Applying The Kelly Criterion to The Stock Market

A Bet-Hedging Analysis for The Case of Continuous Outcomes

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Introduction

You are put in a room with an infinitely wealthy opponent. As an experiment, they point to the coin on the table in front of them and tell you that it is biased to land heads 60% of the time. Every time it lands heads, you receive double the amount you bet on it. Every time it lands tails, you lose your bet. You have $100 in your pocket, and you're allowed to leave whenever with however much you have. How much will you bet?

Originally developed by John Kelly (1956), the Kelly Criterion is a formula used to determine the optimal bet sizing for such a game. As Kelly proved, as long as the bet is favorable, it is in the gambler's best interest to gamble a fraction f^* of their bankroll on each bet, when p is their chance of winning, following

$$f^* = 2p - 1$$

In this paper, I will continue the work of Kelly, by attempting to determine an f^* for a bet with continuous outcomes, such as an investment in a stock. After developing a formula for this continuous case using Thorp (1992), I will attempt to continuously approximate the gamble using Geometric Brownian Motion (GBM), which is representative of a stock price. Upon building intuition for GBM, I will call on Thorp (2006) to determine how a gambler/investor can find the optimal fraction of their bankroll to bet under such a circumstance. Putting my result to use, I will apply historical market data to the developed Kelly model to determine an f^* for the S&P500. I will then caveat this result to complete my discussion with a brief exploration into The Black Swan Theory and Fat Tails (Taleb 2001) and why they indicate a need for the "fractional Kelly."

1 Background and My Interest

Progressing my interest in probability, I heard about the Kelly criterion through the above biased-coin riddle on a podcast. Quickly humbling me, a self-proclaimed smart gambler and investor, I had absolutely zero clue how much to bet. It turns out, however, that I was not alone. The riddle that puzzled me puzzled other finance and economic students in Haghani and Dewey's 2016 study. As their study described, even these quantitative individuals wanting to work in finance had zero idea how much to bet. I ran with this hypothetical and started quizzing my friends. To my dismay, no one knew the correct answer. How could the Kelly criterion, such an important piece of gambling and investing, be unknown to such a demographic? I even quizzed my stats teacher, and he had zero clue. Understanding its importance, and the fact that it's relatively unknown, I held it in the back of my mind and grappled with the Kelly criterion in my day-to-day life while attempting to card count in casinos, win money against my friends in poker, and now, invest in the stock market. I figured with how common it is for me to think about it, it was finally time for me to understand it in greater detail and spread the love for such a beautiful formula around. Thus, I chose to explore it in this paper.

The literature regarding the Kelly criterion is rather comprehensive. The applications of the formula have been explored in sports betting (Thorp 2006), lawsuits (Barnett 2010), and biological evolution (Childs 2010), just to name a few. The formula has also been dissected and modified in a variety of different ways for different circumstances: facing a fluctuating winning percentage, different payoff odds, unknown winning chances, maximizing economic utility instead of pure profit, and even, as I will discuss in this paper, continuous payoffs and the stock market. However, with all this information, there is also a lot of noise. As I sought to explore the Kelly criterion in detail, I struggled to find the what should be common applications I was looking for in simple terms. Even after discovering two of Thorp's papers on the Kelly criterion and the stock market, I was still left grasping at hurried proofs and quick explanations. It is evident Thorp wrote his paper for the already understanding eyes, where in this, I seek to distill the continuous Kelly criterion down in its simplest form because I believe the Kelly criterion is not just for the mathematically elite; it's something everyone must understand.

2 Research Questions

Prompting my desire to discover more about the widely applicable Kelly criterion, I found myself asking the same types of questions:

Can we apply the Kelly criterion to a bet with outcomes following a continuous distribution?
Can we rigorously derive these results and apply them to the stock market?
Can we say something about the fluctuation of stock prices mathematically?
Can we apply the Kelly criterion to such a continuously changing bet?
How do Taleb's (2001) proposed concepts of fat tails and black swans apply to our results?

