

# Chapter XV - Making Simple Decisions

## 15.1 Combining Beliefs and Desires in an Uncertain Context

In decision-making under uncertainty, an agent must consider both **beliefs** (its knowledge or beliefs about the world) and **desires** (the goals or objectives it wishes to achieve). These components are combined to help the agent make rational decisions in uncertain environments.

### Key Concepts:

- **Beliefs:** The agent's knowledge about the current state of the world, often represented probabilistically.
- **Desires:** The agent's goals or objectives, which guide its actions. These are often captured as utility functions or goals that the agent seeks to maximize.
- **Uncertainty:** The agent's beliefs may be uncertain, and its desires may conflict with the uncertainties, requiring a framework to make decisions that balance the two.

### Example:

A robot in a factory has a belief about where certain parts are located (e.g., in which section of the warehouse). Its desire is to retrieve those parts efficiently, but the robot must also account for potential errors in its belief about part locations.

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## 15.2 Basics of Utility Theory

**Utility Theory** provides a framework for modeling rational decision-making under uncertainty. The central idea is that an agent can assign a **utility** (a numerical value) to each possible outcome, representing the agent's preference for that outcome.

### Key Concepts:

- **Utility:** A measure of the satisfaction or benefit derived from a particular outcome. Higher utility represents a more desirable outcome.
- **Rational Decisions:** A decision is considered rational if it maximizes expected utility, which is the weighted sum of utilities of all possible outcomes, where each outcome is weighted by its probability.

- **Expected Utility:** The expected utility is the sum of utilities of all possible outcomes, weighted by their probabilities. This is used to make decisions when outcomes are uncertain.

### Example:

If a person is deciding whether to invest in the stock market, the utility function could model the person's satisfaction with different levels of return. The decision would involve choosing the option that maximizes the expected utility of returns.

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## 15.3 Utility Functions

A **Utility Function** is a mathematical function that represents an agent's preferences over different outcomes. It assigns a utility value to each possible outcome, and higher values indicate more preferred outcomes.

### Key Concepts:

- **Monotonicity:** A utility function is monotonic if more desirable outcomes are associated with higher utility values.
- **Risk Aversion:** A decision-maker is risk-averse if they prefer outcomes with less uncertainty for a given expected utility.
- **Risk Seeking:** A decision-maker is risk-seeking if they prefer outcomes with more uncertainty for a given expected utility.

### Example:

Consider a simple utility function  $U(x) = \log(x)$  for wealth. This function models diminishing returns to wealth, meaning that the agent derives less additional satisfaction from each additional unit of wealth.

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## 15.4 Multi-Attribute Utility Functions

In real-world decision-making, an agent often has to make choices based on multiple attributes or factors. A **Multi-Attribute Utility Function** is an extension of utility theory that allows the agent to consider multiple criteria simultaneously.

### Key Concepts:

- **Attributes:** The different factors or characteristics of the outcomes that the agent cares about (e.g., cost, quality, time).
- **Weighting:** Each attribute is often assigned a weight based on its importance relative to the others.
- **Aggregation:** The utility of each outcome is calculated by combining the utilities of the individual attributes, often using a weighted sum.

### Example:

When buying a car, an agent might consider multiple attributes such as price, fuel efficiency, and safety. A multi-attribute utility function would combine these attributes into a single utility value, allowing the agent to compare different cars.

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## 15.5 Decision Networks

A **Decision Network** is a graphical model that represents decisions, uncertainties, and utilities in a decision-making problem. Decision networks combine **decision nodes**, **chance nodes**, and **utility nodes** to model decision-making processes.

### Key Concepts:

- **Decision Nodes:** Represent decisions the agent must make.
- **Chance Nodes:** Represent uncertain events that affect the outcomes of decisions.
- **Utility Nodes:** Represent the utility associated with different outcomes.

### Example:

A decision network can model a healthcare decision, such as whether to undergo a medical treatment. The decision node would represent the decision (treatment or not), chance nodes would model uncertain outcomes (e.g., success or failure), and utility nodes would capture the patient's satisfaction with each outcome.

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## 15.6 The Value of Information

The **Value of Information** (Vol) refers to the benefit gained from acquiring additional information before making a decision. It quantifies how much better an agent's decision can be when additional information reduces uncertainty.

## Key Concepts:

- **Expected Value of Information:** The expected improvement in decision-making after acquiring additional information.
- **Decision Trees:** A common tool for calculating the value of information, which helps agents determine whether gathering more information is worth the cost.
- **Perfect vs. Imperfect Information:** Perfect information fully resolves uncertainty, while imperfect information only partially resolves it.

## Example:

In a business context, before making a large investment, a company might decide whether to gather market research. The value of this research would depend on how much it helps reduce uncertainty about the potential success of the investment.

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## 15.7 Decision-Theoretic Expert Systems

**Decision-Theoretic Expert Systems** combine artificial intelligence with decision theory to help automate complex decision-making processes. These systems use utility theory, decision networks, and probability to make decisions in uncertain environments.

## Key Concepts:

- **Expert Systems:** Computer programs designed to simulate the decision-making ability of a human expert in a specific domain.
- **Decision Theory:** The application of utility theory and probability to optimize decision-making.
- **Automated Decision Support:** These systems assist human decision-makers by providing recommendations based on a thorough analysis of available information.

## Example:

An expert system in healthcare could recommend a course of treatment for a patient based on their symptoms, medical history, and available treatment options, considering the uncertainty and possible outcomes of each option.

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## Exercises

1. **Utility Function Exercise:** Consider a decision-maker who has two options: a guaranteed payout of \$100 and a gamble that offers a 50% chance of winning \$200 and a 50% chance of winning \$0. Assuming a logarithmic utility function  $U(x) = \log(x)$ , compute the expected utility of the gamble and the guaranteed payout, and determine which option the decision-maker should choose.
2. **Multi-Attribute Decision Making Exercise:** Imagine you are buying a laptop. The two attributes you care most about are performance (measured by processor speed) and price. The performance has a weight of 0.7, and the price has a weight of 0.3. Use the following values to calculate the total utility for two laptops:
  - Laptop A: Performance = 4.5 GHz, Price = \$800
  - Laptop B: Performance = 3.5 GHz, Price = \$750 Assume a linear utility function for each attribute:  $U(\text{Performance}) = \text{Performance}/5$  and  $U(\text{Price}) = 1 - (\text{Price} - 500)/1000$ .
3. **Decision Network Exercise:** Construct a decision network for a person deciding whether to invest in a stock, where the decision node is the choice to invest or not, a chance node represents the uncertainty in the stock's performance (good or bad), and a utility node represents the expected profit or loss from the investment.
4. **Value of Information Exercise:** Given a decision problem where you have the option to gather additional market research before investing in a product, calculate the value of this information. Assume you can estimate the probability of success in the market with and without the research and compare the expected utility of the decisions with and without the information.