

1 Introduction

In the next part of the course, we will be working with the **Scheme** programming language. In addition to learning how to write Scheme programs, we will eventually write a Scheme interpreter in Project 4!

Scheme is a dialect of the **Lisp** programming language, a language dating back to 1958. The popularity of Scheme within the programming language community stems from its simplicity – in fact, previous versions of CS 61A were taught in the Scheme language.

2 Primitives

Scheme has a set of *atomic* primitive expressions. Atomic means that these expressions cannot be divided up.

```
scm> 123
123
scm> 123.123
123.123
scm> #t
True
scm> #f
False
```

Unlike in Python, the only primitive in Scheme that is a false value is `#f` and its equivalents, `false` and `False`. The `define` special form defines variables and procedures by binding a value to a variable, just like the assignment statement in Python. When a variable is defined, the `define` special form returns a symbol of its name. A procedure is what we call a function in Scheme!

The syntax to define a variable and procedure are:

- `(define <variable name> <value>)`
- `(define (<function name> <parameters>) <function body>)`

Questions

2.1 What would Scheme display?

```
scm> (define a 1)
```

a

```
scm> a
```

1

```
scm> (define b a)
```

b

```
scm> b
```

1

```
scm> (define c 'a)
```

c

```
scm> c
```

a

3 Call Expressions

To call a function in Scheme, you first need a set of parentheses. Inside the parentheses, you specify an operator expression, then zero or more operand subexpressions (remember the spaces!).

Operators may be symbols, such as `+` and `*` or more complex expressions, as long as they evaluate to procedure values.

```
scm> (- 1 1)           ; 1 - 1
0
scm> (/ 8 4 2)         ; 8 / 4 / 2
1
scm> (* (+ 1 2) (+ 1 2)) ; (1 + 2) * (1 + 2)
9
```

Evaluating a Scheme function call works just like Python:

1. Evaluate the operator (the first expression after the `()`), then evaluate each of the operands.
2. Apply the operator to those evaluated operands.

When you evaluate `(+ 1 2)`, you evaluate the `+` symbol, which is bound to a built-in addition function. Then, you evaluate 1 and 2, which are primitives. Finally, you apply the addition function to 1 and 2.

Questions

- 3.1 What would Scheme display?

```
scm> (+ 1)
```

1

```
scm> (* 3)
```

3

```
scm> (+ (* 3 3) (* 4 4))
```

25

```
scm> (define a (define b 3))
```

a

```
scm> a
```

b

```
scm> b
```

3

4 Special Forms

There are certain expressions that look like function calls, but *don't* follow the rule for order of evaluation. These are called *special forms*. You've already seen one — `define`, where the first argument, the variable name, doesn't actually get evaluated to a value.

4.1 If Expression

Another common special form is the **if** form. An **if** expression looks like:

(if <condition> <then> <else>)

where <condition>, <then> and <else> are expressions. First, <condition> is evaluated. If it evaluates to #t, then <then> is evaluated. Otherwise, <else> is evaluated.

Remember that only #f is a false-y value (also **False** for our interpreter); everything else is truth-y.

```
scm> (if (< 4 5) 1 2)
1
scm> (if #f (/ 1 0) 42)
42
```

4.2 Boolean Operators

Much like Python, Scheme also has the boolean operators **and**, **or**, and **not**. In addition, **and** and **or** are also special forms because they are short-circuiting operators.

```
scm> (and 25 32)
```

```
32
```

```
scm> (or 1 2)
```

```
1
```

Questions

4.1 What would Scheme display?

```
scm> (if (or #t (/ 1 0)) 1 (/ 1 0))
```

```
1
```

```
scm> (if (> 4 3) (+ 1 2 3 4) (+ 3 4 (* 3 2)))
```

```
10
```

```
scm> ((if (< 4 3) + -) 4 100)
```

```
-96
```

```
scm> (if 0 1 2)
```

```
1
```

4.3 Lambdas and Defining Functions

Scheme has lambdas too! The syntax is

```
(lambda (<PARAMETERS>) <EXPR>)
```

Like in Python, lambdas are function values. Also like in Python, when a lambda expression is called in Scheme, a new frame is created where the parameters are bound to the arguments passed in. Then, <EXPR> is evaluated in this new frame. Note that <EXPR> is not evaluated until the lambda function is called.

