale value corresponding to the maximum optical power measured by the amplitude modulation experiment is set to inner grayscale, and the grayscale value corresponding to the minimum optical power is set to outer grayscale as shown in Figure 6.

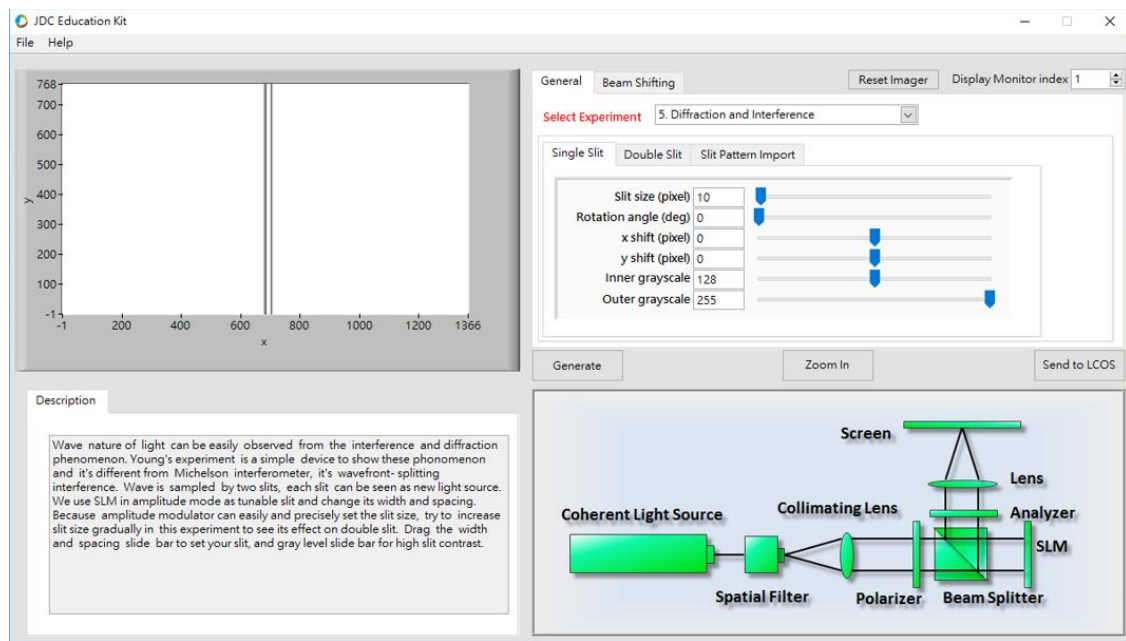


Figure 6

4. Set up the CCD and adjust the CCD position until the image is clear and at the center of the window.
5. Analyze the slit diffraction image by matlab to calculate the wavelength of the laser.
Calculate the percentage error by comparing with the standard value of the laser wavelength.

(3) Double-slit

1. Select the double slit sheet in the EDK software and enter the slit spacing and width. The other settings are the same as the single slit settings as shown in Figure 7.
2. Analyze the slit diffraction image by matlab to calculate the wavelength of the laser.
Calculate the percentage error by comparing with the standard value of the laser wavelength

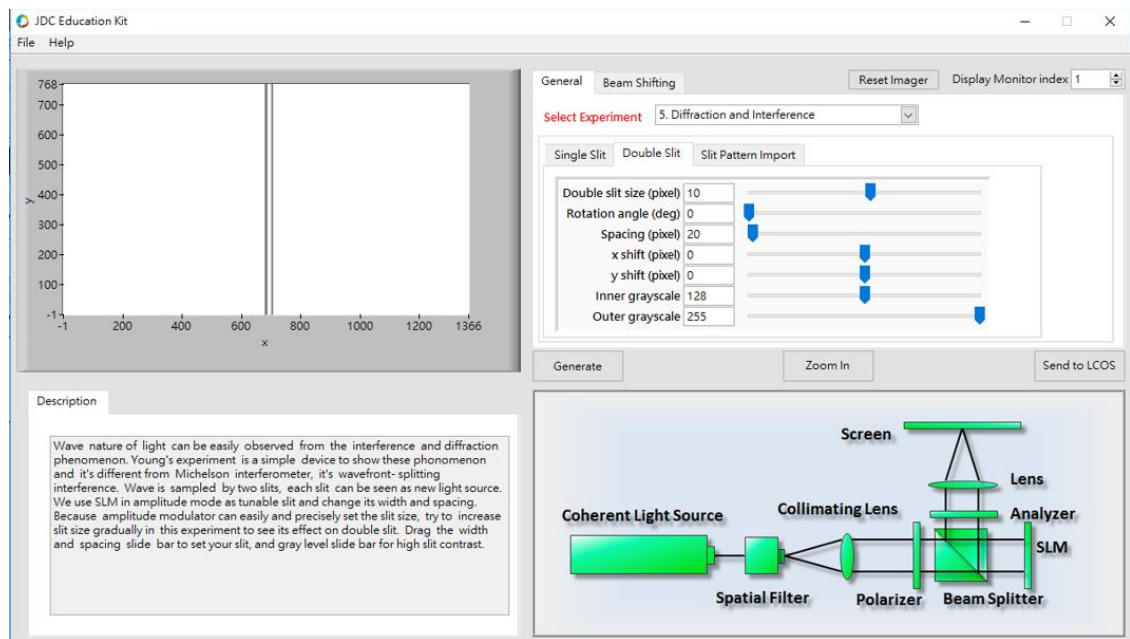


Figure 7

5. Questions :

- (1) If we use other kinds of light source instead of monochromatic light source, what will happen?
Discuss the results.
- (2) If we use green light laser instead, what will happen? Try to discuss the reasons which cause different results.
- (3) Compare diffraction patterns of laser whose light goes through single-slit and double- (multi-) slit. Explain.