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GLENLAIR
DALBEATTIE,
Nov. 13, 1867.

Dear Tait

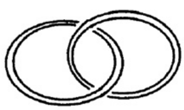
If you have any spare copies of your translation of Helmholtz on "Water Twists" I should be obliged if you could send me one.

I set [sic] the Helmholtz dogma to the Senate House in '66, and got it very nearly done by some men, completely as to the calculation, nearly as to the interpretation.

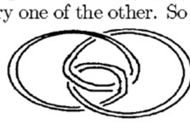
Thomson has set himself to spin the chains of destiny out of a fluid plenum as M. Scott set an eminent person to spin ropes from the sea sand, and I saw you had put your calculus in it too. May you both prosper and disentangle your formulae in proportion as you entangle your worbles. But I fear the simplest indivisible whirl is either two embracing worbles or a worble embracing itself.

For a simple closed worble may be easily split and the parts separated

but two embracing worbles preserve each others solidarity thus



though each may split into many, every one of the one set must embrace every one of the other. So does a knotted one.



yours truly
J. CLERK MAXWELL

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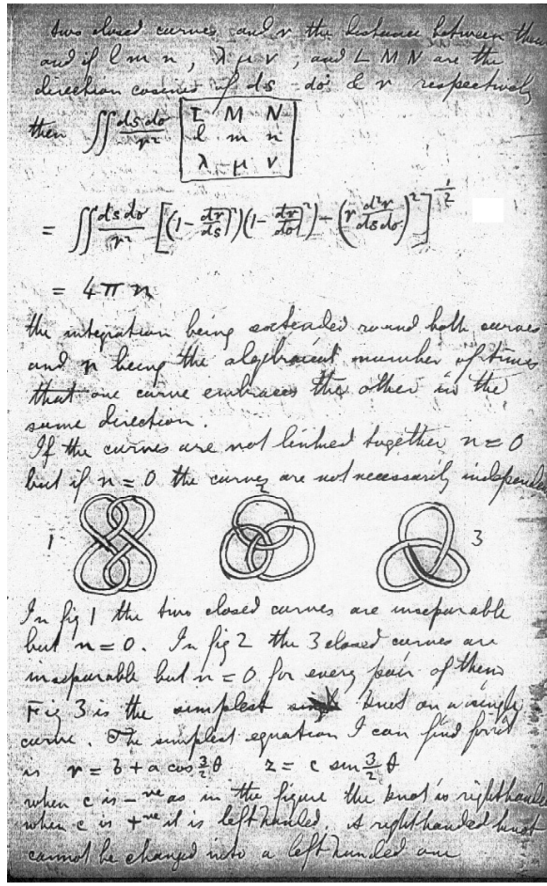


two closed curves and r the distance between them and if l m n, λ μ ν, and L M N are the direction cosines of ds dσ & r respectively

then $\iint \frac{ds\,d\sigma}{r^2} \boxed{\begin{matrix} L & M & N \\ l & m & n \\ \lambda & \mu & \nu \end{matrix}}$

$= \iint \frac{ds\,d\sigma}{r^2}\left[\left(1-\frac{dr}{ds}^2\right)\left(1-\frac{dr}{d\sigma}^2\right)-\left(r\frac{d^2r}{ds\,d\sigma}\right)^2\right]^{\frac{1}{2}}$

$= 4\pi n$

the integration being extended round both curves and n being the algebraical number of times that one curve embraces the other in the same direction.

