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THE SQUARE-LOOP FERRITE CORE AS A CIRCUIT-ELEMENT

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SUMMARY

A quantitative theory is presented to explain the shape of the output waveforms when square-loop ferrite cores are switched. This is based on the qualitative theory of Menyuk and Goodenough.[3] Various useful equations arising from the theory are deduced, and it is shown that the agreement with experiment is reasonable. The theory indicates which parameters of the material need to be known in order to predict its behaviour.

LIST OF SYMBOLS

μ_r = Residual permeability.
$\mathrm{F} = \mathrm{F}(r) = \mathrm{F}(B)$ = Distribution function.
v = Radial velocity of domain wall.
t = Time.
p = Probability of an element ($d\theta$) reaching radius r.
α, a = Arbitrary areas.
ρ = Density of domain centres.
H_m = Magnetizing force.
H_c = Coercive force.
$H = H_m - H_c$.
B = Flux density.
B_m = Saturation flux density.
X_0 = Loss parameter of the material: $dB/dt = X_0\mathrm{F}H$, ohms/m.
k_1, k_2 = Constants.
T = Switching time.
i_0 = Applied current.
i_c = Coercive current.
$i = i_0 - i_c$.
V_{max} = Peak output voltage per turn.
R_0 = Loss resistance of a core: $V_{max} = R_0 i$.
R = Instantaneous loss resistance = $R_0\mathrm{F}$.
Φ = Magnetic flux.
Φ_m = Saturation flux.

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(1) INTRODUCTION

It is found impossible to describe the behaviour of a square-loop ferrite core in terms of an equivalent circuit composed of linear elements, owing to the essential non-linearity of the material. The major property of the core to be taken into account when it is switched at high speeds is its residual loss, which makes the core itself appear resistive rather than inductive. The effect of this residual loss will be considered in some detail, and a method of calculating the behaviour of any particular core circuit will be described. This method does, in fact, predict the behaviour of a core with more than sufficient accuracy for the design of switching circuits, although it does not take into account the higher-order effects which are relevant when considering the suitability of the material for storage.

(2) D.C. PROPERTIES

The properties of the material at zero and low frequencies are described completely by its well-known hysteresis loop. An idealized hysteresis loop is shown in Fig. 1. The origin has

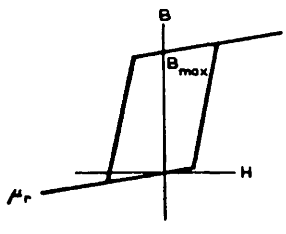

Fig. 1.—Idealized hysteresis loop.

been chosen so that in one of the remanent states B is zero. This is convenient because B_{max} then represents the change in B when the core is fully switched, and because in practice the core is never likely to be completely demagnetized. Two values of the permeability of the material are of interest, namely the residual permeability (μ_r), which is small, and the permeability on the steep part of the loop, which is so large that it may, as is shown later, be considered infinite. The values of the field at the knee of the curve and at the saturation point are not vastly different, owing to the high permeability in this region, and so both these values will be loosely referred to as the 'coercive force'. A more precise and convenient definition of this quantity will be given later.

So long as the fields applied do not exceed the coercive force, the material behaves as an ordinary linear magnetic material of permeability μ_r. This causes any winding on a core to have an inductance (with a mutual inductance to any other winding), and this effect of the residual permeability is often of importance when the core is not actually being switched. While the core is being switched, the effect becomes of secondary importance, and in the following discussion it will be neglected.

(3) A.C. PROPERTIES

(3.1) Losses

An ordinary metallic ferromagnetic material has an eddy-current loss due to currents in the material itself. It also has a 'residual loss', which is a fundamental property of the material, but this is usually negligible because of the much larger eddy-current effect. In the case of a ferrite, the resistivity is so high that eddy currents can be entirely neglected, so that the residual loss becomes the dominant factor.

If a core is switched by a step function of current applied to a winding, then, in principle, the output voltage should be a spike of infinite amplitude lasting for an infinitesimal time. In practice, it is observed to be of finite amplitude and to last for a finite length of time, i.e. longer than the rise time of the current (see Fig. 2). Thus the core is giving a voltage output while its input current is constant, which implies a resistance rather than an inductance. The time taken to switch the core is found to decrease as the current is increased. In order to switch it in about a microsecond, the applied field must be about twice the coercive force and is therefore much larger than the difference between the field at the knee of the hysteresis loop and that at the saturation point. It is for this reason that the slope on the steep part of the loop can be considered infinite, since it plays a very small part in determining the output voltage when the core is switched at these high speeds; in the rest of the paper it will be assumed to be infinite.

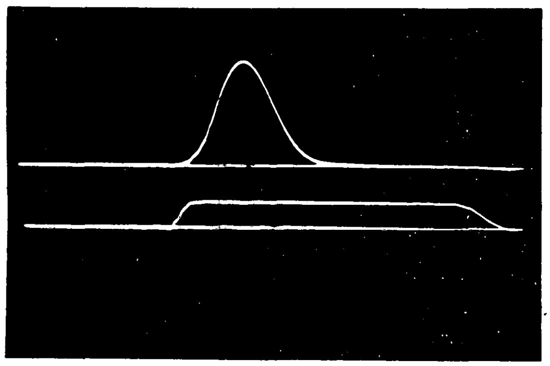

Fig. 2.—Core output waveform.

