Deep Active Inference with Predictive Coding Network

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1 Introduction

Active inference is a normative framework for modelling the sentient behaviour of both artificial and biological agents. This principle is derived from the free energy principle, as introduced by Friston et al. (2006), and posits that perception, action, planning, decision-making, and learning can all be understood as an organism optimising two complementary functionals: variational free energy and expected free energy.

Variational free energy measures how well an agent's internal model explains its sensory inputs. It is an information-theoretic quantity that serves as an upper bound on the negative log-likelihood of surprise. Under the assumption that neural processes can be described by maximising Bayesian model evidence or equivalently, minimising variational free energy, the free energy principle provides a neuroscientifically plausible process theory. This theory suggests that the brain operates through a hierarchy of generative models that predict sensory data, with learning driven by the minimisation of the prediction error between expected and observed sensory inputs.

This emphasis on prediction error leads to the concept of Predictive Processing, where prediction error serves as a general, unsupervised training signal that enhances our understanding of the world. Active Inference extends this idea by incorporating action. There are two primary ways to minimise prediction error: updating internal models to better align with sensory data (perception), and taking actions to align sensory data with prior predictions (action). This dual approach allows Active Inference to unify action and perception under a single formalism, enabling simultaneous optimisation by minimising variational free energy.

Expected free energy, on the other hand, evaluates potential future actions based on prior preferences, guiding decision-making and planning (Da Costa et al., 2020). Together, these two functionals provide a comprehensive framework for understanding how agents interact with their environment to minimise surprise and achieve their goals.

2 Related Work

Da Costa et al. (2020) described the application of active inference in discrete state-spaces from the ground up. When reading about active inference and its application in modelling agents. One can get stuck on how to apply the principles or whether the pursuit of applying these principles is a worthy course of action in the first place, the practical vs the biological approach. Da Costa et al. (2020) bridges this gap and explains the practical applications of active inference with motivations from the neural process theory that underlies the framework.

Smith et al. (2022) then provides an introduction and practical overview of how one could go about applying the framework to empirical data. Smith et al. (2022) provides an introduction to the formulation of Partially Observable MDPs as well as how this can be applied using variational (Winn et al., 2005) and marginal message passing (Parr et al., 2019).

The practical implementation of active inference in Smith et al. (2022) is using so called tabular methods. In a POMDP one can factorise the model into different components which correspond to likelihood of an observation given current beliefs about the current environment, the A matrix. We can then describe the transition dynamics of the environment (B matrix) which explains the likelihood of being in a certain state in the next time-step given current beliefs about the state and a policy. We then use the C matrix to denote the prior distribution over observations, which encodes the agents preferences. Naturally, when dealing with larger states spaces holding these matrices becomes computationally expensive.

(Millidge, 2020) introduces a model of 'deep' active inference which uses neural networks as function approximations for the different components of the generative model that have been described. This gave rise to very good results against traditional reinforcement algorithms, Deep Q-Learning and Actor-Critic, in the OpenAI Gym environments. This is particularly interesting since the environments used were MDP's not POMDP's which would generally suit the reinforcements learning algorithms because of their direct learning of the state value pairs, in the case of q-learning and critic network, as well as direct policy networks in the actor. The notion of direct here is due to the fact that deriving the model such as in (Millidge, 2020) yields epistemic terms in the network to evaluate certain actions.

This model was then extended by Himst and Lanillos (2020) to incorporate POMDPs. This was done by using a Variational Autoencoders (VAE) as so called observations models. The VAE's used in the model use an encoder network to map the previous 4 pixel observations in the environment to a latex space. The decoder network then produced observations from this latent space.

Fountas et al. (2020) - Review their formulation of the deep model

The approaches taken to deep active inference so far have included using neural networks to learning certain components with the objective function of some variant of free energy, depending on the paper. Though this is a very interesting venture in the case of practical applications of active inference. The approach now loses its biologically plausible approach since neural networks learning through back-propagation.

3 Predictive Coding

3.1 Energy Based Models

3.2 Predictive Coding Networks

3.3 Predictive Coding as Variational Inference

3.4 Approximating Backpropagation

3.5 Beyond Backpropagation

4 Active Inference and MDPs

4.1 Generative Models

Active inference presents a unified theory for modelling sentient behaviour. The framework presents perception, learning and actions can all be seen as optimising two complementary functionals. (Da Costa et al., 2020) That is, variational free energy, which measures the fit between an agents internal model and sensory observations. And expected free energy, which scores the future courses of action in relation to prior preferences. (Da Costa et al., 2020)

To formulate the respective functionals we will first need to introduce the notion of KL-divergence. KL-divergence can be understood as the difference between two respective distributions.

Definition 1 (KL Divergence) *Let q, and p be two distributions defined over Ω. The KL-divergence between then is defined as:*

$$\begin{aligned} D_{KL}\left[q(x) \| p(x)\right] &= \mathbb{E}_{q(x)}\left[\log \frac{q(x)}{p(x)}\right] \\ &= \sum_{x \in \Omega} q(x) \log \frac{q(x)}{p(x)} \end{aligned}$$

The formulation clearly suits our needs for a divergence measure between two distributions. Furthermore, let us consider some joint distribution over all states, action and observations in a POMDP which will be called the generative model of our agent. This generative model is a model of the generative process that creates sensory data that the agent then observes.

Definition 2 (POMDP Generative Model)

$$p(s_t, o_t, a_t, s_{t-1}, a_{t-1}) = p(o_t|s_t)\ p(a_t|s_t)\ p(s_t|s_{t-1}, a_{t-1})\ q(s_{t-1}, a_{t-1})$$

Note that the model need only consider the state and observation of the timestep before by invoking the Markov Property.

Throughout the text, $p(\cdot)$ will be used to denote the true generative model and $q(\cdot)$ will be used to denote the variational posterior distribution.

When an agent observes some sensory input how can it update its beliefs of the state it is currently in.

4.2 Perception

4.3 Learning

4.4 Action

5 Deep Active Inference with Predictive Coding Network

5.1 Deep Active Inference

5.2 Model and Environments

6 Results

7 Discussion

8 Conclusion

References

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