Transfer Summary

- 'Density' not number for flux.
- Need nondimensionalising to be complete: not just equation but also domain.
- Ideas: get system of simultaneous equations for ϕ^0, ϕ^1, ϕ^2 etc. and solve. Then can deal with errors down to ϵ^3 (e.g. 10^{-15} with $\epsilon = 10^{-5}$ as $\phi = \phi^0 + \epsilon\phi^1 + \epsilon^2\phi^2 + O(\epsilon^3)$).
- Talked about solving in boundary layer in a similar way: so carry out an expansion near the boundary and prove it is 'asymptotic' (prove that the ratio of successive terms in the expansion is ϵ). This will be a different expansion to before: where we maybe need three terms in the central part of the domain to capture the accuracy we want (called the "outer" expansion for some reason...), we might need two or four terms in the boundary layer to capture the same accuracy (called the "inner" expansion...). Then all three expansions are combined to make a full system.
- Jonathan suggested two books that would be useful to learn more about perturbation theory and asymptotics. These are [1] (placed hold on this one) and [4] (I have got this one). He said he was happy for me to talk to him about this work and any problems.
- Jonathan also said the large values of ϵ should be used too, just for interest and comparison.
- Jonathan mentioned how Theorem 4.6 looked like it might be mirroring the asymptotic derivation of the diffusion equation in some form. Could draw parallels between the two, in particular between truncating the Neumann series and using only the first 3 powers of ϵ.
- Adrian suggested some books where the Fourier Integral Theorem might be found: [3] and [6].
- Need to finish work in Chapter 5: proof of the symmetry of $PT^{-1}\Sigma$, and results about its eigenvalues. This would round that off better. Adrian asked how the discretisation had been chosen to preserve symmetry? I did not know, but said that numerical results suggested that it had been preserved. He mentioned it might be easier using finite elements (in particular for 2D and 3D). His opinion

was that this was important to conclude before moving to programming the 2D code so that we could guarantee symmetry in the 2D discrete form.

- Drawing everything together with a good Krylov method chapter and one on preconditioning with DSA could be a good thesis core. Need to start making this progress though, [Jonathan] maybe 2D and 3D programs not so important, but better to try diffusion with continuous cross-sections &/or working on 2D and 3D versions of Theorem 4.6?
- Emphasised getting perturbation theory work solid. This doesn't appear to be achieved clearly in the literature and so would be a good addition to a thesis (and the literature). "Original" could be the link to the operator form, plus any other things.

General comments:
The consensus was that I was behind where I should be at 24 months, but if the more recent work pace was maintained then should be ok. Definitely focus on solidifying the asymptotics work into a rigorous chapter (maybe with help and suggestions from Jonathan). Adrian said that with Alastair and Ivan's backgrounds there would be plenty of support and knowledge to boost the Krylov/ preconditioning work. Adrian said that having multiple areas of interest would be helpful, as it would allow breaks from each if/when progress slowed. Adrian mentioned that there had been a large amount of initial learning required from engineering literature which could have delayed progress. I added that a recent book found (The Handbook of Nuclear Engineering [2]) in particular Volume 1, Chapter 5 [5] was proving very helpful in adding to understanding. I also think my work ethic after christmas was poor. To counter this, my plan is that if I think things are moving slowly during a day, or I don't know how to proceed, I will write some recent work up in a semi-formal way. I have found this to be a good way to spawn new ideas or refresh my thinking. This will also hopefully be helpful in the long run too.

References

[1] Carl M. Bender and Steven A. Orszag. *Advanced Mathematical Methods for Scientists and Engineers: Asymptotic Methods and Perturbation Theory (v. 1).* Springer, 1999.

[2] Dan Gabriel Cacuci, editor. *Handbook of Nuclear Engineering, (5 Volume set).* Springer, 2010.

[3] Avner Friedman. *Partial Differential Equations.* Holt,Rinehart & Winston of Canada Ltd, 1969.

[4] E. J. Hinch. *Perturbation Methods (Cambridge Texts in Applied Mathematics).* Cambridge University Press, 1991.

[5] A. K. Prinja and E. W. Larsen. *Handbook of Nuclear Engineering, (5 Volume set)*, chapter 5. General Principles of Neutron Transport. Volume I of Cacuci [2], 2010.

[6] Elias M. Stein. *Singular Integrals and Differentiability Properties of Functions. (PMS-30).* Princeton University Press, 1971.