It is found experimentally that, if the peak output voltage is plotted against the switching current, a straight line is obtained as in Fig. 3, provided that the rise time of the current is short compared with the switching time. The coercive force will be defined, for the purpose of this paper, to be the intercept of this line on the current (or magnetizing force) axis (H_c in Fig. 3).

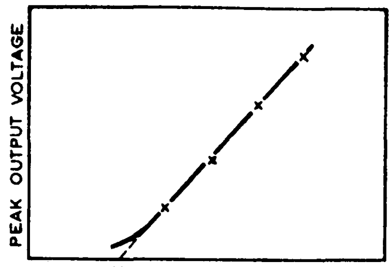

Fig. 3.—Variation of peak output voltage with applied field.

This definition has also been used by Karnaugh,[1] who called it the 'pseudo-coercive force'. It is a quantity which is precisely determined and of considerable use, being somewhat larger than the coercive force as usually defined, namely the field necessary to reduce the magnetization from remanence to zero.

(3.2) Approximate Equivalent Circuit

It is found that the graph of Fig. 3 is accurately a straight line, limited only at large currents by stray capacitances of the windings. The slope of this line, when current is plotted against voltage, has the dimensions of resistance, and is of the order of half an ohm for typical cores. The properties of the core when it is being switched can be approximately represented by a fictitious resistor of this value, connected to a fictitious single-turn winding, the core being regarded as a perfect transformer, provided that the magnetizing force exceeds the coercive value, and that the coercive force is subtracted from the magnetizing force before the output voltage is calculated. This equivalent circuit for a core was first suggested, in connection with metal cores, by Sands.[2]

Although this simple representation of a core is adequate for rough calculations, it does not give a true picture of its behaviour. The equivalent circuit implies that when a current step of amplitude i_0 is applied, the output should be a square voltage-pulse of amplitude $R(i_0 - i_c)$. This is roughly what is observed with metal cores, which is to be expected, since their losses are due to the resistance of the material, which the fictitious resistor R should be able to represent fairly well. With ferrites, however, the output pulses are observed to be far from square, and are in fact more nearly triangular, as is shown in Fig. 2. An explanation of this phenomenon will now be given.

(4) THEORY OF SWITCHING

(4.1) Shape of Output Waveform: Qualitative Theory

(4.1.1) Domain Wall Movement.

A qualitative explanation of the phenomenon has been given by Menyuk and Goodenough.[3] Their theory applies to both metal and ferrite cores, the only differences being that, for the metal core, there is an extra term due to the eddy-current loss and the saturation magnetization is greater, which leads to a different switching time. They suppose that, when the material is magnetized in one direction, small domains of reverse magnetization exist at the grain boundaries, and also at voids in the material in the case of ferrites, or that such reverse domains are formed by the application of a small reverse field, the latter mechanism tending to give a squarer hysteresis loop. The boundaries between such domains and the adjacent ones are 180° Bloch walls—the directions of magnetization on opposite sides of such a wall being 180° apart. It is proposed that the reversal of magnetization takes place by the movement of these

Bloch walls in a direction at right angles to that of magnetization. A picture is presented of cylindrical or ellipsoidal domains of reverse magnetization growing by increasing their diameter, until they either reach the grain boundaries or meet with other such growing domains.

Menyuk and Goodenough then go on to suggest that a Bloch wall will not move until a certain minimum field has been applied, and that when it does the only significant retarding force is a viscous one, which is due in part to the eddy-current loss, if any, and in part to the spin-relaxation effect. (The latter occurs because any attempt to rotate an electron spin by applying a magnetic field will, by classical theory, cause the electron to precess, so that if it is required to turn it over quickly, a large field must be applied.) The viscous force implies that the velocity of the Bloch wall is proportional to the amount by which the applied field exceeds the minimum field necessary for motion to start.

(4.1.2) Voltage Output.

The voltage output from the core is proportional to the rate of change of flux within it, and therefore to the rate of change of cross-sectional area of the growing reverse domains. Since the radius of these domains increases at a constant rate for a constant applied field, and since the cross-sectional area is proportional to the square of this radius, the output voltage initially increases with time. Later, when the growing domains begin to coalesce, the rate of increase of the area diminishes again, and thus accounts for the roughly triangular shape of the observed output waveform. The following equation [eqn. (6) of Reference 3] is proposed:

$$\frac{d\Phi}{dt} = \frac{16 I_s (\cos\theta)^3}{\beta} \mathrm{F}(r)(H_m - H_0) \quad . \quad . \quad (1)$$

where I_s is the saturation magnetization of the material, β is the viscous damping parameter, H_m is the applied field, H_0 is the critical field mentioned, and θ is the angle between the easy direction of magnetization and the field. $\mathrm{F}(r)$ is a distribution function which takes account of the variations in the rate of change of the area as the radius of the domains increases. The form of $\mathrm{F}(r)$ is not given, but it will be calculated in the following Section.

The above is an outline of Menyuk and Goodenough's theory, and the experimental evidence quoted in their paper, and in another paper,[4] indicates that this outline probably represents the true mechanism of the process. They also give some further quantitative detail of the probable magnitudes of some of the quantities involved, in an effort to predict in what direction improvement of the material is likely to lie. The correctness or otherwise of these details does not affect the validity of the basic outline, neither does it affect what follows.

(4.1.3) Effect of θ.

