

## Introduction

Once built the graph, I **compute the metrics** we discussed in class and **draw some conclusions on the type of the underlying network**. This is the first paper of three. In the second I will analyze the robustness and in the last one the social contagion scenario.

From: Stanford Large Network Dataset Collection  
(<http://snap.stanford.edu/data/ego-Facebook.html>)

Social networks : online social networks, edges represent interactions between people

# My real Network Analysis

**Name:** Facebook

**Type:** Graph

**Number of nodes:** 4039 (named from 0 to 4038)

**Number of edges:** 88234

**Average degree:** 43.6910

**Density:** 0.010820, so we can see that our graph is not dense: it's **sparse** because  $p \rightarrow 0$ .

**is directed:** False

**is complete:** False

**Diameter (it is the maximum eccentricity):** 8

**top 7 nodes with highest-betweenness centrality (runn. time : ~3min.):**

1. **107**: 0.48077531149557645
2. **1684**: 0.33812535393929544,
3. **3437**: 0.23649361170042005,
4. **1912**: 0.22967697101070242,
5. **1085**: 0.14943647607698152,
6. **0**: 0.14672864694039878,
7. **698**: 0.115768513859876

**top 7 nodes with highest-closeness centrality (runn. time : ~3min.):**

1. **107**: 0.45969945355191255
2. **58**: 0.3974018305284913,
3. **428**: 0.3948371956585509,
4. **563**: 0.3939127889961955,
5. **1684**: 0.39360561458231796,
6. **171**: 0.37049270575282134,

7. **348**: 0.36991572004397216

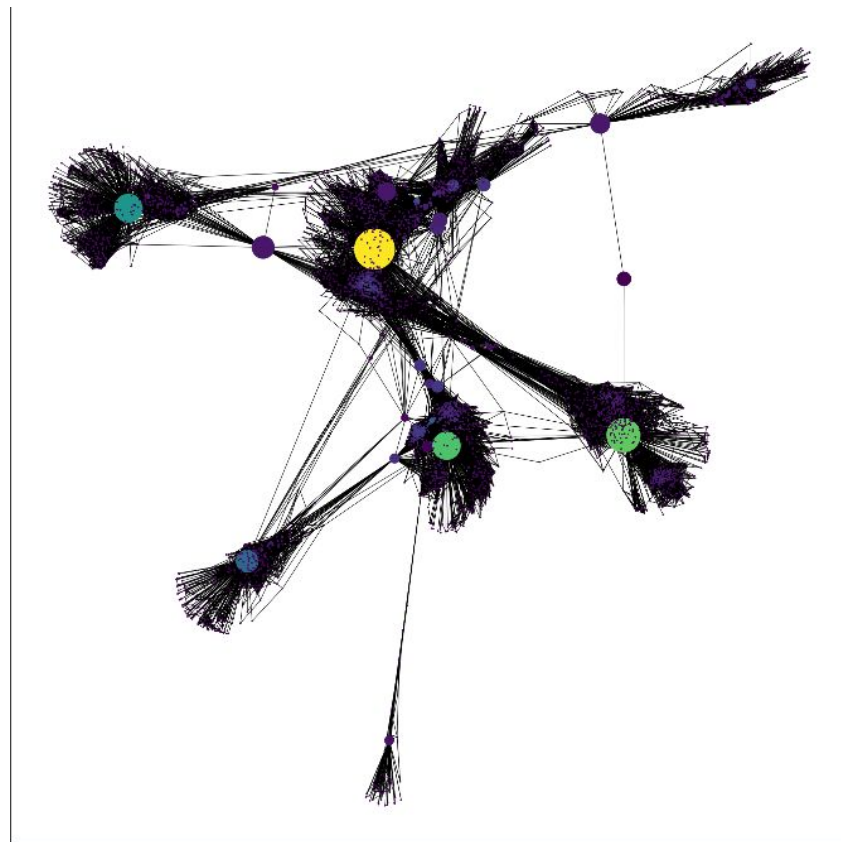
**top 7 nodes with highest-degree (runn. time : ~15sec.):**

