Looking at DSA again

6/7/2012

Start with 3D transport eqns

~~3D transport eqns~~

$$\Psi = \Psi(r, \underset{\sim}{\Omega}), \quad r \in V, \quad \underline{\Omega} \in S^2$$

important difference from fyans work: this is angular dependence that is needed in DSA.

$$\underbrace{\underset{\sim}{\Omega} \cdot \nabla \Psi + \sigma_t \Psi}_{T\Psi} - \frac{1}{4\pi} \int_{S^2} \sigma_s(r, \underbrace{\Omega \cdot \Omega'}_{\text{"isotropic"}}) \Psi(r, \Omega') \, d\Omega'$$

(Lewis & Miller P11)

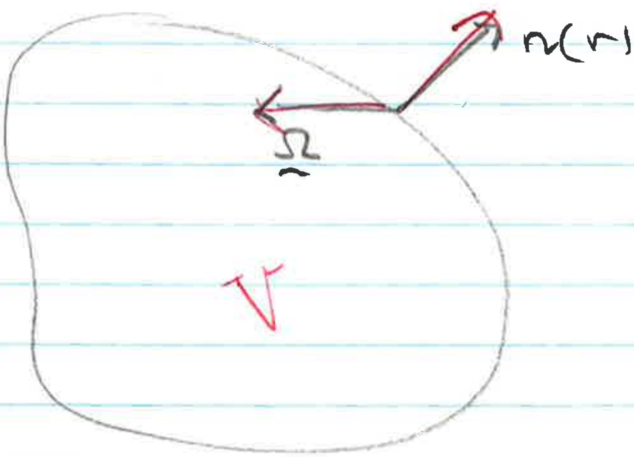
$$= \lambda \frac{\nu \sigma_f(r)}{4\pi} \int_{S^2} \Psi(r, \Omega') \, d\Omega' \qquad (1)$$

(Fission cross section is not angular dept)

On boundary

$$\Psi(r, \Omega) = 0 \quad \text{when} \quad \underset{\sim}{n}(r) \cdot \Omega < 0$$

[No angular dependence in fission cross section]



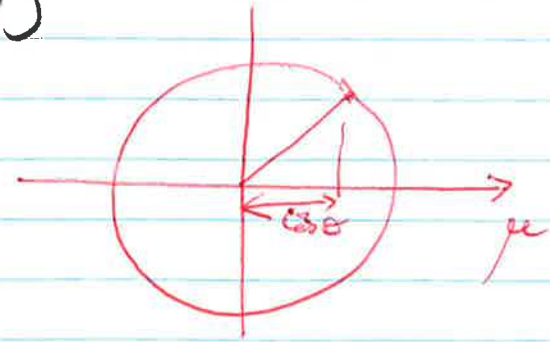
$$(T - S)\Psi = \lambda F \Psi$$

Typical inverse iteration

$$((T - S) - sF)\Psi^{(new)} = F \Psi^{(old)}$$

s ↑ shift

Preconditioner At least a good approximation of $(T - S)^{-1}$



1D slab geometry

$$\Psi(r,\Omega) \longrightarrow \psi(r,\mu) \qquad \mu = \cos\theta$$

$\boxed{\text{BCs}}$

$\psi(0,\mu) = 0 \qquad \mu > 0$

$\psi(1,\mu) = 0 \qquad \mu < 0$

$r \in [0,1]$

$\mu \in [-1,1]$

(*)

so only one is used & is so ~~that means~~ that we get an initial value problem. i.e. we specify a condition at the start of the propagation, not at the end.

~~eq (1) becomes~~ $\cancel{\psi \frac{d\psi}{dx} + \sigma_t \psi = \Sigma}$

$$T\Psi \to \mu \frac{d\psi}{dr} + \sigma_t(r)\,\psi$$

$$S\Psi(r,\mu) \to \sum_{n=0}^{\infty} (2n+1)\, P_n(\mu)\, \sigma_{s,n}(r)\, \phi_n(r) \qquad (A)$$

where $$\phi_n(r) = \frac{1}{2}\int_{-1}^{1} P_n(\mu')\, \psi(r,\mu')\, d\mu' \qquad (B)$$

$$F\Psi \to \nu\sigma_f(r)\, \phi_0(r)$$

Note Ⓐ, Ⓑ $\Longrightarrow$

~~in 1D~~

$$\times P_{n'}(\mu')$$

$$S\psi(r,\mu) = \frac{1}{2}\int_{-1}^{1} \underbrace{\left[\sum_{n=0}^{\infty} (2n+1) P_n(\mu) P_n(\bar{\mu}') \, \sigma_{s,n}(r)\right]}_{\text{Separable expansion of } \sigma_s(r,\Omega\cdot\Omega') \text{ in 1D}} \psi(r,\bar{\mu}') \, d\bar{\mu}'$$

$\mu = \cos\theta, \quad \mu' = \cos\theta'$

(Need to understand better)

Consider operator

$$S\psi(r,\mu) = \sum_{n=0}^{\infty} (2n+1) P_n(\mu) \sigma_{s,n}(r) \phi_n(r)$$

~~This map~~

~~$\psi(r,\mu)$~~

Maps $\psi(r,\mu)$ to a new function of (r,μ) by first computing the ~~components of~~ coordinates ϕ_n of ψ in Legendre series expansion.