3 The Continuous Kelly

In the case of a continuous set of outcomes, profits follow the model

$$V_n = V_{n-1}(1 + fX)$$

Where V_n is our profits after n trials, f the fraction bet, and X the random variable representing the multiple applied to the bet. Similarly, our profits can be modeled as

$$V_n = V_o \prod_{i=1}^{n}(1 + fX_i)$$

Thorp (2006) mentions Kelly (1965) and uses the fact that

$$V_o e^{nln\frac{V_n}{V_o}\frac{1}{n}} = V_n$$

To indicate that to find f^*, one needs to maximize the growth rate coefficient

$$G(f) = \ln\left(\frac{V_n}{V_o}\right)^{\frac{1}{n}}$$

Or rather, the expected value of it

$$g(f) = EG(f) = E\ln\left(\frac{V_n}{V_o}\right)^{\frac{1}{n}} = E\ln(1 + fX) = \int_x \ln(1 + fX)\cdot f_X(x)\,dx$$

However, since $g(f) = \frac{1}{n}(E\ln V_n - \ln V_o)$, it is equivalent to max $E\ln V_n$, the log of our profits, or similarly, $E\ln\frac{V_n}{V_o}$, as we will do later. It is important to note the formula achieved above is the same formula used in the binary case, just instead of evaluating the expected value with two outcomes, an integral is used to evaluate the expected value for all possible outcomes.

3.1 Example

To understand this process, let's look at an example. You want to buy a stock. The price is currently \$100. You expect that in a year it will be worth anywhere from \$50 to \$170 with the same probability for each outcome. How much of your \$100 should you invest to maximize long-term profits?

Mathematically, we can model this with the above equation where X represents the multiplier we apply to our investment. In this example, X $\sim$ Uniform(-.5, .7): our worst case result is we lose 50% of the investment, and best case, we win 70%. So we maximize

$$g(f) = \int_x \ln(1 + fX)\cdot f_X(x)\,dx = \int_{-.5}^{.7}\ln(1 + fX)\cdot\frac{1}{1.2}\,dx = \frac{1}{1.2}\int_{-.5}^{.7}\ln(1 + fX)dx$$

Taking the derivative and applying computational techniques

$$g'(f) = \frac{1}{1.2}\int_{-0.5}^{0.7}\frac{X}{1 + fX}\,dx = \frac{1}{1.2f}\left(\int_{-0.5}^{0.7}1\,dx - \int_{-0.5}^{0.7}\frac{1}{1 + fX}\,dx\right)$$

Which, when set equal to 0 gives us

$$1.2 = \frac{1}{f}\ln\left(\frac{1 + 0.7f}{1 - 0.5f}\right)$$

Thus, $f^* \approx .85$, indicating the optimal investment in this security is \$85.

3.2 General Case

Interestingly, this $f^* \approx .85$ is far greater than what the binary case with the same expected value would produce. In fact, given the standard biased coin Kelly experiment above, the gambler would need a 92.5% chance of winning to justify betting $f^* = .85$. Such a binary bet has an expected payoff of 1.85, far greater than the expected payoff (1.1) of this continuous case. This difference follows from the fact that in this continuous case, it is impossible to lose all of an investment in one trial, allowing the investor to be more aggressive. However, what if the investment multiple ranged from -1 to 2.7 (giving an expected payoff equal to 1.85 and the possibility that the investor loses it all)? To understand this case, and any general case that follows a uniform distribution of outcomes, let's maximize the above $g(f)$ for X $\sim$ Uniform(a, b). This results in:

$$\frac{1}{b-a} = \frac{1}{f}\ln\left(\frac{1+fb}{1+fa}\right)$$

Looking at X $\sim$ Uniform(-1, 2.7), we determine $f^* \approx .86$, almost the same as the binary case with the same expected value. Let's look at some other examples:

1) $X \sim Uniform(-.5, .5)$ yields $f^* = 0$.
2) $X \sim Uniform(-.6, .5)$ yields $f^* \approx -.5$.
3) $X \sim Uniform(-1, 2)$ yields $f^* \approx .7$.
4) $X \sim Uniform(-.5, 1)$ yields $f^* \approx 1.4$.
5) $X \sim Uniform(-.2, .4)$ yields $f^* \approx 3.6$.
6) $X \sim Uniform(-.2, .3)$ yields $f^* \approx 2.5$.