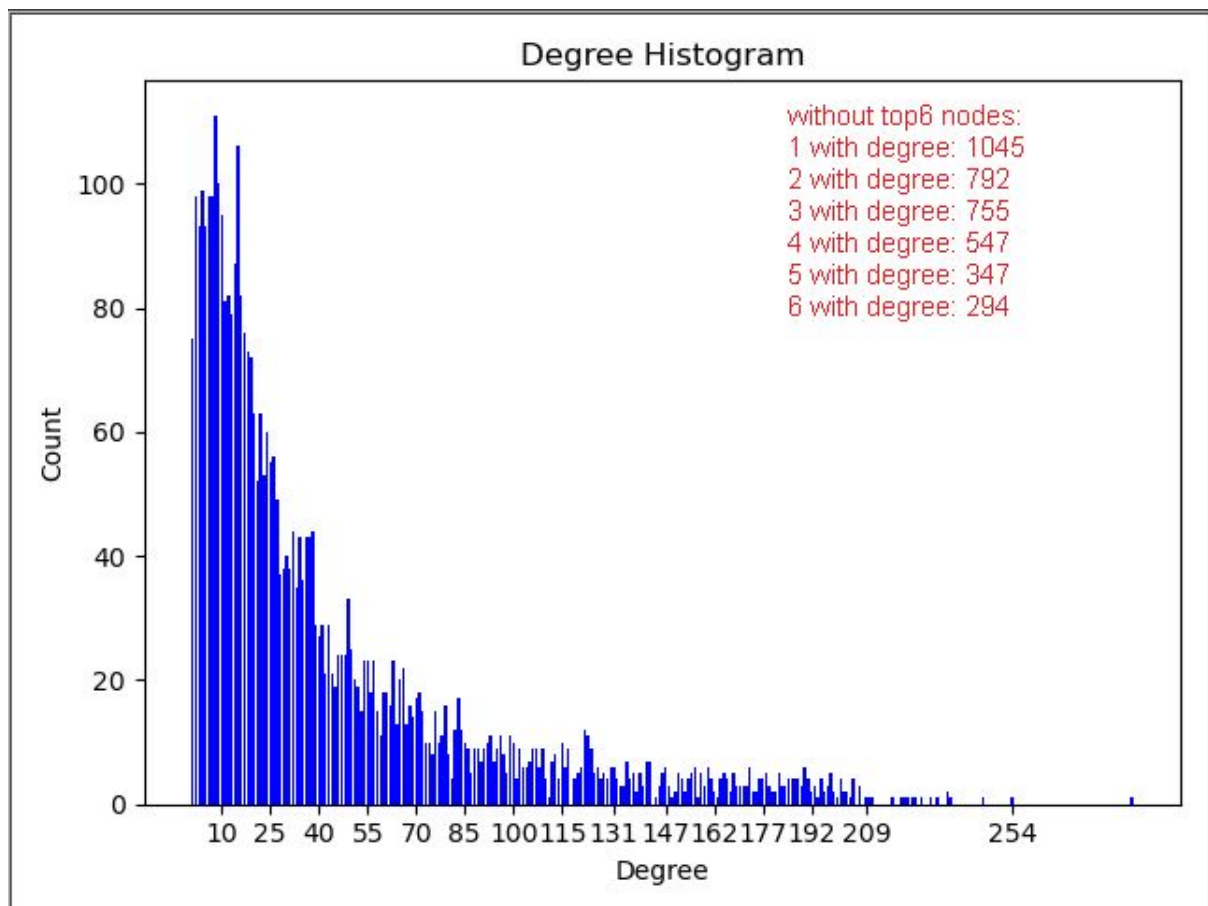
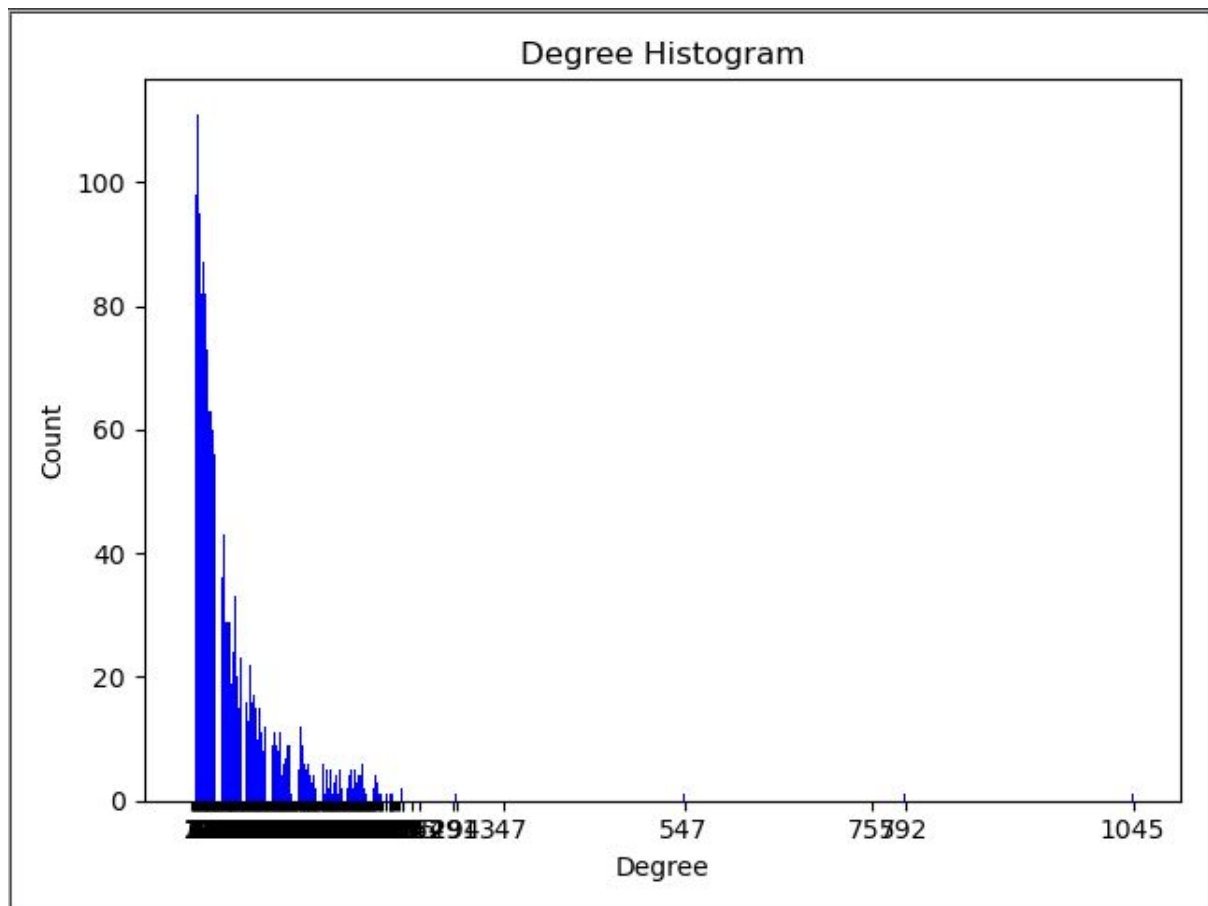
1. node: **107** degree:1045
2. node: **1684** degree:792
3. node: **1912** degree:755
4. node: **3437** degree:547
5. node: **0** degree:347
6. node: **2543** degree:294
7. node: **2347** degree:291