Diversion - Legendre poly expansion

$\{P_n\}$ orthogonal wrt $\langle f, g\rangle = \int_{-1}^{1} fg$

$$\langle P_n, P_n \rangle = \frac{2}{2n+1}$$

$$\psi(r,\mu) = \sum_{n=0}^{\infty} \frac{\langle \psi(r,\cdot), P_n \rangle}{\langle P_n, P_n \rangle} P_n(\mu)$$

$$= \sum_{n=0}^{\infty} \left[\frac{(2n+1)}{2} \int_{-1}^{1} P_n(\mu')\, \psi(r,\mu')\, d\mu' \right] P_n(\mu)$$

$$= \sum_{n=0}^{\infty} (2n+1) P_n(\mu)\, \phi_n(r)$$

This series converges in L_2 (wrt μ)

when $\psi(r,\cdot) \in L_2$ (why?)

(Orthogonal Polynomials T. Rivlin)

So knowing $\{\phi_n\}_{n=0}^{\infty}$ is equivalent to knowing ψ. Q If ψ satisfies b.c.

$$\left.\begin{aligned} \psi(0,\mu) &= 0, \quad \mu > 0 \\ \psi(1,\mu) &= 0 \quad \mu < 0 \end{aligned}\right\}$$ what condition does $\phi_n(r)$ satisfy

§.

Now consider approx of $(\mathcal{T}-S)^{-1}$, ie

Solution of

$$(\mathcal{T} - S)\,\psi = q \quad \text{for any } q.$$

~~P_N approx^n~~

Take inner product of each side with

$P_{n'}$ (see prev. eqn (B) on p 2)

$$\frac{1}{2}\int_{-1}^{1} \mu \frac{d\psi}{dx}(r,\mu')\, P_{n'}(\mu')\, d\mu'$$

$$+\ \sigma_t(r)\frac{1}{2}\int_{-1}^{1} \psi(r,\mu')\, P_{n'}(\mu')\, d\mu'$$

by multiplying form at top of p3 by $P_{n'}(\mu')$

$$-\sum_{n=0}^{\infty}\left[\frac{(2n+1)}{2}\underbrace{\int_{-1}^{1} P_n(\mu')\, P_{n'}(\mu')\, d\mu'}_{=0 \text{ iff } n=n'}\right]\sigma_{s,n}(r)\,\phi_n(r)$$

$$= \frac{1}{2}\int_{-1}^{1} q(r,\mu')\, P_{n'}(\mu')\, d\mu'$$

$$\Rightarrow \quad \frac{1}{2}\int_{-1}^{1} \mu' \frac{d\psi}{dx}(r,\mu') P_{n'}(\mu')\, d\mu'$$

$$+ \frac{\sigma_t(r)}{2} \phi_{n'}(r)$$

$$- \sigma_{s,n'}(r)\, \phi_{n'}(r) = \frac{1}{2}\langle q(r,\cdot), P_n(\cdot)\rangle \quad \text{A}$$

This holds for all $n' = 0, 1, 2, \ldots$

Also first term above

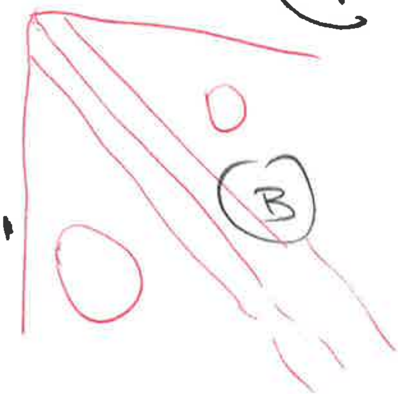
$$= \frac{1}{2}\frac{d}{dr}\int_{-1}^{1} \mu' P_{n'}(\mu')\, \psi(r,\mu')\, d\mu'$$

recurrence relation for Legendre polynomials.

$$= \frac{1}{2}\frac{d}{dr}\left[\int_{-1}^{1}\left[\frac{n'}{2n'+1} P_{n'-1}(\mu') + \frac{n'+1}{2n'+1} P_{n'+1}(\mu')\right] \psi(r,\mu')\, d\mu'\right]$$

$\cancel{\Rightarrow \frac{1}{2}}$

$$= \frac{n'}{2n'+1}\frac{d}{dr}\phi_{n'-1} + \frac{n'+1}{2n'+1}\frac{d}{dr}\phi_{n'+1}$$



insert (B) into (A) gives infinite sequence of first order differential equns. for $\{\phi_n(r)\}_{n=0}^{\infty}$

Truncation to terms only in $n=0,1$ and rearranging gives ~~diffusion~~ DSA approx^n to $(I - S)^{-1}$

It would be easy(?) to include term sF (see page 1) to get an approximation of $(I - S - sF)^{-1}$ instead.

[Exercise!]

$\boxed{Q}$ If ψ solves original problem with BC (*) what conditions does ϕ_0 satisfy?