In a polycrystalline material, the directions of easy magnetization of the grains are not necessarily parallel to the applied field, which accounts for the term in θ in eqn. (1). The effect of this cannot be large, however, since the ferrite materials in question have cubic structures, so that any given direction cannot be far from one of the three possible directions of easy magnetization. An attempt to take account of this effect in a theory will lead to much complication; it will probably be invalid because of the fields due to poles set up in the material wherever grains with different orientations meet, and because of the stresses which exist in ferrite toroids and which, by symmetry, must be either tangential or radial, and therefore either parallel or perpendicular to the applied field. Either way will tend to align the easy directions of magnetization, either parallel to the applied field or perpendicular to it, depending upon the sign of the magnetostriction coefficient, and this will probably invalidate any attempt to bring θ into the theory. In any case, the only effect of having the direction of easy magnetization not parallel to the field would be to reduce the velocity of the Bloch walls, and this would not affect the shape of the output waveform basically. In the following treatment, therefore, the effect of θ will be neglected.

(4.2) Quantitative Theory

(4.2.1) Calculation of F(r).

It is assumed that reversal of magnetization takes place by the growth of cylindrical domains which are parallel to the applied field. The centres from which these domains start are scattered at random throughout the material. At time $t = 0$, all the domains are assumed to be of zero radius, at which time a field H_m is applied. The domains then expand with radial velocity proportional to $(H_m - H_c)$, where H_c is the coercive force.

It is necessary to consider only a typical cross-section of the material perpendicular to the field, so that the domains become circles. Let the radius of a domain be r, and let the domain walls move outward with velocity v [proportional to $(H_m - H_c)$]. The area inside a domain, A, is πr^2;

therefore $$\frac{dA}{dt} = 2\pi r \frac{dr}{dt} = 2\pi r v \quad . \quad . \quad . \quad . \quad . \quad (2)$$

It is now necessary to consider the annihilation of domain walls by the coalescence of the expanding domains (see Fig. 4).

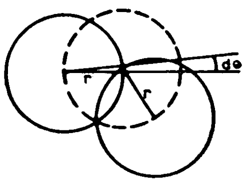

Fig. 4.—Intersection of two domain walls.
The applied field is perpendicular to the paper.

Consider an element of domain wall subtending an angle $d\theta$ at the centre of the domain. If, when the domain reaches a radius r, this element meets another domain wall, also of radius r, the centre of this second domain must lie on a circle of radius r (dotted in Fig. 4), centred at the point of intersection. If the centre of a second domain had lain within this circle, the element $d\theta$ of the first domain would never have got this far. Therefore the probability that this element reaches a radius r is the same as the probability of finding no other domain centre within this circle of radius r (or within any other circle of radius r, since domain centres are scattered at random). Let this probability be p. Then p also represents the proportion of such elements which survive until they reach a radius r. Thus the rate of change of the area enclosed by the circles, divided by the number of circles, is given by

$$\frac{dA}{dt} = 2\pi r v p \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

It is now necessary to calculate the probability p. Consider an area α, and within it an area a. If a particle is put in α, the probability of finding it in a is a/α, and the probability of not finding it in a is $(1 - a/\alpha)$, so that, if n particles are put in α, the probability of finding no particle in a is $(1 - a/\alpha)^n$.

If ρ is the density of the particles, $\rho = n/\alpha$, and $\alpha = n/\rho$.

Therefore the probability of finding no particles is

$$p = (1 - a\rho/n)^n$$

Now $\lim_{n \to \infty} p = \varepsilon^{-a\rho}$. Thus, if a is the area of a circle of radius r,

$$p = \varepsilon^{-\pi r^2 \rho} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Combining eqns. (3) and (4),

$$\frac{dA}{dt} = 2\pi r v \varepsilon^{-\pi r^2 \rho} \quad . \quad . \quad . \quad . \quad . \quad (5)$$

But v is proportional to $(H_m - H_c)$, and the rate of change of flux density ($= dB/dt$) is proportional to dA/dt,

Therefore $$\frac{dB}{dt} \propto (H_m - H_c) r \varepsilon^{-\pi r^2 \rho} \quad . \quad . \quad . \quad (6)$$

so that the required distribution function is given by

$$\mathrm{F}(r) = K r \varepsilon^{-\pi r^2 \rho} \quad . \quad . \quad . \quad . \quad . \quad (7)$$

where K is an arbitrary constant.

Since, however, the value of B at time t, being proportional to A, is a function of r, it is more convenient for practical purposes to express the distribution function F(r) as a function of B. (Note that $B = 0$ when the material is fully saturated in one direction.) This function is shown, in Appendix 8.1, to be

$$\mathrm{F} = \mathrm{F}(B) = \sqrt{(2\varepsilon)}\sqrt{\left[\log_\varepsilon\left(\frac{B_m}{B_m - B}\right)\right]}\left(\frac{B_m - B}{B_m}\right) \quad . \quad (8)$$

The variation of F with B/B_m is shown in Fig. 5.

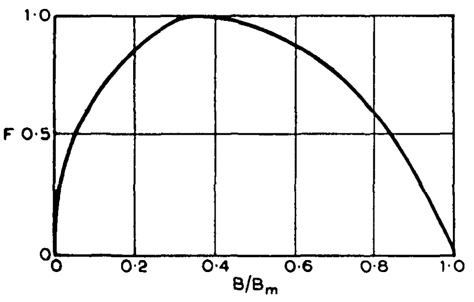

Fig. 5.—Variation of F with B/B_m.