**node with max-degree** is **107** with degree=1045 and clustering=0.049038 that is under the average\_clustering!

My graph have the same characteristics of a **scale-free** network, in particular we can see that it follows a **power-law** distribution. The most notable characteristic in a scale-free network is the relative commonness of vertices with a degree that greatly exceeds the average.

In addition, I found some **hubs**: people with degree  $\gg$  Average degree (=43.6910).





In graph theory, a **clustering coefficient** is a measure of the degree to which nodes in a graph tend to cluster together. Evidence suggests that in most real-world networks, and in particular social networks, **nodes tend to create tightly knit groups characterised by a relatively high density of ties**; this likelihood tends to be greater than the average probability of a tie randomly established between two nodes (Holland and Leinhardt, 1971;<sup>[1]</sup> Watts and Strogatz, 1998<sup>[2]</sup>).

Two versions of this measure exist: the global and the local. We calculated the **local** that gives an indication of the embeddedness of single nodes.

**average\_clustering:** 0.605547

**top 5 nodes with Low-clustering coefficient:**

1. **3437**: 0.032230414314509376,
2. **0**: 0.04196165314587463,
3. **1684**: 0.044774546986936364,
4. **107**: 0.049038479165520905,
5. **3980**: 0.0853302162478083.

**top 5 nodes with High-clustering coefficient:**

1. **595**: 0.9883040935672515,
2. **3919**: 0.9848484848484849,
3. **3639**: 0.9848484848484849,
4. **3668**: 0.9818181818181818,
5. **576**: 0.978021978021978.

In conclusion, we can say that the **node 107** is the most important node followed by node **1684**.

*Giacomo Usai*

# NOTE

Examples of questions you can answer are the following:

1. Does the graph have the same characteristics of a random or a power-law network?
2. Which are the most important nodes, with respect to a given centrality measure?
3. Are the paths short with respect to the size of the network?
4. Is the network dense?
5. And so on

Some measures provide indicators to know.

The importance of a node or an area in the network.

The distance among nodes or areas in the network Cohesion degree of an area in the network.

A **geodesic path (shortest path)** is a path between two vertices such that no shorter paths exist.

