4/21 $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = e^1 \cdot e^{(x-1)}$

$$= e \cdot \sum \frac{(x-1)^n}{n!}$$

$$e^{(x-1)} = \sum_{n=0}^{\infty} \frac{(x-1)^n}{n!}$$

$\Downarrow$

$$e^x = \sum_{n=0}^{\infty} C_n (x-1)^n$$

$$f(x) = e^x = \sum_{n=0}^{\infty} \frac{f^{(n)}(1)}{n!} (x-1)^n$$

$$f'(1) = e^x \Big|_{x=1} = e$$

$$f''(1) = e^x \Big|_{x=1} = e$$

$\vdots$

$\Downarrow$

$$\sum_{n=0}^{\infty} \frac{e}{n!} (x-1)^n$$

power series of $f(x) = e^x$ in $x-1$

$$= \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

power series of e^{x^2} in x

$$e^{x^2} = \sum_{n=0}^{\infty} \frac{(x^2)^n}{n!} = \sum_{n=0}^{\infty} \frac{x^{2n}}{n!}$$

$$\sum_{n=0}^{\infty} \frac{(x-1)^n}{n!} = e^{x-1}$$

$$\neq e^x$$

$$e^x = e^{1+(x-1)} = e^1 \cdot e^{x-1}$$

$f(x) = \sin x$ (power series in $x - \frac{\pi}{6}$)

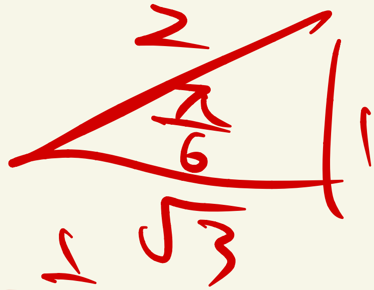
$\Downarrow$

(Taylor series expansion

of $f(x) = \sin x$

at $x = \frac{\pi}{6}$)

$$f(x) = \sin x \quad \text{at} \quad x = \frac{\pi}{6}$$



$$f\left(\frac{\pi}{6}\right) = \frac{1}{2}$$

$$f'\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2} \qquad f''\left(\frac{\pi}{6}\right) = -\frac{1}{2}$$

$$f'''\left(\frac{\pi}{6}\right) = \frac{-\sqrt{3}}{2}$$

$$\sin x = \sum_{n=0}^{\infty} \frac{f^{(n)}\left(\frac{\pi}{6}\right)}{n!}\left(x - \frac{\pi}{6}\right)^n$$

$$= \frac{1}{2} + \frac{\frac{\sqrt{3}}{2}\left(x - \frac{\pi}{6}\right)}{1!} - \frac{\frac{1}{2}\left(x - \frac{\pi}{6}\right)^2}{2!} - \frac{\frac{\sqrt{3}}{2}\left(x - \frac{\pi}{6}\right)^3}{3!} \cdots$$

$$= \frac{1}{2}\cos\left(x - \frac{\pi}{6}\right) + \frac{\sqrt{3}}{2}\sin\left(x - \frac{\pi}{6}\right)$$

$$= \sin x = \sin\left(\frac{\pi}{6} + x - \frac{\pi}{6}\right)$$

$$= \underbrace{\sin\frac{\pi}{6}}_{=\frac{1}{2}} \cos\left(x - \frac{\pi}{6}\right) +$$

$$\sin\left(x - \frac{\pi}{6}\right) \cdot \cancel{\cos\frac{\pi}{6}} \quad \frac{\sqrt{3}}{2}$$

$$\#$$

$\cos(\sqrt{x})$

$=$

$$\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$$