If the curves are not linked together n = 0
but if n = 0 the curves are not necessarily independent

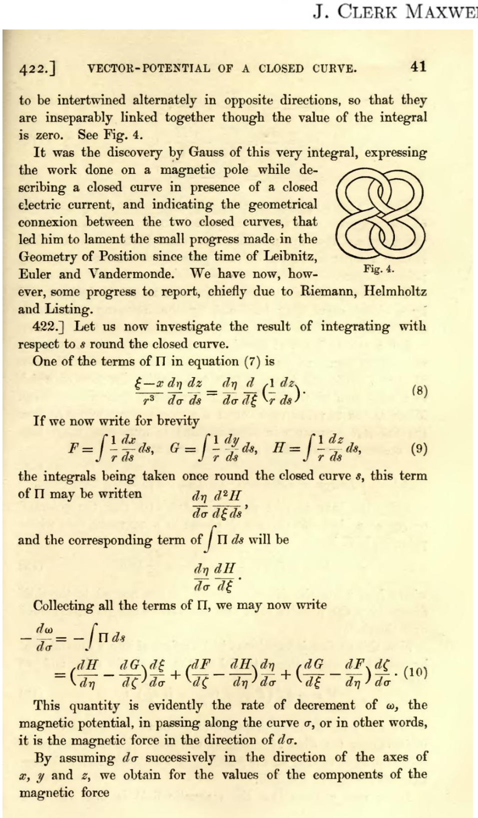
1 3

In fig 1 the two closed curves are inseparable but n = 0. In fig 2 the 3 closed curves are inseparable but n = 0 for every pair of them

Fig 3 is the simplest knot on a single curve. The simplest equation I can find for it is $r = b + a\cos\frac{3}{2}\theta$ $z = c\sin\frac{3}{2}\theta$

when c is −ve as in the figure the knot is righthanded when c is +ve it is lefthanded. A righthanded knot cannot be changed into a lefthanded one

c



422.] VECTOR-POTENTIAL OF A CLOSED CURVE. 41

to be intertwined alternately in opposite directions, so that they are inseparably linked together though the value of the integral is zero. See Fig. 4.

It was the discovery by Gauss of this very integral, expressing the work done on a magnetic pole while describing a closed curve in presence of a closed electric current, and indicating the geometrical connexion between the two closed curves, that led him to lament the small progress made in the Geometry of Position since the time of Leibnitz, Euler and Vandermonde. We have now, however, some progress to report, chiefly due to Riemann, Helmholtz and Listing.

Fig. 4.

422.] Let us now investigate the result of integrating with respect to s round the closed curve.

One of the terms of Π in equation (7) is

$$\frac{\xi - x}{r^3}\frac{d\eta}{d\sigma}\frac{dz}{ds} = \frac{d\eta}{d\sigma}\frac{d}{d\xi}\left(\frac{1}{r}\frac{dz}{ds}\right). \quad (8)$$

If we now write for brevity

$$F = \int \frac{1}{r}\frac{dx}{ds}ds, \quad G = \int \frac{1}{r}\frac{dy}{ds}ds, \quad H = \int \frac{1}{r}\frac{dz}{ds}ds, \quad (9)$$

the integrals being taken once round the closed curve s, this term of Π may be written $\frac{d\eta}{d\sigma}\frac{d^2H}{d\xi\,ds}$,

and the corresponding term of $\int \Pi\,ds$ will be

$$\frac{d\eta}{d\sigma}\frac{dH}{d\xi}.$$

Collecting all the terms of Π, we may now write

$$-\frac{d\omega}{d\sigma} = -\int \Pi\,ds$$

$$= \left(\frac{dH}{d\eta} - \frac{dG}{d\zeta}\right)\frac{d\xi}{d\sigma} + \left(\frac{dF}{d\zeta} - \frac{dH}{d\eta}\right)\frac{d\eta}{d\sigma} + \left(\frac{dG}{d\xi} - \frac{dF}{d\eta}\right)\frac{d\zeta}{d\sigma}. \quad (10)$$

This quantity is evidently the rate of decrement of ω, the magnetic potential, in passing along the curve σ, or in other words, it is the magnetic force in the direction of $d\sigma$.

By assuming $d\sigma$ successively in the direction of the axes of x, y and z, we obtain for the values of the components of the magnetic force

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