The **diameter of a network** is the length of the longest shortest-path between any pairs of vertices.

The simplest centrality measure is **node degree** and it can be applied to directed and undirected graphs:

1. **indegree**: number of links entering node  $i$ ;
2. **outdegree**: number of links leaving node  $i$ ;
3. **degree**: number of links of node  $i$ . Provides an indication of the ability of a node engaging in a direct relationship with the other nodes;

node degree=The total number of links of the node;

The maximum number of possible edges in a simple undirected network is  $\frac{1}{2} (N)(N-1)$

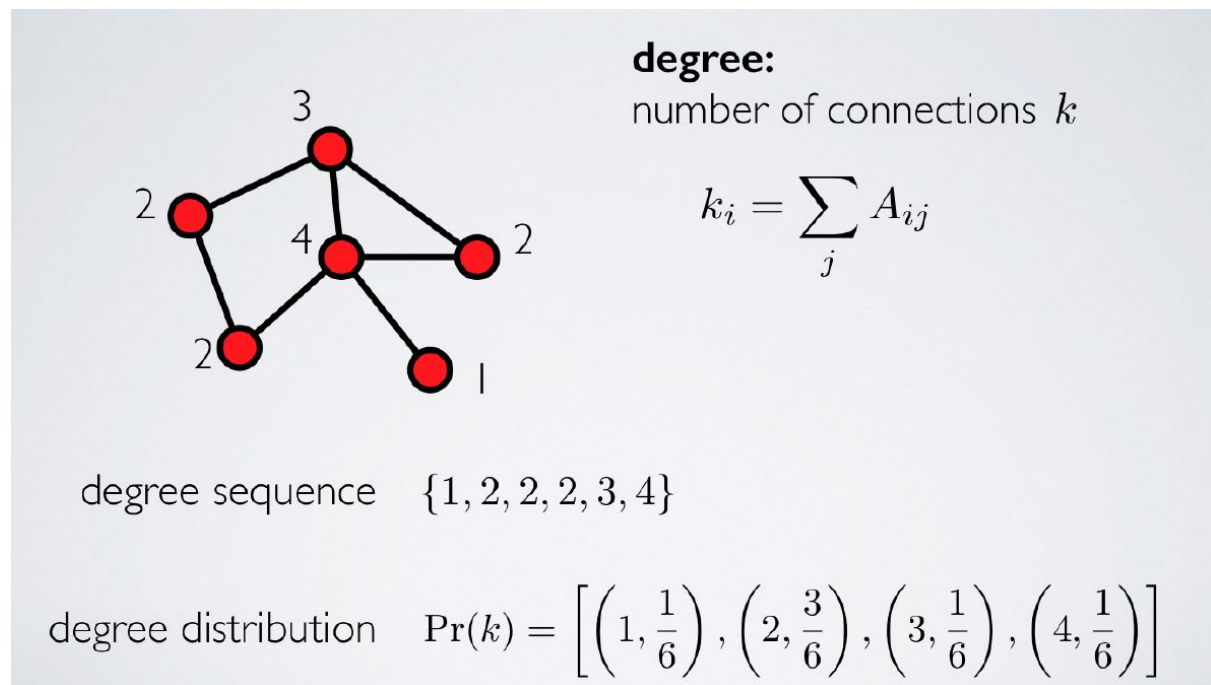
The density  $p$  of a network is the fraction of these edges really present in the network. So, it's simply:  $\text{\#edges}/\text{\#max\_number of possible edges}$ .  **$0 \leq p \leq 1$  always.**

If ( $p \rightarrow \text{constant}$  when  $N \rightarrow \infty$ ) **dense** network;

else If ( $p \rightarrow 0$  when  $N \rightarrow \infty$ ) **sparse** network.

**Degree distribution  $p_k$** : Provides the probability that a randomly selected node

in the network has degree  $k$ . Nodes' link/maxLink.



### Betweenness centrality of a node $v$

is the sum of the fraction of all-pairs shortest paths that pass through  $v$ .

So, the node's size it's related with the node's betweenness centrality. In practice: how many pairs of individuals would have to go through you in order to reach one another in the minimum number of hops? **Their removal may disrupt communication.**

- [Nodes with high betweenness have more important.](#)

**Closeness** measures the mean distance of a vertex to other vertices:

- [Nodes with high closeness have better access to information or more direct influence on other vertices.](#)

**Closeness centrality** of a node  $u$  is the reciprocal of the sum of the shortest path distances from  $u$  to all  $n - 1$  other nodes. Since the sum of distances depends on the number of nodes in the graph, closeness is normalized by the sum of minimum possible distances  $n - 1$ .