From eqns. (6) and (7),

$$\frac{dB}{dt} \propto \mathrm{F}(H_m - H_c)$$

Let $(H_m - H_c) = H$, and let

$$dB/dt = X_0 \mathrm{F} H \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

Then X_0 is a fundamental parameter of the material, depending on the velocity of the domain walls for a given field H, on the density of the reverse domain centres ρ, and on the saturation flux density B_m.

(4.2.2) **Constant Field Case.**

A case of particular interest arises when the applied field H_m does not vary with time, so that v is constant. This corresponds to a step function of current applied to the core.

Now $r = vt$, so that eqn. (5) becomes

$$\frac{dA}{dt} = 2\pi v^2 t \varepsilon^{-\pi v^2 t^2 \rho}$$

and therefore

$$\frac{dB}{dt} = k_1 (H_m - H_c)^2 t \varepsilon^{-k_2 (H_m - H_c)^2 t^2}$$
$$= k_1 H^2 t \varepsilon^{-k_2 H^2 t^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

where k_1 is a constant depending on the saturation flux density and on the ratio of v to $(H_m - H_c)$, and k_2 is a constant depending on the same ratio and on ρ.

This, therefore, is the form of the output pulse with a step-current input, and it will be seen to be at least similar to the observed waveform (Fig. 2). A detailed comparison of the predicted and observed waveforms will be given later.

The maximum value of dB/dt, which is, of course, proportional to the peak output voltage, occurs when F has its maximum value of unity.

Therefore, from eqn. (9),

$$\left(\frac{dB}{dt}\right)_{max} = X_0 H \quad . \quad . \quad . \quad . \quad . \quad (11)$$

Thus the value of X_0 may be determined easily be measuring the peak output voltage of a core for various currents, and plotting a grapn such as that of Fig. 3.

(4.2.3) **Switching Time.**

Eqn. (10) shows that it takes an infinite time for the material to saturate completely. Any definition of switching time must therefore be arbitrary. In the past, many workers have defined switching to be complete when, with a step function of current applied, the output voltage has decreased to one-tenth of its peak value. This definition is obviously unsatisfactory, since it cannot be applied when the current is not constant during switching. A more logical way is to define switching to be complete when B/B_m reaches some suitable value, which will be chosen, for convenience, to correspond to the instant when the output voltage has decreased to one-tenth of its peak value in the constant-current case. It is shown in Appendix 8.2 that, at this time,

$$B/B_m = 0{\cdot}978 \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

and, in Appendix 8.3, that in the constant-current case the actual switching time is given by

$$T = 1{\cdot}67 \frac{B_m}{X_0 H} \quad . \quad . \quad . \quad . \quad . \quad (13)$$

It should be noted that the switching time thus defined is a function of the applied current, and is not the same as the 'switching time of the material'—a term used when describing the properties of a material to be used for storage purposes, when the current is fixed by other considerations.

(4.2.4) **Behaviour of a Core.**

So far, we have considered the properties of the material in bulk. Consider now a core of mean radius r and cross-sectional area A. Let a current of i_0 ampere-turns flow through a single-turn winding. Let the coercive current be i_c, and let $i = i_0 - i_c$. In Appendix 8.4 it is shown that the peak output voltage per turn, when i is a step function, is given by

$$V_{max} = \frac{2 X_0 A i}{r} = R_0 i \quad . \quad . \quad . \quad . \quad . \quad (14)$$

where $R_0 = 2X_0 A/r$. R_0, which has the dimensions of resistance, is obviously the slope of the line in Fig. 3. It also follows that X_0 should be measured in ohms per metre (unrationalized).

Hence, for a single core, eqns. (8), (9) and (13) become

$$F(\Phi) = \sqrt{(2\varepsilon)}\sqrt{\left[\log\left(\frac{\Phi_m}{\Phi_m - \Phi}\right)\right]\left(\frac{\Phi_m - \Phi}{\Phi_m}\right)} \quad . \quad (15)$$

$$\frac{d\Phi}{dt} = R_0 F i \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

$$T = 1\cdot 67\frac{\Phi_m}{R_0 i} \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

The output voltage is therefore, in general, given by

$$V = R_0 F i = Ri \quad . \quad . \quad . \quad . \quad . \quad (18)$$

where $$R = R_0 F \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

This implies that the current equivalent circuit for a core is as described in Section 3.2, except that the fictitious resistor R is now variable, of magnitude $R = R_0F$. It is suggested that the quantity R_0 should be termed the 'loss resistance' of the core.

(4.2.5) Energy Required to Switch a Core.

If a core is switched by a constant field H_m, the energy required per unit volume is

$$\frac{1}{4\pi}B_m H_m = \frac{1}{4\pi}B_m(H_c + H)$$

$$= \frac{1}{4\pi}B_m\left(H_c + 1\cdot 67\frac{B_m}{X_0 T}\right) \text{ from eqn. (13)}$$

$$= \frac{1}{4\pi}H_c B_m + \frac{1\cdot 67}{4\pi}\frac{B_m^2}{X_0 T} \quad . \quad . \quad . \quad . \quad . \quad (20)$$

The first term represents the hysteresis loss, and the second term the residual loss.

(4.3) Experimental Evidence

(4.3.1) General Experimental Evidence.

The fact that a linear relationship exists between the peak voltage from a core and the input current (Fig. 3) supports the above theory, but it is necessary to show also that, with a step-function input, the output voltage is of the predicted form [eqn. (10)], and that, in other cases, the application of eqn. (8) yields the correct result.

(4.3.2) Experimental Details.