In these examples, we find some interesting results. In 1, as expected, when our expectation is equal to 0, we don't bet. However, rather surprisingly, in 2, when the expected value is negative, we obtain a negative f^*, corresponding to shorting the security. All 3, 4, and 5 have the same expected value, as the variance goes down, however, Kelly tells us to bet more, even sometimes exceeding $f^* > 1$. This is equivalent to buying the security on the margin or using debt to leverage our investment. Finally, even bets with lower expected values (compare 6 to 4), can have a greater f^* if they have a small enough variance, implying a better investment as determined by the Kelly criterion.

3.3 One Step Further

However, stocks don't follow this perfect uniform probability distribution for their prices after a year. Rather, it's very random. As we will see later, such prices can be modeled with a random walk called Geometric Brownian Motion. However, for now, sticking to the discrete-time jump of one year, let's look at what happens when the stock price no longer follows a uniform density. Take, for instance, the probably more accurate model, the normal distribution. What if the return rate for a stock after a year could be represented by $X \sim Normal(\mu, \sigma^2)$ where μ is equal to the expected value of the stock, and σ^2 the variance. It's actually, as you may be able to guess, a very easy change from the uniform case. We still seek to maximize

$$g(f) = E(\ln(1+fX)) = \int_x \ln(1+fX) \cdot f_X(x)\,dx$$

Plugging in for $f_X(x)$

$$g(f) = \int_x \ln(1+fX) \cdot \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}\,dx$$

The only problem is that now, our range for x is $(-\infty, \infty)$ implying that it's possible to lose more than our investment, which, as we know, is not possible when buying a stock long. Thus, we truncate our bounds to be between A and B, such that, $A > -1$. Thorp (1992) did this by setting $A = \mu - 3\sigma$, $B = \mu + 3\sigma$, and rescaling the distribution so the integral still equals one, which works for most stocks of note. For more details on this, look at Thorp's paper. For now, we will move on to a more accurate model for the price of stocks.

4 Kelly for Continuous Stock Fluctuations

As hinted at earlier, the notion of looking at a discrete time jump and the resulting outcomes, although providing valuable insight for long-term investors, may be a bit simplistic. In reality, stocks are in constant flux. We now seek to find a Kelly optimal bet for this real-world case.

4.1 Geometric Brownian Motion

To model the randomness of a stock price, we need a fitting stochastic process. Immediately, thinking of a random process to model natural fluctuations, we might seek to apply Standard Brownian Motion, defined by summing independent trials of a normal random variable to create a random walk. However, such a model falls short for a couple of reasons. One, Standard Brownian Motion would allow for stock prices to go negative. Obviously, in the real world, this does not make sense. Two, Standard Brownian Motion sums independent trials, whereas, in the stock market, we know that prices benefit from compounding growth. They are not simply a random number added to the previous day's price. A more fitting model is Brownian Motion's big brother, Geometric Brownian Motion (GBM), defined as its log being Standard Brownian Motion. This means that instead of summing independent trials, stripping away the concept of compounding growth, GBM multiplies independent trials and is a more accurate measure of stock price. Mathematically, GBM is defined by the stochastic differential equation

$$dS_t = \mu S_t \, dt + \sigma S_t \, dW_t$$

where S_t is the asset price at time t, μ is the drift coefficient, σ is the volatility coefficient, and W_t is standard Brownian motion. Put more simply however, μ is the growth we expect from the stock, σ is the variation we expect from the stock, and W_t is just the random process that paired with σ makes the price constantly fluctuate. Here's an image with several realizations of this random process:

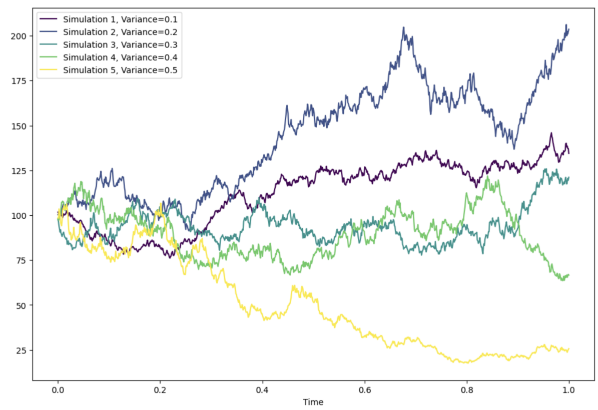

Figure 1: GBM w/ $\mu = .25$ and different variations

Now, to confirm that the stochastic differential equation (SDE) above represents GBM, $\ln S_t$ should give us Standard Brownian Motion. Applying Ito's lemma and subbing in for dS_t

$$d \ln S_t = \frac{1}{S_t} dS_t - \frac{1}{2S_t^2} dS_t^2 = \frac{1}{S_t}(\mu S_t \, dt + \sigma S_t \, dW_t) - \frac{1}{2S_t^2}(\mu S_t \, dt + \sigma S_t \, dW_t)^2$$

Simplifying the expression, using $dW_t^2 = dt$ and removing terms that approach 0 slower than dt

$$d \ln S_t = (\mu dt + \sigma dW_t) - \frac{1}{2}(\sigma^2 dt) = (\mu - \frac{\sigma^2}{2})dt + \sigma dW_t$$

Thus, we achieve our desired result of Brownian Motion, which has an SDE equal to the type we obtained where S_t is no longer multiplied by μ and σ to achieve dS_t. In simpler terms, the change in S_t, following Standard Brownian Motion, is equal to a sum of μ and σ instead of a product of them with S_t. However, for this paper, the exact understanding of the above SDEs is not necessary, and rather, the concept of GBM representing the random walk of a product of random trials and the corresponding image is far more important.

4.2 Continuous Approximation

With such an understanding in mind, let's develop a Kelly optimal f^* for an investment in a stock that follows GBM. At every moment in time, take $\mathrm{P(X} = \mu + \sigma) = \mathrm{P(X} = \mu - \sigma) = 0.5$. Thus, profits can now be modeled by

$$V_n = V_{n-1}\left(1 + (1-f)r + fX\right)$$

Where the new variable r is equal to the risk-free return rate granted by the market (aka, an investment in treasury bills returning 5 percent every year). We include this, because as in the real world, the money we don't invest, can be put to compound in something safe, and thus, should be reflected in our model. We again wish to maximize $g(f)$

$$g(f) = E\ln(1 + (1 - f)r + fX)$$

However now, we divide the interval into n different independent steps and replace μ with $\frac{\mu}{n}$, σ^2 with $\frac{\sigma^2}{n}$, and r with $\frac{r}{n}$. Now our continuous movement can be modeled by $P(X_i = \frac{\mu}{n} + \frac{\sigma}{\sqrt{n}}) = P(X_i = \frac{\mu}{n} - \frac{\sigma}{\sqrt{n}})$ = 0.5 for X_i = 1, 2, ..., n. Then, representing GBM with a product to simulate random movements

$$\frac{V_n}{V_o} = \prod_{i=1}^{n}\left(1 + (1 - f)\,\frac{r}{n} + fX_i\right)$$

Again though, we seek to maximize $g(f)$, which is equivalent to $\ln\frac{V_n}{V_0}$

$$g(f) = E\ln\frac{V_n}{V_o} = E\ln\prod_{i=1}^{n}\left(1 + (1 - f)\,\frac{r}{n} + fX_i\right)$$

Continuing with computations:

$$g(f) = E\sum_{i=1}^{n}\ln\left(1 + (1 - f)\,\frac{r}{n} + fX_i\right) = \sum_{i=1}^{n}E\ln\left(1 + (1 - f)\,\frac{r}{n} + fX_i\right) = nE\ln\left(1 + (1 - f)\,\frac{r}{n} + fX_i\right)$$

Subbing in $X_i = \frac{\mu}{n} + \frac{U\sigma}{\sqrt{n}}$, where U is either -1 or 1, for ease of computation.

$$\frac{g(f)}{n} = E\ln\left(1 + (1 - f)\,\frac{r}{n} + f(\frac{\mu}{n} + \frac{U\sigma}{\sqrt{n}})\right)$$