Like in Python, lambda functions are also values! So you can do this to define functions:

```
scm> (define (square x) (* x x)) ; Create function square using define special form
```

```
square
```

```
scm> (define square (lambda (x) (* x x))) ; Equivalently, bind the name square to a lambda function
```

```
square
```

```
scm> (square 4)
```

```
16
```

`let` is another special form based around `lambda`. The structure of `let` is as follows:

```
(let ( (<SYMBOL1> <EXPR1>)
      ...
      (<SYMBOLN> <EXPRN>) )
  <BODY> )
```

This binds the results of evaluating expressions 1 through n to their associated symbols, creating temporary variables. Finally, the body of the `let` is evaluated.

This special form is really just equivalent to:

```
( (lambda (<SYMBOL1> ... <SYMBOLN>) <BODY>) <EXPR1> ... <EXPRN>)
```

Think of the temporary variables as being the parameters of a lambda function. Then, the arguments are the values of the expressions, which we bind to the temporary variables by calling the lambda.

Consider the following example:

```
(let ((x 1)
      (y 2))
  (+ x y))
```

This is equivalent to:

```
((lambda (x y) (+ x y)) 1 2)
```

Questions

- 4.1 Write a function that returns the factorial of a number.

```
(define (factorial x)

  (if (< x 2)
      1
      (* x (factorial (- x 1)))))
```

- 4.2 Write a function that returns the n^{th} Fibonacci number.

```
(define (fib n)
  (if (or (= n 0) (= n 1))
      n

      (+ (fib (- n 1)) (fib (- n 2)))))
```

5 Pairs and Lists

To construct a (linked) list in Scheme, you can use the constructor `cons` (which takes two arguments). `nil` represents the empty list. If you have a linked list in Scheme, you can use selector `car` to get the first element and selector `cdr` to get the rest of the list. (`car` and `cdr` don't stand for anything anymore, but if you want the history go to http://en.wikipedia.org/wiki/CAR_and_CDR).

```
scm> nil
()
scm> (null? nil)
#t
scm> (cons 2 nil)
(2)
scm> (cons 3 (cons 2 nil))
(3 2)
scm> (define a (cons 3 (cons 2 nil)))
a
scm> (car a)
3
scm> (cdr a)
(2)
scm> (car (cdr a))
2
scm> (define (len a)
  (if (null? a)
      0
      (+ 1 (len (cdr a)))))
len
scm> (len a)
2
```

If a list is a “good looking” list, like the ones above where the second element is always a linked list, we call it a **well-formed list**. Interestingly, in Scheme, the second element does not have to be a linked list. You can supply something else instead, creating a **malformed list**. The difference is shown with a dot:

```
scm> (cons 2 3)
(2 . 3)
scm> (cons 2 (cons 3 nil))
(2 3)
scm> (cdr (cons 2 3))
3
scm> (cdr (cons 2 (cons 3 nil)))
(3)
```

In general, the rule for displaying a pair is as follows: use the dot to separate the `car` and `cdr` fields of a pair, but if the dot is immediately followed by an open

parenthesis, then remove the dot and the parenthesis pair. Thus, `(0 . (1 . 2))` becomes `(0 1 . 2)`

There are many useful operations and shorthands on lists. `list` takes zero or more arguments and returns a list of its arguments.

This typically behaves much like quoting a list, except that quoting will not evaluate the list you have quoted which can result in some different outcomes.

```
scm> (list 1 2 3)
(1 2 3)
scm> '(1 2 3)
(1 2 3)
scm> (car '(1 2 3))
1
scm> (equal? '(1 2 3) (list 1 2 3))
#t
scm> '(1 . (2 3))
(1 2 3)
scm> '(define (square x) (* x x))
(define (square x) (* x x))
scm> square ; We didn't actually define square above because of the quote
Error
scm> (list (cons 1 2))
((1 . 2))
scm> '((cons 1 2))
((cons 1 2))
```

`=`, `eq?`, `equal?`

- `=` can only be used for comparing numbers.
- `eq?` behaves like `==` in Python for comparing two non-pairs (numbers, booleans, etc.). Otherwise, `eq?` behaves like `is` in Python.
- `equal?` compares pairs by determining if their `cars` are `equal?` and their `cdrs` are `equal?` (that is, they have the same contents). Otherwise, `equal?` behaves like `eq?`.

```
scm> (define a '(1 2 3))
a
scm> (= a a)
Error
scm> (equal? a '(1 2 3))
#t
scm> (eq? a '(1 2 3))
#f
scm> (define b a)
b
scm> (eq? a b)
#t
```


Questions

- 5.1 Write a function which takes two lists and concatenates them.

