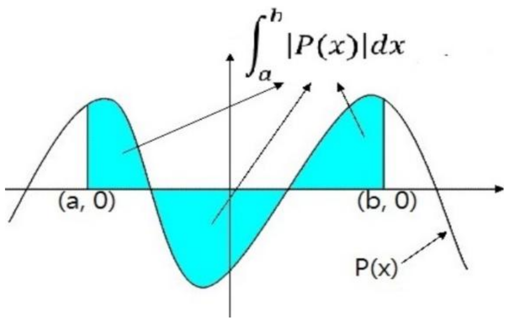
Programming Practice: Integrals

1. Consider a degree n real polynomial $P(x)=c_nx^n+c_{n-1}x^{n-1}+\ldots+c_2x^2+c_1x+c_0$, the coefficient of the highest degree c_n cannot be zero and two points on the x-axis, (a, 0) and (b, 0) such that a<b. Assume the curve in the following figure is P(x). In the following figure, the blue shaded area covered by polynomial P(x) and the x-axis between interval (a, 0) and (b, 0) is the integral $\int_a^b |P(x)|dx$.



The area can be computed using an approximation approach that divides the interval of (a, 0) and (b, 0) into $T=2^t$ intervals evenly, where 0<t. Let $(p_1, 0)$, $(p_2, 0)$, ..., $(p_T, 0)$ be the middle points of the divided intervals. The approximation value of of the blue shaded area covered by the polynomial curve and the X-axis is the definite integral of the following formula and can be computed using the Riemann sum (http://en.wikipedia.org/wiki/Riemann_sum) approximation:

$$\int_a^b |P(x)|dx \approx A_t = \frac{(b-a)}{2^t}\sum_{i=1}^{2^t}|P(p_i)|.$$

Compute A_0, A_1, …, A_{t-1}, and A_t until $|A_t-A_{t-1}|<\varepsilon$, where ε is a small error value, e.g., 0.000001, and then A_t is area the the definite integral. Write a C program to process the following steps:

a. Input an integer n, $0 \le n \le 10$ as the highest degree of polynomial P(x).
b. Input two real numbers, a and b, such that a<b and $b-a \le 5$;
c. Randomly generate n+1 real numbers, between -1 and 1 (including), as coefficients c_n, c_{n-1}, ..., c_2, c_1, c_0;
d. Output polynomial P(x) and [a, b];
e. Compute the area covered by P(x) and the X-axis between (a, 0) and (b, 0); at the end of each iteration, output the number of partitioned intervals in this iteration, the size of the interval, and the approximation value of the area up to 6 digits after the decimal point;
f. Output the final value of the area.

Program solution: riemann_sum.c. Program execution example:

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命令提示字元

D:\>riemann_sum
Enter the degree of polynomial P(x): 5
Enter two real numbers a and b such that 0<b-a<=5: 2.5 6.3
Polynomial P(x):
-0.6690 X^5 + 0.1970 X^4 - 0.9880 X^3 - 0.4790 X^2 + 0.9060 X + 0.0490

Interval [a, b]: [2.5000, 6.3000]
Number of intervals: 1, interval size: 3.800000, area: 4251.637142
Number of intervals: 2, interval size: 1.900000, area: 6218.878052
Number of intervals: 4, interval size: 0.950000, area: 6769.677662
Number of intervals: 8, interval size: 0.475000, area: 6911.064402
Number of intervals: 16, interval size: 0.237500, area: 6946.641514
Number of intervals: 32, interval size: 0.118750, area: 6955.550193
Number of intervals: 64, interval size: 0.059375, area: 6957.778263
Number of intervals: 128, interval size: 0.029687, area: 6958.335337
Number of intervals: 256, interval size: 0.014844, area: 6958.474609
Number of intervals: 512, interval size: 0.007422, area: 6958.509427
Number of intervals: 1024, interval size: 0.003711, area: 6958.518132
Number of intervals: 2048, interval size: 0.001855, area: 6958.520308
Number of intervals: 4096, interval size: 0.000928, area: 6958.520852
Number of intervals: 8192, interval size: 0.000464, area: 6958.520988
Number of intervals: 16384, interval size: 0.000232, area: 6958.521022
Number of intervals: 32768, interval size: 0.000116, area: 6958.521031
Number of intervals: 65536, interval size: 0.000058, area: 6958.521033
Number of intervals: 131072, interval size: 0.000029, area: 6958.521033

The number of intervals: 131072
Area of polynomial P(x) between (2.5000, 0.0) and (6.3000, 0.0): 6958.521033

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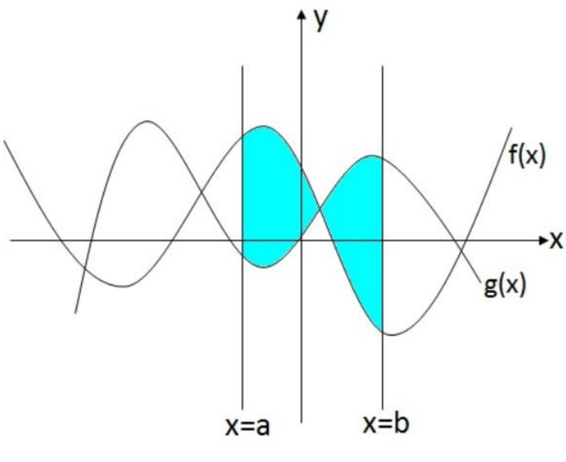
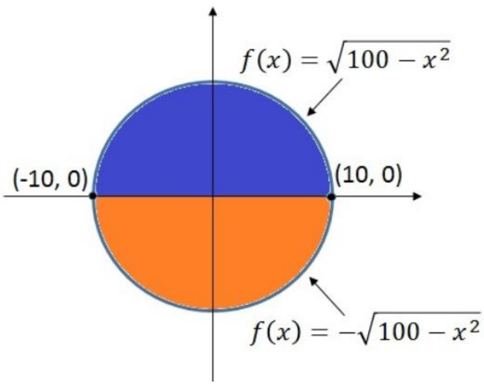
2. Suppose f(x) and g(x) are two continuous functions. The area between two vertical lines x=a and x=b and the curves between f(x) and g(x) is the definite integral $\int_a^b |f(x) - g(x)|dx$ as shown in the following figure. This area can be computed using Riemann sum approximation.



Write a C program to compute the area of the blue shade. The solution program defines two functions **double** upper_circle(**double**) and **double** lower_circle(**double**). These two functions are the upper-half and lower-half of the circle of radius 10 with the origin as the center as shown in the following figure:



Program solution riemann_sum_two_curves.c.