The experiment was carried out with 3 mm cores in grade D2 material. Two input windings and a 'read' winding (all of 10 turns) were put upon the core. A 'reset' pulse of 2·5 ampere-turns was applied to one input, and a pulse of variable amplitude, with a rise time of 0·2 microsec, to the other. The output was applied to a double-beam oscillograph, and the trace was photographed. The voltage across a 100-ohm resistor carrying the current pulse was applied to the other beam. Fig. 2 is one of these photographs. In this Figure the current appears to rise before the voltage: this is due to the delay in the amplifier, and also to the finite rise-time of the current and the coercive force of the core. The finite rise-time of the current also causes the voltage waveform to be concave upwards at the start, whereas the theory predicts that the voltage should rise almost linearly at first, and then become concave downwards. That this is the correct explanation is clear, since the effect is only marked when the switching time of the core is comparable with the rise time of the current.

The bandwidth of the amplifier, which is a limiting factor in this experiment, was greater on the ranges with less gain, so that it was necessary to use rather a small trace on the screen and, in doubtful cases, to photograph the same waveform with different gain settings. The photographs finally selected were considered to be a fair representation of the true waveform, but owing to the small traces, the accuracy of the experiment could not be very high.

The photographs were traced on to graph paper in an enlarger, and were measured. Fig. 6 shows the resulting traces taken

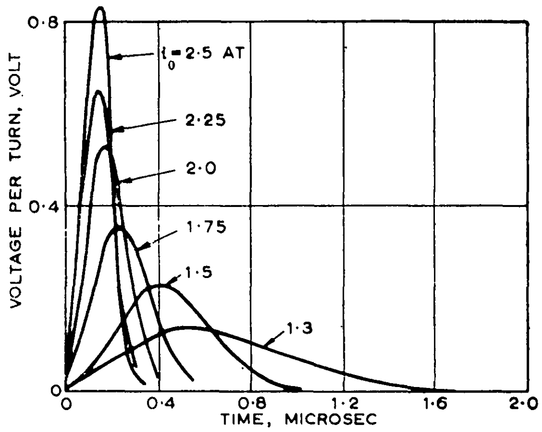

Fig. 6.—Observed output waveforms (3 mm core).

with input currents varying between 2·5 and 1·3 ampere-turns. (At the higher current, the switching time is comparable with the rise time of the current, and the lower current is only slightly higher than the coercive value.) A plot of peak voltage versus current produced values of R_0 and i_c for the core [see Section (4.2.4)]; the area under the curves gave Φ_m.

The curves were then normalized by multiplying the voltages by $\frac{0\cdot 428}{R_0(i_0 - i_c)}$, and the times by $\sqrt{\left(\frac{\varepsilon}{2}\right)}\frac{R_0(i_0 - i_c)}{\Phi_m}$ (0·428 being the peak value of the function $x\varepsilon^{-x^2}$). If the theory is correct, all the normalized curves should fit the function $x\varepsilon^{-x^2}$. Owing to the finite rise-time of the current, with the resulting error at the beginning of the output waveforms, it was necessary to take as the origin of the time axis the instant when the voltage reached its maximum value. The results are plotted in Fig. 7, together with the function $x\varepsilon^{-x^2}$. It will be seen that the discrepancy is greater for the initial voltages, and especially at the larger input currents, which is almost certainly due to the current rise-time. The scatter of the curves amongst themselves is within the limits of the experimental error.

It will be seen from Fig. 7 that the observed voltage waveforms fit the theory reasonably well, allowing for the limitations mentioned above. There appears to be a tendency for the observed values to be less than those predicted in the region $x = 1$. This may be due to overshoot on the part of the amplifier, but it may be a genuine effect. When the input current approaches the coercive value, the agreement is not so good. This is probably due to higher-order effects not taken account of in the theory, which one would expect to be more noticeable when the current is only slightly greater than the coercive value.

(4.3.3) Currents other than Step Functions.

That the theory still holds when the current applied to the core is not a step function can best be shown by loading the core with some impedance, the actual current applied then being the difference between the step function and the current drawn by this impedance. Fig. 8 shows the form of the output voltage, measured as in the previous case, with a 46-ohm resistor across one of the 10-turn windings. The current step was 164 mA in 10 turns. The theoretical waveform for this case, which was