Cleaning up this equation further, Taylor expanding the ln, using $\ln(1 + x) = x - \frac{x^2}{2} + ...$, and dropping $O(n^{\frac{-3}{2}})$ and smaller terms since they converge to 0 faster than $O(n^{-1})$ terms as n goes to infinity (which we send n to as to make it approximately continuous)

$$\frac{g(f)}{n} = E\left(\frac{r}{n} - \frac{fr}{n} + \frac{f\mu}{n} + \frac{fU\sigma}{\sqrt{n}} - \left(\frac{(\frac{r}{n} - \frac{fr}{n} + \frac{f\mu}{n} + \frac{fU\sigma}{\sqrt{n}})^2}{2}\right)\right) = E\left(\frac{r}{n} - \frac{fr}{n} + \frac{f\mu}{n} + \frac{fU\sigma}{\sqrt{n}} - \frac{f^2U^2\sigma^2}{2n}\right)$$

Now, by evaluating the expected value when P(U = -1) = P(U = 1) = 0.5, we arrive at

$$\frac{g(f)}{n} = .5\left(\frac{r}{n} - \frac{fr}{n} + \frac{f\mu}{n} + \frac{f\sigma}{\sqrt{n}} - \frac{f^2\sigma^2}{2n}\right) + .5\left(\frac{r}{n} - \frac{fr}{n} + \frac{f\mu}{n} - \frac{f\sigma}{\sqrt{n}} - \frac{f^2\sigma^2}{2n}\right) = \frac{r}{n} - \frac{fr}{n} + \frac{f\mu}{n} - \frac{f^2\sigma^2}{2n}$$

Implying that

$$g(f) = r + f(\mu - r) - \frac{f^2\sigma^2}{2}$$

Now, $g(f)$, as n approaches infinity, is the instantaneous growth rate, finding its maximum

$$g'(f) = \mu - r - f\sigma^2 = 0 \rightarrow f^* = \frac{\mu - r}{\sigma^2}$$

Isn't that just beautiful. To reiterate, μ is the instantaneous drift, r is the risk-free return rate, and σ is the volatility of our security.

4.3 The S&P500

Now let's do something with the information we just uncovered. Historically, the S&P500 has returned on average 10.62% a year for the last 100 years with a deviation of roughly 0.15. Running a simulation, this would look something like:

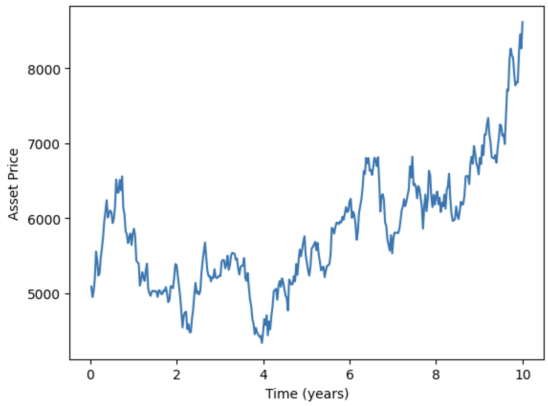

Figure 2: Simulation of S&P500 over next 10 years

Plugging into our f^* formula, granted our risk-free return rate is somewhere around 5%, we determine

$$f^* = \frac{.1062 - .05}{.15^2} \approx 2.5$$

Wow.

5 The Fractional Kelly

Given such a result, and the difference between the math and actual investor behavior, it's important to analyze the reason for the discrepancy. Most people aren't leveraging their S&P500 investments using $1.50 of debt for every $1.00 invested. This is the case for a couple of obvious reasons. Primarily, humans act more based on emotions than complicated math. It's a little bit scary to be so leveraged in the S&P500. One big crash and all of a sudden your world comes tumbling down. The minuscule extra gains probably don't offset the stress most people will have about this. Also, many people have other investments with better returns to put their money in, maybe in their own business or to go on a vacation with family. At the very least, however, this large Kelly optimal f^* should leave readers at least contemplating the power of compound growth and simple investments in the S&P500 as the Kelly fraction indicates it's quite a favorable bet.