Notice that simply calling `(cons a b)` would not work because it will create a deep list.

```
(define (concat a b)
```

```
  (if (null? a)
      b
      (cons (car a) (concat (cdr a) b))))
```

```
scm> (concat '(1 2 3) '(2 3 4))
(1 2 3 2 3 4)
```

- 5.2 Write a function that takes an element `x` and a non-negative integer `n`, and returns a list with `x` repeated `n` times.

```
(define (replicate x n)
```

```
  (if (= n 0)
      nil
      (cons x (replicate x (- n 1)))))
```

```
scm> (replicate 5 3)
(5 5 5)
```

- 5.3 A **run-length encoding** is a method of compressing a sequence of letters. The list (a a a b a a a) can be compressed to ((a 3) (b 1) (a 4)), where the compressed version of the sequence keeps track of how many letters appear consecutively.

Write a function that takes a compressed sequence and expands it into the original sequence. *Hint:* You may want to use `concat` and `replicate`.

```
(define (uncompress s)

  (if (null? s)
      s
      (concat (replicate (car (car s)) (car (cdr (car s))))
                (uncompress (cdr s)))))
```

```
scm> (uncompress '((a 1) (b 2) (c 3)))
(a b b c c c)
```

- 5.4 Write a function that takes a procedure and applies it to every element in a given list.

```
(define (map fn lst)

  (if (null? lst)
      nil
      (cons (fn (car lst)) (map fn (cdr lst)))))
```

```
scm> (map (lambda (x) (* x x)) '(1 2 3))
(1 4 9)
```

- 5.5 Write a function that takes a procedure and applies to every element in a given nested list.

The result should be a nested list with the same structure as the input list, but with each element replaced by the result of applying the procedure to that element.

Use the built-in `list?` procedure to detect whether a value is a list.

```
(define (deep-map fn lst)

  (cond ((null? lst) lst)
        ((list? (car lst)) (cons (deep-map fn (car lst)) (deep-map fn (cdr lst)))))
        (else (cons (fn (car lst)) (deep-map fn (cdr lst)))))
  )
```

```
scm> (deep-map (lambda (x) (* x x)) '(1 2 3))
(1 4 9)
scm> (deep-map (lambda (x) (* x x)) '(1 ((4) 5) 9))
(1 ((16) 25) 81)
```

6 Extra Questions

- 6.1 Fill in the following to complete an abstract tree data type:

```
(define (make-tree label branches) (cons label branches))
```

```
(define (label tree)
```

```
(define (branches tree)
```

```
(define (label tree) (car tree))
(define (branches tree) (cdr tree))
```

- 6.2 Using the abstract data type above, write a function that sums up the entries of a tree, assuming that the entries are all numbers.

Hint: you may want to use the `map` function you defined above, and also write a helper function for summing up the entries of a list.

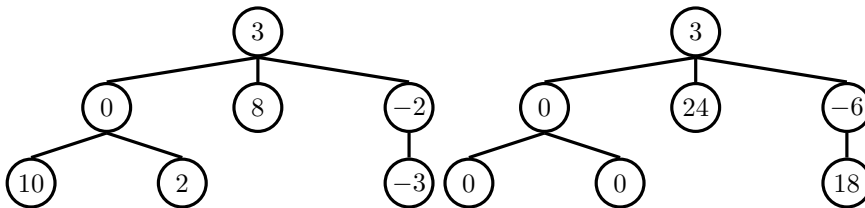
```
(define (tree-sum tree)

  (+ (label tree) (sum (map tree-sum (branches tree))))))
```

```
(define (sum lst)
  (if (null? lst) 0 (+ (car lst) (sum (cdr lst))))))
```

- 6.3 Using the abstract data type above, write a function that creates a new tree where the entries are the product of the entries along the path to the root in the original tree.

Hint: you may want to write a helper function that keeps track of the current product.



```
(define (path-product-tree t)

  (define (path-product t product)
    (let ((prod (* product (label t))))
      (make-tree prod
        (map (lambda (t) (path-product t prod))
          (branches tree))))))

  (path-product t 1))
```