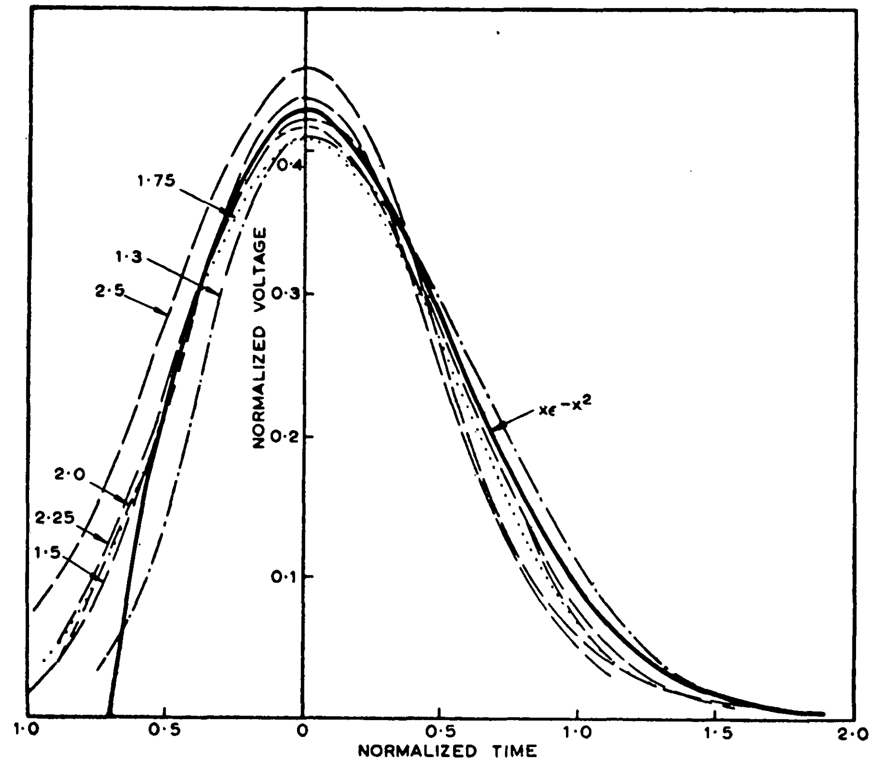

Fig. 7.—Comparison of observed and theoretical waveforms.

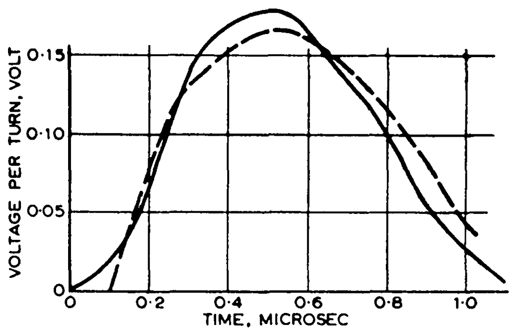

Fig. 8.—Output waveform with resistive load.

——— Observed waveform.
– – – – Calculated waveform.

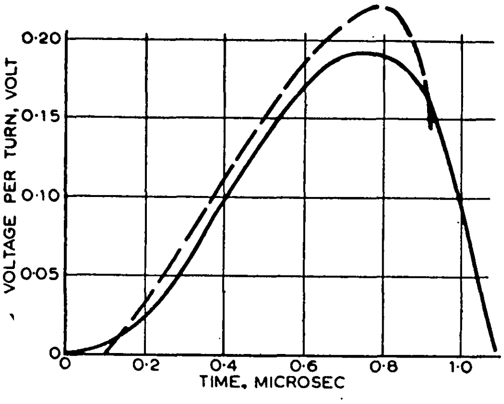

Fig. 9.—Output waveform with capacitive load.

——— Observed waveform.
– – – – Calculated waveform.

obtained by solving eqn. (15) numerically, using the Edsac, is also shown in Fig. 8. It will be seen that the agreement is reasonable.

Fig. 9 shows the results of a similar experiment using a $0\cdot009\,\mu$F condenser across 10 turns.

(4.3.4) The Effect of not Switching a Core fully.

The above theory has assumed that, before switching, the core has previously been saturated in the reverse direction. If it has only been partially switched in that direction, it is found that R_0 is reduced when the core is subsequently switched forwards. The extent of this reduction cannot be predicted from the theory, but it has been examined experimentally.

A 3 mm core was partially set by a current pulse of variable amplitude, and the flux change which occurred was measured

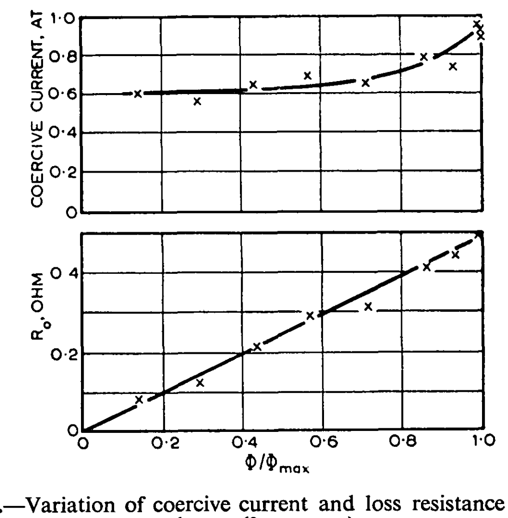

Fig. 10.—Variation of coercive current and loss resistance with flux change (3 mm core).

with a Miller integrator. It was reset with current steps of various known amplitudes, and the peak output voltage was measured at this time with a diode and condenser circuit followed by a slide-back voltmeter. (This method of measurement removes the uncertainty due to the poor frequency response of the amplifier when an oscillograph is used to measure peak voltages.) For each value of the flux change, the output voltage was plotted against the switching current to obtain values for R_0 and the coercive current, i_c. The values obtained are shown plotted against Φ/Φ_{max} in Fig. 10, from which it will be seen that R_0 is directly proportional to the flux change. However, i_c does not follow any such simple law, but it does exhibit a marked increase as saturation is approached. The scatter of the points in the i_c plot is rather great, since i_c is measured as the intercept on the current axis when the output voltage is plotted against current, a method which does not lead to great accuracy.

(5) CONCLUSIONS

A quantitative theory has been presented which explains the observed output waveforms from cores switched at high speeds. In view of the simplifying assumptions made, it is not exact, but it is quite adequate to assist in the design of switching circuits, and the qualitative concepts contained in it will be of use to the engineer in understanding the waveforms that he sees.