5.1 Black Swans

Perhaps you're sane and don't need to be convinced to not follow this math perfectly and buy the S&P500 on such an extreme margin. Perhaps, however, you're a firm believer in acting optimally and don't care about the extra stress. Here's another, more mathematical explanation, for why you should reconsider. In Fooled by Randomness (2001), Nassim Taleb introduces the Black Swan Theory. Following a story that gives it its name, the black swan theory illustrates the fault of relying on historical data to guide future actions. Taleb describes black swans as events that given their extreme rarity have never happened and seem impossible, but are actually more than possible. When, given enough time, these black swans inevitably occur, they have extreme impacts both due to their magnitude and also because people don't prepare for "impossible events." Take the 2008 Crash and COVID-19 as two recent examples. Here's how people can get into trouble by not considering the possibility of black swans. They look at historical data and presume it's a good representation of what normally occurs, so they base their bets/decisions on that. Then, a black swan that completely opposes events of the past happens, and it blows everything up.

5.2 Fat Tails

Black Swans give rise to another Taleb concept, fat tails, or when rare events have a proportionally extreme impact. Take the concept of wealth distribution in the United States, taking a random sample of 100 people, you'd find that the average net worth is probably around $150k. However, if you somehow included Warren Buffett in your sample, you'd find that the average net worth of your sample is around $1.3 billion. "Outliers" create almost all of the results, we can't remove them. Apply this same idea to the stock market. In our above example, we took the last 100 years of data, similar to our sample of 100 people. Imagine, however, we missed the Warren Buffett of stock volatility (more extreme than both 2008 and the great depression), and all of a sudden, in the coming years, such an event occurs. You'd lose more than everything being so over-leveraged. This is why we don't rely on past data to guide our future actions. If you're ever in charge of building a flood wall around a city, don't build it to the historical highest flood level unless you want your whole city destroyed when the next historical high occurs. The same applies when investing in stocks. Prepare for the worst.

5.3 The Dangers of Overbetting

Thorp (2006) understood this intuitively and worked it into his paper on the Kelly criterion. When we plug in example drift coefficients and volatility, they were estimates based on historical data. Following the garbage in garbage out principle of models, if these estimates are poor, our model is poor. Thus, we need to exercise caution when using it. Especially since over-betting, as compared to under-betting, can lead to complete ruin.

5.3.1 Example

Recall our formula for instantaneous growth rate

$$g(f) = r + f(\mu - r) - \frac{f^2\sigma^2}{2}$$

Thorp (2006) explains that if $g(f) < 0$, profits almost surely go to 0. Thus, if an investor accidentally bets at a fraction f that results in $g(f) < 0$, they will almost surely lose all their money, even if the investment is favorable. Before, we set $\mu = .1062$, $\sigma = .15$, and $r = .05$ to model the S&P500 and determine $f^* \approx 2.5$. Let's assume, however, that the historical data is a poor representation of the S$P500. Instead, exercising caution, let's assume that in the coming years, following such an uptick in the market, returns won't be so great: $\mu = .06$, $\sigma = .16$, and $r = .05$. Plugging in these numbers and our previous f^*, we determine $g(f^*) < 0$. Thus, betting the previously optimal f^* will almost surely lead an investor to go broke. However, as Thorp (2006) recommends, by betting cf^* where c is some fraction between 0 and 1, we can account for variability in the inputs and still achieve profound growth, avoiding extreme loss in the process. This is referred to as the fractional Kelly, commonly used throughout all of its applications. The fractional Kelly is perfect for risk-averse gamblers/investors who want to reap the rewards from the Kelly criterion without accidentally losing everything.

