

Chapter VII - First-Order Logic

7.1 The Revisited Representation

First-Order Logic (FOL) extends propositional logic by introducing a richer syntax for expressing statements about the world. In FOL, you can describe relationships between objects, properties of objects, and even quantify over objects. It allows us to represent more complex and nuanced knowledge than propositional logic.

Key Concepts in FOL:

1. **Objects:** Entities in the world that we want to reason about (e.g., John, car, dog).
2. **Predicates:** Properties or relations that apply to objects (e.g., "is a dog", "lives_in", "is_brother_of").
3. **Functions:** Mapping objects to other objects (e.g., "father_of(x)").
4. **Quantifiers:**
 - **Universal Quantifier (\forall):** Indicates that a statement applies to all objects in a domain (e.g., "For all x, x is a student").
 - **Existential Quantifier (\exists):** Indicates that there exists at least one object for which the statement is true (e.g., "There exists an x such that x is a teacher").

Example of FOL Representation:

- **Proposition:** "John is a student."
- **FOL Representation:** `Student(John)`
- **Proposition:** "All students are human."
- **FOL Representation:** `$\forall x \text{ (Student}(x) \rightarrow \text{Human}(x))$`

This allows for much more expressive and general representations of knowledge than propositional logic.

7.2 Syntax and Semantics of First-Order Logic

In FOL, the **syntax** defines the formal rules for writing valid statements, and the **semantics** defines the meaning of those statements.

Syntax of FOL:

- **Terms:** A term can be a constant (e.g., `John`), a variable (e.g., `x`), or a function applied to terms (e.g., `father_of(John)`).
- **Atomic Sentences:** An atomic sentence consists of a predicate and terms (e.g., `Likes(John, Mary)`).
- **Quantified Sentences:** A sentence that includes quantifiers (e.g., $\forall x \exists y (\text{Likes}(x, y))$).

Semantics of FOL:

- **Domain (D):** The set of objects that the predicates and functions refer to. For example, the domain could be "all people" in a human-related problem.
- **Interpretation:** A mapping of the constants, variables, predicates, and functions to the domain. For example, "John" might refer to a specific person in the domain, and "`Likes(x, y)`" could map to whether person `x` likes person `y`.
- **Truth Value:** A sentence is either true or false based on the interpretation. The interpretation evaluates the sentence according to the domain and the relationships defined in the model.

7.3 Using First-Order Logic

FOL is used to represent and reason about knowledge in a more structured and powerful way compared to propositional logic. It is commonly used in fields like **knowledge representation**, **natural language processing**, and **automated reasoning**.

Using FOL in AI:

1. **Knowledge Representation:** FOL allows us to represent real-world facts in a form that machines can understand. For example, a database of employee information can be represented using FOL to describe the relationships between employees, departments, and projects.
2. **Inference and Reasoning:** FOL allows reasoning over knowledge bases by applying logical rules to derive conclusions from facts. For instance, if we know that all students are humans and John is a student, we can infer that John is a human using **modus ponens**.
3. **Automated Theorem Proving:** FOL can be used to prove theorems automatically by using inference systems that apply logical rules to generate new knowledge or check the validity of a statement.

Example:

Given the knowledge base:

1. $\forall x (\text{Student}(x) \rightarrow \text{Human}(x))$ (All students are humans).

2. `Student(John)` (John is a student).

We can infer that:

- `Human(John)` (John is a human) through **modular inference**.
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7.4 Knowledge Engineering in First-Order Logic

Knowledge Engineering is the process of designing and creating knowledge bases, inference engines, and AI systems that can reason about the world. In FOL, knowledge engineering involves modeling the world using FOL representations and ensuring the knowledge base is well-structured and complete.

Steps in Knowledge Engineering for FOL:

1. **Knowledge Acquisition:** Gathering information from experts, databases, or documents to populate the knowledge base.
2. **Modeling with FOL:** Representing the acquired knowledge using FOL statements, predicates, and relationships.
3. **Knowledge Base Design:** Structuring the knowledge base to make inference and reasoning efficient, typically using ontologies and logical hierarchies.
4. **Automated Reasoning:** Using inference algorithms to extract useful information from the knowledge base or answer queries.

Challenges in Knowledge Engineering:

- **Complexity:** As the knowledge base grows, managing and structuring large amounts of information becomes more complex.
- **Uncertainty and Incompleteness:** Real-world knowledge is often incomplete or uncertain, which can be difficult to represent in FOL. Techniques like **probabilistic logic** or **non-monotonic reasoning** are used to handle these cases.

Example:

Consider a medical knowledge base for diagnosing diseases. Using FOL, we can model symptoms and diseases with predicates like `HasSymptom(patient, symptom)` and `HasDisease(patient, disease)`. Inference rules allow the system to deduce potential diagnoses based on observed symptoms.

Exercises

1. FOL Representation of a Simple Statement:

- Write the FOL representation for the following sentence: "If it is raining, then the ground is wet."

2. Knowledge Base Construction:

- Construct a simple knowledge base for a small university system using FOL. Represent facts about students, courses, and professors (e.g., John is a student, John is enrolled in Math101, and Math101 is taught by Prof. Smith).

3. Inference in FOL:

- Given the following knowledge base:

1. $\forall x (\text{Student}(x) \rightarrow \text{EnrolledIn}(x, \text{Math101}))$

2. $\text{Student}(\text{John})$

What can be inferred using FOL?

4. Design a Knowledge Base in FOL for a Family Tree:

- Create a knowledge base representing family relationships (e.g., parent-child, sibling, etc.) and demonstrate how you would infer family relations.

5. Real-World Application of FOL:

- Think of a real-world domain (e.g., weather forecasting, healthcare, or legal reasoning). Write a few FOL sentences to represent knowledge in that domain, and suggest how automated reasoning could be applied.