It is now apparent that the behaviour of these materials can be adequately described for most purposes by four parameters, namely:

μ_r = Residual permeability.
B_m = Maximum flux density. (If μ_r is significantly different from unity, B_m should be the remanent rather than the saturation value.)
H_c = Pseudo-coercive force as defined in Section 3.1.
X_0 = Loss parameter.

The engineer designing circuits needs to know the values of these four quantities for the material that he is using, and the extent to which they are likely to vary between different samples. This information, other than the mean values of the remanence and the ordinary coercive force, is not at present obtainable from manufacturers' literature. This situation is to be regretted, and it is hoped that it will be remedied in the future.

(6) ACKNOWLEDGMENTS

I am grateful to the Director of the University Mathematical Laboratory, Cambridge, for providing the facilities for this research, and to Ferranti Ltd. for a maintenance grant.

(7) REFERENCES

(1) KARNAUGH, M.: 'Pulse-Switching Circuits using Magnetic Cores', *Proceedings of the Institute of Radio Engineers*, 1955, **43**, p. 570.
(2) SANDS, E. A.: 'The Behaviour of Rectangular Hysteresis-Loop Magnetic Materials under Current Pulse Conditions', *ibid.*, 1952, **40**, p. 1246.
(3) MENYUK, N., and GOODENOUGH, J. B.: 'Magnetic Materials for Digital Computer Components—I. A Theory of Flux Reversal in Polycrystal Ferromagnetics', *Journal of Applied Physics*, 1955, **26**, p. 8.
(4) MENYUK, N.: 'Magnetic Materials for Digital Computer Components—II. Magnetic Characteristics of Ultra Thin Molybdenum-Permalloy Cores', *ibid.*, 1955, **26**, p. 692.

(8) APPENDICES

(8.1) *To express* F(r) *as a function of B.*

Now
$$\frac{dA}{dr} = \frac{dt}{dr}\frac{dA}{dt} = \frac{1}{v}\frac{dA}{dt} = 2\pi r\varepsilon^{-\pi r^2\rho} \text{ from eqn. (5)}$$

Therefore
$$A = 2\pi\int_0^r r\varepsilon^{-\pi r^2\rho}dr = \frac{1}{\rho}\Big[1 - \varepsilon^{-\pi r^2\rho}\Big]_0^r$$

Since, when switching is complete, $A = A_{max} = 1/\rho$,

$$\frac{A}{A_{max}} = \frac{B}{B_m} = 1 - \varepsilon^{-\pi r^2\rho}$$

where B_m is the saturation flux density.

Therefore
$$\varepsilon^{-\pi r^2\rho} = \frac{B_m - B}{B_m}$$

whence
$$\pi r^2\rho = \log_\varepsilon\left(\frac{B_m}{B_m - B}\right)$$

so that
$$r = \frac{1}{\sqrt{(\pi\rho)}}\sqrt{\left[\log_\varepsilon\left(\frac{B_m}{B_m - B}\right)\right]}$$

and the required function is

$$\mathrm{F}(B) = \mathrm{F}(r) = Kr\varepsilon^{-\pi r^2\rho} \text{ from eqn. (7)} = \frac{K}{\sqrt{(\pi\rho)}}\sqrt{\left[\log_\varepsilon\left(\frac{B_m}{B_m - B}\right)\right]}\left(\frac{B_m - B}{B_m}\right)$$

K is an arbitrary constant. It is convenient to choose K so that the maximum value of F is unity.

Let
$$\frac{B_m}{B_m - B} = y$$

Then
$$F = \frac{K}{\sqrt{(\pi\rho)}}\sqrt{\log_\varepsilon y}\left(\frac{1}{y}\right)$$

and
$$\frac{d\mathrm{F}}{dy} = \frac{K}{\sqrt{(\pi\rho)}}\,\frac{1 - 2\log_\varepsilon y}{2y^2\sqrt{(\log_\varepsilon y)}}$$

For maximum F, $\log_\varepsilon y = \frac{1}{2}$, i.e. $y = \sqrt{\varepsilon}$.

Therefore
$$\mathrm{F}_{max} = \frac{K}{\sqrt{(\pi\rho)}}\,\frac{1}{\sqrt{(2\varepsilon)}} = 1 \text{ by hypothesis}$$

so that
$$\frac{K}{\sqrt{(\pi\rho)}} = \sqrt{(2\varepsilon)}$$

and
$$\mathrm{F}(B) = \sqrt{(2\varepsilon)}\sqrt{\left[\log_\varepsilon\left(\frac{B_m}{B_m - B}\right)\right]}\left(\frac{B_m - B}{B_m}\right) \quad . \quad (8)$$

(8.2) *To find B/B_m when the output voltage has dropped to one-tenth of its peak value, the input current being constant.*

At this time, $\mathrm{F} = \frac{1}{10}$

From eqn. (8), putting $\frac{B_m}{B_m - B} = y$

$$\frac{1}{10} = \frac{\sqrt{(2\varepsilon \log_\varepsilon y)}}{y}$$

whence $$\frac{1}{200\varepsilon} = \frac{\log_\varepsilon y}{y^2}$$

which gives $$y = 45 \cdot 5$$

and $$B/B_m = 0 \cdot 978 \quad \ldots \ldots \quad (12)$$

(8.3) *To find the switching time in the constant-current case.*

Eqn. (10) states that $\frac{dB}{dt} = k_1 H^2 t \varepsilon^{-k_2 H^2 t^2}$

Therefore $$\frac{d^2 B}{dt^2} = (k_1 H^2 - 2k_1 k_2 H^4 t^2)\varepsilon^{-k_2 H^2 t^2}$$