6 Conclusion

Originally, I sought to determine whether the Kelly criterion was applicable to bets with a continuous array of outcomes. I did this with the end goal of applying the Kelly criterion to the stock market. Upon following the same steps as the binary case and maximizing the expected value of the log of the geometric growth rate, several interesting results, that didn't come up in the binary case, were analyzed: when $f^* > 1$, $f^* < 0$, and when bets with a lower expected value produced a greater f^*. After these findings, a more accurate model for stock prices using Geometric Brownian Motion was introduced. Then, using GBM to continuously approximate a stock's price, a Kelly optimal f^* was determined based on a stock's drift and variation rate. Historical numbers from the S&P500 were plugged into the equation, $f^* = \frac{\mu - r}{\sigma^2}$, and we determined the Kelly optimal bet is $f^* \approx 2.5$, indicating an investor should buy the S&P500 on the margin. Given the seemingly insane nature of this proposal, I then introduced The Black Swan Theory (Taleb 2001) to caveat the prior result and warn investors of the dangers of overbetting. This was followed by a look into fat tails and eventually, the Kelly criterion itself, analyzing how a fluctuation in the inputs can lead to what was once the optimal bet being a sure way to go broke.

6.1 Discussion

In setting out to determine a Kelly optimal bet for the continuous case, I did not expect to progress to where I did. I thought, given the difficulties of finding many papers on the subject, the continuous case would be vague and hard to prove. However, that was not the case at all. Rather, stumbling on Thorp (2006), I immediately understood the simple formula and derivation. What I did not expect to learn, however, was the continuous approximation Kelly mentioned in the same paper. It took several rereads to even understand what he was getting at. I didn't understand why, after developing the Kelly criterion for the continuous case, he did a "continuous approximation." Once I realized that by continuous approximation he meant gambling on a bet that continuously moved (i.e. the stock market) and not just a continuous array of outcomes, I was hooked. He mentioned the idea that the model he proposed followed log-normal diffusion. Unaware of the concept, I looked it up, and to my delight, found that it was also called Geometric Brownian Motion, tangent to a topic of already great interest to me, Brownian Motion. I had heard that stocks could be modeled with Brownian Motion before, and that made sense, what I was unaware of, however, was that the type of Brownian Motion they were modeled by was geometric. Thus, I figured given the nature of the students in our class, many interested in finance, it would be wise to include an explanation of GBM, in which I had to do a small dive into Ito calculus to derive, something I found rather fascinating.

6.2 Next Steps

Finding it rather fascinating, to expand on the current scope of this project, I'd like to dive further into the concept of Geometric Brownian Motion. Although I feel like I currently have a decent grasp on it, I'm sure I'm just scratching the surface. Applying Ito's lemma, I realized I'm unaware of many things regarding stochastic processes and I'm determined to learn more about SDEs in the future.

In terms of the Kelly Criterion, I also feel as if I've barely scratched the surface. As I mentioned at the start, I was roped into the idea of the Kelly criterion upon hearing about a study where, when asked a bet-hedging question, finance students had no idea what to do. Even my stats teacher in high school was lost. Understanding its power, I thought it was absurd more people weren't aware of the Kelly criterion, and thus, I sought to be more aware myself. I'd love to continue diving into the fractional Kelly, which I only briefly mentioned at the end. It's closely related to the idea of black swans and fat tails, and that realm of extreme value statistics has been an interest of mine as of late. In the future, I'd like to construct a model that indicates the optimal fractional Kelly given a presumed range of errors for drift rate and variation. I only briefly mentioned how a slight error in inputs could lead an investor to go broke, but that was a contrived example to prove a point without analyzing what would instead be the best fraction to bet.

Continuing, I'm also interested in soon exploring my original aim, bet-hedging under power law/Pareto distributions with undefined variance. Everything I constructed in this paper relied on the fact that the random variable had a defined variation and mean, what if it didn't? It follows that there would still be some optimal bet, but what would that bet be? Does it again involve maximizing the expected value of the log of wealth or the geometric growth rate, or is there some new Kelly-unrelated process involved?

Finally, I'd like to explore how to apply this approximately continuous Kelly criterion to options and other derivatives from the original security analyzed in this paper. For an option, the downside is capped. At most, you lose the amount you paid for it, no more. How can I work that into this model? What about futures? What about swaps? How can I apply any of this information to make money? I think writing this paper left me with more questions than answers. However, such questions I'm more than willing to explore.

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Links to Code

Simulation for different values of f^* in original continuous case (not included in paper)
Simulation for GBM
Simulation for S&P500
Desmos Graph for Derived Equations (really fun to play around with)