Propagation Assignment: Part 1

Starting with the Maxwell equations, one can derive a 1D propagation model using two equations for the time evolution of the electric, $E_y(x,t)$, and magnetic, $B_z(x,t)$, fields of a laser. We assume that the field propagates in the x-direction while the electric and magnetic fields are polarized in the y and z directions. The field propagates at a speed v through the medium. The two propagation equations are:

$$\frac{\partial B_z}{\partial t} = -\frac{\partial E_y}{\partial x} \tag{1}$$

$$\frac{\partial E_y}{\partial t} = v^2 \frac{\partial B_z}{\partial x} \tag{2}$$

If you take the x-partial-derivative of Eq. (1), then you can decouple E_y from B_z and get:

$$\frac{\partial^2 E_y}{\partial t^2} = v^2 \frac{\partial^2 E_y}{\partial x^2} \tag{3}$$

To solve this equation numerically, we'll assume that we know the electric field as a function of x at two times, t_n and t_{n-1}, separated by a time step Δt; then solve for the electric field at the time t_{n+1}. If we discretize the x-axis into pixels of size Δx, each labeled by the index i (x_i) then it's helpful to express the electric field as

$$E_y(x_i, t_n) \longrightarrow E_i^n \tag{4}$$

Next, you must finite difference both sides of Eq. (3):

$$\frac{\partial^2 E_y}{\partial t^2} \simeq \frac{E_i^{n+1} - 2E_i^n + E_i^{n-1}}{\Delta t^2} \tag{5}$$

$$\frac{\partial^2 E_y}{\partial x^2} \simeq \frac{E_{i+1}^n - 2E_i^n + E_{i-1}^n}{\Delta x^2} \tag{6}$$

Insert these into Eq. (3) and solve algebraically for E_i^{n+1}. That's how you solve for the electric field at the next time step.
For your first two fields in time, start with:

$$E_y(x_i, t_0) = \exp\left[\frac{(t_0 - x_i/v + x_p/v)^2}{\tau_w^2}\right] \cos(kx_i - \omega t_0), \tag{7}$$

$$E_y(x_i, t_1) = \exp\left[\frac{(t_1 - x_i/v + x_p/v)^2}{\tau_w^2}\right] \cos(kx_i - \omega t_1), \tag{8}$$

where $k = 2\pi/\lambda$ and $\omega = 2\pi f$. Take the wavelength $\lambda = 800$ nm and the frequency $f = v/\lambda$. Take the so-called "pulsewidth" τ_w (which is not a width in meters, but time measure of how long the flash of light lasts) to be 5 fs. The variable x_p is where you want the peak of the pulse to be along your x-grid initially. Finally, let the speed $v = c$, where c is the vacuum speed of light.

For this calculation to work, your chosen Δx should be small enough to easily resolve each light wave oscillation in a wavelength (i.e. $\Delta x \leq \lambda/10$). Also, to be numerically stable, make sure your chosen time step $\Delta t < \Delta x/v$.

Make sure this calculation works for propagation in a vacuum. After that, select some portion near the end of your x-axis where the pulse will enter a dielectric, and change the value of v there to $c/1.5$. What happens to the light after it hits this medium?

In part 2 of the assignment, we'll focus on solving Eqs. (1) and (2) together using the Finite Difference Time Domain method.