For maximum $\frac{dB}{dt}$, $1 = 2k_2 H^2 t^2$

which gives $$t = \frac{1}{H\sqrt{(2k_2)}}$$

Therefore $$\left(\frac{dB}{dt}\right)_{max} = \frac{k_1}{\sqrt{(2k_2)}} \frac{H}{\sqrt{\varepsilon}} \quad \ldots \quad (21)$$

But, from eqn. (11), $\left(\frac{dB}{dt}\right)_{max} = X_0 H$

Therefore $$\frac{k_1}{\sqrt{(2k_2)}} \frac{1}{\sqrt{\varepsilon}} = X_0 \quad \ldots \ldots \quad (22)$$

Integrating eqn. (10), we obtain

$$B_m = k_1 H^2 \int_0^\infty t \varepsilon^{-k_2 H^2 t^2} dt$$

$$= \frac{k_1}{2k_2} \quad \ldots \ldots \quad (23)$$

From eqns. (22) and (23),

$$\frac{B_m}{X_0} = \sqrt{\frac{\varepsilon}{2k_2}} \quad \ldots \ldots \quad (24)$$

Switching is defined to be complete when the output voltage has dropped to one-tenth of its peak value, at which time, from eqns. (10) and (21),

$$\frac{1}{10} \frac{k_1}{\sqrt{(2k_2)}} \frac{H}{\sqrt{\varepsilon}} = k_1 H^2 t \varepsilon^{-k_2 H^2 t^2}$$

Therefore $$\frac{1}{10\sqrt{(2\varepsilon)}} = \sqrt{(k_2)} H t \varepsilon^{-k_2 H^2 t^2}$$

$$= x\varepsilon^{-x^2}$$

where $x = \sqrt{(k_2)}Ht$, so that $x = 1 \cdot 95$, and the switching time is given by

$$t = \mathrm{T} = \frac{1 \cdot 95}{\sqrt{(k_2)}H}$$

$$= 1 \cdot 95 \sqrt{\left(\frac{2}{\varepsilon}\right)} \frac{B_m}{X_0 H} \text{ from eqn. (24)}$$

$$= 1 \cdot 67 \frac{B_m}{X_0 H} \quad \ldots \ldots \quad (13)$$

(8.4) *To calculate the output voltage of a core.*

It follows from Ampère's theorem that $2\pi r H_m = 4\pi i_0$.

Therefore $H_m = 2i_0/r$, and likewise $H_c = 2i_c/r$ and $H = 2i/r$. Let the flux in the core be Φ ($\Phi = 0$ for one state of saturation).

Then $$\Phi = AB$$

Eqn. (11) then becomes $\frac{1}{A}\left(\frac{d\Phi}{dt}\right)_{max} = X_0 \frac{2i}{r}$

so that the peak output voltage per turn is

$$V_{max} = \left(\frac{d\Phi}{dt}\right)_{max}$$

$$= \frac{2X_0 A i}{r}$$

$$= R_0 i \quad \ldots \ldots \quad (14)$$

where $R_0 = 2X_0 A/r$.

Note that if X_0 is expressed in rationalized units, $V_{max} = R_0 i$, where $R_0 = X_0 A/2\pi r$.

Note*

Since writing the monograph, I have come across a paper by Haynes,† in which he performs a similar calculation assuming ellipsoidal domains rather than cylindrical ones. He thus arrives at an output waveform for the constant-current case which is of the form $x^2\varepsilon^{-x^3}$, as opposed to $x\varepsilon^{-x^2}$. The cylindrical approximation is more likely to be correct if the ellipsoids are so eccentric that their major axes are large compared with the dimensions of the grains of the material whilst their minor axes are still small. The ellipsoids are then truncated by the grain boundaries and therefore approximate to cylinders.

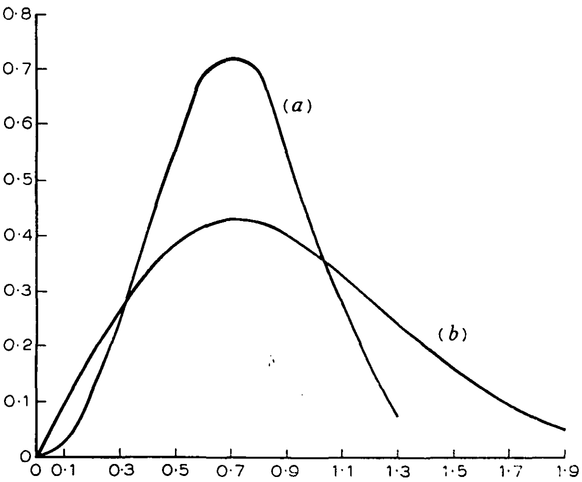

Fig. 11

(a) $2 \cdot 83x^2\varepsilon^{-1 \cdot 89x^3}$
(b) $x\varepsilon^{-x^2}$

Fig. 11 shows the two curves, normalized to have the same area and the same peaking time, and I think it will be agreed that the curve for $x\varepsilon^{-x^2}$ appears more like the shape of the observed output waveforms, as shown in Fig. 7. One therefore deduces that the ellipsoidal domains must be highly eccentric indeed.

* The note was received 24th July, 1959.

† HAYNES, M. K.: 'Model for Nonlinear Flux Reversals of Square-Loop Polycrystalline Magnetic Cores', *Journal of Applied Physics*, 1958, 29, p. 472.