

We have the transport equation

$$\mathcal{T}\psi - \Sigma\phi = \epsilon Q, \quad (1)$$

where

$$\mathcal{T} = \mu \frac{\partial}{\partial x} + \frac{1}{\epsilon} I, \quad (2a)$$

$$\Sigma = \left[ \frac{1}{\epsilon} - \epsilon \right] I. \quad (2b)$$

By Neumann series we know  $(I - X)^{-1} = \sum_{n=0}^{\infty} X^n$ , and applying this to (2a) we get

$$\begin{aligned} \mathcal{T}^{-1} &= \epsilon \left( I + \epsilon \mu \frac{\partial}{\partial x} \right)^{-1}, \\ &= \epsilon \left( I - \epsilon \mu \frac{\partial}{\partial x} + \epsilon^2 \mu^2 \frac{\partial^2}{\partial x^2} - \dots \right). \end{aligned} \quad (3)$$

Now from (1) we can see

$$\begin{aligned} \psi - \mathcal{T}^{-1}\Sigma\phi &= \epsilon \mathcal{T}^{-1}Q, \\ \Rightarrow \mathcal{P}\psi - \mathcal{P}\mathcal{T}^{-1}\Sigma\phi &= \epsilon \mathcal{P}\mathcal{T}^{-1}Q, \\ \Rightarrow (I - \mathcal{P}\mathcal{T}^{-1}\Sigma)\phi &= \epsilon \mathcal{P}\mathcal{T}^{-1}Q. \end{aligned} \quad (4)$$

Working with the left hand side of (4) and using (3) and (2b) we find

$$\begin{aligned} (I - \mathcal{P}\mathcal{T}^{-1}\Sigma)\phi &= \left( I - \epsilon \mathcal{P} \left( I - \epsilon \mu \frac{d}{dx} + \epsilon^2 \mu^2 \frac{d^2}{dx^2} - \dots \right) \left[ \frac{1}{\epsilon} - \epsilon \right] \right) \phi, \\ &= \phi - [1 - \epsilon^2] \left( \mathcal{P}\phi - \epsilon \frac{d}{dx} \phi \mathcal{P}\mu + \epsilon^2 \frac{d^2}{dx^2} \phi \mathcal{P}\mu^2 - \dots \right). \end{aligned}$$

If we note that  $\mathcal{P}\phi = \phi$ , and that  $\mathcal{P}\mu = 0$  and  $\mathcal{P}\mu^2 = \frac{1}{3}$ , then we find

$$(I - \mathcal{P}\mathcal{T}^{-1}\Sigma)\phi = -\frac{\epsilon^2}{3} \frac{d^2}{dx^2} \phi + \epsilon^2 \phi + \frac{\epsilon^4}{3} \frac{d^2}{dx^2} \phi, \quad (5)$$

and so

$$\frac{1}{\epsilon^2} (I - \mathcal{P}\mathcal{T}^{-1}\Sigma)\phi = -\frac{1}{3} \frac{d^2}{dx^2} \phi + \phi + O(\epsilon^2). \quad (6)$$